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COURSE SCALLOPING AMPLITUDES IN STANDARD VOR BEARING INDICATIONS

By

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I

INTRODUCTION

This is the fourth Interim Report on Contract No. DOT-FA72WA-2882, "Application of the Large Gradient VOR Antenna," and covers the period 1 March 1973 to 31 May 1973.

The theory of course scalloping amplitude in standard VOR bearing indications has been discussed in our third Interim Report [1]. The present report discusses the numerical results for the course scalloping amplitudes in the bearing indications of a standard VOR system under various situations. The scatterer producing the scalloping errors is assumed to be isotropic and located at various heights above a perfectly conducting planar earth. Numerical results for the scalloping amplitudes are obtained when the VOR ground station uses standard and large gradient antennas with variable gradients. The effects of the height of the VOR transmitting antenna above ground on the scalloping amplitudes are investigated in detail.

II

COURSE SCALLOPING AMPLITUDE EXPRESSION

The geometry of the VOR ground station antenna, the isotropic scatterer and the observation point located at a stationary aircraft are shown in Fig. 1. The VOR antenna is located at $(Z_0, 0, 0)$, the scatterer at (d_1, θ_1, ϕ_1) and the ground plane is at $z = 0$; the coordinates of the observation point are (r, θ, ϕ). Sometimes it may be found convenient to represent the location of the scatterer by the parameters H and D as shown in Fig. 1. Note that

$$\tan \theta_1 = \frac{D}{H} . \qquad (1)$$

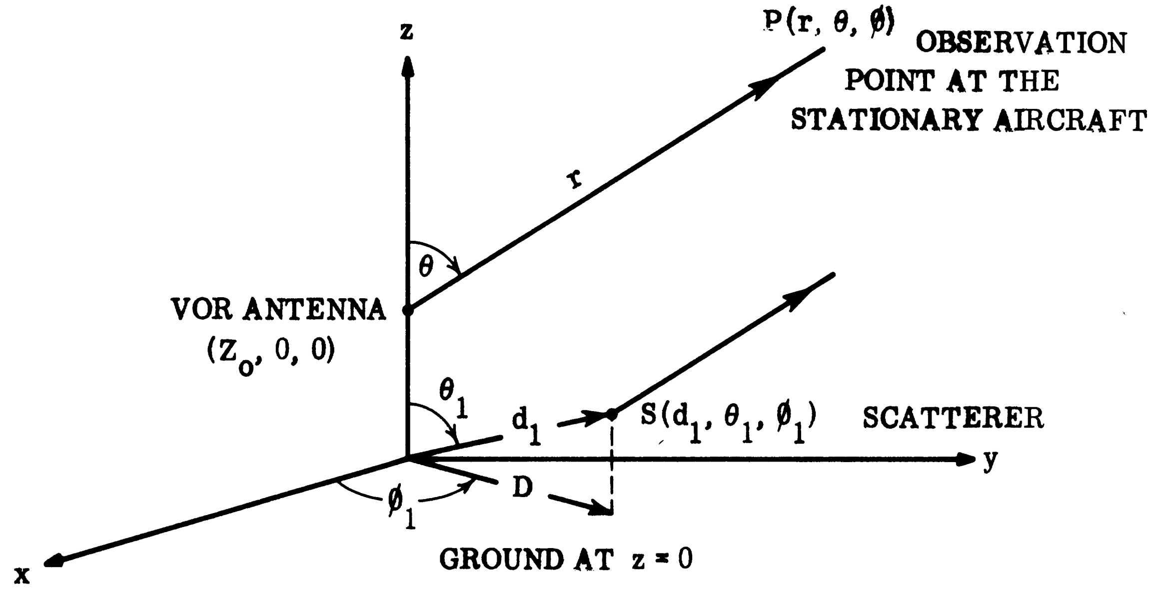

Fig. 1: Location of the VOR transmitting antenna, the scatterer and the observation point at the stationary aircraft.

It is assumed that at the observation point an ideal receiver receives the signal transmitted by the VOR ground station.

It can be shown [1] that in the presence of a scatterer the course scalloping amplitudes in the bearing indications of a standard VOR receiver at the observation point P are given by:

$$S_{\frac{1}{2}} = \mp \tan^{-1} \left[\frac{2A \left| \frac{S_s^T(\theta_1)}{S_s^T(\theta)} \right| \sin(kH\cos\theta)\sin(\phi - \phi_1)}{1 \pm 2A \left| \frac{S_s^T(\theta_1)}{S_s^T(\theta)} \right| \sin(kH\cos\theta)\cos(\phi - \phi_1)} \right], \tag{2}$$

where,

$k = 2\pi/\lambda$ is the propagation constant in free space,

A is the generalized scattering coefficient for the scatterer and is, in general, a function of the coordinates of the scattering object and the observation point,

$S_S^T(\theta)$ is the side-band mode elevation plane complex pattern of the VOR transmitting antenna located above ground.

$S_s^T(\theta)$ is related to the corresponding free space pattern of the VOR antenna by the following:

$$S_s^T(\theta) = S_s(\theta)e^{-ikZ_o\cos\theta} - S_s(\pi - \theta)e^{ikZ_o\cos\theta}, \tag{3}$$

where $S_s(\theta)$ is the side-band mode elevation plane complex pattern of the VOR transmitting antenna in free space.

Equation (2) indicates that during an orbital flight around the VOR station, the maximum bearing errors, i.e., the scalloping amplitudes as observed in the aircraft receiver will lie between the limits S_1 and S_2. We emphasize here the following basic assumptions made in obtaining Eq. (2): (i) the carrier and the side-band mode fields are in RF phase for all θ, (ii) the scatterer is located in the far zone of the VOR antenna. The general effects of the different parameters on the scalloping amplitudes are discussed in [1] and will not be repeated here.

For simplicity we shall assume that the scatterer is isotropic, i.e., A is a constant and is independent of the coordinates of the scatterer and the observation point.

III

FREE SPACE PATTERN CHARACTERISTICS OF THE VARIOUS VOR ANTENNAS

In the present section we discuss the pertinent characteristics of the free space patterns of the various VOR antennas used during the investigation of the scalloping amplitudes. The antennas used are the standard four-loop array above a 52' diameter counterpoise (or the standard VOR antenna), the double parasitic loop counterpoise antenna (DPLC antenna) and stacked arrays of VOR four-loop arrays. Detailed discussion of these antennas and the patterns produced by them when located in free space and above ground may be found in [2], [3]. The important free space side-band mode elevation plane pattern characteristics of the above antennas operating at 109 MHz are shown in Table I.

TABLE I

FREE SPACE SIDE-BAND MODE ELEVATION PLANE PATTERN CHARACTERISTICS OF VOR ANTENNAS AT 109 MHz

Property*	Standard VOR	DPLC	Stacked Arrays 1	2	3	4
θ_{max}	60^o	60^o	74^o	74^o	73^o	74^o
α_f	9.47 dB	16.50 dB	8.88 dB	7.75 dB	6.64 dB	5.72 dB
$\alpha_g/6^o$	3.05 dB	21.22 dB	16.93 dB	10.20 dB	6.74 dB	5.22 dB

* $\theta = \theta_{max}$ is the position of the principal maximum

$$\alpha_f = \frac{\text{field strength at } \theta = \theta_{max}}{\text{field strength at } \theta = \pi/2\,(\text{horizon})}$$

α_g = field gradient at the horizon per 6^o.

Observe that the antennas listed in Table I have different field gradients. The four stacked arrays are obtained by using four different excitations appropriate for the indicated field gradients [3]. Further information about the antennas and their patterns may be found in the references cited.

IV
VARIATION OF S_1 AND S_2

For orbital flights Eq. (2) indicates that S_1 evaluated at $\pi + (\phi - \phi_1) = S_2$ evaluated at $(\phi - \phi_1)$ and vice versa. This implies that in order to study the scalloping amplitude variation during an orbital flight it is sufficient to calculate S_1 (or S_2) in the range $0 \leq (\phi - \phi_1) \leq 2\pi$ or calculate S_1 and S_2 in the range $0 \leq (\phi - \phi_1) \leq \pi$.

It can be seen from Eq. (2) that for $A \ll 1$ and $2 \left| \frac{S_s^T(\theta_1)}{S_s^T(\theta)} \right| < 1$, the scalloping amplitude is directly proportional to the scattering coefficient A. Figure 2 shows the variation of S_2 in the direction $(\phi - \phi_1) = 60^o$, $\theta = \theta_{min} = 78.5^o$, as a function of A. The results apply to the case of a VOR system using a standard VOR antenna located 15' above ground; an isotropic scatterer is located at a distance D = 1000' and at a height H = 7.2' above ground. The height of the scatterer H is chosen such that for the observation angle $\theta = \theta_{min} = 78.5^o$, $kH \cos \theta_{min} = \pi/2$, where θ_{min} is the direction of the minimum closest to the horizon in the elevation plane pattern of the antenna. This has been done to remove direct dependence of S_2 on H (see Eq. (2)). For the range of values of A shown in Fig. 2, the scalloping amplitude is found to increase almost linearly with the increase of the scattering coefficient.

The complete variation of the scalloping amplitude during an orbital flight is shown in Fig. 3(a) for a standard VOR antenna located 50' above ground and for different heights of an isotropic scatterer located at different heights above ground. The scatterer is located 1000' away from the VOR antenna. The scalloping amplitudes shown in Fig. 3(a) are S_1 in the range $0 \leq (\phi - \phi_1) \leq \pi$ and S_2 in the range $\pi \leq (\phi - \phi_1) \leq 2\pi$. The orbital flight is assumed to take place at an elevation angle such that it corresponds to the minimum nearest to the horizon $(\theta = 90^o)$ of the elevation plane of the antenna. Similar results are shown in Fig. 3(b) when the elevation angle of the flight path corresponds to the first maximum

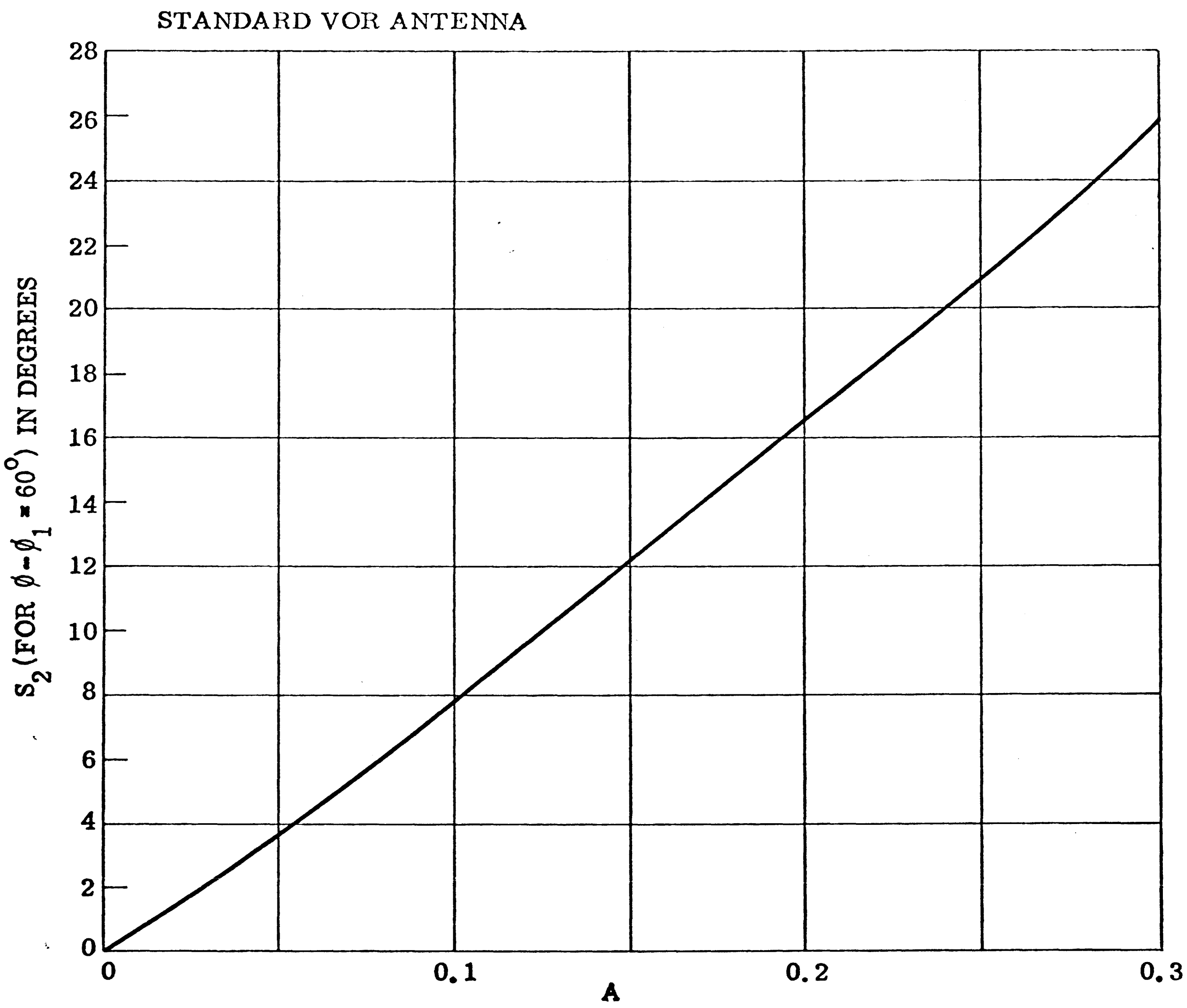

Fig. 2: Scalloping amplitude S_2 in the direction $\phi - \phi_1 = 60^o$, $\theta = \theta_{min} = 78.5^o$ as a function of A. $H = 7.2'$, $Z_o = 15'$ and $D = 1000'$.

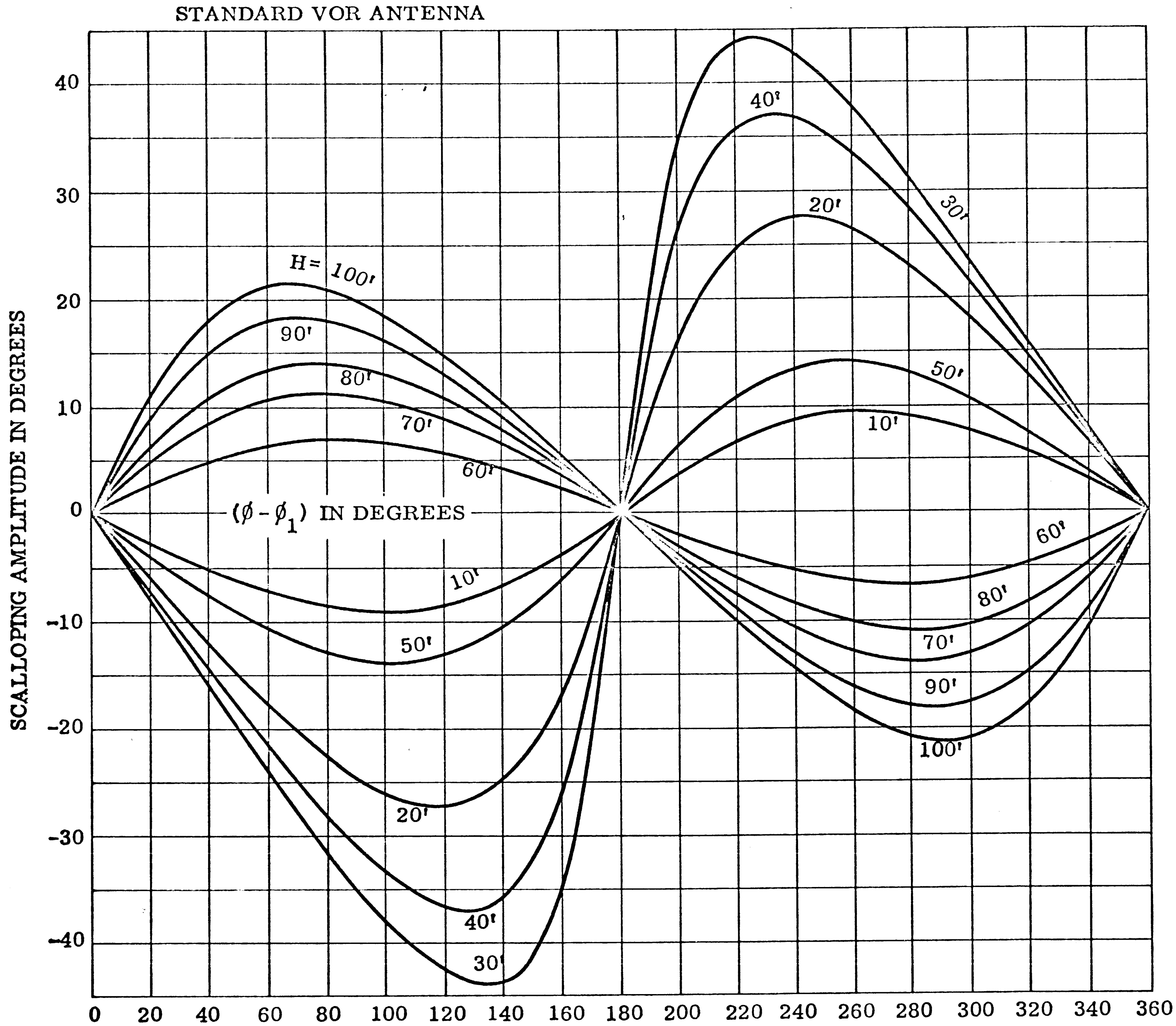

Fig. 3(a): Scalloping amplitude in the direction of the first minimum in the pattern as a function of $(\phi - \phi_1)$ for the standard VOR antenna. $Z_o = 50'$, $D = 1000'$, $A = 0.1$.

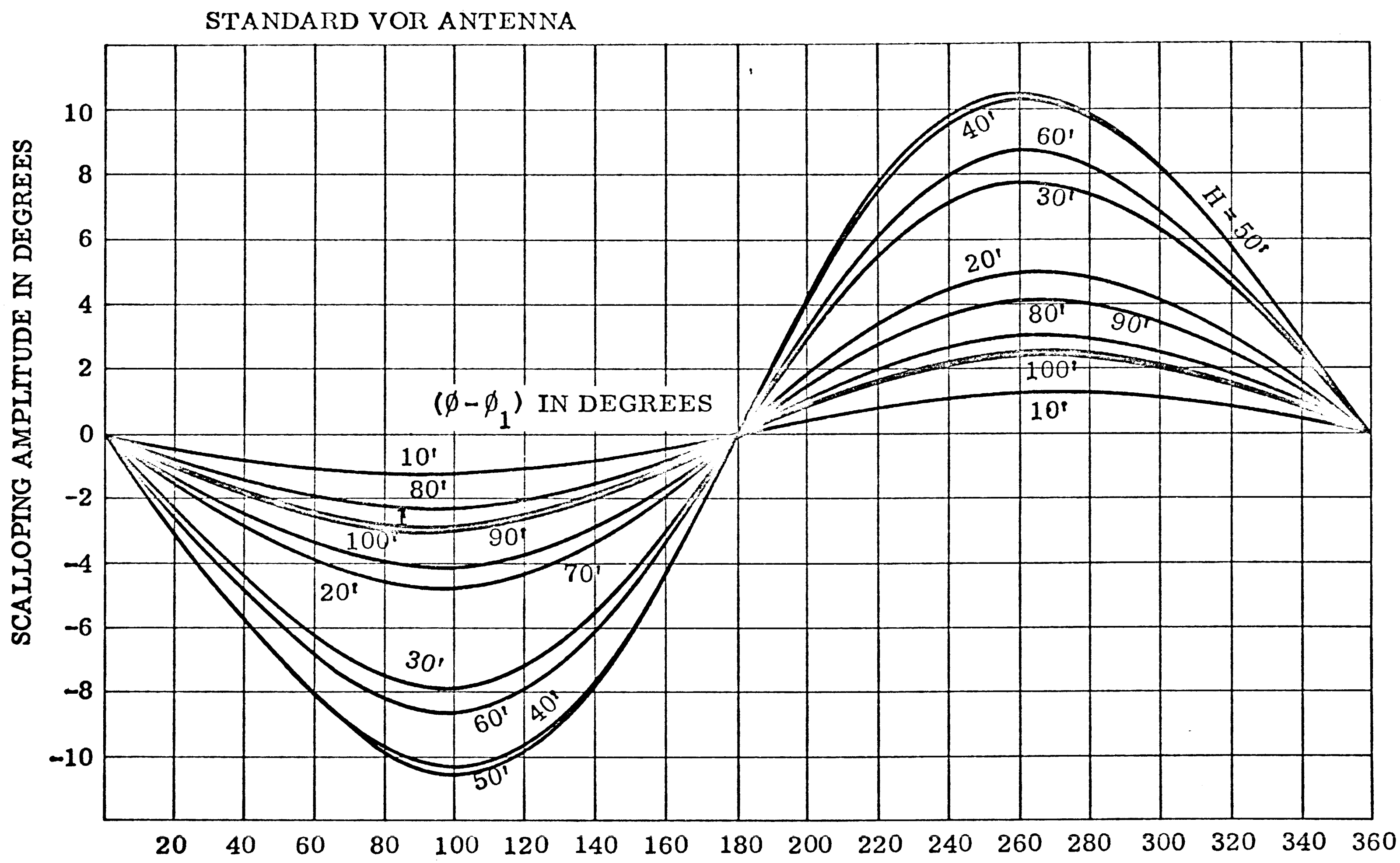

Fig. 3(b): Scalloping amplitude in the direction of the first maximum in the pattern as a function of $(\phi - \phi_1)$ for the standard VOR antenna. $Z_o = 50'$, $D = 1000'$ and $A = 0.1$.

(nearest to the horizon) in the elevation plane pattern of the antenna. For the range of parameters considered in Figs. 3(a) and 3(b) it is found that the maximum scalloping amplitude is about four times larger in the direction of the first minimum in the pattern. For the case of a large gradient VOR DPLC antenna located 50' above ground, similar scalloping results are shown in Fig. 4 for orbital flight at an elevation angle which corresponds to the first minimum in the elevation plane pattern of the antenna. As compared with Fig. 3(a) the results in Fig. 4 indicate that the maximum scalloping amplitudes are less for the large gradient antenna.

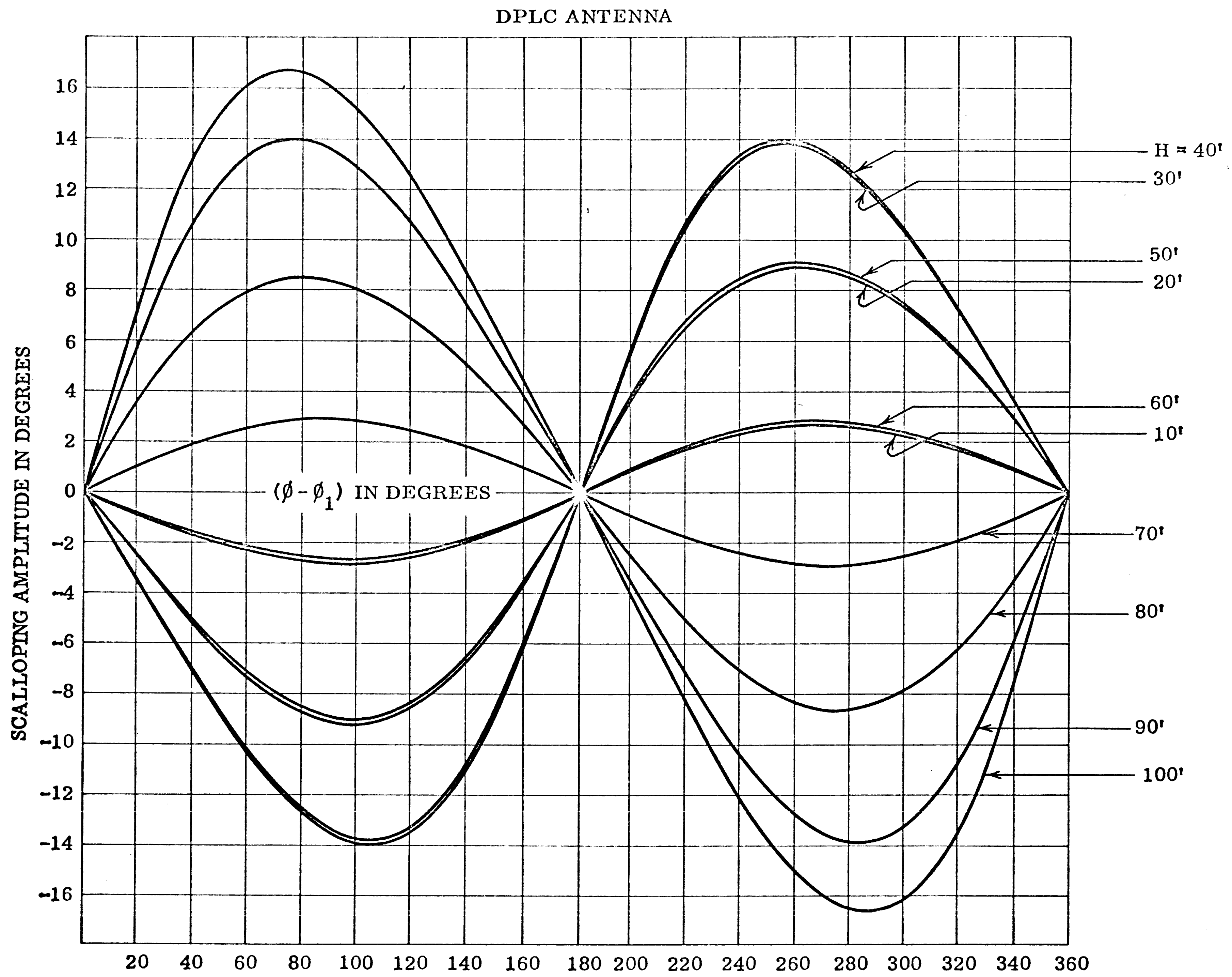

Fig. 4: Scalloping amplitude in the direction of the first minimum in the pattern as a function of $(\phi - \phi_1)$ for the DPLC antenna. $Z_0 = 50'$, $D = 1000'$ and $A = 0.1$.

V

SCALLOPING AMPLITUDE VARIATION WITH FIELD GRADIENT

In this section we discuss the results obtained for scalloping amplitudes with a VOR antenna having different values of the field gradient. The antenna chosen is the five-bay stacked array [2]. The excitations are chosen such that the field gradients in the free space elevation plane patterns of the antenna are as given in Table 1. The patterns produced by such antennas located above ground are discussed in [2].

In general, for a scattering object located at a height below that of the antenna, the observed scalloping amplitudes decrease with the increase of the field gradient of the antenna. Figures 5 and 6 show the scalloping amplitude vs. $(\phi - \phi_1)$ for an antenna with variable field gradients and for observation angles (θ) corresponding to the first minimum and the first maximum respectively in the elevation plane patterns of the antenna. The variation with the field gradient is found to be more pronounced in Fig. 5. Once again we observe that the maximum scalloping amplitude is larger in the direction of the pattern minimum.

In order to present the results in a more meaningful way we define the following parameters characterizing the scalloping amplitudes:

average maximum scalloping amplitude

$$S = \frac{|S_{1\,max}| + |S_{2\,max}|}{2} ,$$

scalloping improvement coefficient

$$\rho = \frac{\text{S for the test antenna}}{\text{S for the standard VOR antenna}} .$$

The average maximum scalloping amplitude S and the scalloping improvement coefficient ρ as functions of the field gradient are shown in Fig. 7. For the range of the parameters shown the average maximum scalloping corresponding to the first

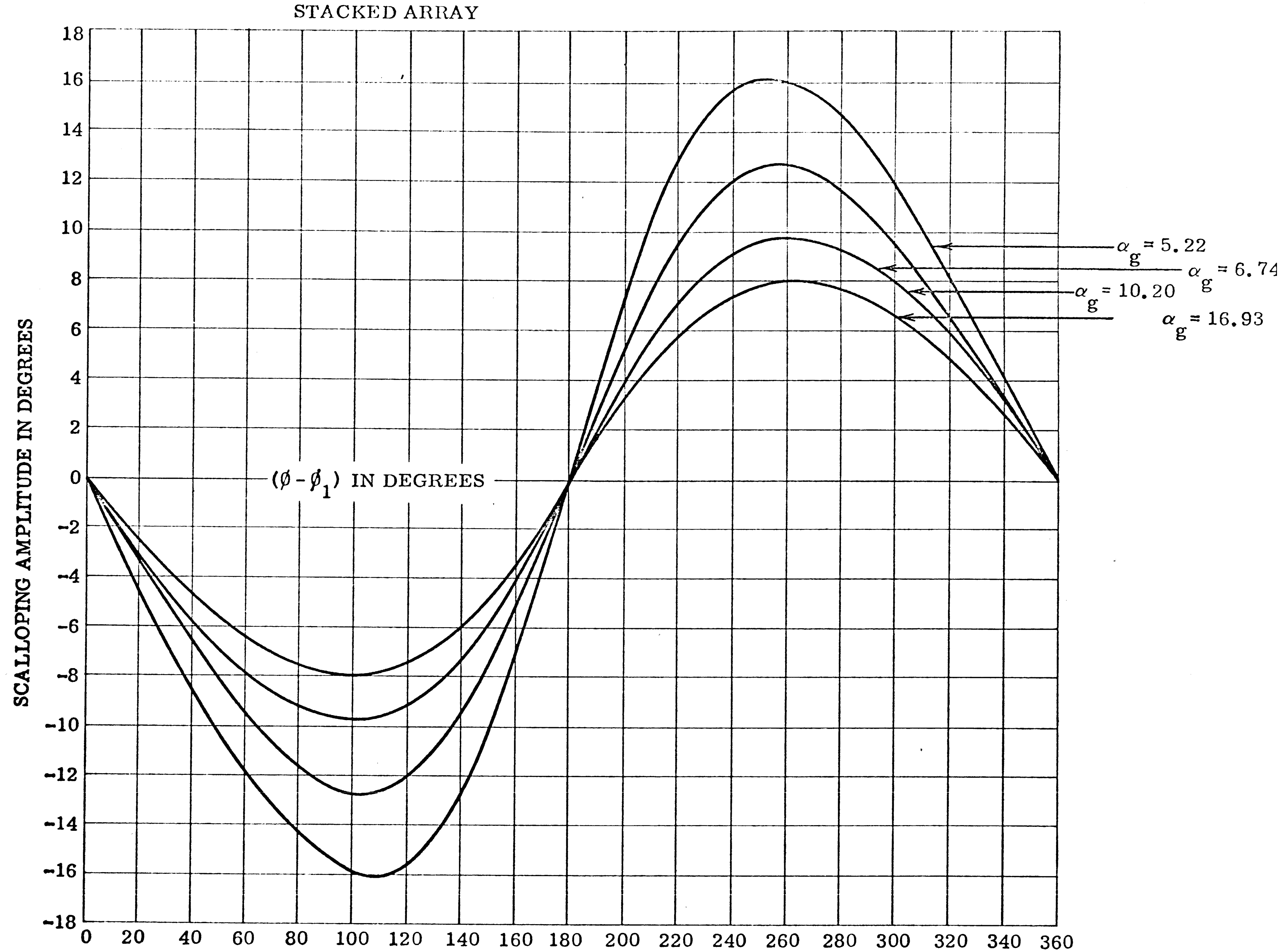

Fig. 5: Scalloping amplitude in the direction of the first minimum in the pattern as a function of ($\phi - \phi_1$) for the stacked array antenna. Z = 200', H = 60', D = 1000' and A = 0.02.

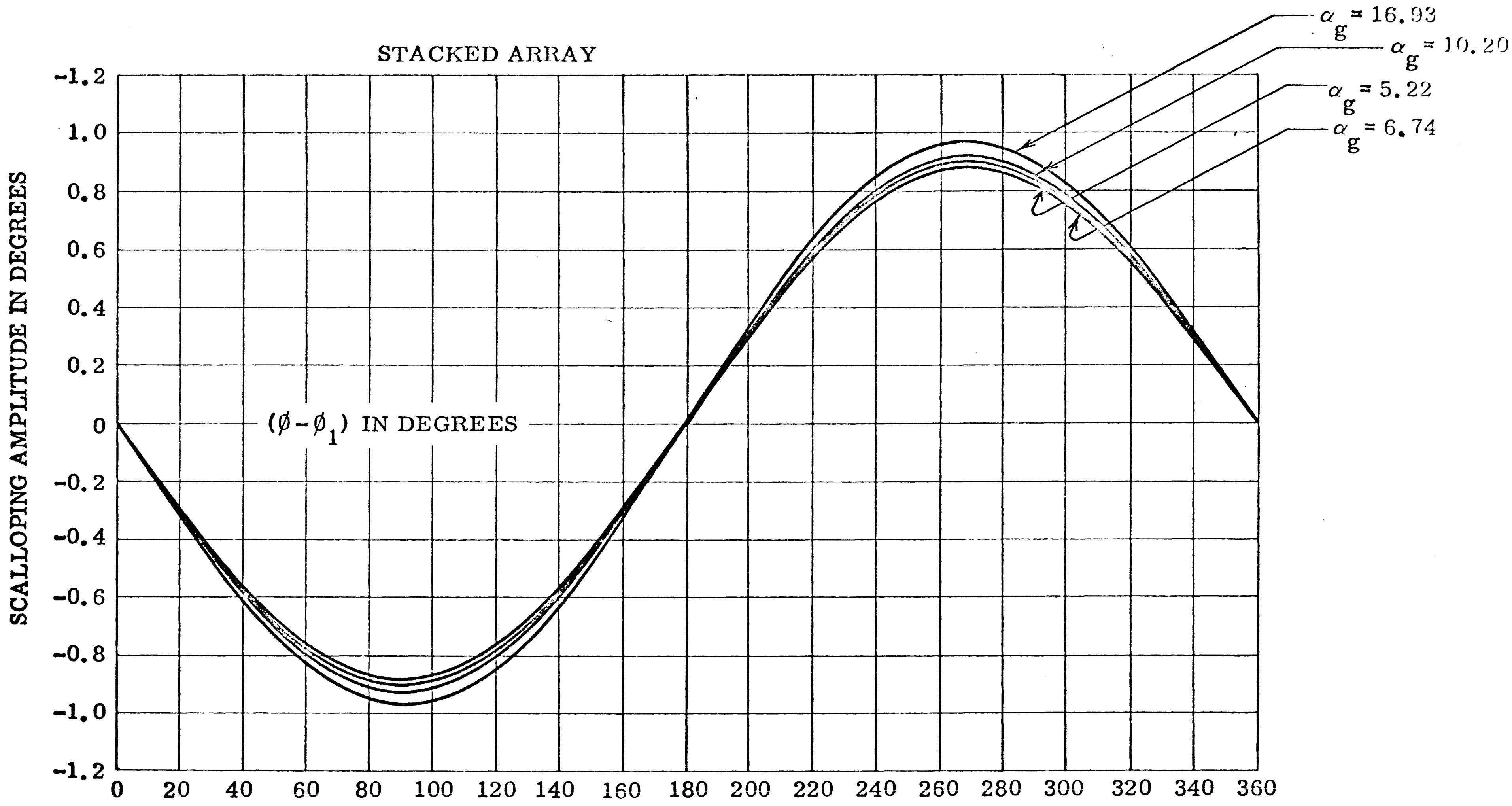

Fig. 6: Scalloping amplitude in the direction of the first maximum in the pattern as a function of $(\phi - \phi_1)$ for the stacked array antenna. $Z_o = 200'$, $H = 60'$, $D = 1000'$ and $A = 0.02$.

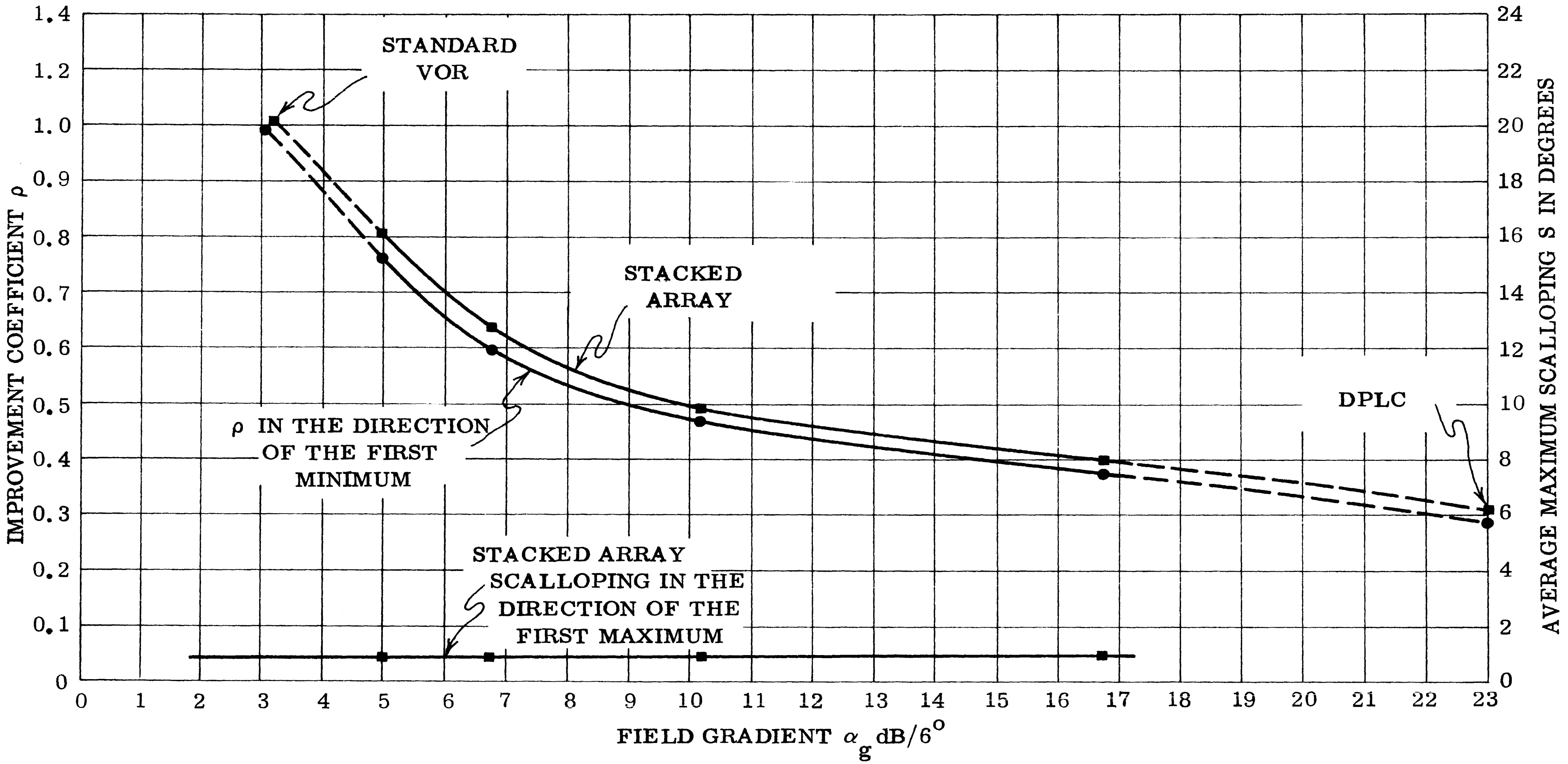

Fig. 7: Improvement coefficient ρ and average maximum scalloping as functions of the field gradient α_g. $Z_o = 200'$, $H = 60'$, $D = 1000'$ and $A = 0.02$.

minimum in the pattern is found to decrease continuously with the increase of the field gradient. The dotted extrapolated extensions of this curve at its two ends terminate at the corresponding values for the standard VOR and the DPLC antennas respectively. The average maximum scalloping amplitude corresponding to the first maximum in the pattern is found to be independent of the field gradient for the range of parameters considered in Fig. 7. The improvement coefficient ρ as a function of the field gradient, shown in Fig. 7, has been obtained for the observation angle corresponding to the first minimum in the pattern. This curve is again extrapolated to the standard VOR and DPLC antenna values.

VI

VARIATION OF S AND ρ WITH RELATIVE LOCATION OF THE ANTENNA AND THE SCATTERER

The variations of the average maximum scalloping amplitude S and the scalloping improvement coefficient ρ as functions of the relative location of the antenna and the scatterer above ground are discussed in the present section. Results have been obtained for (see Fig. 1 for notation) D = 500', 1000', 2000' and 3000' and for H = 25', 50', 75' and 100'. The antennas considered are the standard VOR and the DPLC antennas. In all cases the antenna heights have been varied from Z_o = 15' to Z_o = 500'. In the following sections we first discuss the reulsts obtained when the scatterer is located at a distance D = 1000' from the antenna.

Results for D = 1000'

Average maximum scalloping amplitude vs. antenna height for two observation angles and for different heights of the scatterer are shown in Figs. 8(a) and 8(b) for a standard VOR antenna. In general, for a given scatterer height the average maximum scalloping S is found to be an oscillating function of the height of the antenna above ground. This is indicative of the image effects on the field radiated by the antenna which in turn affects the scalloping. The average value of S tends to increase with the increase of the scatterer height. This may be attributed to the fact that in this particular case with D = 1000' the field incident at the scatterer increases with the height of the scatterer. It should also be observed from Eq. (2) that for a given antenna height the scalloping goes to zero if the height of the scatterer is such that $H = \frac{n\lambda}{2\cos\theta}$, where n = 1, 2,..., This is due to the image effects on the scattered field at the observation point. The corresponding results for DPLC antenna are shown in Figs. 9(a) and 9(b).

The ratio of the average maximum scalloping amplitudes observed at the first maximum and the first minimum angles in the pattern of the standard VOR antenna are shown in Fig. 10. The corresponding results for the DPLC antenna

STANDARD VOR ANTENNA

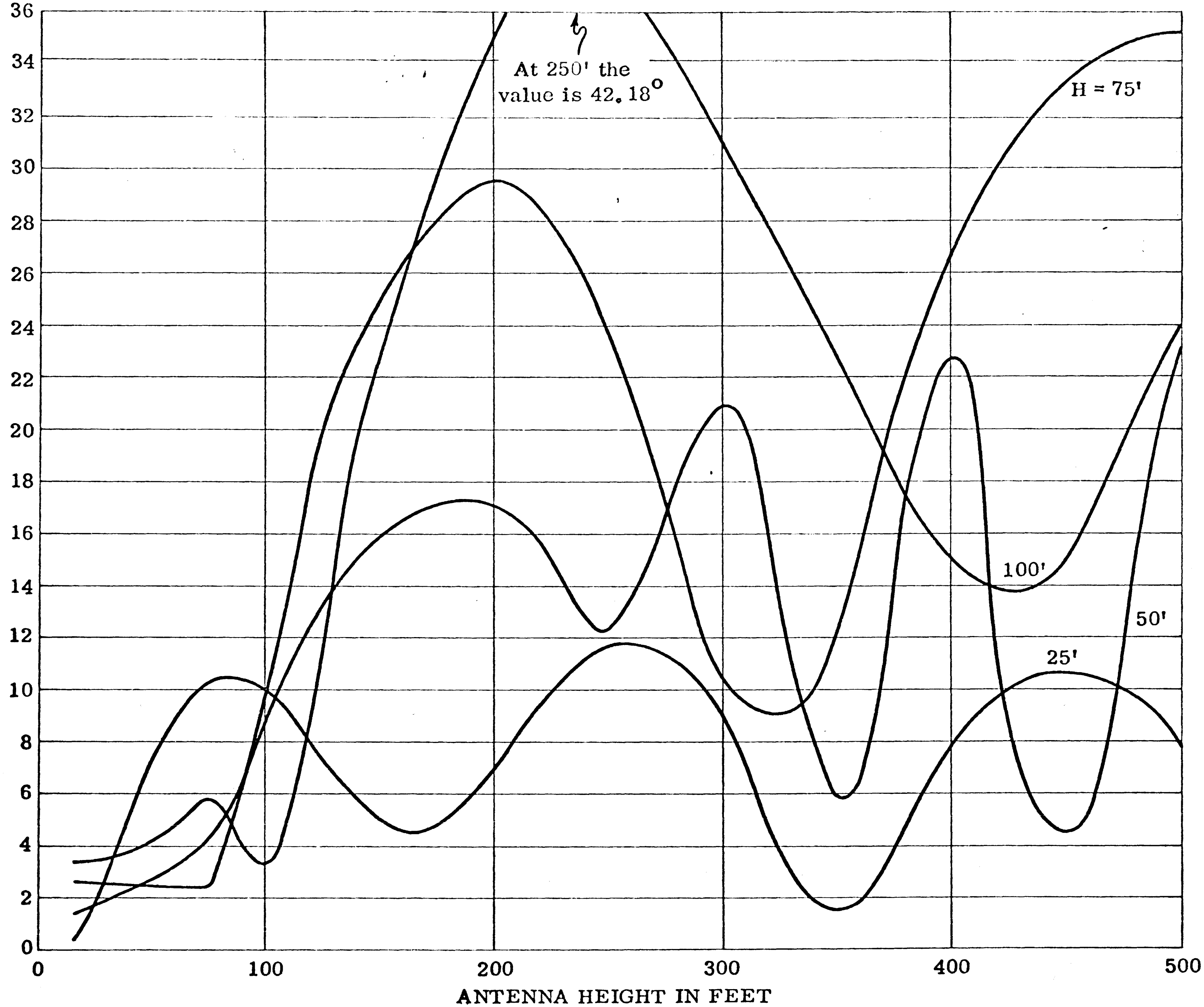

Fig. 8(a): Average maximum scalloping as a function of standard VOR antenna height with scatterer height as the parameter. Observation angle is in the direction of the first minimum. D = 1000', A = 0.02.

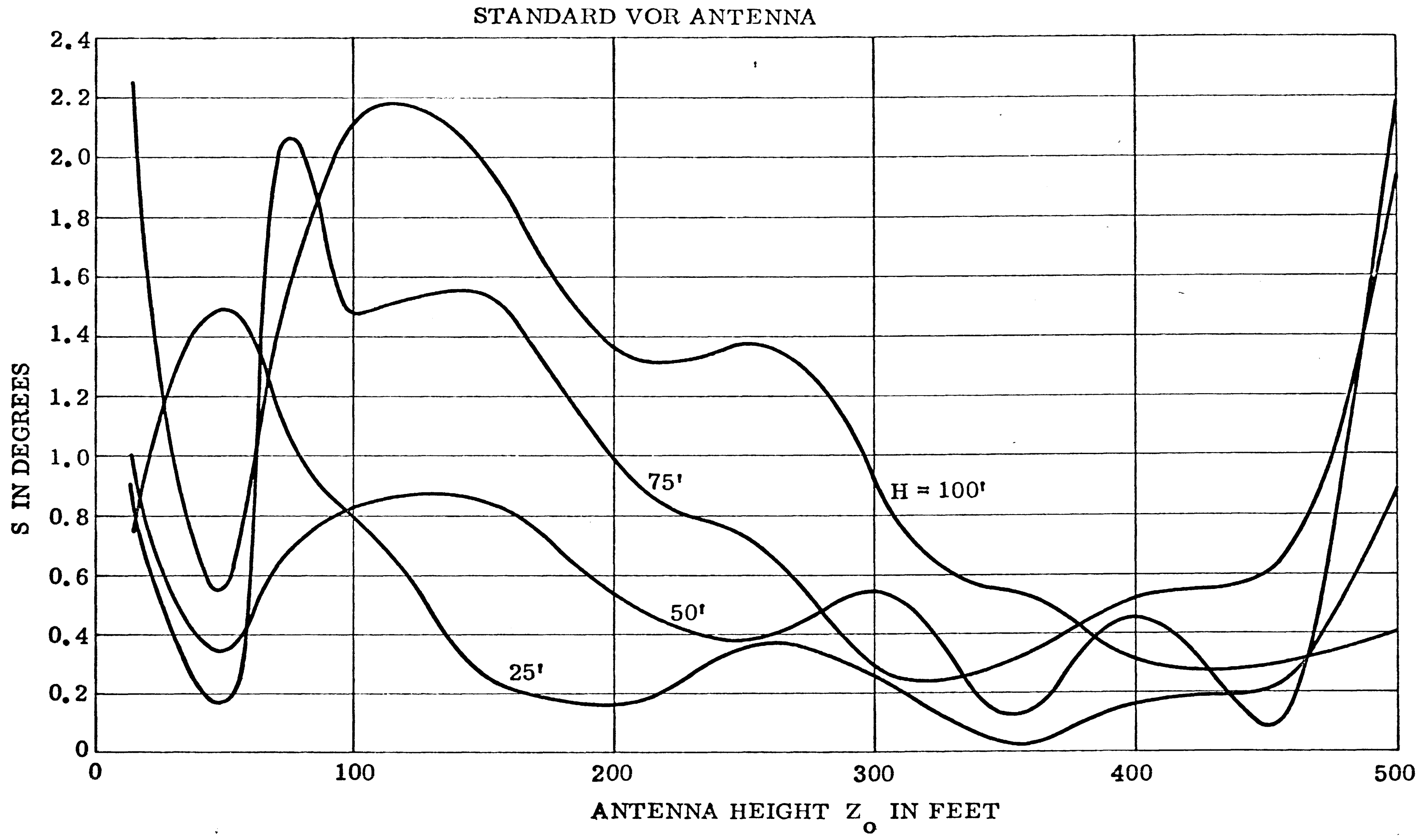

Fig. 8(b): Average maximum scalloping as a function of standard VOR antenna height with scatterer height as the parameter. Observation angle is in the direction of the first maximum. D = 1000', A = 0.02.

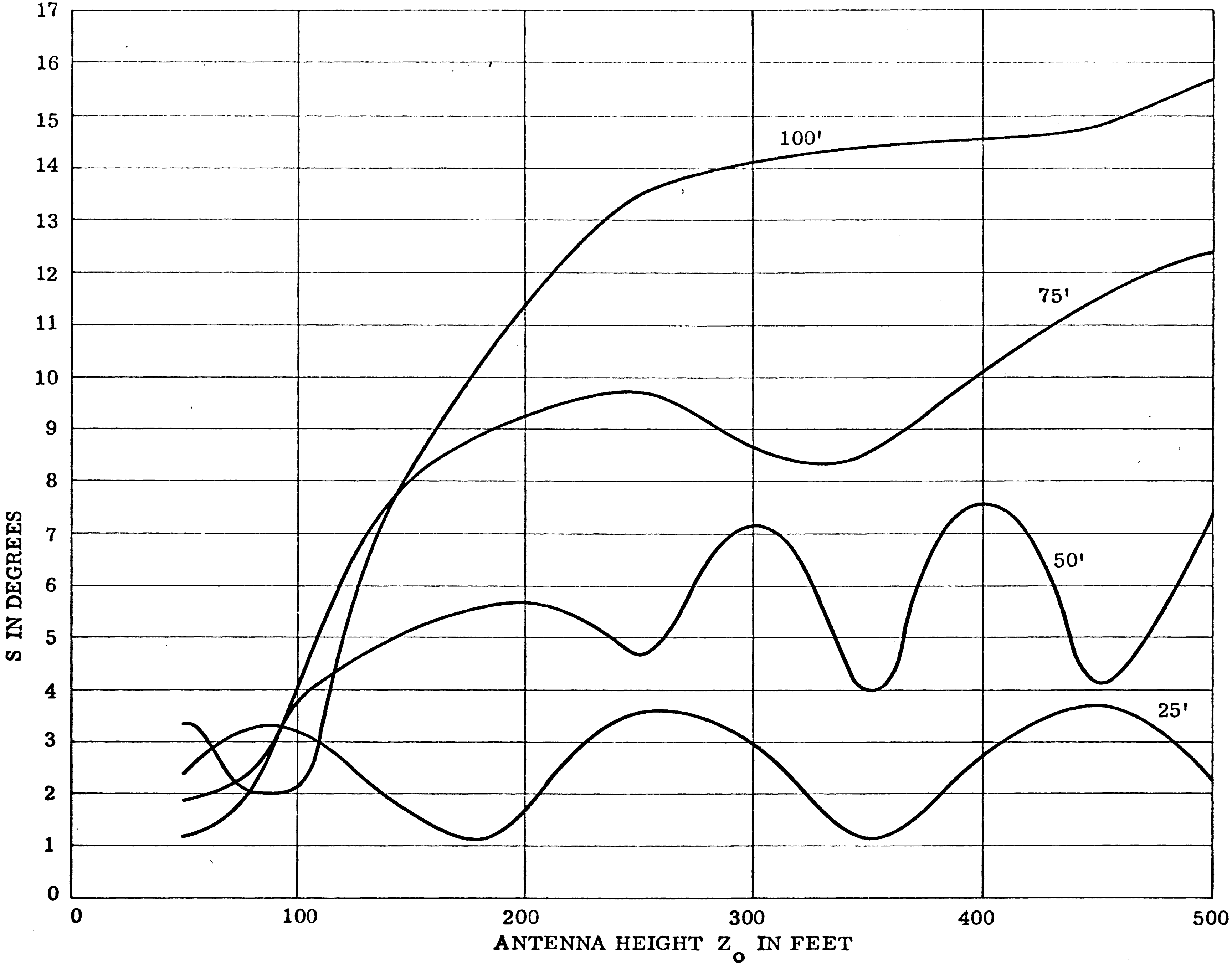

Fig. 9(a): Average maximum scalloping as a function of DPLC antenna height with scatterer height as the parameter. Observation angle is in the direction of the first minimum. D = 1000', A = 0.02.

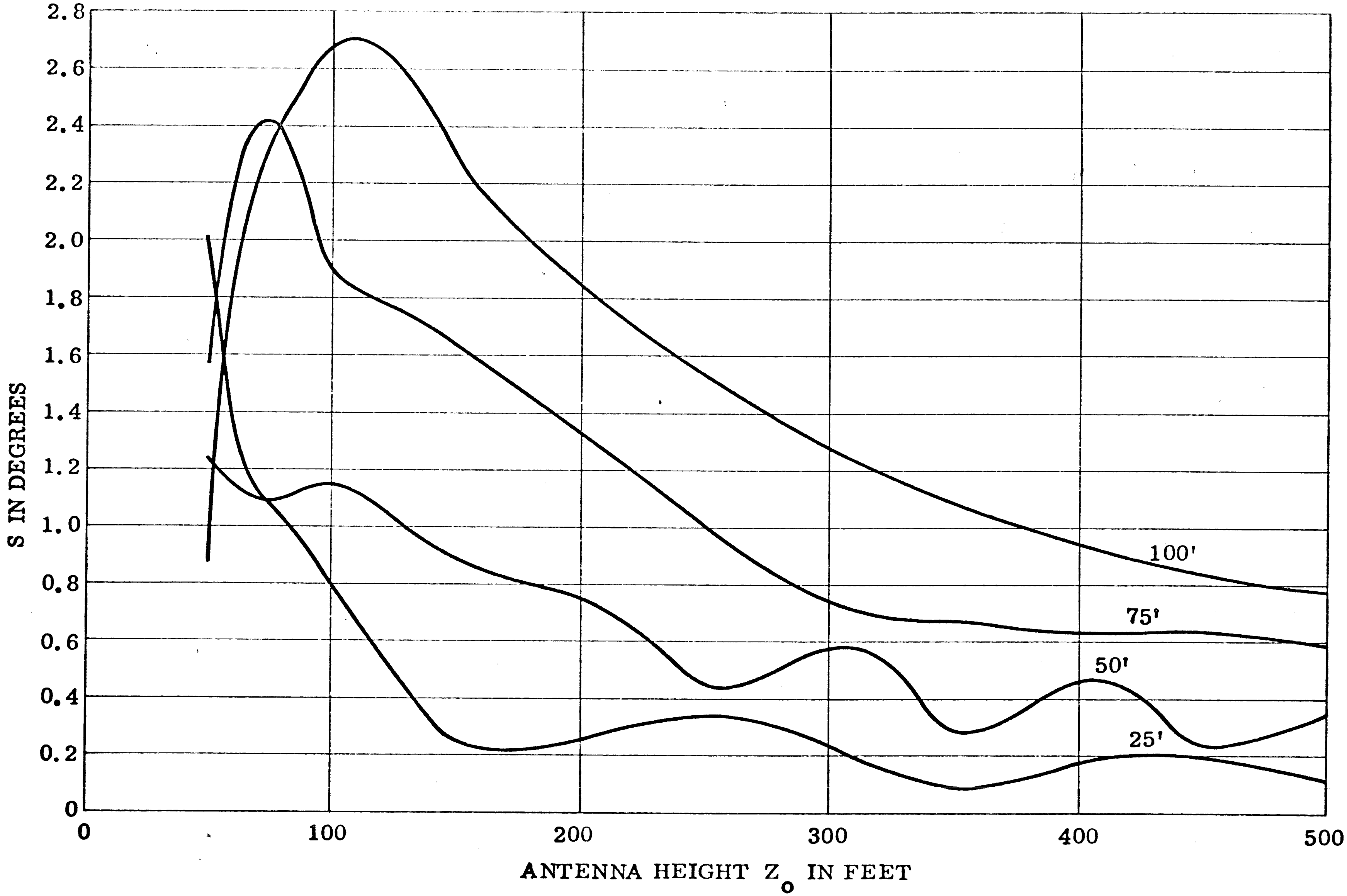

Fig. 9 (b): Average maximum scalloping as a function of DPLC antenna height with scatterer height as the parameter. Observation angle is in the direction of the first maximum. D = 1000', A = 0.02.

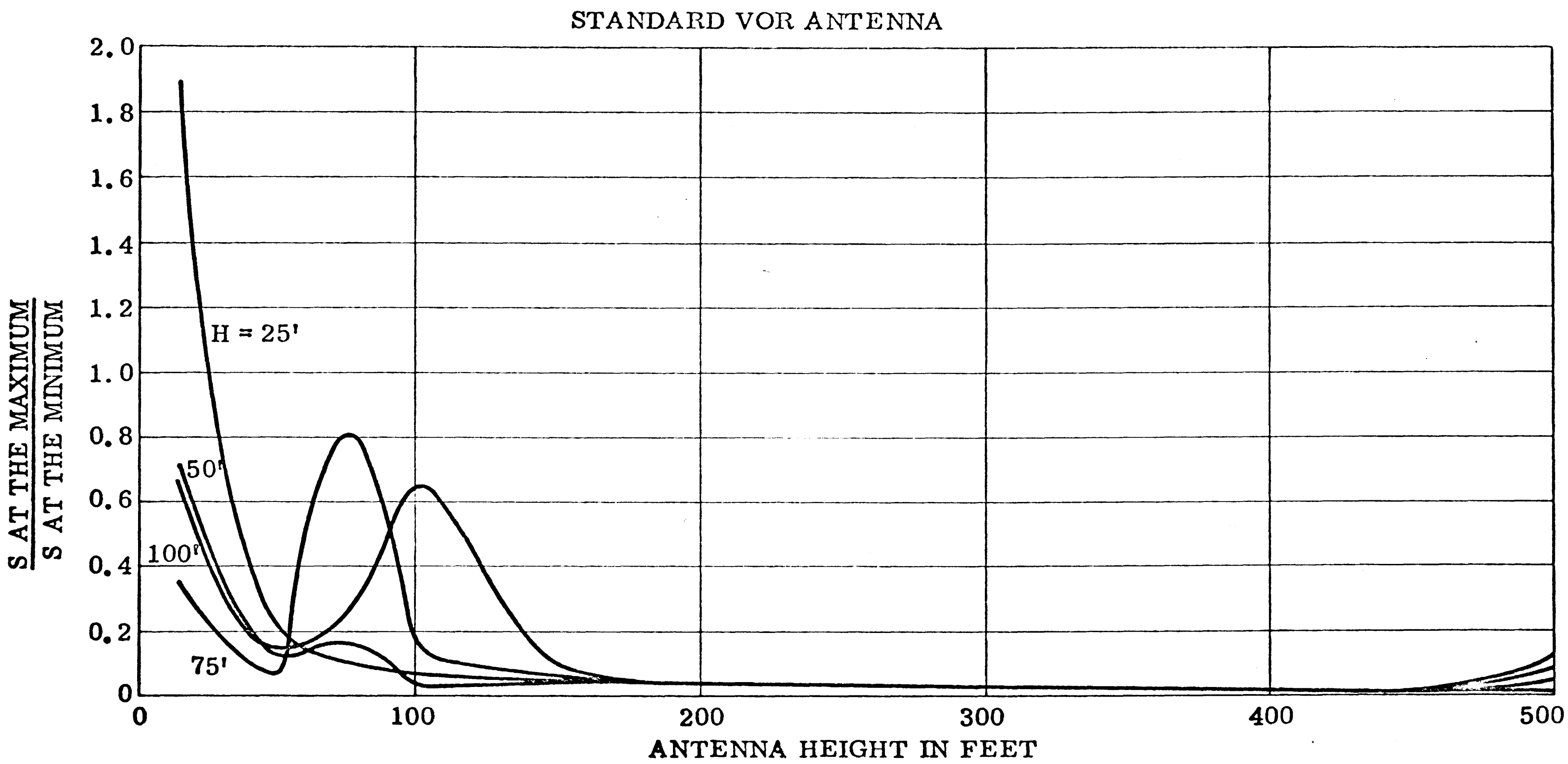

Fig. 10: $\frac{\text{S at the pattern maximum}}{\text{S at the pattern minimum}}$ as a function of the standard VOR antenna height with scatterer height as the parameter. D = 1000', A = 0.02.

are shown in Fig. 11. The results shown in Figs. 10 and 11 indicate that except for some isolated points, the average maximum scalloping amplitudes are in general larger at the observation angles corresponding to the first minimum in the pattern of the antenna. However, in some cases, depending on the particular combination of H and Z_o the scalloping may be larger at the observation angle corresponding to the pattern maximum.

The improvements in scalloping obtained by using a DPLC antenna for different heights of the antenna and the scatterer are shown in Figs. 12 and 13. Observe that for the range of parameters used, the DPLC antenna in general reduces the scalloping amplitude except for H = 100', Z_o = 412' in Fig. 12.

Results for D = 500', 2000' and 3000'

Similar results for D = 500', 2000' and 3000' are shown in Figs. 14-16. The elevation of the flight path is assumed to correspond to the first minimum in the elevation plane pattern of the antenna. The general behavior of the results is similar to that of the results for D = 1000'.

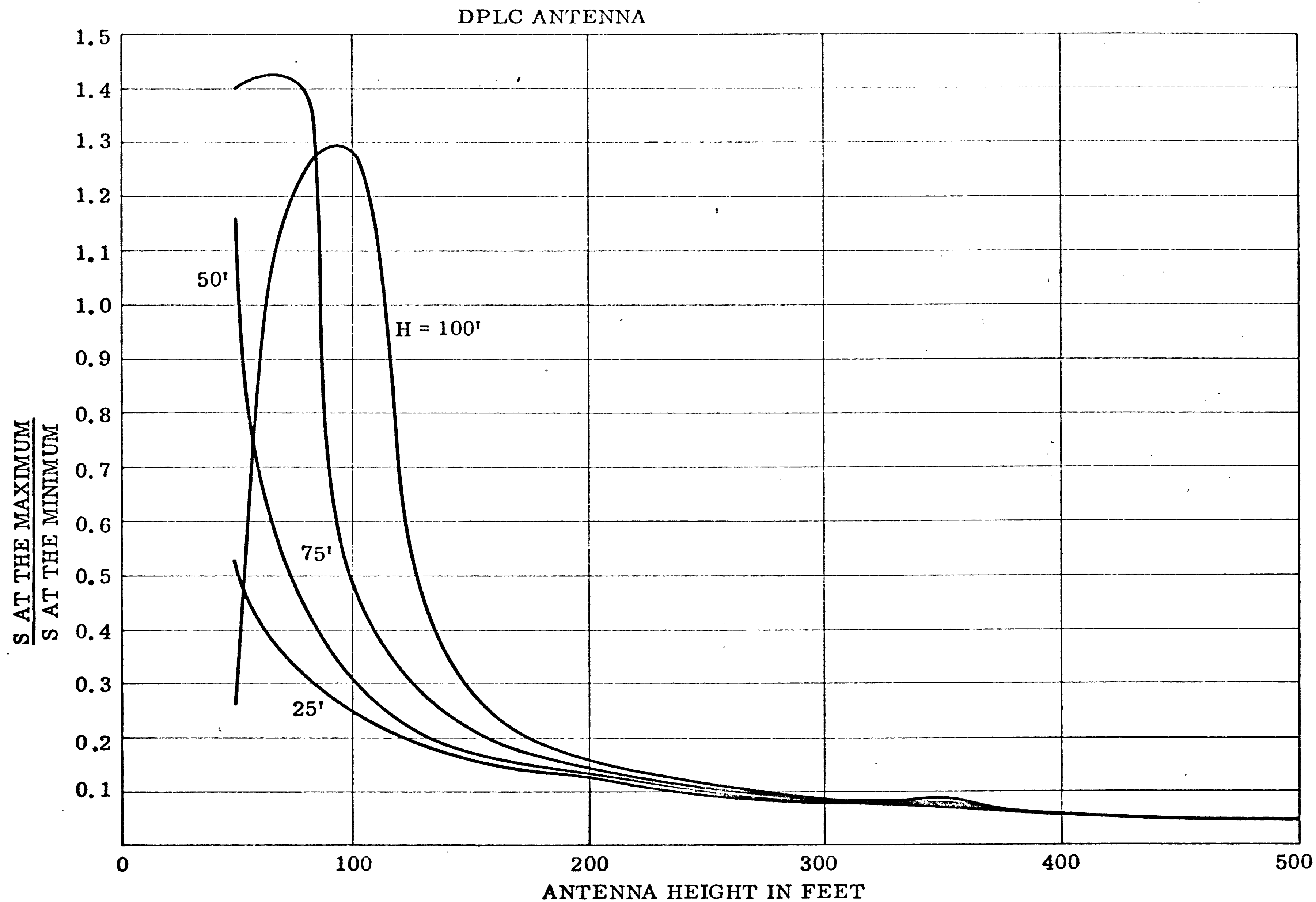

Fig. 11: $\frac{\text{S at the pattern maximum}}{\text{S at the pattern minimum}}$ as a function of the DPLC antenna height with scatterer height as the parameter. D = 1000', A = 0.02.

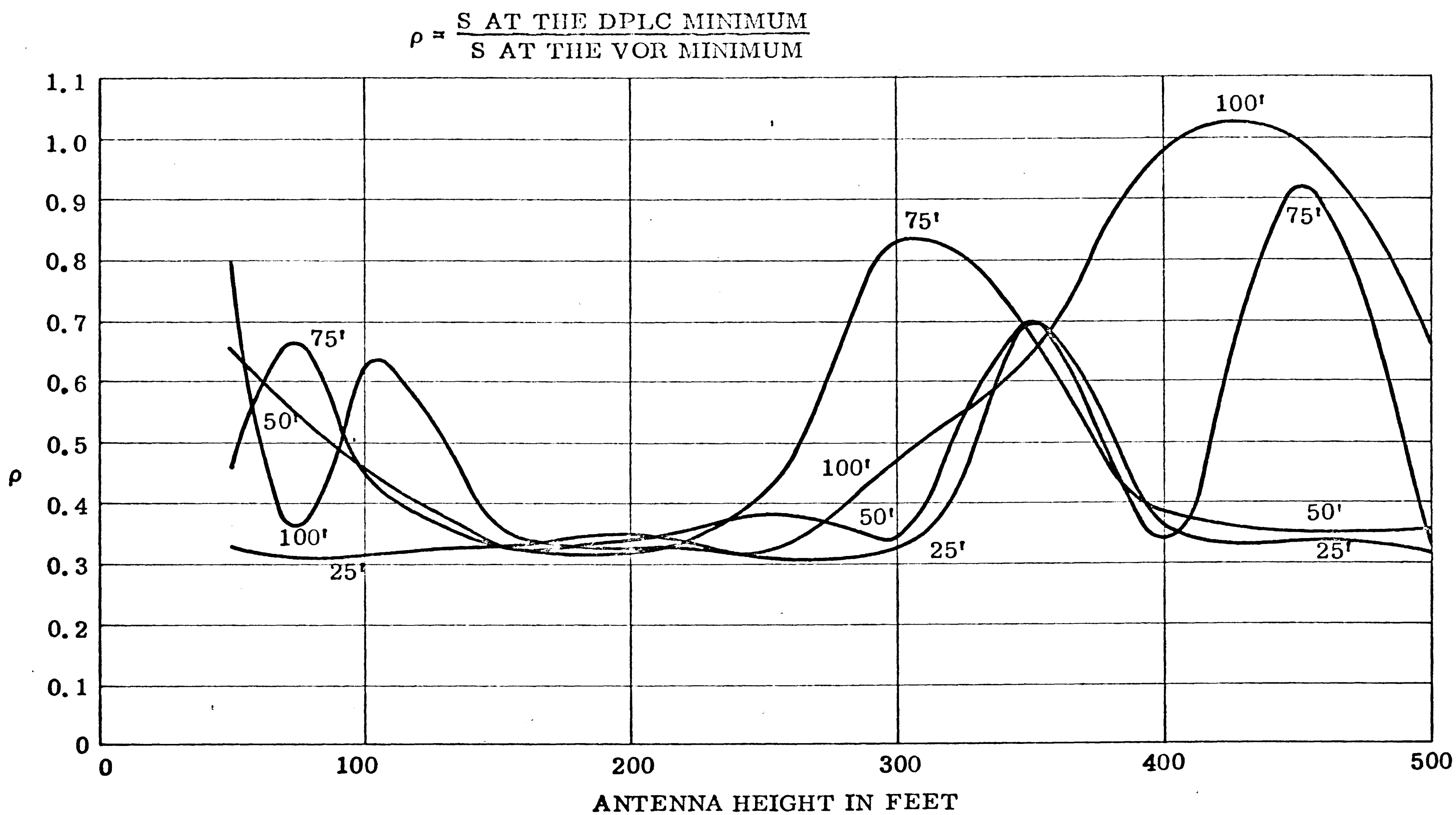

Fig. 12: The DPLC antenna improvement coefficient ρ as a function of the antenna height with scatterer height as the parameter. Observation angle is in the direction of the first pattern minimum. $D = 1000'$, $A = 0.02$.

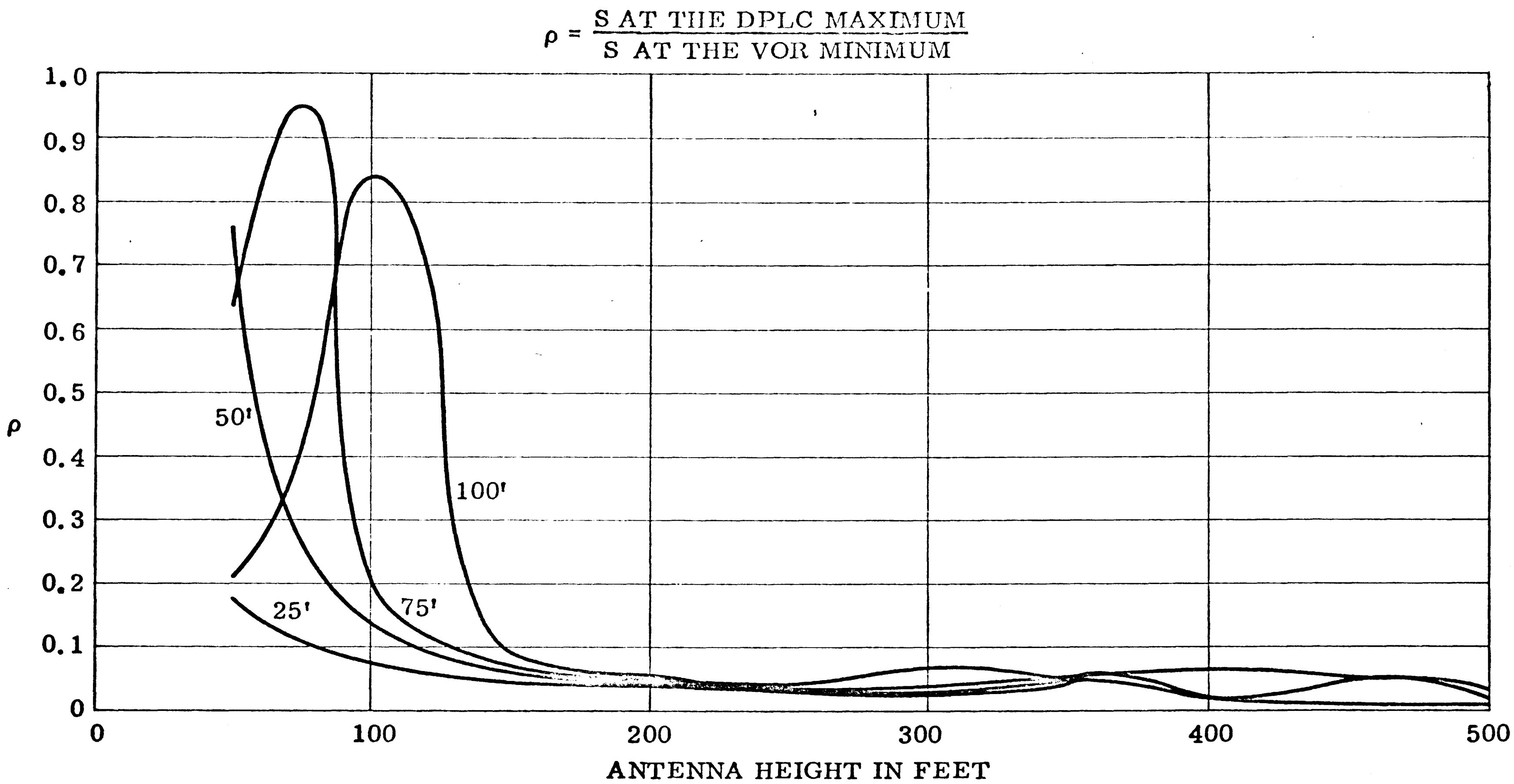

Fig. 13: The DPLC antenna improvement coefficient ρ as a function of the antenna height with scatterer height as the parameter. Observation angle is in the direction of the first pattern maximum. $D = 1000'$, $A = 0.02$.

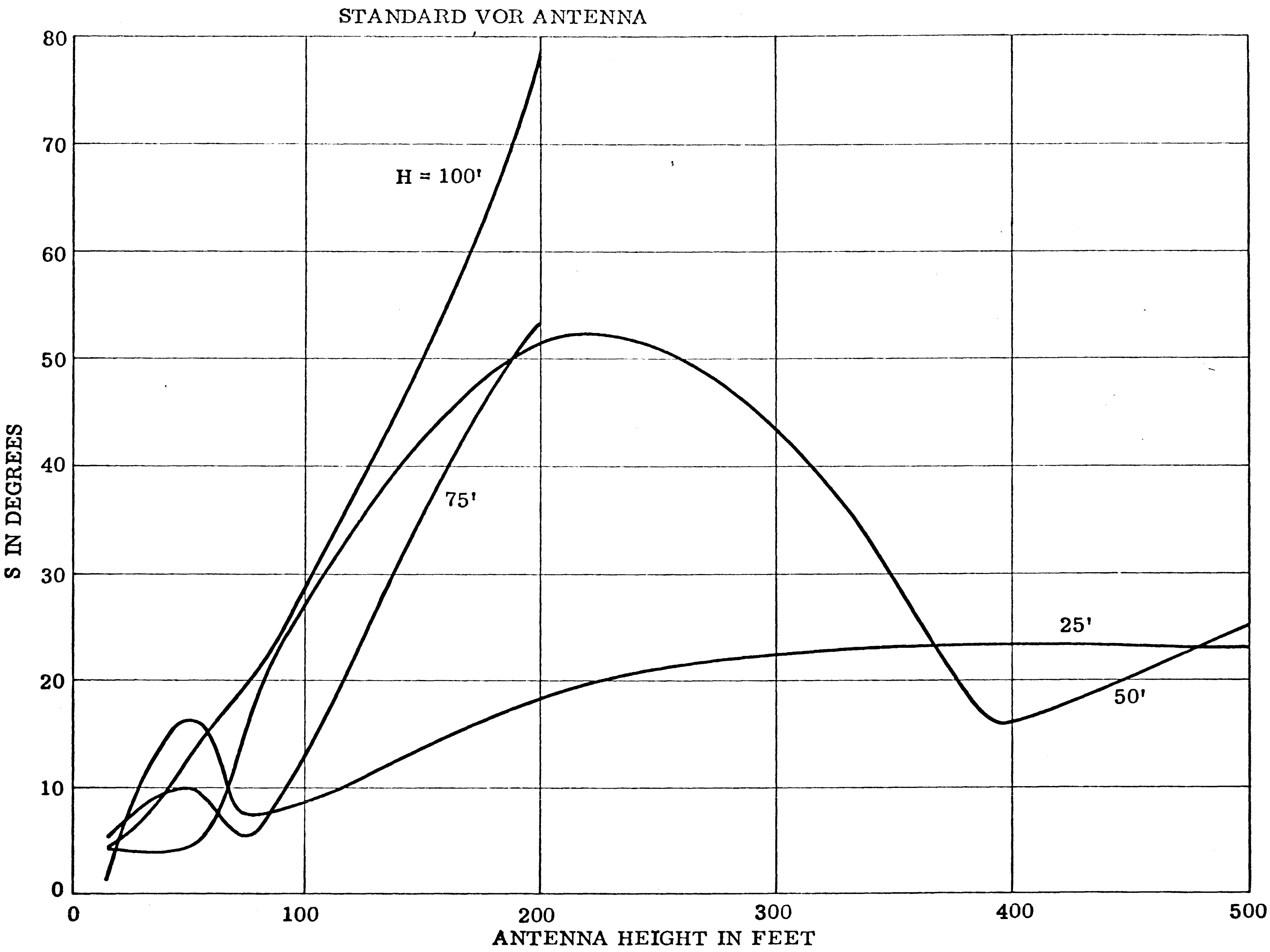

Fig. 14(a): Average maximum scalloping as a function of standard VOR antenna height with scatterer height as the parameter. Observation angle is in the direction of the first minimum. D = 500', A = 0.02.

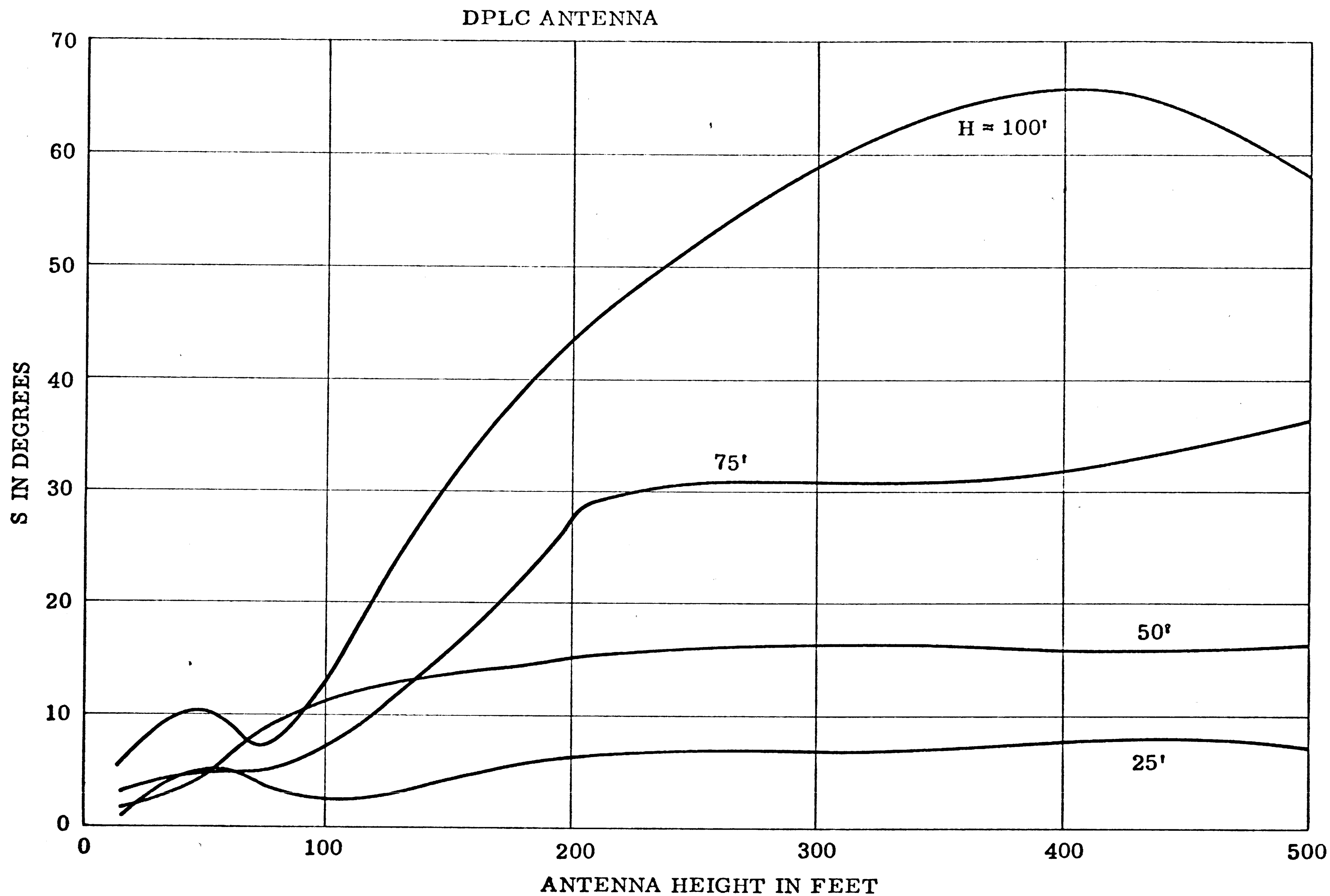

Fig. 14(b): Average maximum scalloping as a function of the DPLC antenna height with scatterer height as the parameter. Observation angle is in the direction of the first minimum. D = 500', A = 0.02.

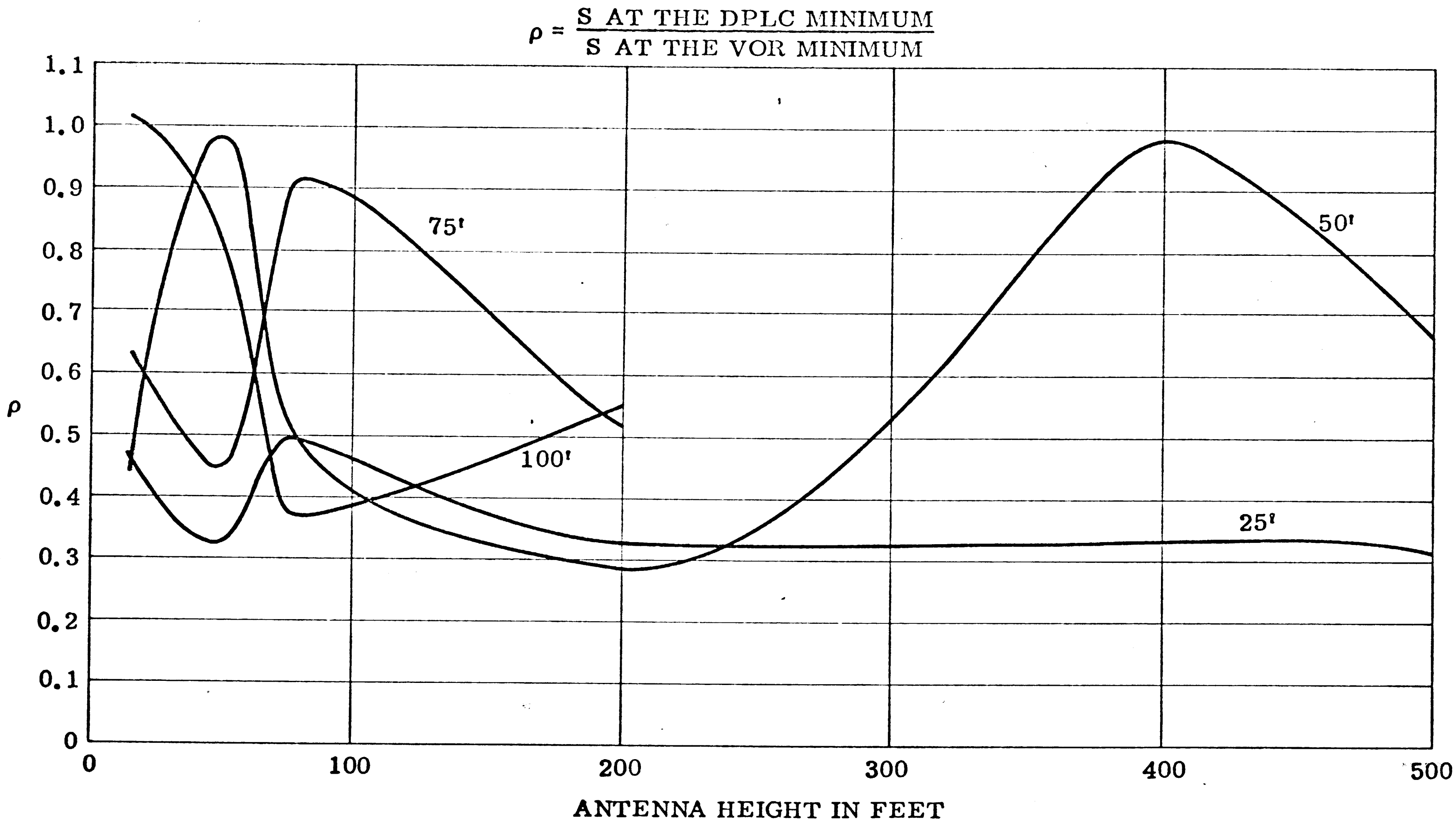

Fig. 14(c): The DPLC antenna improvement coefficient ρ as a function of the antenna height with scatterer height as the parameter. Observation angle is in the direction of the first pattern minimum. $D = 500'$, $A = 0.02$.

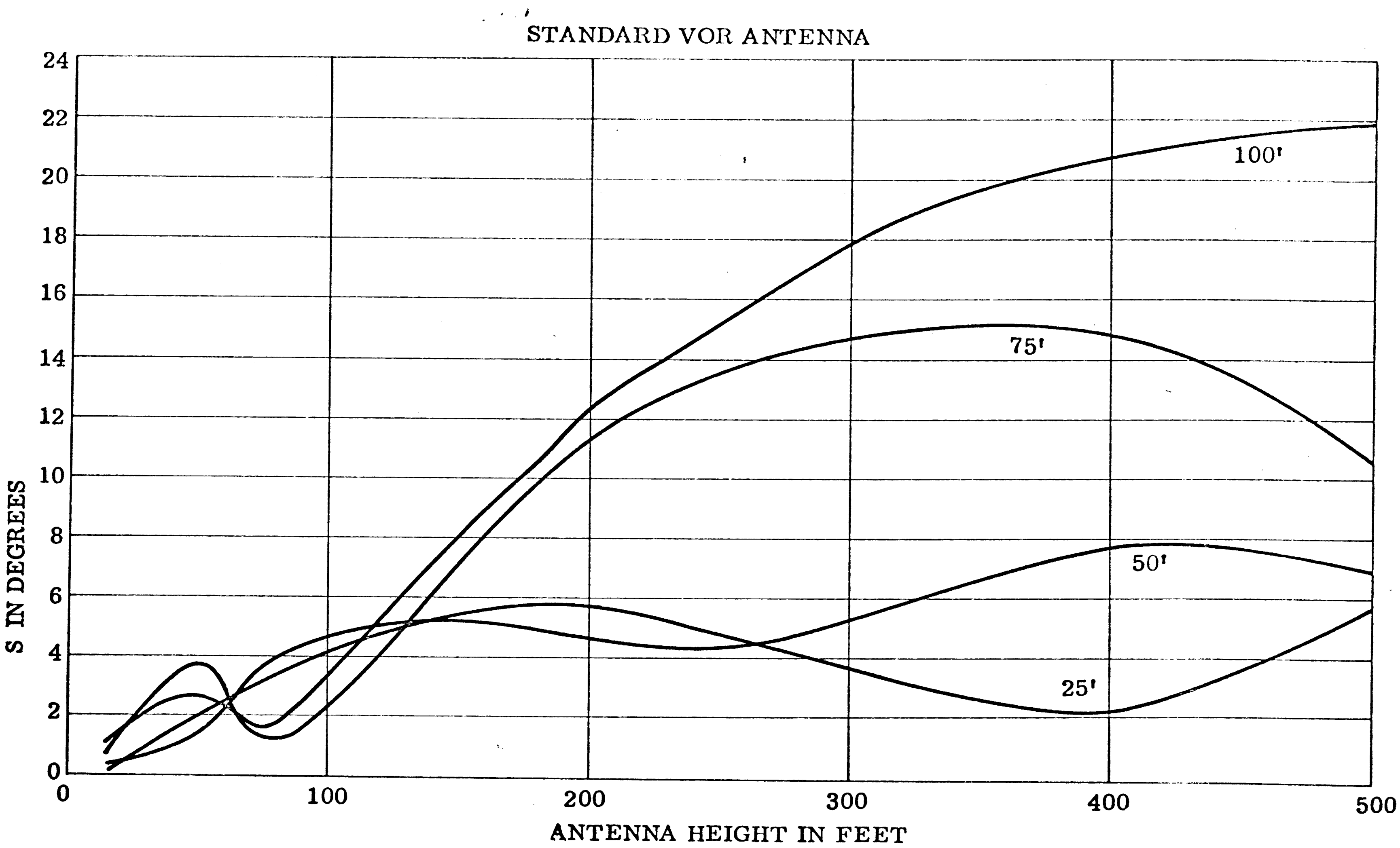

Fig. 15(a): Average maximum scalloping as a function of standard VOR antenna height with scatterer height as the parameter. Observation angle is in the direction of the first pattern minimum. D = 2000', A = 0.02.

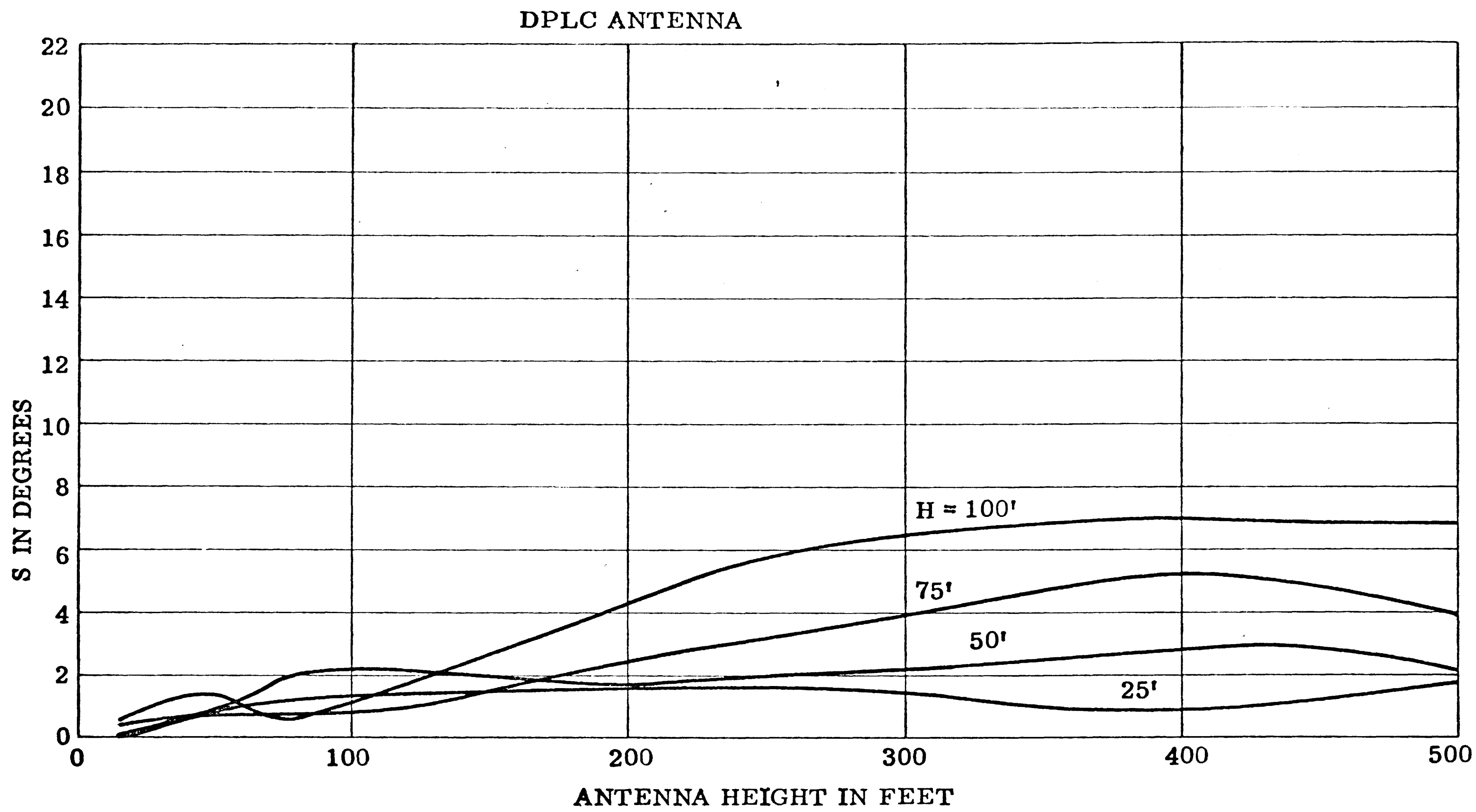

Fig. 15(b): Average maximum scalloping as a function of the DPLC antenna height with scatterer height at the parameter. Observation angle is in the direction of the first pattern minimum. D = 2000', A = 0.02.

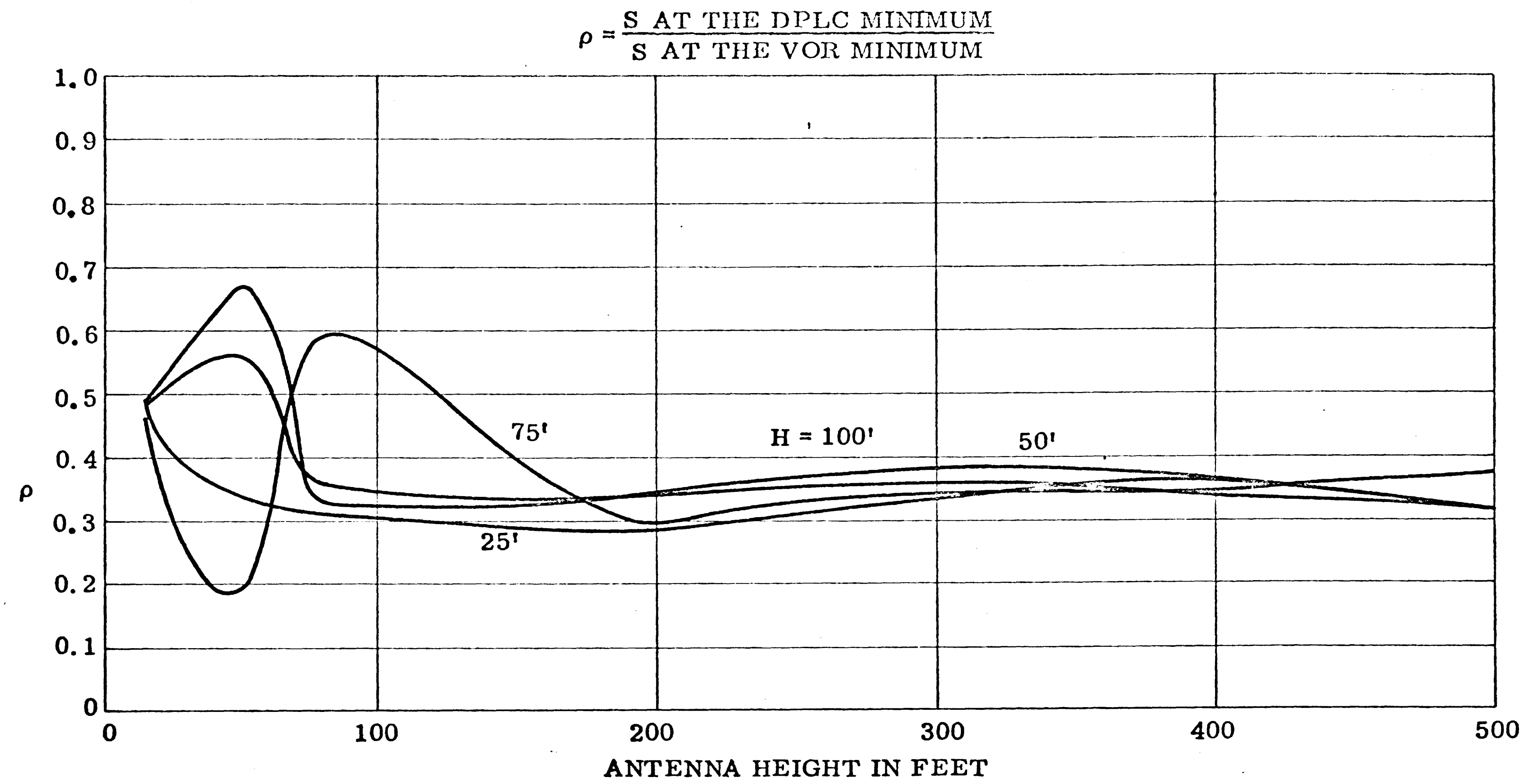

Fig. 15(c): The DPLC antenna improve coefficient ρ as a function of the antenna height with scatterer height as the parameter. Observation angle is in the direction of the first pattern minimum. D = 2000', A = 0.02.

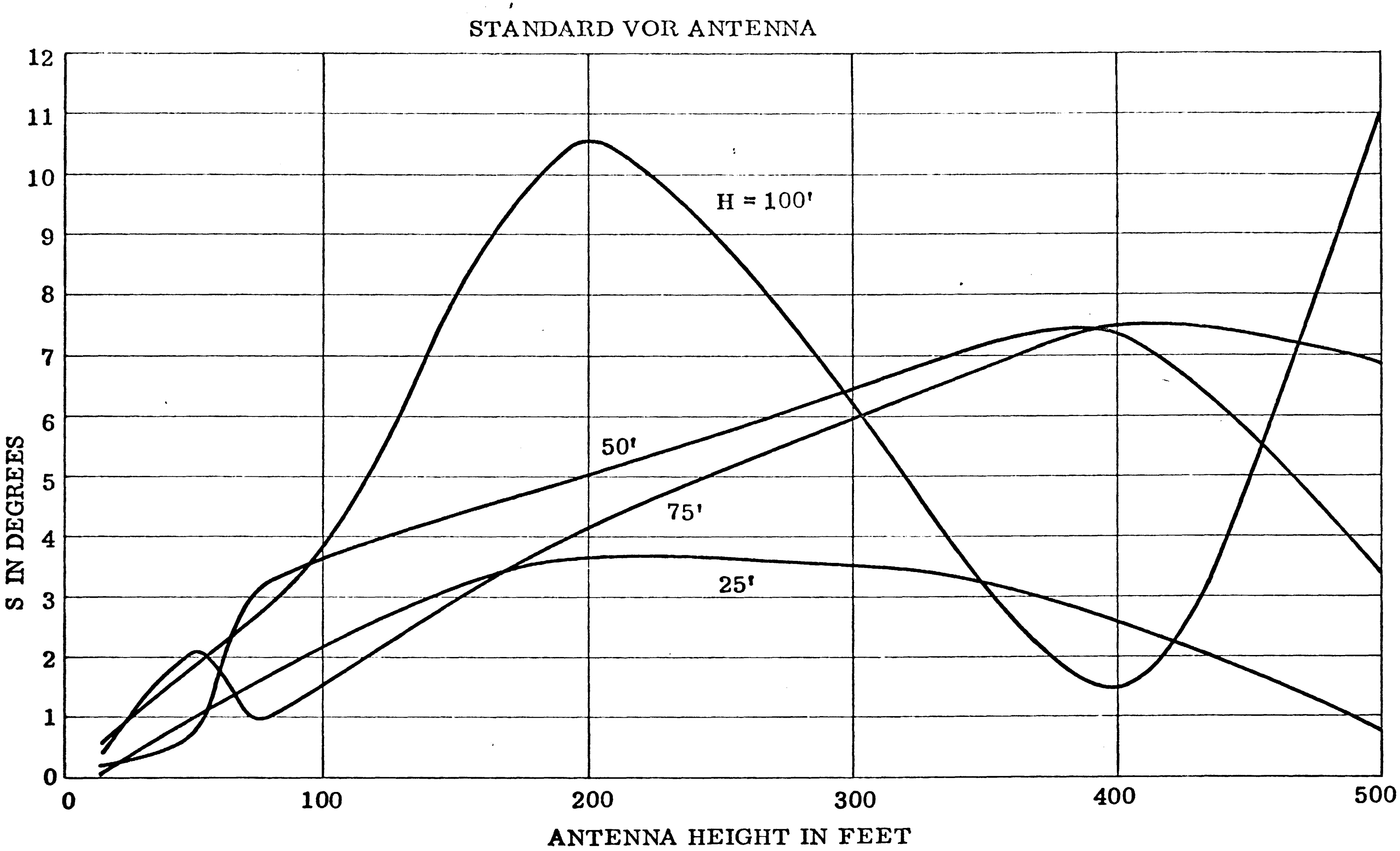

Fig. 16(a): Average maximum scalloping as a function of the standard VOR antenna height with scatterer height as the parameter. Observation angle is in the direction of the first pattern minimum. D = 3000', A = 0.02.

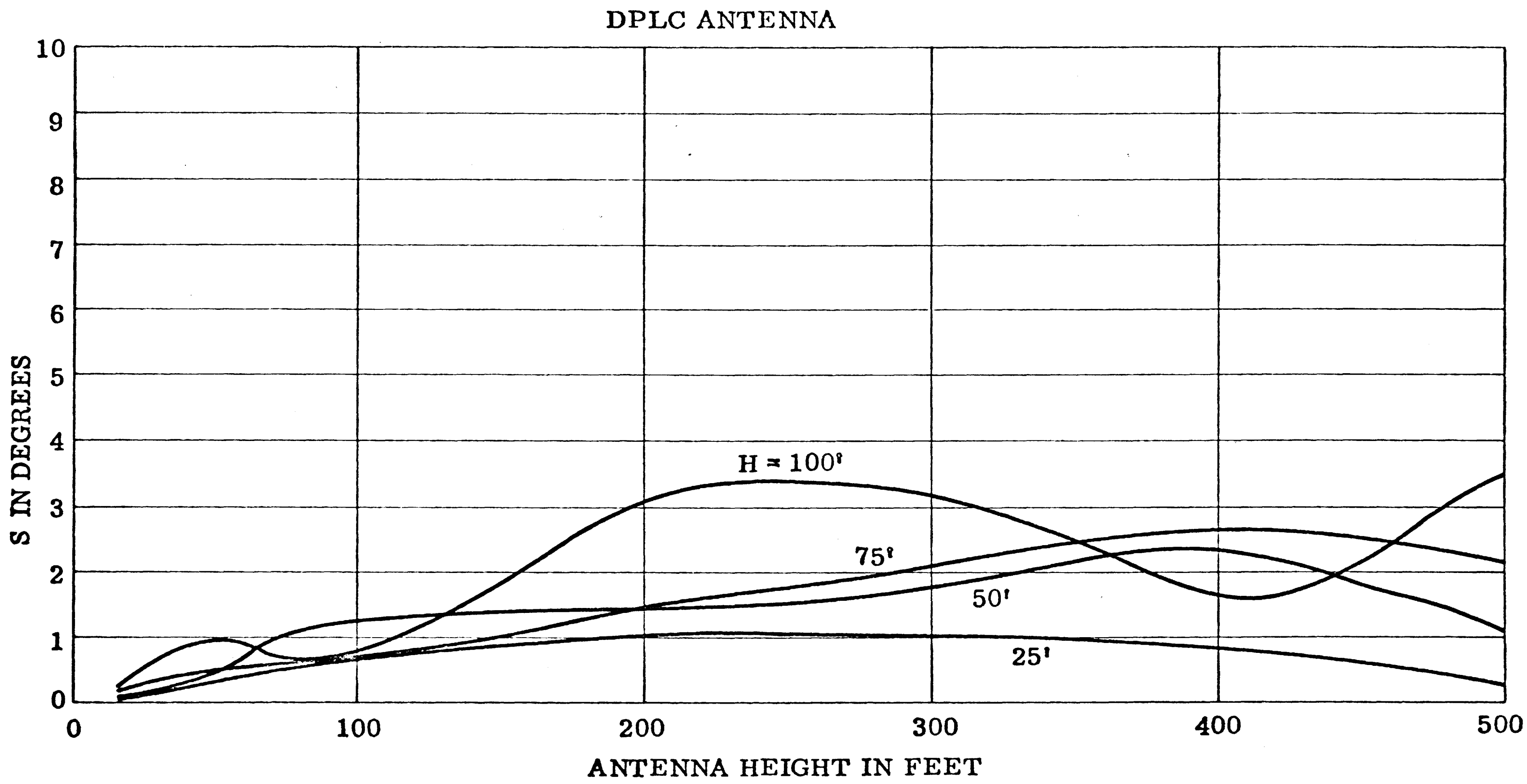

Fig. 16 (b): Average maximum scalloping as a function of the DPLC antenna height with scatterer height as parameter. Observation angle is in the direction of the first pattern minimum. D = 3000', A = 0.02.

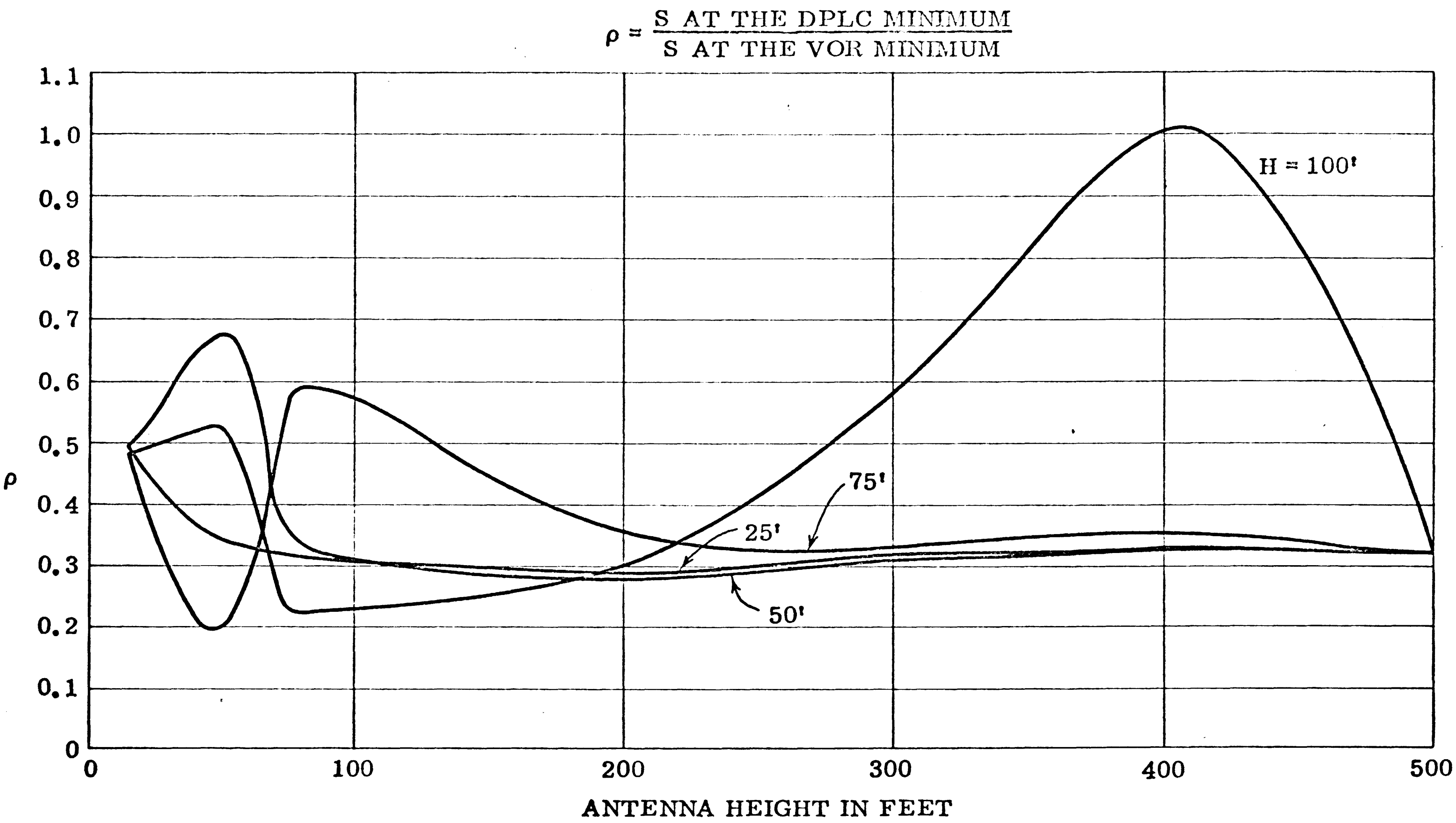

Fig. 16(c): The DPLC antenna improvement coefficient σ as a function of the antenna height with scatterer height as the parameter. Observation angle is in the direction of the first pattern minimum. D = 3000', A = 0.02.

VII

SCALLOPING AMPLITUDE AS A FUNCTION OF D

In this section we discuss the variation of the scalloping amplitude as a function of distance D of the scatterer from the antenna. The scalloping amplitude given by Eq. (2) depends on D through the parameter A and θ_1. In general, the magnitude of the scattering coefficient varies inversely with the distance D. In order to bring out the dependence of S on D we define a modified scattering coefficient A' such that

$$A' = \frac{A \times 1000}{D} \quad , \tag{4}$$

where the distance D is expressed in feet. Note that A' is normalized such that at D = 1000', A' = A. We shall assume that A = 0.02, as before. With this modified definition of the scattering coefficient we calculate S_1 by using Eq. (2). The results obtained for the standard VOR antenna located 15' above ground are shown in Fig. 17. Corresponding results for the DPLC antenna located at Z_o = 15' and 50' are shown in Figs. 18(a) and 18(b). In all cases, the elevation of the flight path corresponds to the first minimum in the pattern of the antenna. In general the scalloping amplitude decreases with increase of D. The cross-over points in some cases are attributed to the image effects on the scattered field at the observation point, i.e., due to the special combination of the values of H and θ.

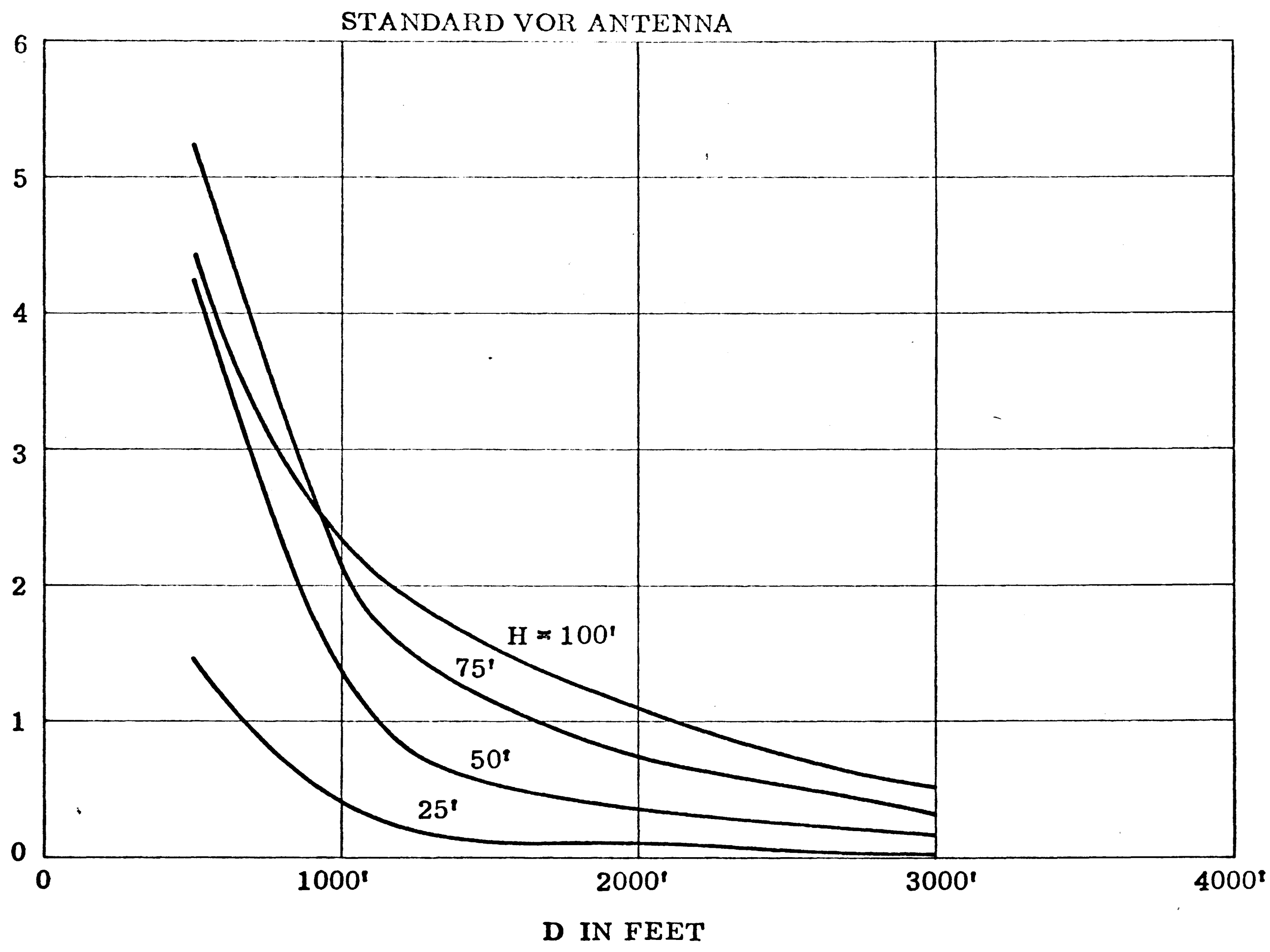

Fig. 17: Average maximum scalloping as a function of D with the scatterer height as the parameter. Observation angle is in the direction of the first pattern minimum. $Z_o = 15'$, VOR antenna.

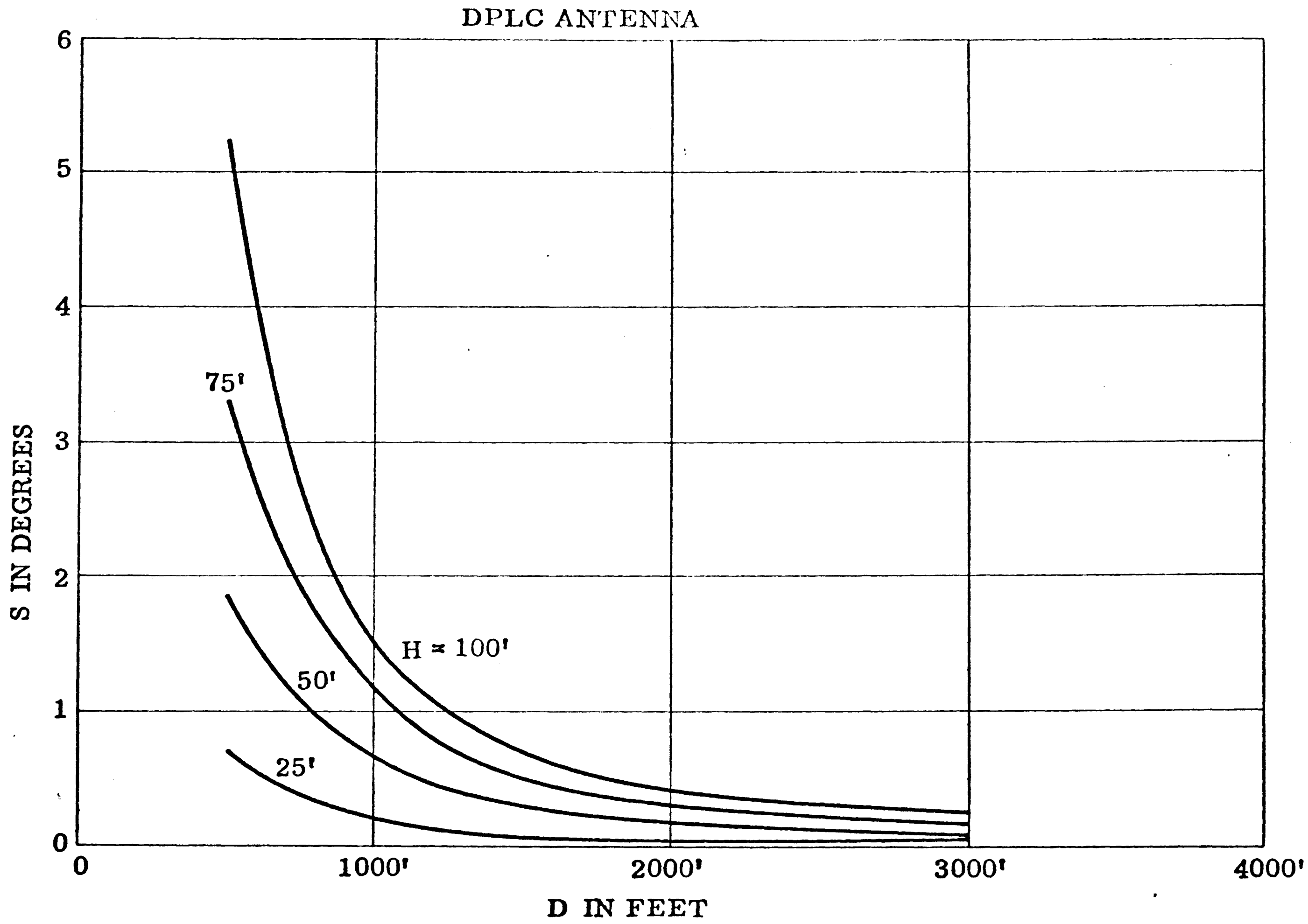

Fig. 18(a): Average maximum scalloping as a function of D with the scatterer height as the parameter. Observation angle is in the direction of the first pattern minimum. $Z_o = 15'$. DPLC antenna.

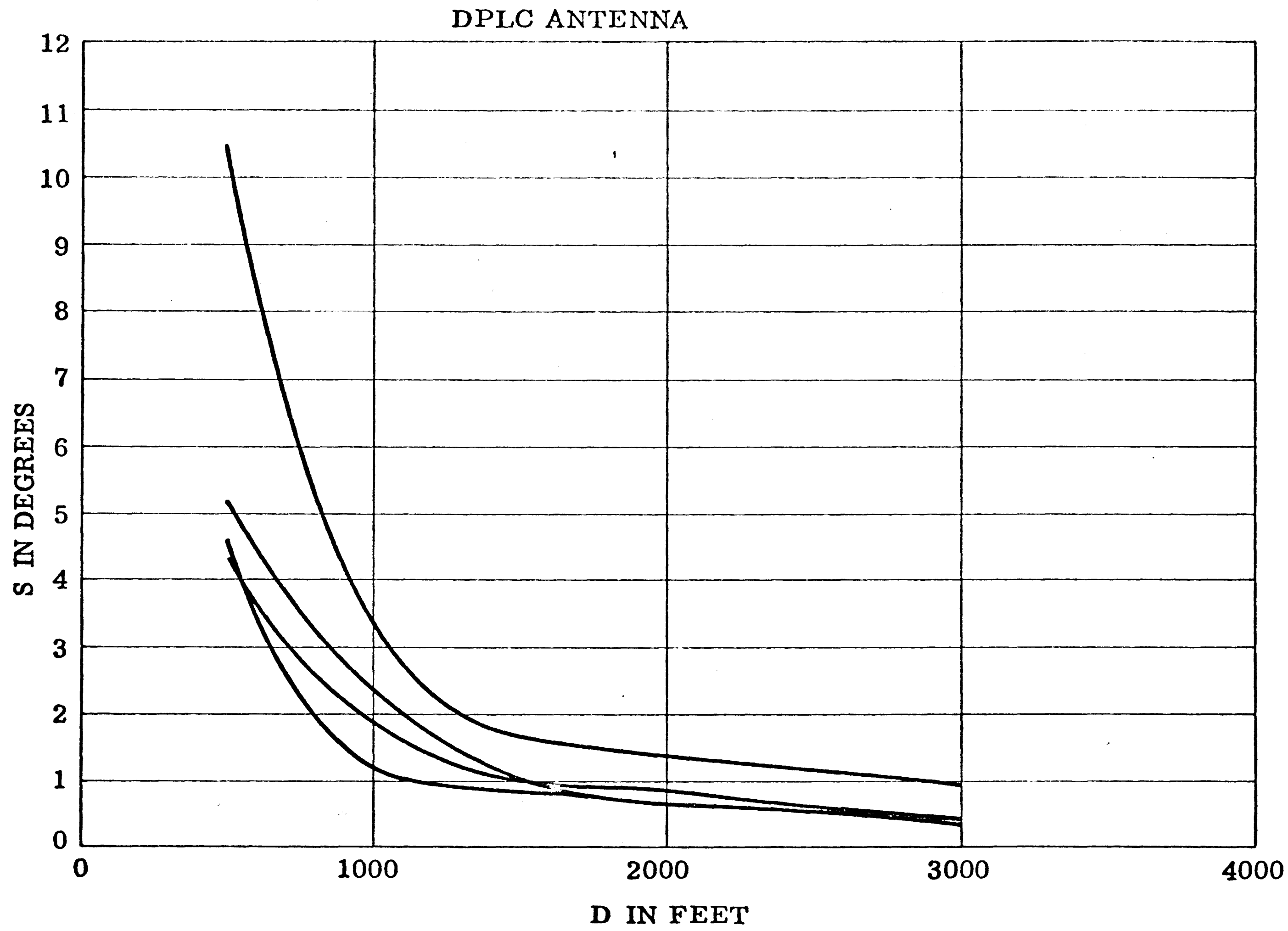

Fig. 18 (b): Average maximum scalloping as a function of D with the scatterer height as the parameter. Observation angle is in the direction of the first pattern minimum. $Z_o = 50'$, DPLC antenna.

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