011764-507-M

21 September 1973

MEMO TO T.B.A.Senior

FROM Sharad R. Laxpati

SUBJECT Scattering from an impedance half-plane at edge-on incidence.

The problem of electromagnetic scattering of a half-plane from an impedance half-plane at normal as well as oblique incidence has been solved (Senior, 1952, 1960). Several authors (Malyughinetz, 1960; Lebedev et al, 1963) have obtained analytical solutions for the general geometry of an impedance wedge. None of these studies, however, have presented any data on the effect of the magnitude of the impedance on backscattering or on the diffraction pattern. Bowman (1967) applied Malyughinetz's result to derive approximate (asymptotic) results for the problem of diffraction by a wide strip. He has obtained numerical results for the strip for a number of surface impedances, which to a first order approximate the characteristic of the absorbers utilized in the experimental results.

Recent work on the relationship between impedance boundary conditions on a sheet and an impedance sheet has led to a conclusion that for the case of a half-plane with an E-polarized, edge-on incident wave, the scattered field for the boundary condition specified by a surface impedance ηz_o are the same as that for a resistive sheet of resistance $R_s = \eta z_o/2$. (See memo 011764-504-M, and its revision). Siginificance of such a close connection is of course, that the numerical results for an impedance surface can be used to verify and/or design resistive coatings for radar cross-section reduction. From numerical

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standpoint, the integral equations for an impedance boundary condition are more convenient to program, since the same equations may be used for both E- and H-polarization.

Senior (memo 011764-501-M) recognizing the close relationship between the impedance surface and resistive sheet, has obtained some asymptotic results for the half-plane. He has obtained the back-scattered diffraction coefficient for an impedance half-plane, normalized to that of a perfectly conducting half-plane, for edge-on incidence. The results are valid for small and large η. These results are not valid for the range of η for which it is expected to provide optimum reduction of radar returns for both polarizations. This memo reports the numerical results for the impedance half-plane obtained from the exact expression. The analytical results of Malyughinetz (1960) are used instead of the results of Senior (1952) because of the numerical expediency. The normalized back-scattered as well as bistatic diffraction coefficients are obtained for edge-on incidence and E-polarization.

Back-Scattered Diffraction Coefficient

Malyughinetz (1960) has shown that for a plane wave of unit amplitude incident at an angle α with the impedance half-plane, the scattered far field at an angle θ with the half-plane is given by

$$E_z^s \sim \frac{1}{4\pi i}\sqrt{\frac{2\pi}{kr}}\, e^{ikr - i\pi/4}\, U(\chi, \theta, \alpha) \tag{1}$$

where

$$U(\chi, \theta, \alpha) = \frac{\sin \alpha/2}{\psi(\pi - \alpha)}\left\{\frac{\psi(-\theta)}{\sin\theta/2 + \cos\alpha/2} + \frac{\psi(2\pi-\theta)}{\sin\theta/2 - \cos\alpha/2}\right\} \tag{2}$$

$$\psi(\beta) = \psi_\pi(\beta + \pi + \chi)\,\psi_\pi(\beta + \pi - \chi)\,\psi_\pi(\beta - \pi - \chi)\,\psi_\pi(\beta - \pi + \chi) \tag{3}$$

$$\psi_\pi(\beta) = \exp.\left\{ -\frac{1}{8\pi}\int_0^\beta \frac{\pi \sin v - 2\sqrt{2}\,\pi \sin(v/2) + 2v}{\cos v}\, dv \right\} \tag{4}$$

and

$$\cos \chi = 1/\eta \quad . \tag{5}$$

For edge-on incidence, $\alpha = \pi$ and for back-scattering $\theta = \pi$. The normalized back-scattered coefficient is then given by

$$\frac{P_E(\eta)}{P_E(0)} = \frac{U(\chi, \pi, \pi)}{U(i\infty, \pi, \pi)} \quad . \tag{6}$$

Bowman (1967) has listed in the appendix, properties and alternate definitions for function $\psi_\pi(\beta)$. We shall use them and refer to them using his equation numbers. Since the function $\psi_\pi(\beta)$ is an even function β, we obtain from equations (2) and (3),

$$U(\chi, \pi, \pi) = \frac{2\,\psi(\pi)}{\psi(0)} \quad . \tag{7}$$

Use of equation (A.11) and considerable algebraic manipulation lead to

$$\frac{\psi(\pi)}{\psi(0)} = \frac{8 \cos \chi}{(1 + \sqrt{2}\cos \chi/2)^4} \left[\frac{\psi_\pi(\chi)}{\psi_\pi(\pi/2)} \right]^8 \quad . \tag{8}$$

Use of equations (A.9) and (A.3), once again followed by algebraic simplifications, gives

$$U(\chi, \pi, \pi) = 2 \; e^{-2\left[B(\chi) + b\right]} \tag{9}$$

where

$$B(\chi) = \frac{1}{\pi}\int_0^{\chi} \frac{v\,d\,v}{\cos v} \quad . \tag{10}$$

and $b = \frac{2K}{\pi}$, $K = 0.9159656....$ (Catalan's constant).

Since $B(i\,\infty) = -\,b$, (from equation A.6), the back-scattered coefficient of euqation (6) is

$$\frac{P_E(\eta)}{P_E(0)} = \exp\left\{-\,2\,\left[B(\chi) + b\right]\right\} . \tag{11}$$

Equation (11) was used in numerical calculation for the back-scattered coefficient. Integral in equation (10) was evaluated using 32 point Gauss quadrature formula. $B(\chi)$ was found to have at least 5 digit accuracy. All the calculations were performed with single precision arithmetic on IBM machine. Figure 1 shows a plot of the normalized diffraction coefficient as a function of η for E-polarization. Using the duality, the scale for the impedance η for H-plarization is also indicated. A comparison with the asmyptotic results indicate that for large η the asymptotic results are fairly accurate and in general, they do predict the trend of the results fairly well.

Bistatic Diffraction Coefficient

Although the ecpressions for the normalized bistatic coefficient are more complicated, they are still relatively simple to evaluate numerically. From the definition of the diffraction coefficient and equation (1) we have,

$$\frac{P_E(\eta, \theta)}{P_E(0, \theta)} = \frac{U(\chi, \theta, \pi)}{U(i\infty, \theta, \pi)} \quad . \tag{12}$$

From equation (2),

$$U(\chi, \theta, \pi) = \frac{1}{\sin\theta/2}\left[\frac{\psi(-\theta) + \psi(2\pi - \theta)}{\psi(0)}\right] \quad .$$

Use equation (A.14) and then (A. 16) in the expression for $\psi(2\pi-\theta)$. After some trigonometric manipulations, above equation reduces to

$$U(\chi, \theta, \pi) = \frac{2\cos\chi}{\sin\theta/2\,(\sin\theta + \cos\chi)}\;\frac{\psi(-\theta)}{\psi(0)} \quad .$$

Now the ratio of two ψ functions can be expressed in a form appropriate for numerical computation by means of equations (A.11) and (A.4). This leads to

$$U(\chi, \theta, \pi) = \frac{2}{\sin\theta/2\,(\eta\sin\theta + 1)} \exp\left\{\frac{1}{4}\left[2A(\chi) - A(\chi+\theta) - A(\chi-\theta)\right]\right\} \tag{13}$$

where

$$A(\chi) = \frac{1}{\pi}\int_0^{\chi} \frac{\pi\sin v - 2v}{\cos v}\, dv \quad . \tag{14}$$

Use of equation (13) in equation (12) leads to the following expression for normalized bistatic diffraction coefficient.

$$\frac{P_E(\eta,\theta)}{P_E(0,\theta)} = \frac{1}{1+\eta\sin\theta}\exp\left\{\frac{1}{4}\Big[2A(\chi) - A(x+\theta) - A(\chi-\theta)\Big] + \right.$$

$$\left. - \frac{1}{4}\Big[2\ A(i\,\infty) - A(i\,\infty+\theta) - A(i\,\infty-\theta)\Big]\right\} \qquad (15)$$

Equation (15) is used to calculate the desired diffraction coefficient as a function of θ over the range 0 to 180°. The integral (14), in its derived forms as shown in the apprndix, was evaluated using Gauss quadrature methods. The integrals were accurate to at least 3 digits. The results for the normalized coefficient are not surprising. The value of the diffraction coefficient increases monotonically from its value in backscattered direction ($\theta = \pi$) toward the forward scatter direction ($\theta = 0$) where it reaches a value of 1 for all values of the impedance η in the range of 10^{-2} to 10^{2}.

References

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Appendix

Various Formulas Employed for Numerical Evaluation of the Exponents in Equation (15)

Note that for n < 1, χ is imaginary whereas for $\eta > 1$, χ is real. It is found convenient to evaluate the exponent in a combined form $A(\chi) - 1/2\, A(\chi + \theta) - 1/2\, A(\chi - \theta)$ instead of individual integrals when χ is imaginary. We also note that $A(\chi) = A(-\chi)$ for χ real. The formulas shown below are obtained by standard procedures of change of variable, etc., and some special integrals discussed in work by Bowman (1963).

(A) χ Real - let $\chi = \alpha$.

(1) $$A(\alpha) = \frac{1}{\pi}\int_0^{\alpha} \frac{\pi \sin v - 2v}{\cos v}\, dv, \quad 0 < \alpha < \pi/2 .$$

(2) $$A(\pi/2) = -\ln 2 + b$$

(3) $$A(\alpha) = 4b + \frac{1}{\pi}\int_0^{\pi-\alpha} \frac{\pi \sin v - 2\pi + 2v}{\cos v}\, dv, \quad \frac{\pi}{2} < \alpha \leq \pi .$$

(4) $$A(\alpha) = 4b + \int_0^{\alpha - \pi} \frac{\pi \sin v + 2\pi + 2v}{\cos v}\, dv, \quad \pi < \alpha < \frac{3\pi}{2} .$$

(b) χ Imaginary - let $\chi = i\beta$

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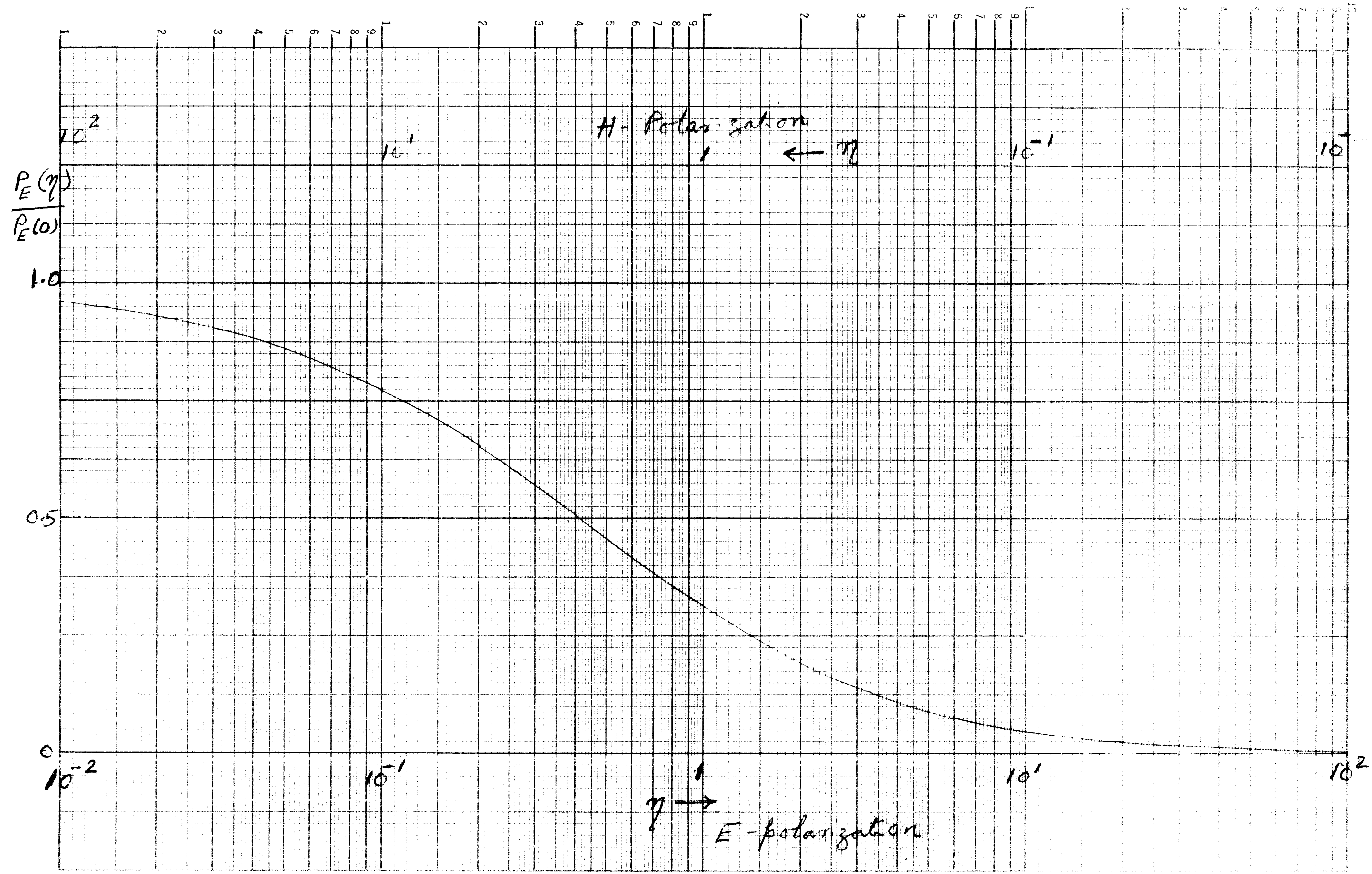
$$\left[A(i\beta) - \frac{1}{2}A(i\beta+\theta) - \frac{1}{2}A(i\beta-\theta)\right] = -\frac{1}{\pi}\int_0^\theta \frac{\pi \sin u - 2u}{\cos u}\, du -$$

$$-\frac{1}{\pi}\int_0^\beta \left\{ \frac{\pi \sinh v - 2v}{\cosh v} + \right.$$

$$\left. - \frac{\pi \sinh v \cosh v - 2\theta \sin\theta \sinh v - 2v \cos\theta \cosh v}{\cos^2\theta + \sinh^2 v} \right\} dv \ .$$

Note that for $\beta = \infty$, the second integral in the above expression may be evaluated using Gauss-Laguerre quadrature formula. However, for large β the integral approaches 0. It can be truncated to a finite upper limit and evaluated by Gauss-quadrature method. The results are found to be accurate to 4 digits if the upper limit of 20 is used.



H- Polarization
← η
10^{2}
10^{1}
1
10^{-1}
$P_E(\eta) / P_E(0)$
1.0
0.5
0
10^{-2}
10^{-1}
1
10^{1}
10^{2}
η →
E-polarization

Figure 1.