011764-506-M

20 September 1973

MEMO TO T. B. A. Senior

FROM Sharad R. Laxpati

SUBJECT Revisions of Memo 011764-504-M
dated 11 September 1973

(1) In Section I, $\hat{n}$ appearing in Equations (4a) and (4b) should be $\hat{n}'$. Also the definition of r should be

$$\underline{r} = \underline{\rho}' - \underline{\rho} \ .$$

(2) The conclusions (b) and (c) of Section III are incorrect. The error arises due to improper comparison between Equations (4b) and (9). These two equations cannot be compared as they stand since the definition of the currents in two cases are not identical. In order to compare the two, they must first be transformed in terms of the fields on the surface. Equation (4b) then reads

$$Y_o E_z^{inc}(s) = Y_o E_z(s) - \frac{1}{4} \int_C H_s(s') H_o^{(1)}(k r) \, d(ks') + \frac{i Y_o}{4} \int_C E_z(s') \, (\hat{n}' \cdot \hat{r}) \, H_1^{(1)}(k r) \, d(ks') \ . \quad (1)$$

Here $H_s(s)$ is the transverse, surface magnetic field. Similarly, rewriting

DISTRIBUTION

Hiatt/File
Knott
Laxpati
Senior

11764-506-M = RL-2244

equation (9) in terms of the axial surface field (E_z) leads to

$$Y_o E_z^{inc}(s) = Y_o E_z(s) + \frac{1}{4}\int_C \frac{E_z(s')}{R_s(s')} H_o^{(1)}(kr)\, d(ks') \tag{2}$$

Equations (1) and (2), valid for a closed surface C can now be compared. We recognize that, in general, we do not have a simple analogy. The reason for this is discussed in (a) part of the conclusion of the memo of 9-11-73. The above equations simply bare the differences.

(a) Impedance Half-plane at Edge-on Incidence

Fortunately, all is not lost. Surprisingly, if one considers, in the above, a special geometry — that of a half-plane, there appears an interesting analogy. Let us first reduce the above equations to forms appropriate for this geometry, using the work of Senior (1952). We have for the impedance half-plane located at $y = 0,\ x > 0$, the equation for the electric field E_z as (equation (6) of Senior (1952))

$$E_z = E_z^{inc} + \frac{1}{4}\int_0^\infty \left[-\frac{k}{\eta}\left\{E_z^+(x') + E_z^-(x')\right\} - i\left\{E_z^+(x') - E_z^-(x')\right\}\frac{\partial}{\partial y}\right] H_o^{(1)}(kr)\, dx' \quad . \tag{3}$$

Note that we have rewritten currents I_1 and I_2 in terms of the tangential fields at $y = \pm 0$. The integral equation for the sum field (corresponds to $I_2(x')$, and equation (11) of Senior (1952)) is as follows:

$$Y_o E_z^{inc}(x) = -\frac{Y_o}{2}\left[E_z^+(x) + E_z^-(x)\right] +$$

$$-\frac{1}{4}\int_0^\infty \frac{\left[E_z^+(x') + E_z^-(x')\right]}{\eta Z_o} H_o^{(1)}(k|x-x'|)\, d(kx') \tag{4}$$

Equation for the difference field (corresponding to $I_1(x')$) can be obtained from equation (12) of Senior (1952). However, for the case of interest for comparison with the resistive half-plane, that of edge-on incidence, the contribution of the current $I_1(x')$ to the scattered field is zero. This can be readily seen from equations (32) and (33) of Senior (1952), wherein $\alpha = \pi$. Hence we will restrict our discussion to edge-on incidence. The expression for the scattered field, from equation (3) is,

$$E_z^s = -\frac{1}{4}\int_0^{\infty} \frac{\left[E_z^+(x') + E_z^-(x')\right]}{\eta} H_0^{(1)}(kr)\ dx' \tag{5}$$

(b) Resistive Half-plane

In a separate memo (011764-505-M) it has been shown that the integral equation (2) derived for a closed surface C, remains unchanged for an open surface in case of a resistive sheet. (In actuality, the resistive sheet is a special case of the more general one considered there.) Hence the integral equation for a resistive half-plane is

$$Y_o E_z^{inc}(x) = Y_o E_z(x) + \frac{1}{4}\int_0^{\infty} \frac{E_z(x')}{R_s} H_0^{(1)}(k|x - x'|)\ d(kx') . \tag{6}$$

Since for the resistive half-plane, $E_z^+(x) = E_z^-(x) = E_z(x)$, above equation can be rewritten to look similar to equation (4) for the impedance half-plane. The difference is a minus sign in front of E_z^{inc}. This, however, is of no real consequence, since this results in an absolute phase error of 180^o. Equation for the scattered field for the resistive half plane, for all angles of incidence is

$$E_z^s = \frac{Z_o}{4} \int_0^{\infty} \frac{E_z(x')}{R_s} H_o^{(1)}(kr)\, d(kx') \qquad (7)$$

Equation (7) for the scattered field is similar to equation (5) of impedance half-plane, once again but for the 180^o phase difference.

From the above similarities between equations (4) and (6) and between (5) and (7) we conclude:

For edge on incidence ($\alpha = \pi$), for the case of a semi-infinite sheet with impedance boundary conditions, the diffraction coefficient for E-polarization is identical to that for the case of a semi-infinite resistive sheet of resistance $R_s = \frac{\eta Z_o}{2}$, η being the surface impedance on sheet.

A dual of this will hold for a magnetic resistance sheet.

mfk