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A Study of Some Electromagnetic Problems  
Relating to EMP Technology

This is the final report under Air Force Office of Scientific Research Grant No. 77-3465 bearing the above title covering the period from 30 September 1977 to 28 February 1979.

The research covers two main topics: the equivalent circuit representations of radiating systems and certain aspects of sensor characteristics. The detailed treatments have been written up and are attached as the appendices to this report. The following is a brief summary of the research results.

For the equivalent circuit problem the development and the testing of a numerical technique for rational functions have been accomplished. A network modelling leading to a canonical ladder configuration is then formulated. Finally the equivalent circuit representation of a thin biconical antenna is constructed based on these methods. The research shows that transfer functions involved in radiating and scattering problems can be represented by such an equivalent circuit representation. In contrast to the equivalent circuit representation suggested by C.E. Baum [Single Port Equivalent Circuits for Antenna and Scatterers, Interaction Note 295, Air Force Weapons Laboratory, March, 1976] our method does not require explicit knowledge of the poles of these transfer functions.

The second appendix treats the responses of a short dipole and a small loop placed in a right-angle conducting corner. The work is intended to correlate the open-circuit voltage of a sensor with the local surface charge density or surface current density on a scatterer. The right-angle corner is a simple structure for which an exact formulation of the problem can be given. The results show the exact relationship between the open circuit

voltage and the local surface charge or surface current density on the corner. The appendix also contains an investigation of the impedance functions of these probes taking into consideration the proximity effect of the corner.

The research on the equivalent circuit representation was presented at the Nuclear EMP Meeting held at the University of New Mexico, June 6-8, 1978. The paper is entitled "A Network Model for the Biconical Antenna," by C.B. Sharpe and C.J. Roussi. Our work on the sensor research has just been completed and will be submitted to the Air Force Weapons Laboratory for consideration of publication as a note in the Sensor Series.

## APPENDIX A

Equivalent Circuit Representation of  
Radiation Systems

by

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## Abstract

In this report the biconical antenna is treated as a representative scattering system. It is shown that at its input terminals the biconical antenna can be modeled by a transmission line terminated in a canonical LC ladder network. The real and imaginary parts of the input impedance of the biconical antenna serve as useful test functions for studying the approximation of complex functions of frequency by rational functions. An effective algorithm for this purpose was implemented and evaluated. It is also shown that over a limited domain in the complex frequency plane the poles and zeros of the system function can be recovered via the rational approximation.

## 1. Introduction

In developing equivalent circuits for radiating systems we have divided the problem into two basic parts: the development of a rational function approximation technique and the development of lumped network synthesis procedures appropriate to the system in question. Of course, the first part must serve as a basis for the second. Typically the rational function will represent in analytic form the transfer admittance of a system obtained by experimentally measuring the amplitude and phase of the surface current as a function of frequency at some point on the scattering object with reference to the incident electric field at some reference plane. In the general case all we can say about the transfer function is that its poles must all lie in the closed left half-plane. The zeros may lie in either half-plane. Among the parameters in the problem are the polarization and aspect of the incident field, the location and orientation of the current probe on the object and, of course, the shape of the scattering body. An important question which remains to be answered is the nature of the dependence of the poles and zeros of the transfer function on these parameters.

In order to explore the above question, much of our initial effort has been devoted to the development and testing of a numerical approximation technique for rational functions. This technique and its application in several representative approximation problems is described in Section 3. As a preliminary exercise, a network modelling problem leading to a canonical ladder configuration was also investigated and completed. This work is presented in the next section. Both of the above studies were centered around the biconical antenna. One reason for this approach is that the input impedance of the biconical antenna exhibits many of the frequency response characteristics of more general scattering structures, and is, therefore, a useful vehicle for test purposes.

## 2. A Network Model for the Biconical Antenna

The biconical antenna offers an interesting example on which to test network modeling techniques because an exact analytical expression for the input impedance is available. Tai<sup>[1]</sup> has shown that the input impedance at the center of the biconical antenna can be represented by a section of uniform line terminated in a frequency-dependent admittance  $Y_t(\beta l)$ . This equivalent circuit is illustrated in Figure 1, where

terminated in a resistance. The results of the  $Y_t$  approximation are shown in Figure 9 and Figure 10.

Another test was performed on data describing the input impedance of the biconical antenna as shown in Figure 11. This example was approximated by a ninth-order numerator and tenth-order denominator. The results are presented in Figure 12 and Figure 13. It is seen by comparing these results with Figure 11 that the approximated imaginary part of  $Z_{in}$  fails to fit the data near  $\beta l=0$ . This is due to a pole at zero which the data contained that the rational approximating function could not accommodate due to its chosen structure. This could have been corrected by changing the data and reinserting the pole later, a technique described by Levy [5].

The poles and zeros of the approximating function were extracted by standard techniques and compared with those found by Tai and Cho via a grid search. The result is shown in Figure 14. The poles and zeros reflect the closeness of the fit over the approximating range.

### 3.3 Conclusion

The rational function approximation method employed here has the advantage of being able to produce an analytic representation of data that is amenable to linear transform methods of solution. Furthermore, the implementation of the method is straightforward and computationally efficient. For the accuracy achieved here, the typical run took 2 CPU seconds (Amdahl 470) and cost \$.50.

As a final remark, a more nearly mini-max approximation could be obtained by incorporating Lawson's algorithm [2] in the iterative procedure but our investigations to date do not indicate that this will be necessary.



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APPENDIX B