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MEMO TO: File

FROM: T. B. A. Senior

SUBJECT: Assessment of the Far Field Criterion, I: Circular Cylinder

In radar cross section studies the scattering cross section is customarily defined for a plane wave incident on the target and for the scattered field observed at an infinitely large distance away. It is obvious that neither of these idealisations can be achieved in practice, and if the scattering is measured using transmitting and receiving antennas a finite distance R from the target, there will be some effects attributable to the finite value of R; but if R is chosen large enough, say, $R \geq R_0$, it is expected that these errors can be reduced to a level which is acceptable. This, then, is the basis for the far field criterion in scattering work.

If the transmitter is sufficiently far from the target, the illuminating field will be a spherical wave diverging from the phase center of the transmitting antenna, and this differs from the idealisation of an incident plane wave in the progressive amplitude decay as a function of distance and the curvature of the wavefront. Under most circumstances the latter would appear to be the dominant effect and a criterion for an acceptable distance of the transmitter from the target can now be obtained by imposing an upper limit on the phase variation of the illuminating field over the lateral extent of the target. In backscattering work it is customary to use the same antenna for reception and transmission and the scattering part of the problem can likewise be considered in terms of the

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phase deviations. If this antenna is small compared with the lateral dimension D of the target, the range R can be connected with the one-way phase deviation Φ over the target using the formula

$$R = \frac{\pi D^2}{4\lambda\Phi} \quad , \tag{1}$$

and the far field criterion

$$R \geq R_o = \frac{2D^2}{\lambda} \tag{2}$$

now follows by demanding that the phase deviation not exceed $\frac{\pi}{8}$ ($= 22\frac{1}{2}^o$) one-way. A discussion of this criterion in its various guises has been given by Kouyoumjian and Peters (1965).

In many experiments, particularly those carried out at high frequencies, it may be hard to achieve even the minimum range R_o, and for lack of any practical alternative it is then necessary to carry out the measurements at closer range. As is well known, some of the consequences of markedly violating the requirement $R \geq R_o$ are a filling-in of the nulls in the scattering pattern, a broadening of the lobes and a reduction in the levels of the peaks. In extreme cases, the major lobes may assume a hump-backed shape or may be split. Such grosser effects can be illustrated using the physical optics approximation to the scattering.

Most of these defects will not be evident at the larger ranges, but even if $R \geq R_o$, there will still be some differences from the ideal pattern for $R = \infty$. Unfortunately, the nature and magnitudes of these differences are unknown, and though they may well be insignificant for most practical purposes, they could be vital if (for example) measured data, carefully acquired, were used to test the output from a computer program. This application of measured data is of growing importance as the complexity of computer programs increases. It is therefore of interest to examine theoretically the effect of finite R on the amplitude and phase of the backscattered field.

An analysis of this type can be carried out only for a specific choice of scattering object and the choice that is made will almost certainly affect the magnitude of the effects observed. There will, in addition, be frequency and polarisation effects, but since the far field criterion is most difficult to satisfy at high frequencies, it would seem desirable to concentrate on objects at least a wavelength or two in dimension.

Two bodies have been selected for consideration: a right circular cylinder of radius a and a strip (or ribbon) of width d viewed normal to its face. Physical reasoning would suggest that whereas the effective width for the cylinder is less than 2a, becoming more so as ka increases, that for the strip remains d at all frequencies, and the errors attendant on any one choice of R/R_0 should therefore be less for the cylinder than for the strip. Both bodies are two dimensional and are illuminated by a cylindrical wave diverging from a line source parallel to the generators. The fields are observed back at the source. The present Memorandum is concerned only with the results for the circular cylinder.

A perfectly conducting circular cylinder of radius a is illuminated by a line source at a distance ρ from, and parallel to, the axis z of the cylinder. The scattered field observed back at the same point is then

$$E_z^s = -\sum_{n=0}^{\infty} \epsilon_n \frac{J_n(ka)}{H_n^{(1)}(ka)} \left\{ H_n^{(1)}(k\rho) \right\}^2$$

$$H_z^s = -\sum_{n=0}^{\infty} \epsilon_n \frac{J_n'(ka)}{H_n^{(1)'}(ka)} \left\{ H_n^{(1)}(k\rho) \right\}^2 \tag{3}$$

for E and H polarisations, respectively, where a time factor $e^{-i\omega t}$ has been assumed and suppressed. For sufficiently large $k\rho$, the Hankel functions of order $k\rho$ can be replaced by the leading terms of their asymptotic expansion, viz.

$$H_n^{(1)}(k\rho) \sim \sqrt{\frac{2}{\pi k\rho}}\,(-i)^n\, e^{ik\rho - i\,\pi/4}$$

in which case

$$E_z^s \sim -\frac{2i}{\pi k\rho}\, e^{2ik\rho}\, P_E(0)$$

$$H_z^s \sim -\frac{2i}{\pi k\rho}\, e^{2ik\rho}\, P_H(0) \tag{4}$$

where

$$P_E(0) = -\sum_{n=0}^{\infty} \epsilon_n (-1)^n \frac{J_n(ka)}{H_n^{(1)}(ka)}$$

$$P_H(0) = -\sum_{n=o}^{\infty} \epsilon_n (-1)^n \frac{J_n'(ka)}{H_n^{(1)'}(ka)} \tag{5}$$

are the far field amplitudes (see Bowman et al, 1969) for plane wave incidence and scattering in the backwards direction.

These amplitudes could (in principle) be determined experimentally by measuring E_z^s and H_z^s at a sufficiently large range ρ and then removing the effect of the space factors using a calibration process. If this same procedure were applied to eqs. (3), the results would differ from $P_E(0)$ and $P_H(0)$ by the complex factors Γ_E and Γ_H respectively, where

$$\Gamma_E = -\frac{i\pi k\rho}{2P_E(0)}\, e^{-2ik\rho} \sum_{n=0}^{\infty} \epsilon_n \frac{J_n(ka)}{H_n^{(1)}(ka)} \left\{H_n^{(1)}(k\rho)\right\}^2$$

$$\Gamma_H = -\frac{i\pi k\rho}{2P_H(0)}\, e^{-2ik\rho} \sum_{n=0}^{\infty} \epsilon_n \frac{J_n'(ka)}{H_n^{(1)'}(ka)} \left\{H_n^{(1)}(k\rho)\right\}^2 \quad . \tag{6}$$

It is of interest to compute these in amplitude and phase (degrees) as functions of $k\rho$ for fixed ka. According to eq. (2) with $D = 2a$, the far field distance is

$$\rho_o = \frac{8a^2}{\lambda} ,$$

implying

$$k\rho_o = \frac{4}{\pi}(ka)^2$$

and it is convenient to display $\Gamma_{E,H}$ as functions of γ, where

$$\gamma = \frac{k\rho}{(ka)^2} . \tag{7}$$

Since $\rho = a$ corresponds to a line source at the surface of the cylinder, we require that $\gamma > \frac{1}{ka}$. Comparison of eqs. (1) and (7) also shows that

$$\gamma = \frac{1}{2\Phi} , \tag{8}$$

so that γ is merely the reciprocal of the two way difference between the phase associated with the top and bottom of the cylinder and that corresponding to the center. When $\gamma = 1$, $\Phi = \frac{1}{2}\,(\simeq 28^o)$, whereas at the far field distance, $\gamma = \frac{4}{\pi}\,(\simeq 1.273)$.

The calculations turned out to be relatively straightforward. The series expressions on the right hand sides of eqs. (3) and (5) were individually computed and for the values of ka and $k\rho$ of concern to us ($k\rho > ka \geq 5$) it proved adequate to truncate the series at the term $n = 2\,[ka] + 10$. The Hankel functions and their derivatives were calculated by forward recursion. The same procedure was also used for most of the Bessel functions required, but for those functions whose orders were much greater than the argument, a backward recursion scheme proved more effective and was employed. A program listing is given in the Appendix. For each pair of ka and $k\rho$ values, the output consists of Γ_E and Γ_H in amplitude and phase (degrees), as well as the squared moduli of the fields themselves.

Data were obtained for ka = 5.0 (2.5) 25.0 and a variety of $k\rho$ in the range $ka < k\rho \leq 400$. Typical of these are the results shown in Table 1 for ka = 12.5.

Table 1: Date for ka = 12.5

$k\rho$	γ	$\lvert\Gamma_E\rvert$	ϕ_E, deg.	$\lvert\Gamma_H\rvert$	ϕ_H, deg.
400	2.560	1.0156	-0.023	1.0205	0.091
300	1.920	1.0212	-0.000	1.0269	0.111
250	1.600	1.0259	0.002	1.0318	0.145
200	1.280	1.0330	-0.022	1.0394	0.217
150	0.960	1.0448	-0.043	1.0519	0.333
125	0.800	1.0545	-0.055	1.0617	0.423
100	0.640	1.0695	-0.074	1.0762	0.542
90	0.576	1.0780	-0.080	1.0843	0.592
80	0.512	1.0889	-0.099	1.0945	0.647
70	0.448	1.1037	-0.121	1.1076	0.693
60	0.384	1.1245	-0.145	1.1258	0.701
50	0.320	1.1555	-0.176	1.1536	0.631

As expected, with increasing γ, $\lvert\Gamma_E\rvert$ and $\lvert\Gamma_H\rvert$ decrease towards unity and ϕ_E and ϕ_H approach zero, but whereas the amplitudes are monotonic functions, the phases show a very slight oscillation superimposed on a uniform behavior. These oscillations are more apparent for H polarisation and, for some values of ka, produce sign changes in the phase errors as γ increases. $\lvert\Gamma_E\rvert$ and $\lvert\Gamma_H\rvert$ are quite similar to one another; for fixed γ, the (small) difference $\lvert\Gamma_E\rvert - \lvert\Gamma_H\rvert$ is an oscillatory function of ka. In contrast, ϕ_E and ϕ_H are rather different, with the latter exceeding the former by a factor 4 or more, but even ϕ_H is seldom more than a few tenths of a degree. At distances greater than a few radii from the cylinder, the phase errors appear insignificant for most practical purposes.

The amplitude ratios $\lvert\Gamma_E\rvert$ and $\lvert\Gamma_H\rvert$ are plotted as functions of γ in

Figs. 1 and 2. For any fixed value of γ it is at once evident that the errors decrease with increasing ka. In particular, for $\gamma = 1.273$ corresponding to the far field distance $\rho = \rho_0$, the E-polarisation errors decrease from 0.75 dB for ka = 5 to 0.17 dB for ka = 20. If, for example, an error of 1.0 dB were acceptable, the far field criterion would overestimate the distance ρ required for all ka $\geq$ 5, and would do so by an amount which increases with ka. Since, in practice, it may be hard to achieve the far field distance ρ_0, particularly at high frequencies (large ka), such overestimates are important — and wasteful — and it is now of interest to see how $k\rho$ varies with ka for a given amount of error.

As an example, suppose that the maximum permissible error is 0.5 dB. We therefore require $|\Gamma| \leq 1.059$, and by observing the values of γ at which the curves in Figs. 1 and 2 intercept this horizontal line, we obtain the results shown in Table 2. For E-polarisation, ρ_{min} is almost proportional to the radius

Table 2: Minimum Ranges for 0.5 dB Errors

ka	E-polarisation		H-polarisation	
	γ	ρ/a	γ	ρ/a
5.0	1.875	9.4	2.31	11.6
7.5	1.25	9.4	1.62	12.2
10.0	0.925	9.3	1.155	11.6
12.5	0.745	9.3	0.84	10.5
15.0	0.62	9.3	0.62	9.3
17.5	0.53	9.3	0.495	8.7
20.0	0.46	9.2	0.42	8.4
22.5	0.41	9.2	0.37	8.3
25.0	0.365	9.1	0.35	8.8

of the cylinder independently of the frequency. Indeed, for $5 \leq ka \leq 25$, ρ_{min}/a varies only from 9.4 to 9.1, and for an error not exceeding 0.5 dB, it is now

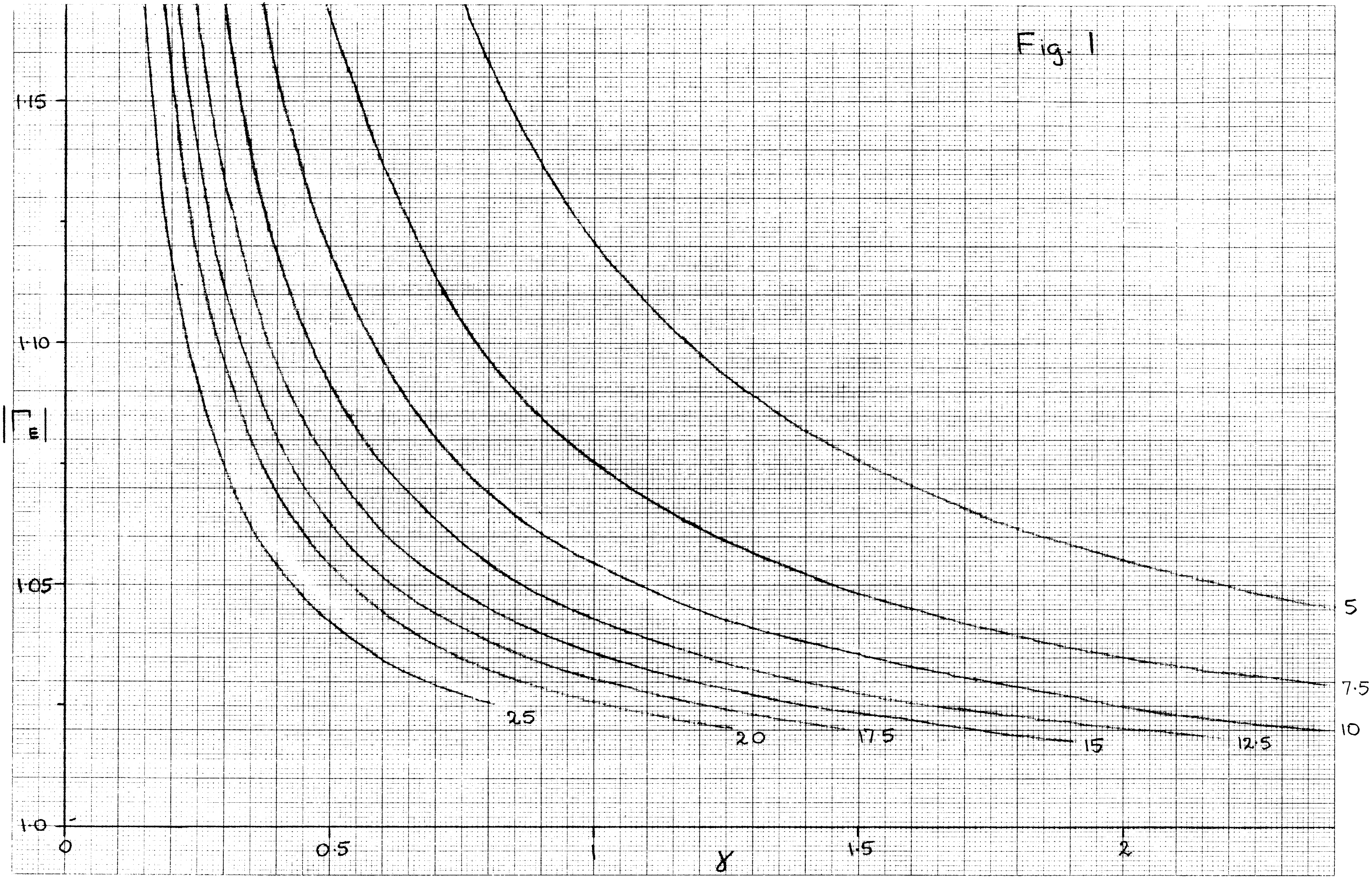
Fig. 1
1·15
1·10
1·05
1·0
|Γ_E|
0
0·5
1
1·5
2
γ
5
7·5
10
12·5
15
17·5
20
25

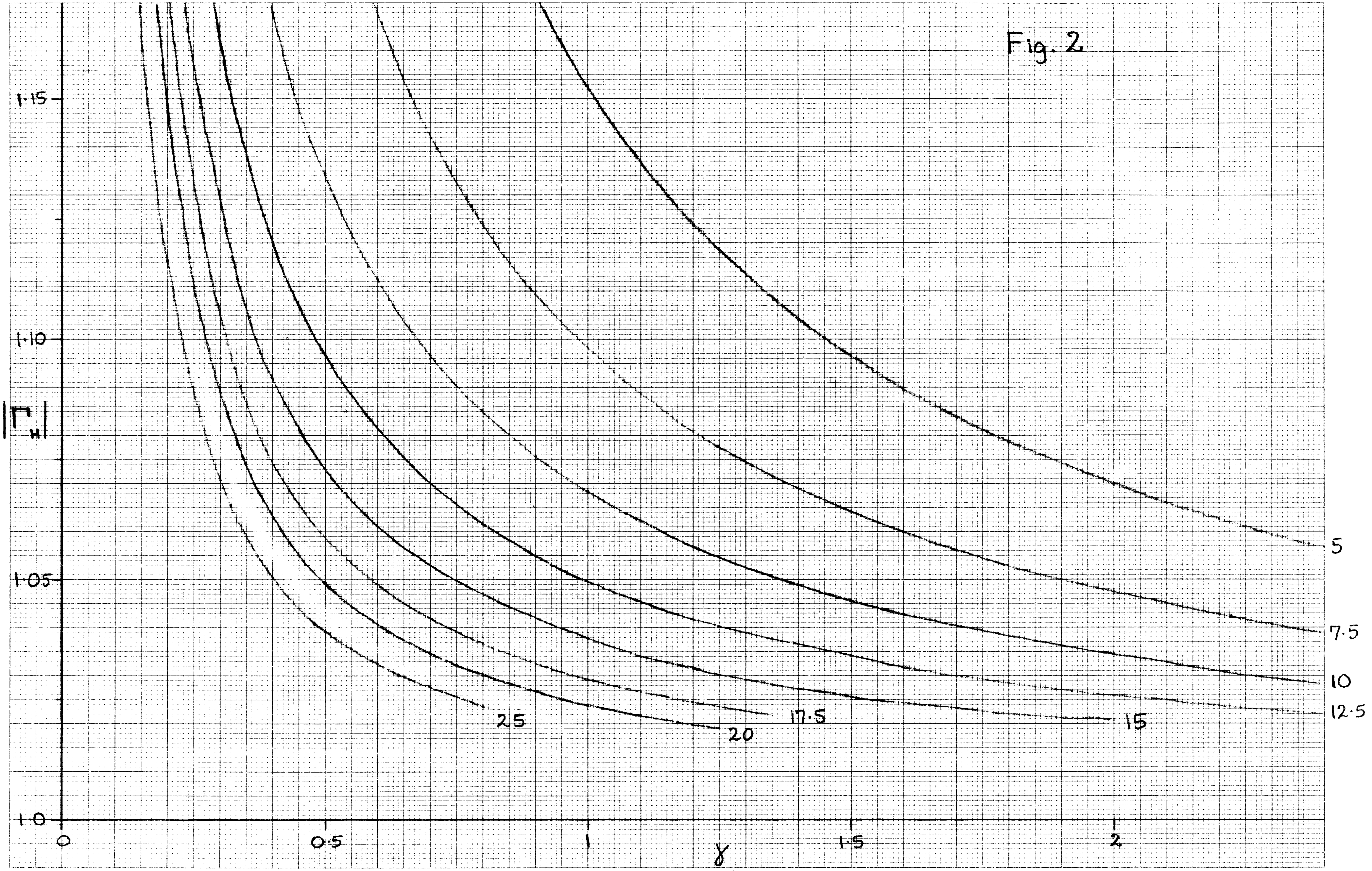

Fig. 2

sufficient to choose

$$\rho \geq 9.4a \tag{9}$$

at all frequencies for which $ka \geq 5$. This is less restrictive than the usual far field criterion (2) if $ka > 7.4$, and becomes ever less restrictive as ka increases. Moreover, it can be reconciled with (2) if an effective radius $\tilde{a}$ is employed in the far field criterion in place of the true radius a, and by comparing (9) and (2) it can be shown that

$$\tilde{a} = \sqrt{1.175\, a\lambda} \quad .$$

A similar result can also be obtained using the concept of Fresnel zones: $\tilde{a}$ is then the radius of the Fresnel zone responsible for the scattering and in the particular case of plane wave incidence, the above value of $\tilde{a}$ corresponds to a zone of depth 0.588λ.

With H polarisation, the values of γ for which the error is 0.5 dB decrease even more rapidly with increasing ka and, as seen from Table 2, ρ_{min}/a also shows a downward trend on which is superimposed an oscillation. The oscillation is almost certainly due to the creeping wave contribution whose magnitude is much greater than for E polarisation and will persist until ka has become large enough for the creeping waves to be a negligible source of scattering. When this is so, the values of ρ_{min} will be indistinguishable from those for E polarisation. For smaller ka, however, ρ_{min} is, in general, larger for H polarisation than for E. A formula which ensures that the errors do not exceed 0.5 dB for all $ka \geq 5$ is

$$\rho \geq 12.2a \tag{10}$$

corresponding to an effective radius

$$\tilde{a} = \sqrt{1.525\, a\lambda} \quad ,$$

but for larger ka, the condition (10) overestimates the minimum range required. Nevertheless, this range is less than that demanded by the usual far-field criterion if $ka > 9.6$.

Although the conditions (9) and (10) are less restrictive than the far field criterion (2) when ka is large, the reverse is true for cylinders whose radii are less than or comparable to the wavelength. To use (2) could then entail errors which are unacceptably large. Thus, for E-polarisation, the error at the far field distance $\rho = 8a^2/\lambda$ (implying $\gamma = 1.273$) is 0.49 dB for ka = 7.5, but 0.76 dB when ka = 5.0, and increases rapidly with decreasing ka. The comparable values for H-polarisation are 0.64 and 0.95 dB, and it is therefore desirable to exceed the standard far field distance when working with targets of resonant size.

<u>References</u>

Bowman, J. J., T. B. A. Senior and P. L. E. Uslenghi (1969), "Electromagnetic and Acoustic Scattering by Simple Shapes," North-Holland Pub. Co.

Kouyoumjian, R. G. and L. Peters, Jr. (1965), "Range requirements in radar cross-section measurements," Proc. IEEE, 53, 920-928.

Appendix: Computer Program Listing

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      COMPLEX*16 HANP(100),HANA(100),DHANA(100),EZS,HZS
      REAL*8 BESP(100),BESA(100),NEUP(100),NEUA(100),BESAA(2)
      REAL*8 DBESA(100)
      REAL*8 XE,YE,XH,YH,PE,PH,GAMAE,GAMAH
      COMPLEX*16 A,APRI,GAME,GAMH
      COMPLEX*16 EZA,HZA
      REAL*8 E,H,EA,HA
      REAL KP,KA,K1
      PIE=3.14159265
      READ(5,1) KP,KA
    1 FORMAT(2F15.5)
      NMAX=INT(2.*(KA+.49)+10.)
      CALL BESJ(KA,0,BESAA(1),1.0E-11,IER)
      CALL BESJ(KP,0,BESP(1),1.0E-11,IER)
      CALL BESJ(KP,1,BESP(2),1.0E-11,IER)
      CALL BESY(KA,0,NEUA(1),IER)
      CALL BESY(KA,1,NEUA(2),IER)
      CALL BESY(KP,0,NEUP(1),IER)
      CALL BESY(KP,1,NEUP(2),IER)
      BESA(NMAX+1)=.14D-16
      BESA(NMAX+2)=.3D-17
      J1=NMAX+1
      DO 2 I=1,NMAX
      I1=J1-I
    2 BESA(I1)=(2.*I1*BESA(I1+1)/KA)-BESA(I1+2)
      K1=BESA(1)/BESAA(1)
      DO 20 I=1,NMAX
      BESA(I)=BESA(I)/K1
      BESP(I+2)=(2.0*I*BESP(I+1)/KP)-BESP(I)
      NEUP(I+2)=(2.0*I*NEUP(I+1)/KP)-NEUP(I)
   20 NEUA(I+2)=(2.0*I*NEUA(I+1)/KA)-NEUA(I)
      WRITE(6,101)
  101 FORMAT(////)
      WRITE(6,102) KP, KA
  102 FORMAT(' ','KP=',F8.2,3X,'KA=',F8.2,///)
      NMAX1=NMAX+1
      DO 3 I=1,NMAX1
      HANP(I)=BESP(I)+(0.0,1.0)*NEUP(I)
    3 HANA(I)=BESA(I)+(0.0,1.0)*NEUA(I)
      DO 4 I=1,NMAX
      DBESA(I)=((I-1)*BESA(I)/KA)-BESA(I+1)
    4 DHANA(I)=((I-1)*HANA(I)/KA)-HANA(I+1)
      PHI=0.
      EZS=BESA(1)*HANP(1)*HANP(1)/HANA(1)
      HZS=DBESA(1)*HANP(1)*HANP(1)/DHANA(1)
      A=2.*BESA(1)/HANA(1)
      APRI=2.*DBESA(1)/DHANA(1)
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      DO 6 I=2,NMAX
      EZS=EZS+2.0*BESA(I)*HANP(I)*HANP(I)*COS((I-1)*PHI)/HANA(I)
      HZS=HZS+2.0*DBESA(I)*HANP(I)*HANP(I)*COS((I-1)*PHI)/DHANA(I)
      A=A+4.*((-1.)**(I-1))*BESA(I)/HANA(I)
      APRI=APRI+4.*((-1)**(I-1))*DBESA(I)/DHANA(I)
    6 CONTINUE
      EZS=EZS*KP
      HZS=HZS*KP
      EZA=(0.0,1.0)*CEXP(2.*(0.C,1.C)*KP)*A/PIE
      HZA=(0.0,1.0)*CEXP(2.*(0.0,1.0)*KP)*APRI/PIE
      GAME=EZS/EZA
      GAMH=HZS/HZA
      XE=DREAL(GAME)
      YE=DIMAG(GAME)
      XH=DREAL(GAMH)
      YH=DIMAG(GAMH)
      PE=180.*CATAN(YE/XE)/PIE
      PH=180.*CATAN(YH/XH)/PIE
      GAMAE=CDABS(GAME)
      GAMAH=CDABS(GAMH)
      WRITE (6,104)
      WRITE(6,103) GAMAE,PE,GAMAH,PH
  103 FORMAT(' ',E15.5,3X,F10.3,6X,E15.5,3X,F10.3//)
  104 FORMAT(' ',6X,'GAMMA E',10X,'PHI E',12X,'GAMMA H',10X,'PHI H',/)
      E=CDABS(EZS*DCONJG(EZS))
      H=CDABS(HZS*DCONJG(HZS))
      EA=CDABS(EZA*DCONJG(EZA))
      HA=CDABS(HZA*DCONJG(HZA))
      WRITE(6,105) E, H
  105 FORMAT(' ','CALCULATED FIELDS',5X,'E=',E15.5,3X,'H=',E15.5//)
      WRITE(6,106) EA, HA
  106 FORMAT(' ','ASYMPTOTIC FIELDS',5X,'E=',E15.5,3X,'H=',E15.5//)
      STOP
      END
```