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Three Dimensional Moment Method Simulation of Penetrable Scatterers Consisting of Non-metallic and Circuit Analog Surfaces

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THREE DIMENSIONAL MOMENT METHOD SIMULATION OF PENETRABLE SCATTERERS CONSISTING OF NON-METALLIC AND CIRCUIT ANALOG SHEETS

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Abstract

This report describes the formulation for a mixed integral equation formulation implemented in the code CADRIS. An important aspect of this implementation is the inclusion of Circuit Analog sheet models that is not available in other codes. This capability is combined with modeling capabilities for Resistive sheets, metallic, dielectric and impedance surfaces, and combination of all. The report begins with the introductions of the appropriate integral equations and proceeds to develop their discertization. Several validations are given for PEC and composite scattereres.

1) Surface Integral Equations

Consider the general electromagnetic scattering problem depicted in Fig.1.1.

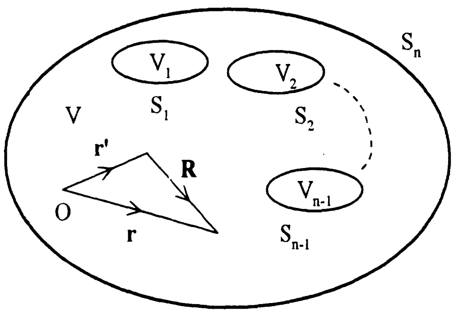


Figure 1.1. Geometry showing the various boundaries

We are interested in a field point **r**, located in a closed volume V or on a regular surface $S_i (i = 1, 2, ..., n)$. Beginning with the vector Green's theorem,

$$\int_V [(\mathbf{Q}.(\nabla' \times \nabla' \times \mathbf{T})) - (\mathbf{T}.(\nabla' \times \nabla' \times \mathbf{Q}))dv'] \\ + \int_{\sum_{i=1}^{n} S_i} ((\mathbf{T} \times \nabla' \times \mathbf{Q}) - (\mathbf{Q} \times \nabla' \times \mathbf{T}))\mathbf{ds}' = 0 \tag{1}$$

where, $\mathbf{Q}(\mathbf{r}), \mathbf{T}(\mathbf{r}) \in C^2; \mathbf{r} \in V, \sum_{i=1}^{n} S_i$. Here the primes refer to the primed(integration) coordinates. To derive the integral equations for the electric and magnetic currents on the surfaces $S_i, i = 1, 2, ..., n$ we set

$\mathbf{T} = \mathbf{E}(\mathbf{r}')$(electric field), and $\mathbf{Q} = \hat{\mathbf{l}}G(\mathbf{r},\mathbf{r}')$ with

$$G(\mathbf{r},\mathbf{r}') = \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}, \quad k = \omega\sqrt{\epsilon\mu} \tag{2}$$

Note that when $\mathbf{r} \rightarrow \mathbf{r}'$ in V, G, ∇G and $\nabla^2 G$ have singularities. To overcome this singularity problem, when integrating G or its derivatives we exclude an infinitesimal sphere of volume $V_\delta \rightarrow 0$ and centered at $\mathbf{r} = \mathbf{r}'$ and shown in Fig.1.2. We deal with the spherical volume V_δ of radius $\delta \rightarrow 0$ separately by invoking the divergence theorem.

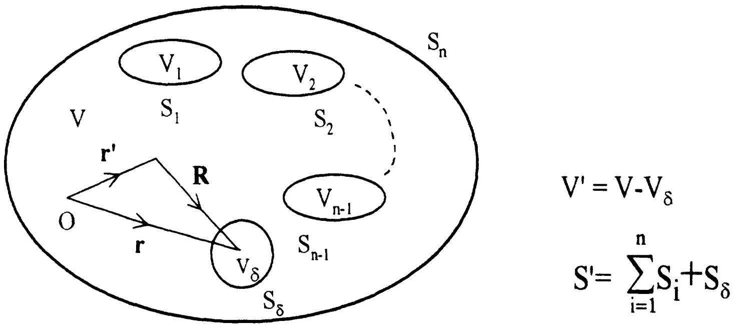


Figure 1.2. Geometry for singularity problem

Next we introduce into(1) Maxwell equations

$$\nabla \times \mathbf{E} + j\omega\mu\mathbf{H} = -\mathbf{M} \tag{3a}$$

$$\nabla \times \mathbf{H} - j\omega\epsilon\mathbf{E} = \mathbf{J} \tag{3b}$$

$$\nabla.(\mu\mathbf{H}) = \rho_m \tag{3c}$$

$$\nabla.(\epsilon\mathbf{E}) = \rho \tag{3d}$$

After some straightforward vector manipulations, (1) can be written as;

$$\begin{aligned} &\int_{V'} [j\omega\mu\mathbf{J}G + \mathbf{M} \times \nabla' G - (\rho/\epsilon)\nabla' G] dv' \\ &+ \int_{\sum_{i=1}^{n} S_i + S_\delta} j\omega\mu(\mathbf{n}' \times \mathbf{H})\Phi - (\mathbf{n}' \times \mathbf{E}) \times \nabla' G - (\mathbf{n}'.\mathbf{E})\nabla' G] ds' = 0 \end{aligned} \tag{4}$$

where we have assumed that V in equation (1) is linear, isotropic and homogeneous.

When $\mathbf{r}'$ is located on one of the $S_i (i = 1, 2, ..)$ surfaces, we proceed to extract the integral singularity noted earlier. Refering to Fig.1.3, we rewrite the surface integrals as,

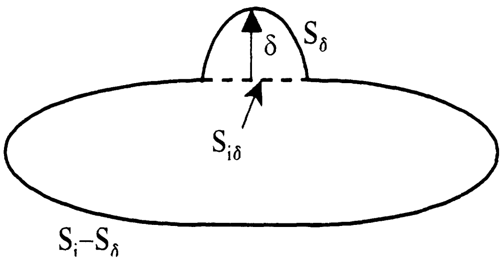


Figure 1.3. Geometry for singularity extraction

$$\int_{S_i + S_\delta} \mathbf{n}' \times \mathbf{E} \times \nabla' G = \lim_{\delta \to 0} \left[\int_{S_i - S_{i\delta}} + \int_{S_{i\delta}} + \int_{S_\delta} \right] \tag{5}$$

$$\lim_{\delta \to 0} \left[\int_{S_{i\delta}} + \int_{S_\delta} \right] = -E(\mathbf{r})[1 - \frac{P}{2\pi}] \tag{6}$$

Here,P is the absolute value of the solid angle subtended by $S_{i\delta}$ at $\mathbf{r}$ in the limit as $\delta \to 0$.

$$P = \left[\begin{array}{l} \pi, \quad \mathbf{r} \in S_i \\ 0, elsewhere \end{array} \right. \tag{7}$$

Based on (6), (4) can now be written as

$$\begin{aligned} \theta(\mathbf{r})\mathbf{E}(\mathbf{r}) = -\Bigg[\int_{V'} [j\omega\mu\mathbf{J}\Phi + \mathbf{M} \times \nabla'\Phi - (\rho/\epsilon)\nabla'\Phi] dv' \\ + \int_{\sum_{i=1}^{n} S_i} j\omega\mu(\mathbf{n}' \times \mathbf{H}\Phi - (\mathbf{n}' \times \mathbf{E}) \times \nabla'\Phi - (\mathbf{n}'.\mathbf{E})\nabla'\Phi ds') \Bigg] \end{aligned} \tag{8}$$

On invoking duality we also obtain the corresponding integral equation for $\mathbf{H}$ as;

$$\begin{aligned} \theta(\mathbf{r})\mathbf{H}(\mathbf{r}) = -\Bigg[\int_{V'} [j\omega\mu\mathbf{M}\Phi - \mathbf{J} \times \nabla'\Phi - (m/\mu)\nabla'\Phi] dv' \\ + \int_{\sum_{i=1}^{n} S_i} -j\omega\mu(\mathbf{n}' \times \mathbf{E}\Phi - (\mathbf{n}' \times \mathbf{H}) \times \nabla'\Phi - (\mathbf{n}'.\mathbf{H})\nabla'\Phi ds') \Bigg] \end{aligned} \tag{9}$$

Here $\int$ denotes the Cauchy Principal Value and $\theta(\mathbf{r})$ can be given as follows.

$$\theta(\mathbf{r}) = \left[\begin{array}{l} 2, \quad \mathbf{r} \in S_i \\ 1, elsewhere \end{array} \right. \tag{10}$$

In (8) and (9) since all the sources are contained in the volume V', this volume integral can be refered to as the 'source term'. If there are no sources in V', this integral will be zero. We will assume that the sources are far from the scatterer and represent the source integral by ($\mathbf{E}^i, \mathbf{H}^i$ which later be set a plane wave incident in the scatterer. Equation (8) and (9) can then be written as

$$\begin{aligned} \theta(\mathbf{r})\mathbf{E}(\mathbf{r}) = \theta \Bigg[\mathbf{E}^i(\mathbf{r}) + \int_{\sum_{i=1}^{n} S_i} [-j\omega\mu(\mathbf{n}' \times \mathbf{H}\Phi \\ + (\mathbf{n}' \times \mathbf{E}) \times \nabla'\Phi + (\mathbf{n}'\mathbf{E})\nabla'\Phi] ds' \Bigg] \end{aligned} \tag{11}$$

$$\theta(\mathbf{r})\mathbf{H}(\mathbf{r}) = \theta\Big[\mathbf{H^i}(\mathbf{r}) + \int_{\sum_{i=1}^{n} S_i} [j\omega\epsilon(\mathbf{n}' \times \mathbf{E})\Phi$$
$$+ (\mathbf{n}' \times \mathbf{H}) \times \nabla'\Phi + (\mathbf{n}'.\mathbf{H})\nabla'\Phi]ds'\Big] \tag{12}$$

Next we rewrite these in terms of surface current densities $(\mathbf{J}, \mathbf{M})$ where

$$\mathbf{J} = \mathbf{n} \times \mathbf{H}\delta(S) \tag{13a}$$

$$\mathbf{M} = \mathbf{E} \times \mathbf{n}\delta(S). \tag{13b}$$

and it follows that

$$\mathbf{n}.\mathbf{E} = \frac{-1}{j\omega\epsilon}\nabla.(\mathbf{n} \times \mathbf{H}) = \frac{-1}{j\omega\epsilon}\nabla.\mathbf{J} \tag{14a}$$

and

$$\mathbf{n}.\mathbf{H} = \frac{1}{j\omega\mu}\nabla.(\mathbf{n} \times \mathbf{E}) = \frac{-1}{j\omega\mu}\nabla.\mathbf{M}. \tag{14b}$$

Sustituting $(13a, b)$ and $(14a, b)$ into (11) and (12) gives the integral representation

$$\theta(\mathbf{r})\mathbf{E}(\mathbf{r}) = \mathbf{E^i}(\mathbf{r}) - \Lambda\mathbf{J} + \Omega\mathbf{M} \tag{15a}$$

$$\theta(\mathbf{r})\mathbf{H}(\mathbf{r}) = \mathbf{H^i}(\mathbf{r}) - \Omega\mathbf{J} - \frac{1}{\eta^2}\Lambda\mathbf{M} \tag{15b}$$

where $\mathbf{J}$ and $\mathbf{M}$ are unknown surface current densities. Here, Λ and Ω are the integro-differantial operators given bye,

$$\Lambda bf\Gamma(\mathbf{r}) = \int_S [j\omega\mu\mathbf{\Gamma}(\mathbf{r}') + \frac{j}{\omega\epsilon}\nabla(\nabla'.\mathbf{\Gamma}(\mathbf{r}'))]G(\mathbf{r} - \mathbf{r}')ds' \tag{16a}$$

and

$$\Omega\mathbf{\Gamma}(\mathbf{r}) = \int_S \mathbf{\Gamma}(\mathbf{r}) \times \nabla G(\mathbf{r} - \mathbf{r}')ds', \tag{16b}$$

and $\eta = \sqrt{\mu/\epsilon}$ is the characteristic impedance of the medium.

2) Formulation for Different Type of Boundary Conditions

2.1) PEC Boundary(Metallic Surfaces)

Different types of surface integral formulations have been developed for these kind of surfaces. We give here the very well-known EFIE(Electric Field Integral Equation), MFIE(Magnetic Field Integral Equation) and CFIE(Combined Field Integral Equation) formulations.

a)EFIE Formulation

Consider the PEC surface depicted in Fig.2.1.

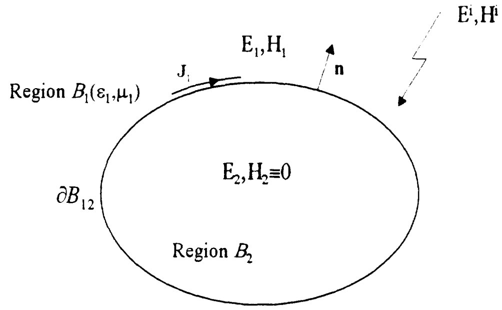


Figure 2.1.Geometry for imposing the PEC boundary condition

Using (15a) and (15b) for the fields outside the PEC surface, we can write,

$$\theta(\mathbf{r})\mathbf{E}(\mathbf{r}) = \mathbf{E^i}(\mathbf{r}) - \Lambda\mathbf{J} + \Omega\mathbf{M} \quad , \quad \mathbf{r} \in \mathcal{B}_1 \cup \partial\mathcal{B}_{12} \tag{17a}$$

$$\theta(\mathbf{r})\mathbf{H}(\mathbf{r}) = \mathbf{H^i}(\mathbf{r}) - \Omega\mathbf{J} - \frac{1}{\eta^2}\Lambda\mathbf{M} \quad , \quad \mathbf{r} \in \mathcal{B}_1 \cup \partial\mathcal{B}_{12}. \tag{17b}$$

$$E_2(\mathbf{r}) = H_2(\mathbf{r}) = 0 \quad , \quad \mathbf{r} \in \mathcal{B}_2 \tag{17c}$$

To construct the integral equation, we note that on the metallic surface,

$$\mathbf{n} \times [\mathbf{E}_1 - \mathbf{E}_2] = \mathbf{n} \times \mathbf{E}_1 = 0 \tag{18a}$$

$$\mathbf{n} \times [\mathbf{H}_1 - \mathbf{H}_2] = \mathbf{n} \times \mathbf{H}_1 = \mathbf{J} \tag{18b}$$

These imply, $\mathbf{M} = 0$ and thus (17a) become,

$$\mathbf{E}_1(\mathbf{r}) = \mathbf{E^i}(\mathbf{r}) - \Lambda\mathbf{J}(\mathbf{r}) \quad , \quad \mathbf{r} \in \mathcal{B}_1 \tag{19a}$$

and

$$\mathbf{n} \times \mathbf{E^i}(\mathbf{r}) = \mathbf{n} \times \Lambda\mathbf{J}(\mathbf{r}) \quad , \quad \mathbf{r} \in \partial\mathcal{B}_{12} \tag{19b}$$

The above are the so called EFIE whose solution gives the unknown current on the surface.

b)MFIE Formulation

From (17b) and (18b) with $\mathbf{M} = 0$, the magnetic field on the boundary $\mathcal{B}_{12}$ is given by

$$\frac{\mathbf{J}(\mathbf{r})}{2} = \mathbf{n} \times \mathbf{H^i}(\mathbf{r}) - \mathbf{n} \times \Omega\mathbf{J}(\mathbf{r}) \quad , \quad \mathbf{r} \in \partial\mathcal{B}_{12} \tag{20}$$

c)CFIE Formulation

The solution of EFIE and MFIE formulation can return non physical results at internal resonant frequencies. In this case we resort to the CFIE to overcome this problem(ref Peterson-Wilton). CFIE combines the EFIE and MFIE equations in a linear fashion as

$$\alpha(EFIE) + (1-\alpha)\eta(MFIE) = CFIE \tag{21a}$$

or

$$\alpha(\mathbf{E}_i(\mathbf{r})) + (1-\alpha)\eta(\mathbf{n}\times\mathbf{H}_i(\mathbf{r})) = (\alpha\Lambda + (1-\alpha)\eta\mathbf{n}\times\Omega + \frac{1}{2}\mathbf{J}(\mathbf{r})) \tag{21b}$$

The coefficient α is arbitrary and possibly complex. Typical α is set to 1/2 but other choices can be made. Basically the CFIE shifts the resonances of the MFIE and EFIE outside the range of interest.

2.2) Resistive Boundary

Consider the Resistive Boundary surface depicted in (Fig.2.2).

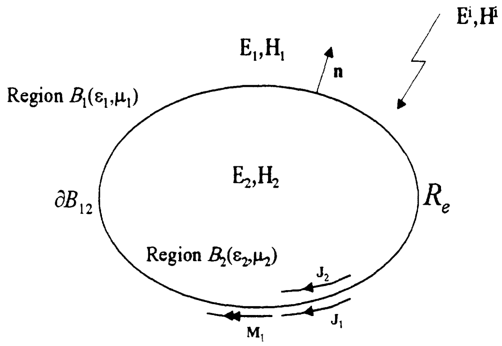


Figure 2.2. Geometry for application of the resistive boundary condition

For a resistive boundary, the boundary condition on $\partial\mathcal{B}_{12}$ are

$$\mathbf{n}\times[\mathbf{E}_1 - \mathbf{E}_2] = 0 \tag{22a}$$

$$\mathbf{n}\times[\mathbf{E}_1 + \mathbf{E}_2] = 2R_e\eta_1\mathbf{n}\times\mathbf{n}\times[\mathbf{H}_1 - \mathbf{H}_2] \tag{22b}$$

$$\mathbf{n}\times\mathbf{E}_1 = -\mathbf{M}_1 \;\; ; \;\; \mathbf{n}\times\mathbf{H}_1 = \mathbf{J}_1 \tag{22c}$$

$$\mathbf{n}\times\mathbf{E}_2 = \mathbf{M}_2 \;\; ; \;\; \mathbf{n}\times\mathbf{H}_2 = -\mathbf{J}_2. \tag{22d}$$

where R_e refers to the normalized surface resistivity in ohms per square. Fields in the regions $\mathcal{B}_1$ and $\mathcal{B}_2$ can be given as

$$\theta_1(\mathbf{r})\mathbf{E}_1 = \mathbf{E}_i - \Lambda_1\mathbf{J}(\mathbf{r}) + \Omega_1\mathbf{M}(\mathbf{r}) \tag{23a}$$

$$\theta_1(\mathbf{r})\mathbf{H}_1 = \mathbf{H}_i - \Omega_1\mathbf{J_1}(\mathbf{r}) + \frac{1}{\eta_1^2}\Lambda_1\mathbf{M_1}(\mathbf{r}) \tag{23b}$$

$$\theta_2(\mathbf{r})\mathbf{E}_2 = -\Lambda_2\mathbf{J_2}(\mathbf{r}) + \Omega_2\mathbf{M_2}(\mathbf{r}) \tag{23c}$$

$$\theta_2(\mathbf{r})\mathbf{H}_2 = -\Omega_2\mathbf{J_2}(\mathbf{r}) + \frac{1}{\eta_2^2}\Lambda_2\mathbf{M_2}(\mathbf{r}) \tag{23d}$$

In $(23a-d)$, indices 1 and 2 are corresponding to the fields and currents in regions $\mathcal{B}_1(\epsilon \rightarrow \epsilon_1, \mu \rightarrow \mu_1)$, and $\mathcal{B}_2(\epsilon \rightarrow \epsilon_2, \mu \rightarrow \mu_2)$, respectively. Using $(22a-d)$ in $(23a-d)$ we obtain

$$\begin{bmatrix} \Lambda_1 + R_e & -\Omega_1 & R_e \\ \Omega_1 & \frac{\epsilon_1}{\mu_1}\Lambda_1 + \frac{\epsilon_1}{\mu_1}\Lambda_2 + \frac{1}{R_e} & -\Omega_2 \\ R_e & \Omega_2 & \Lambda_2 + R_e \end{bmatrix} \begin{bmatrix} \mathbf{J_1} \\ \mathbf{M_1} \\ \mathbf{J_2} \end{bmatrix} = \begin{bmatrix} \mathbf{E_i} \\ \mathbf{H_i} \\ 0 \end{bmatrix} \tag{24}$$

If $(\epsilon_1 = \epsilon_2, \mu_1 = \mu_2)$, (24) reduses to,

$$[\Lambda_1 + 2R_e]\mathbf{J} = \mathbf{E}_i \quad , \quad \mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2. \tag{25}$$

which refers to the case of a resistive sheet boundary in free space.

2.3) Dielectric Boundary

A homogenous penetrable body is depicted in Fig.2.3.

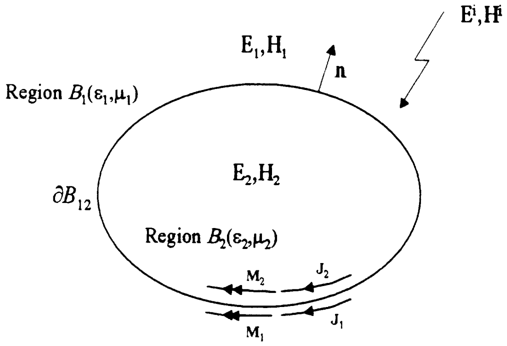


Figure 2.3. Geometry for constructing integral equations for dielectric boundary

Here we will give four different integral equation formulations for dielectric boundaries. For this situation, the expression for $\mathbf{E}_{1,2}$ and $\mathbf{H}_{1,2}$ are given in $(23a-d)$.

A) EFIE formulation:

The problem given in Fig 2.3 can be seperated into two sub-problems each gives the field in one of the regions.

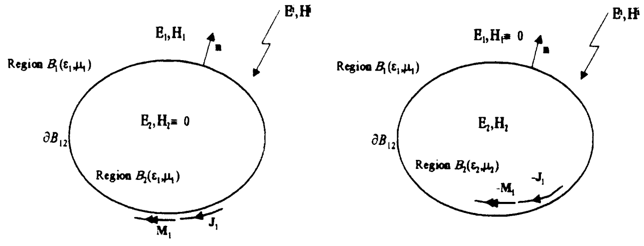


Figure 2.4. Geometry for the application of the EFIE and MFIE for dielectric boundary a) External Problem b) Internal Problem

Boundary conditions for the **E** field for the external and the internal problems are;

$$\mathbf{n} \times \mathbf{E}_1 = 0 \tag{26a}$$

and

$$\mathbf{n} \times \mathbf{E}_2 = 0 \tag{26b}$$

respectively. From $(23a-d)$ and $(26a,b)$, we obtain the EFIE equations,

$$\mathbf{n} \times \mathbf{E}_i = \mathbf{n} \times \Lambda_1 \mathbf{J}_1 - \mathbf{n} \times \Omega_1 \mathbf{M}_1 \tag{27a}$$

$$0 = \mathbf{n} \times \Lambda_2 \mathbf{J}_1 - \mathbf{n} \times \Omega_2 \mathbf{M}_1 \tag{27b}$$

B) MFIE formulation:

Boundary conditions for the Magnetic field for the external and internal problems are;

$$\mathbf{n} \times \mathbf{H}_1 = 0 \tag{28a}$$

and

$$\mathbf{n} \times \mathbf{H}_2 = 0 \tag{28b}$$

respectively. on the surface of the dielectric body. Using (28) in $(23a-d)$, we arrive to MFIE.

$$\mathbf{n} \times \mathbf{H}_i = \mathbf{n} \times \Omega_1 \mathbf{J}_1 + \frac{1}{\eta_1^2} \mathbf{n} \times \Lambda_1 \mathbf{M}_1 \tag{29a}$$

$$0 = \mathbf{n} \times \Omega_2 \mathbf{J}_1 + \frac{1}{\eta_2^2} \mathbf{n} \times \Lambda_2 \mathbf{M}_1 \tag{29b}$$

C) CFIE formulation:

Combining the EFIE and MFIE formulations as outlined in (21), yields

$$\begin{aligned}[\alpha \mathbf{E}^i + (1-\alpha)\eta \mathbf{n} \times \mathbf{H}^i] = (\alpha \Lambda_1 + (1-\alpha)\mathbf{n} \times \Omega_1)\mathbf{J}_1 \\ - (\alpha \Omega_1 + (1-\alpha)\frac{1}{\eta_2^2}\mathbf{n} \times \Lambda_1)\mathbf{M}_1\end{aligned} \tag{30a}$$

$$\begin{aligned}0 = (\alpha \Lambda_2 + (1-\alpha)\mathbf{n} \times \Omega_2)\mathbf{J}_1 \\ - (\alpha \Omega_2 + (1-\alpha)\frac{1}{\eta_2^2}\mathbf{n} \times \Lambda_2)\mathbf{M}_1\end{aligned} \tag{30b}$$

These are the most general integral equations to be solved for $\mathbf{J}_1$ and $\mathbf{M}_1$. They can in general be combined with similar integral equations from other dielectric boundaries for simulating rather complex geometries.

D) PMCHW formulation:

The PMCHW formulation is a special case of the CFIE and is also robust at interior resonant frequencies. The method relies on implying the continuity condition on the surface of the dielectric body. That is,

$$\mathbf{n} \times [\mathbf{E}_1 - \mathbf{E}_2] = 0 \tag{31a}$$

and

$$\mathbf{n} \times [\mathbf{H}_1 - \mathbf{H}_2] = 0. \tag{31b}$$

Employing these conditions to the fields given in (23a-d), we obtain the PMCHW equations.

$$\begin{bmatrix} \Lambda_1 + \Lambda_2 & -\Omega_1 - \Omega_2 \\ \Omega_1 + \Omega_2 & \frac{\epsilon_1}{\mu_1}\Lambda_1 + \frac{\epsilon_2}{\mu_2}\Lambda_2 \end{bmatrix} \begin{bmatrix} \mathbf{J_1} \\ \mathbf{M_1} \end{bmatrix} = \begin{bmatrix} \mathbf{E^i} \\ \mathbf{H^i} \end{bmatrix} \tag{32}$$

2.4) Impedance Boundary

The impedance boundary condition is of the form,

$$\mathbf{n} \times \mathbf{E}_1 = Z\mathbf{n} \times (\mathbf{n} \times \mathbf{H_1}). \tag{33}$$

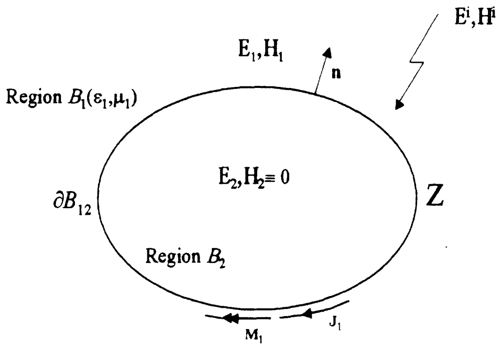


Figure 2.5 Geometry for applying the impedance boundary condition

Substituting (33) into $(23a, b)$, we get the surface integral equation.

$$\begin{bmatrix} \Lambda_1 + \frac{Z}{\eta_1} & -\Omega_1 \\ \Omega_1 & \frac{\epsilon_1}{\mu_1}\Lambda_1 + \frac{\eta_1}{Z} \end{bmatrix} \begin{bmatrix} \mathbf{J_1} \\ \mathbf{M_1} \end{bmatrix} = \begin{bmatrix} \mathbf{E^i} \\ \mathbf{H^i} \end{bmatrix} \tag{34}$$

In (34), $\eta = \sqrt{\frac{\mu_1}{\epsilon_1}}$ represents the characteristic impedance of the surrounding medium.

2.4) CA (Circuit-Analog Boundary)

Consider a thin(penetrable or impenetrable) multilayered sheet(see fig.2.6). Relations between the tangential components on the two sides of the sheet(consistent with duality and reciproity) can be written as

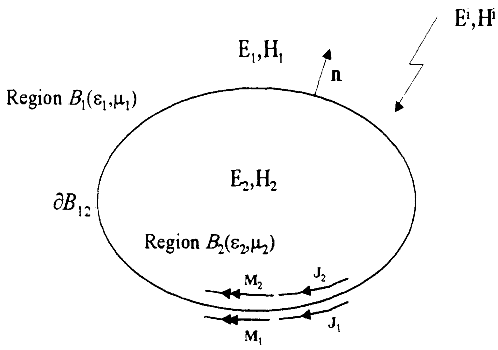


Figure 2.6 Geometry for applying the CA boundary condition

$$\mathbf{n} \times [E^+(\mathbf{r}) + E^-(\mathbf{r})] = \hat{R}_e \mathbf{n} \times \mathbf{n}[H^+(\mathbf{r}) - H^-(\mathbf{r})] - \hat{R}_c \mathbf{n} \times \mathbf{n}[E^+(\mathbf{r}) - E^-(\mathbf{r})] \qquad (35a)$$

$$\mathbf{n} \times [H^+(\mathbf{r}) + H^-(\mathbf{r})] = \hat{R}_m \mathbf{n} \times \mathbf{n}[E^+(\mathbf{r}) - E^-(\mathbf{r})] + \hat{R}_c \mathbf{n} \times \mathbf{n}[H^+(\mathbf{r}) - H^-(\mathbf{r})] \qquad (35b)$$

Here $E^{\pm}(\mathbf{r})$ and $H^{\pm}(\mathbf{r})$ represent the fields on the upper and the lower surfaces of the sheet, respectively; $\hat{R}_e$ and $\hat{R}_m$ are the electric and the magnetic resistivities and $\hat{R}_c$ is a cross coupling term. Rewriting $(35a)$ and $(35b)$ in matrix form, we obtain

$$\begin{bmatrix} \mathbf{E}^- \\ \eta\mathbf{n} \times \mathbf{H}^- \end{bmatrix} = \begin{bmatrix} \chi_{11} & \chi_{12} \\ \chi_{21} & \chi_{22} \end{bmatrix} \begin{bmatrix} \mathbf{E}^+ \\ \eta\mathbf{n} \times \mathbf{H}^+ \end{bmatrix} \qquad (36)$$

where $\chi_{ij}(i,j = 1,2)$ are given by

$$\chi_{11} = [1 + (1/2 - R_c)^2/R_eR_m]/[1 - \{(1/4 - R_c^2)/R_eR_m\}] \qquad (37a)$$

$$\chi_{12} = -1/[R_m[1 - \{(1/4 - R_c^2)/R_eR_m\}] \qquad (37b)$$

$$\chi_{21} = -1/[R_e[1 - \{(1/4 - R_c^2)/R_eR_m\}] \qquad (37b)$$

$$\chi_{22} = [1 + (1/2 + R_c)^2/R_eR_m]/[1 - \{(1/4 - R_c^2)/R_eR_m\}] \qquad (37d)$$

in which $R_n(\hat{R}_e = 2\eta R_e,\ \hat{R}_m = -(2/\eta)R_m,\ \hat{R}_c = 2R_c), (n = e, m, c)$ stand for the normalized resistivities. Assuming that the thin multilayered sheet can be characterized by its reflection and transmission properties, $R_n(n = e, m, c)$ can be determined by relating them to the reflection and transmission coefficients of the sheet. For general layered structure we need two reflection $(\Gamma^{\pm})$ and one transmission coefficient (T). The corresponding reflection and transmission coefficients from $(35a, b)$ are

$$\Gamma^{\pm}(\theta) = (2R_e cos\theta - 2R_m/cos\theta \pm 4R_c)/(4(R_eR_m + R_c^2) + 1 + 2 * R_e cos\theta + 2R_m/cos\theta) \qquad (38a)$$

$$T(\theta) = (4(R_c^2 + R_e R_m) - 1)/(4(R_e R_m + R_c^2) + 1 + 2R_e cos\theta + 2R_m/cos\theta) \quad (38b)$$

where θ stands for the incident angle. At normal incidence ($\theta = 0$) the above can be inverted to yield

$$R_e = [T^2(0) - (1 + \Gamma^+(0))(1 + \Gamma^-(0))]/[\Gamma^+(0)\Gamma^-(0) - (1 - T^2(0))^2]/2 \quad (39a)$$

$$R_m = [T^2(0) - (1 - \Gamma^+(0))(1 - \Gamma^-(0))]/[\Gamma^+(0)\Gamma^-(0) - (1 - T^2(0))^2]/2 \quad (39b)$$

$$R_c = [\Gamma^-(0) - \Gamma^+(0)]/[\Gamma^+(0)\Gamma^-(0) - (1 - T^2(0))^2]/2 \quad (39c)$$

Thus, upon having the reflecyion/transmission coefficients we can extract the corresponding $R_n(n = e, m, c)$ values.

2.4.1) Reduction to simple sheet condition

The transmission line model can be used to relate the above $R_n(n = e, m, c)$ parameters to the resistivity (Z_p) and conductivity (Z_s) values for simple sheets. Equating the reflection and transmission coefficients for the above circuit with those from (4) yields

$$\textbf{series}: \rightarrow \quad R_e \rightarrow \infty, \quad R_m = -Z_s/Z_0, \quad R_c = 0 \quad (40a)$$

$$\textbf{parallel}: \rightarrow \quad R_e \rightarrow \infty, \quad R_m = -Z_s/Z_0, \quad R_c = 0 \quad (40b)$$

and the $\chi_{ij}(i, j = 1, 2)$ matrices reduce to

$$\chi_{ser} = \begin{bmatrix} 1 & Z_s/Z_0 \\ 0 & 1 \end{bmatrix} \quad (41a)$$

and

$$\chi_{par} = \begin{bmatrix} 1 & 0 \\ -Z_0/Z_p & 1 \end{bmatrix}. \quad (41b)$$

When $(41a, b)$ is used in (36), we conclude that a single parallel impedance circuit represents a resistive boundarycondition, whereas a single series imprdance circuit represent a magnetically conductive boundary.

2.4.2) Surface Integral Equation

To construct a surface integral equation let us refer to Fig.2.3.. In this case interior and the exterior fields can be expressed with $(23a - d)$. The relation in between the tangential field components and the surface currents can also be given with $(22c, d)$. Substituting $(23a - d)$ into (36) with the identification that $\mathbf{E_1} = \mathbf{E^+}$, $\mathbf{E_2} = \mathbf{E^-}$, $\mathbf{H_1} = \mathbf{H^+}$, $\mathbf{H_2} = \mathbf{H^-}$ we obtain the integral equation;

$$\begin{bmatrix} \Lambda_1 - \frac{\eta\chi_{22}}{2\chi_{21}} & -\Omega_1 & -\frac{\eta}{2\chi_{21}} & 0 \\ \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 + \frac{\chi_{11}}{2\eta\chi_{12}} & 0 & -\frac{1}{2\eta\chi_{12}} \\ \frac{\eta}{2}(\chi_{12} - \frac{\chi_{11}\chi_{22}}{\chi_{21}}) & 0 & \Lambda_2 - \frac{\eta\chi_{11}}{2\chi_{21}} & -\Omega_2 \\ 0 & \frac{1}{2\eta}(\chi_{21} - \frac{\chi_{11}\chi_{22}}{\chi_{12}}) & \Omega_2 & \frac{1}{\eta_2^2}\Lambda_2 + \frac{\chi_{22}}{2\eta\chi_{12}} \end{bmatrix} \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{M}_1 \\ \mathbf{J}_2 \\ \mathbf{M}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{E}^i \\ \mathbf{H}^i \\ 0 \\ 0 \end{bmatrix} \quad (42)$$

3) Composite Structures

Consider the geometry depicted in Fig. 2.8. We are going to give the surface integral equations for different problems implemented in the code CADRIS.

3.1) Problem 1

In our first problem S_1 and S_2 are dielectric and S_3 and S_4 are the perfectly conducting surfaces. Electric and the magnetic fields can be written as follows in three different region(see Fig. 2.8.).

$$\theta(\mathbf{r})\mathbf{E}_1(\mathbf{r}) = \mathbf{E}^i - \Lambda_1(\mathbf{J}_1 + \mathbf{J}_4) + \Omega_1\mathbf{M}_1 \qquad (43a)$$

$$\theta(\mathbf{r})\mathbf{H}_1(\mathbf{r}) = \mathbf{H}^i - \Omega_1(\mathbf{J}_1 + \mathbf{J}_4) + -\frac{1}{\eta_1^2}\Lambda_1\mathbf{M}_1 \qquad (43b)$$

$$\theta(\mathbf{r})\mathbf{E}_2(\mathbf{r}) = -\Lambda_2(\mathbf{J}_2 - \mathbf{J}_1) + \Omega_2(\mathbf{M}_2 - \mathbf{M}_1) \qquad (43c)$$

$$\theta(\mathbf{r})\mathbf{H}_2(\mathbf{r}) = -\Omega_2(\mathbf{J}_2 - \mathbf{J}_1) + -\frac{1}{\eta_2^2}\Lambda_2(\mathbf{M}_2 - \mathbf{M}_1) \qquad (43d)$$

$$\theta(\mathbf{r})\mathbf{E}_3(\mathbf{r}) = -\Lambda_3(\mathbf{J}_3 - \mathbf{J}_2) + -\Omega_3\mathbf{M}_2 \qquad (43e)$$

$$\theta(\mathbf{r})\mathbf{H}_3(\mathbf{r}) = -\Omega_3(\mathbf{J}_3 - \mathbf{J}_2) - -\frac{1}{\eta_3^2}\Lambda_3\mathbf{M}_2 \qquad (43f)$$

boundary conditions on the surfaces $S_1 - S_4$ can be given as

$$OnS_1 : \mathbf{n} \times [\mathbf{E_1} - \mathbf{E_2}] = 0 \quad ; \quad \mathbf{n} \times [\mathbf{H_1} - \mathbf{H_2}] = 0 \qquad (44a)$$

$$OnS_2 : \mathbf{n} \times [\mathbf{E_2} - \mathbf{E_3}] = 0 \quad ; \quad \mathbf{n} \times [\mathbf{H_2} - \mathbf{H_3}] = 0 \qquad (44b)$$

$$OnS_3 : \mathbf{n} \times \mathbf{E_3} = 0 \; (EFIE) \quad ; \quad \mathbf{n} \times \mathbf{H_3} = \mathbf{J}_3 \; (MFIE) \qquad (44c)$$

$$OnS_4 : \mathbf{n} \times \mathbf{E_1} = 0 \; (EFIE) \quad ; \quad \mathbf{n} \times \mathbf{H_1} = \mathbf{J}_4 \; (MFIE) \qquad (44d)$$

Using $(44a - d)$ in $(43a - f)$, and after some straight forward manipulations we find the surface integral equation as

$$\mathbf{ZI} = \mathbf{V} \qquad (45a)$$

$$\mathbf{Z} = \begin{bmatrix} \Lambda_1 + \Lambda_2 & -\Omega_1 - \Omega_2 & -\Lambda_2 & \Omega_2 & 0 & \Lambda_1 \\ \Omega_1 + \Omega_2 & \frac{1}{\eta_1^2}\Lambda_1 + \frac{1}{\eta_2^2}\Lambda_2 & -\Omega_2 & -\frac{1}{\eta_2^2}\Lambda_2 & 0 & \Omega_1 \\ -\Lambda_2 & \Omega_2 & \Lambda_2 + \Lambda_3 & -\Omega_2 - \Omega_3 & -\Lambda_3 & 0 \\ -\Omega_2 & -\frac{1}{\eta_2^2}\Lambda_2 & \Omega_2 + \Omega_3 & \frac{1}{\eta_2^2}\Lambda_2 + \frac{1}{\eta_3^2}\Lambda_3 & -\Omega_3 & 0 \\ 0 & 0 & -\Lambda_3 & \Omega_3 & \Lambda_3 & 0 \\ \Lambda_1 & -\Omega_1 & 0 & 0 & 0 & \Lambda_1 \end{bmatrix} \qquad (45b)$$

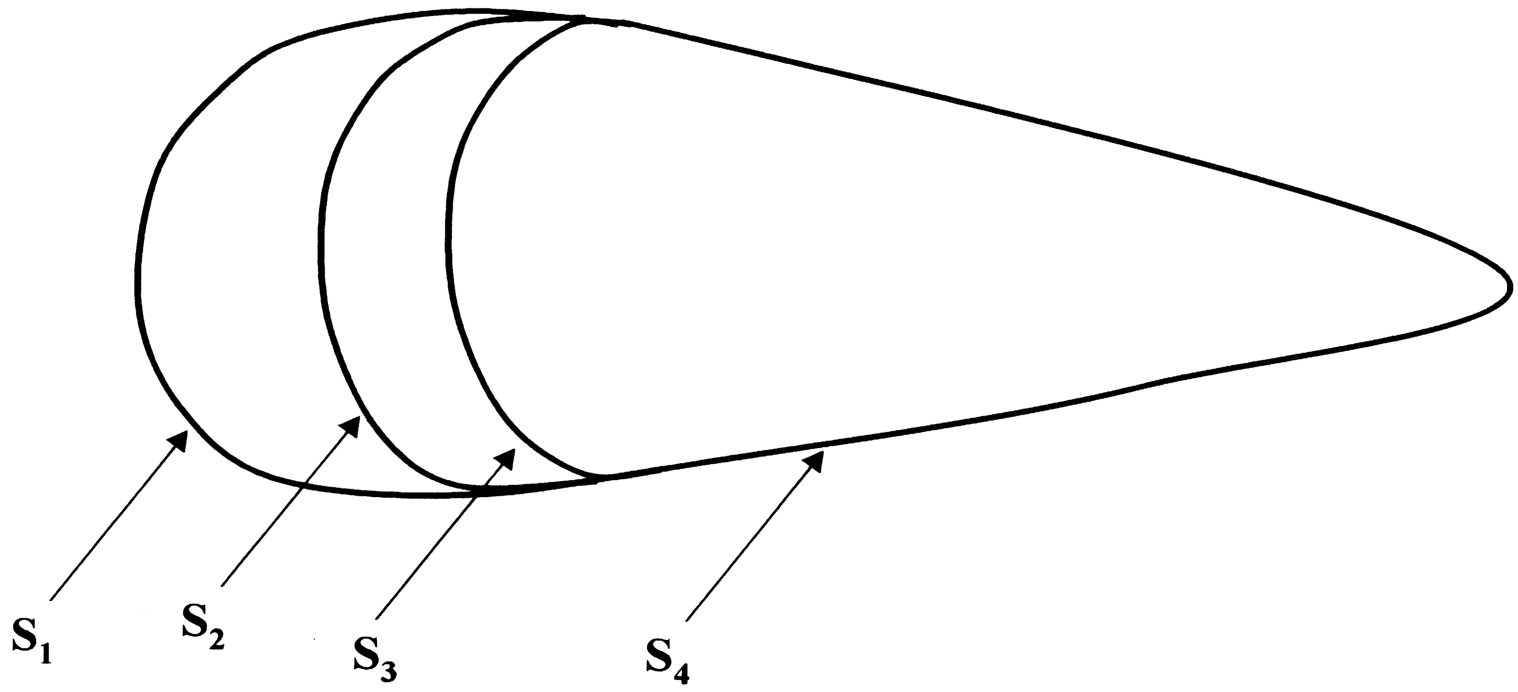


Fig.2.8 General Problem Geometry

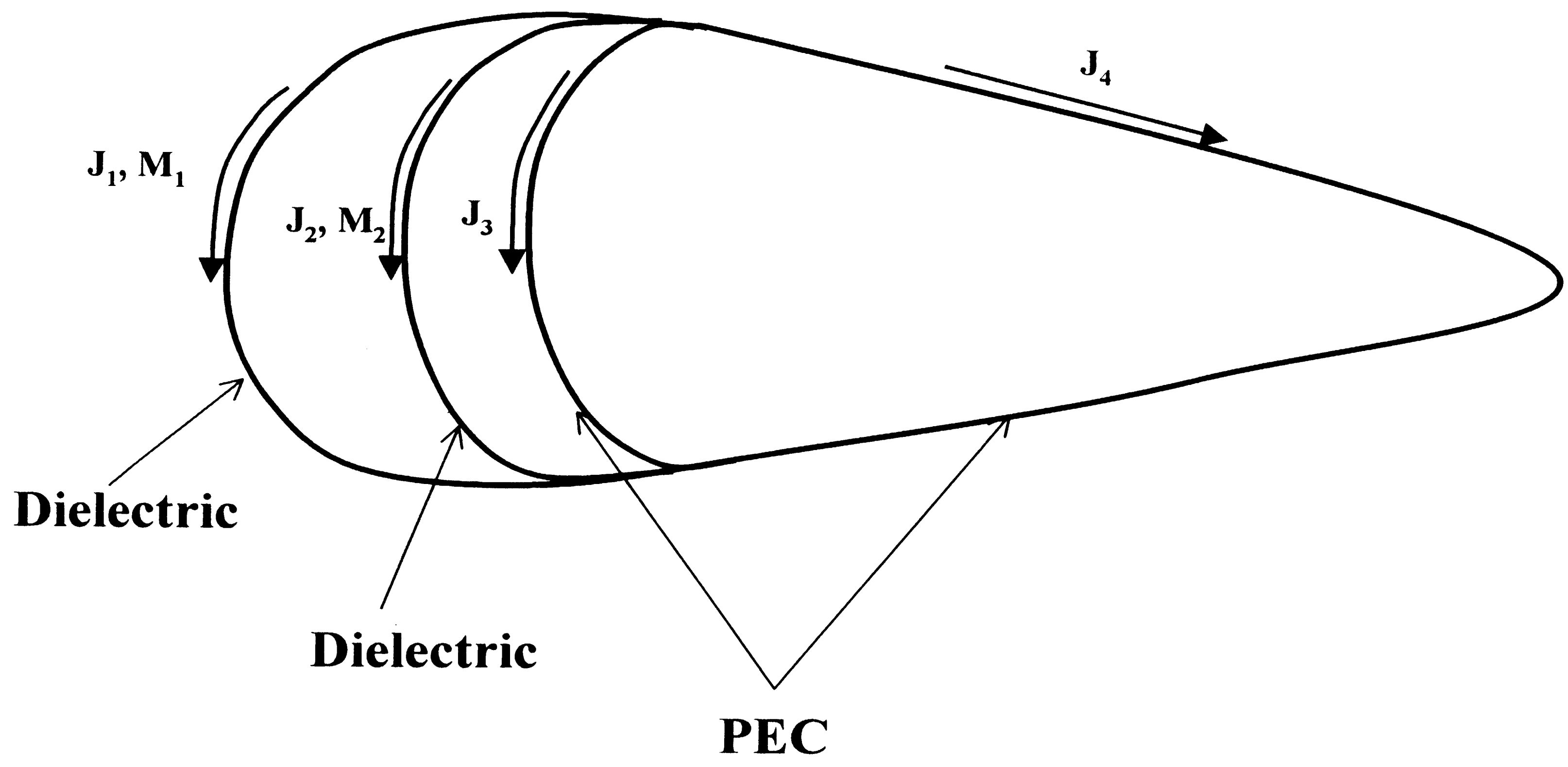


Fig.2.9 Geometry for Problem-1

$$\mathbf{I} = \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{M}_1 \\ \mathbf{J}_2 \\ \mathbf{M}_2 \\ \mathbf{J}_3 \\ \mathbf{J}_4 \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} \mathbf{E}^i(S_1) \\ \mathbf{H}^i(S_1) \\ 0 \\ 0 \\ 0 \\ \mathbf{E}^i(S_4) \end{bmatrix} \tag{45c}$$

Problem 2

In the second problem S_1 is a dielectric, S_2 is a resistive and S_3 and S_4 are the perfectly conducting surfaces.

In this case fields can be as

$$\theta(\mathbf{r})\mathbf{E}_1(\mathbf{r}) = \mathbf{E}^i - \Lambda_1(\mathbf{J}_1 + \mathbf{J}_4) + \Omega_1\mathbf{M}_1 \tag{46a}$$

$$\theta(\mathbf{r})\mathbf{H}_1(\mathbf{r}) = \mathbf{H}^i - \Omega_1(\mathbf{J}_1 + \mathbf{J}_4) + -\frac{1}{\eta_1^2}\Lambda_1\mathbf{M}_1 \tag{46b}$$

$$\theta(\mathbf{r})\mathbf{E}_2(\mathbf{r}) = -\Lambda_2(\mathbf{J}_2 - \mathbf{J}_1) + \Omega_2(\mathbf{M}_2 - \mathbf{M}_1) \tag{46c}$$

$$\theta(\mathbf{r})\mathbf{H}_2(\mathbf{r}) = -\Omega_2(\mathbf{J}_2 - \mathbf{J}_1) + -\frac{1}{\eta_2^2}\Lambda_2(\mathbf{M}_2 - \mathbf{M}_1) \tag{46d}$$

$$\theta(\mathbf{r})\mathbf{E}_3(\mathbf{r}) = -\Lambda_3(\mathbf{J}_3 + \mathbf{J}_{22}) + -\Omega_3\mathbf{M}_2 \tag{46e}$$

$$\theta(\mathbf{r})\mathbf{H}_3(\mathbf{r}) = -\Omega_3(\mathbf{J}_3 + \mathbf{J}_{22}) - -\frac{1}{\eta_3^2}\Lambda_3\mathbf{M}_2 \tag{46f}$$

boundary conditions on the surfaces $S_1 - S_4$ can be given as

$$OnS_1 : \mathbf{n} \times [\mathbf{E_1} - \mathbf{E_2}] = 0 \quad ; \quad \mathbf{n} \times [\mathbf{H_1} - \mathbf{H_2}] = 0 \tag{47a}$$

$$OnS_2 : \mathbf{n} \times [\mathbf{E_2} - \mathbf{E_3}] = 0 \quad ; \quad \mathbf{n} \times [\mathbf{E_2} + \mathbf{E_3}] = 2\eta_1 R\mathbf{n} \times \mathbf{n} \times [\mathbf{H_2} - \mathbf{H_3}] \tag{47b}$$

$$OnS_3 : \mathbf{n} \times \mathbf{E_3} = 0(EFIE) \quad ; \quad \mathbf{n} \times \mathbf{H_3} = \mathbf{J}_3(MFIE) \tag{47c}$$

$$OnS_4 : \mathbf{n} \times \mathbf{E_1} = 0(EFIE) \quad ; \quad \mathbf{n} \times \mathbf{H_1} = \mathbf{J}_4(MFIE) \tag{47d}$$

Using $(47a - d)$ in $(46a - f)$, and after some straight forward manipulations we find the surface integral equation as

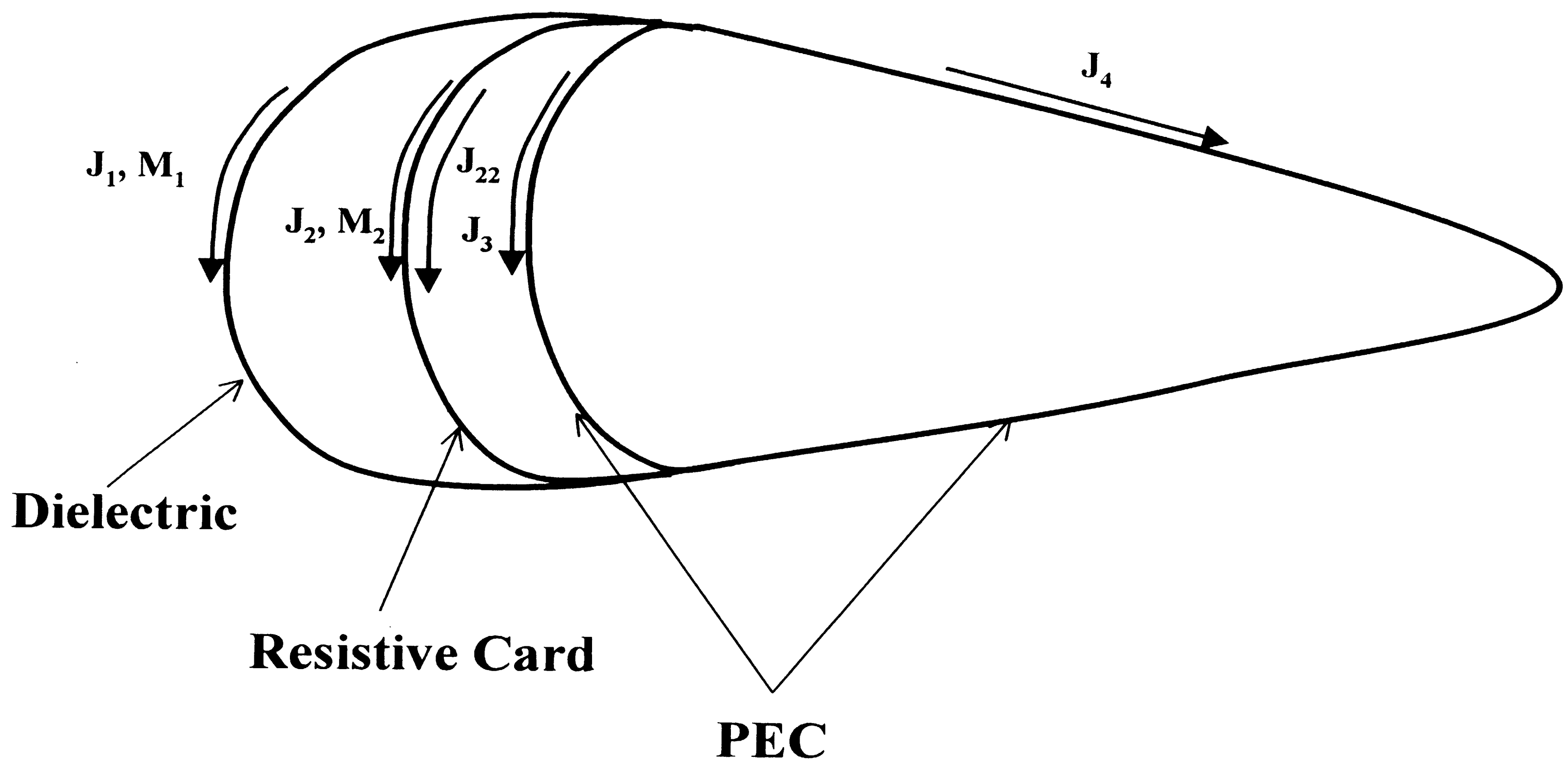


Fig.2.10 Geometry for Problem-2

$$
\mathbf{Z} = \begin{bmatrix}
\Lambda_1+\Lambda_2 & -\Omega_1-\Omega_2 & -\Lambda_2 & \Omega_2 & 0 & 0 & \Lambda_1 \\
\Omega_1+\Omega_2 & \frac{1}{\eta_1^2}\Lambda_1+\frac{1}{\eta_2^2}\Lambda_2 & -\Omega_2 & -\frac{1}{\eta_2^2}\Lambda_2 & 0 & 0 & \Omega_1 \\
-\Lambda_2 & \Omega_2 & \Lambda_2+\frac{\eta_1 R}{2} & -\Omega_2 & \frac{-\eta_1 R}{2} & 0 & 0 \\
-\Omega_2 & -\frac{1}{\eta_2^2}\Lambda_2 & \Omega_2 & \frac{1}{\eta_2^2}\Lambda_2+\frac{1}{\eta_3^2\Lambda_3}+\frac{R}{2\eta_1} & -\Omega_3 & -\Omega_3 & 0 \\
0 & 0 & \frac{\eta_1 R}{2} & \Omega_3 & \Lambda_3+\frac{\eta_1 R}{2} & \Lambda_3 & 0 \\
0 & 0 & 0 & \Omega_3 & \Lambda_3 & \Lambda_3 & 0 \\
\Lambda_1 & -\Omega_1 & 0 & 0 & 0 & 0 & \Lambda_1
\end{bmatrix} \tag{48a}
$$

$$
\mathbf{I} = \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{M}_1 \\ \mathbf{J}_2 \\ \mathbf{M}_2 \\ \mathbf{J}_{22} \\ \mathbf{J}_3 \\ \mathbf{J}_4 \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} \mathbf{E}^i(S_1) \\ \mathbf{H}^i(S_1) \\ 0 \\ 0 \\ 0 \\ 0 \\ \mathbf{E}^i(S_4) \end{bmatrix} \tag{48b}
$$

Problem 3

In the third problem S_1 is a dielectric, S_2 is a CA-boundary and S_3 and S_4 are the perfectly conducting surfaces.

In this case fields can be as

$$\theta(\mathbf{r})\mathbf{E}_1(\mathbf{r}) = \mathbf{E}^i - \Lambda_1(\mathbf{J}_1+\mathbf{J}_4) + \Omega_1\mathbf{M}_1 \tag{49a}$$

$$\theta(\mathbf{r})\mathbf{H}_1(\mathbf{r}) = \mathbf{H}^i - \Omega_1(\mathbf{J}_1+\mathbf{J}_4) + -\frac{1}{\eta_1^2}\Lambda_1\mathbf{M}_1 \tag{49b}$$

$$\theta(\mathbf{r})\mathbf{E}_2(\mathbf{r}) = -\Lambda_2(\mathbf{J}_2-\mathbf{J}_1) + \Omega_2(\mathbf{M}_2-\mathbf{M}_1) \tag{49c}$$

$$\theta(\mathbf{r})\mathbf{H}_2(\mathbf{r}) = -\Omega_2(\mathbf{J}_2-\mathbf{J}_1) + -\frac{1}{\eta_2^2}\Lambda_2(\mathbf{M}_2-\mathbf{M}_1) \tag{49d}$$

$$\theta(\mathbf{r})\mathbf{E}_3(\mathbf{r}) = -\Lambda_3(\mathbf{J}_3+\mathbf{J}_{22}) + +\Omega_3\mathbf{M}_{22} \tag{49e}$$

$$\theta(\mathbf{r})\mathbf{H}_3(\mathbf{r}) = -\Omega_3(\mathbf{J}_3+\mathbf{J}_{22}) - -\frac{1}{\eta_3^2}\Lambda_3\mathbf{M}_{22} \tag{49f}$$

boundary conditions on the surfaces $S_1 - S_4$ can be given as

$$On S_1 : \mathbf{n}\times[\mathbf{E_1}-\mathbf{E_2}] = 0 \quad ; \quad \mathbf{n}\times[\mathbf{H_1}-\mathbf{H_2}] = 0 \tag{50a}$$

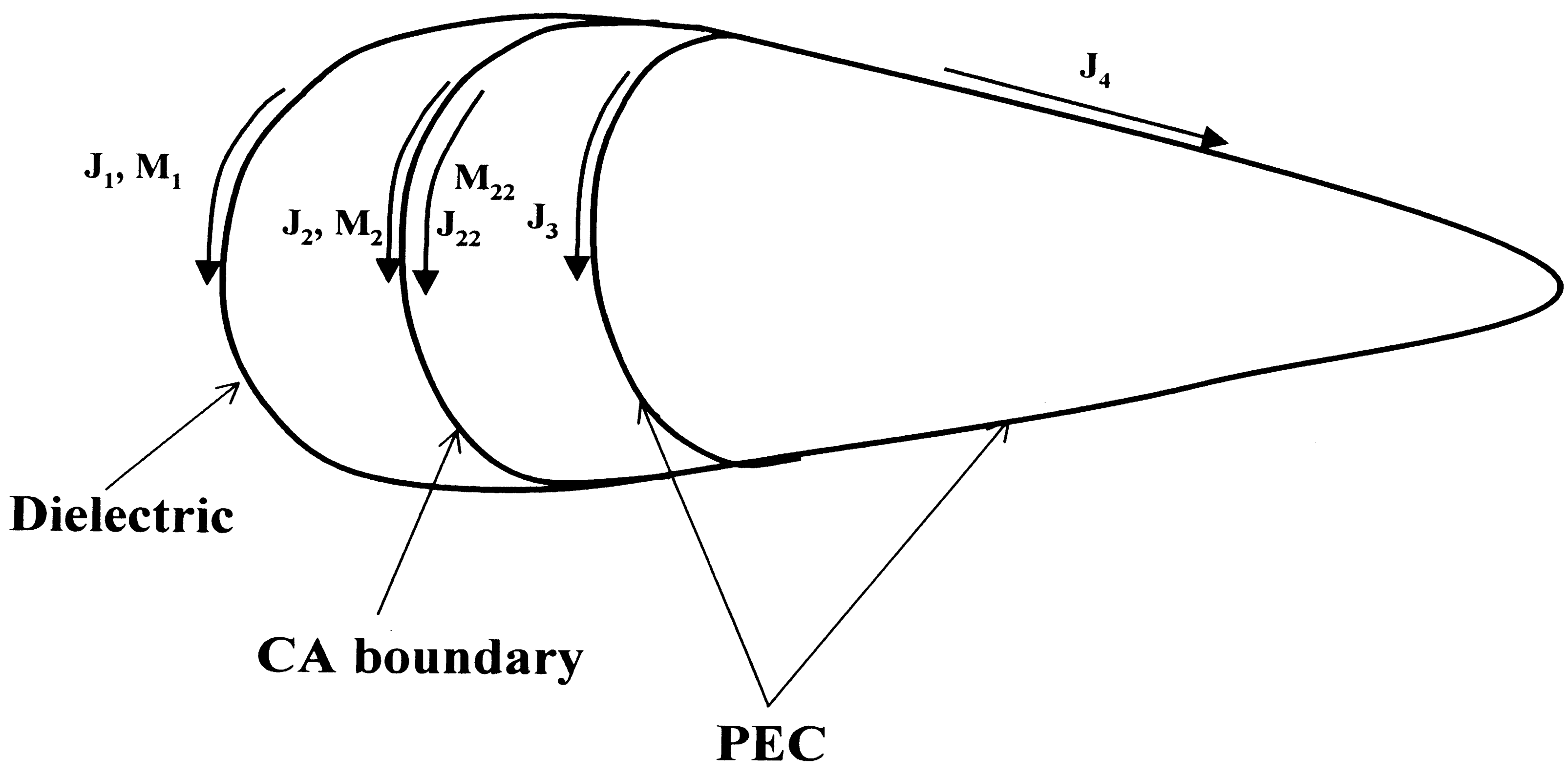


Fig.2.11 Geometry for Problem-3

$$OnS_2 : \begin{bmatrix} \mathbf{E}^- \\ \eta\mathbf{n} \times \mathbf{H}^- \end{bmatrix} = \begin{bmatrix} \chi_{11} & \chi_{12} \\ \chi_{21} & \chi_{22} \end{bmatrix} \begin{bmatrix} \mathbf{E}^+ \\ \eta\mathbf{n} \times \mathbf{H}^+ \end{bmatrix} \tag{50b}$$

$$OnS_3 : \mathbf{n} \times \mathbf{E_3} = 0(EFIE) \quad ; \quad \mathbf{n} \times \mathbf{H_3} = \mathbf{J}_3(MFIE) \tag{50c}$$

$$OnS_4 : \mathbf{n} \times \mathbf{E_1} = 0(EFIE) \quad ; \quad \mathbf{n} \times \mathbf{H_1} = \mathbf{J}_4(MFIE) \tag{50d}$$

Using $(50a - d)$ in $(49a - f)$, and after some straight forward manipulations we find the surface integral equation as

$$\mathbf{Z} =$$

$$\begin{bmatrix}
\Lambda_1 + \Lambda_2 & -\Omega_1 - \Omega_2 & -\Lambda_2 & \Omega_2 & 0 & 0 & 0 & \Lambda_1 \\
\Omega_1 + \Omega_2 & \frac{1}{\eta_1^2}\Lambda_1 + \frac{1}{\eta_2^2}\Lambda_2 & -\Omega_2 & -\frac{1}{\eta_2^2}\Lambda_2 & 0 & 0 & 0 & \Omega_1 \\
-\Lambda_2 & \Omega_2 & \Lambda_2 - \frac{\eta\chi_{22}}{2\chi_{21}} & -\Omega_2 & -\frac{\eta}{2\chi_{21}} & 0 & 0 & 0 \\
-\Omega_2 & -\frac{1}{\eta_2^2}\Lambda_2 & \Omega_1 & \frac{1}{\eta_1^2}\Lambda_2 + \frac{\chi_{11}}{2\eta\chi_{12}} & 0 & -\frac{1}{2\eta\chi_{12}} & 0 & 0 \\
0 & 0 & \frac{\eta}{2}(\chi_{12} - \frac{\chi_{11}\chi_{22}}{\chi_{21}}) & 0 & \Lambda_2 - \frac{\eta\chi_{11}}{2\chi_{21}} & -\Omega_2 & \Lambda_3 & 0 \\
0 & 0 & 0 & \frac{1}{2\eta}(\chi_{21} - \frac{\chi_{11}\chi_{22}}{\chi_{12}}) & \Omega_3 & \frac{1}{\eta_2^2}\Lambda_3 + \frac{\chi_{22}}{2\eta\chi_{12}} & \Omega_3 & 0 \\
0 & 0 & 0 & 0 & \Lambda_3 & -\Omega_3 & \Lambda_3 & 0 \\
\Lambda_1 & -\Omega_1 & 0 & 0 & 0 & 0 & 0 & \Lambda_1
\end{bmatrix} \tag{51a}$$

$$\mathbf{I} = \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{M}_1 \\ \mathbf{J}_2 \\ \mathbf{M}_2 \\ \mathbf{J}_{22} \\ \mathbf{M}_{22} \\ \mathbf{J}_3 \\ \mathbf{J}_4 \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} \mathbf{E}^i(S_1) \\ \mathbf{H}^i(S_1) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \mathbf{E}^i(S_4) \end{bmatrix} \tag{51b}$$

Problem 4

In the fourth problem S_1 and S_2 are dielectric, S_3 is a PEC and S_4 is an impedance surface.

In this case fields can be written as

$$\theta(\mathbf{r})\mathbf{E}_1(\mathbf{r}) = \mathbf{E}^i - \Lambda_1(\mathbf{J}_1 + \mathbf{J}_4) + \Omega_1(\mathbf{M}_1 + \mathbf{M}_4) \tag{52a}$$

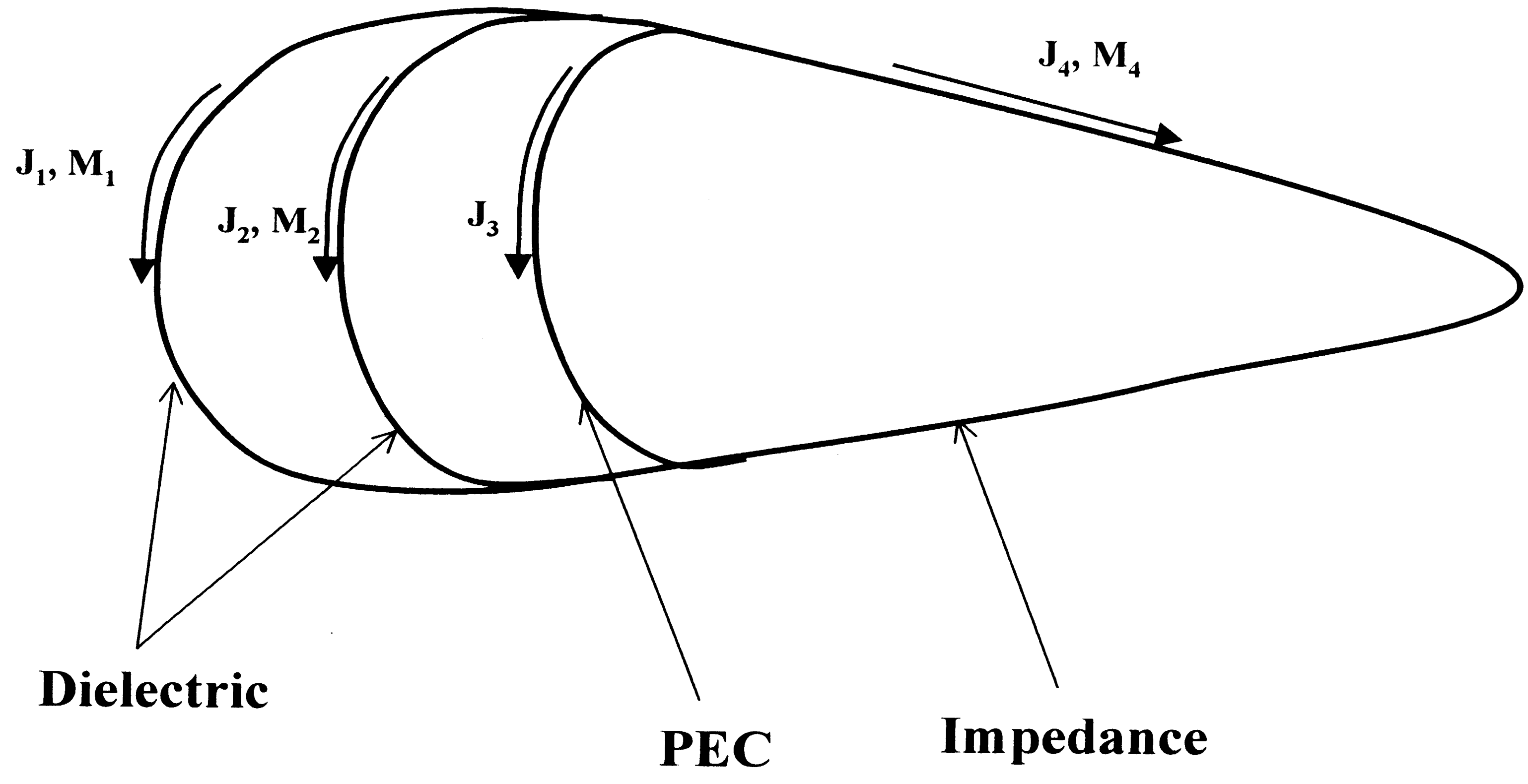


Fig.2.12 Geometry for Problem-4

$$\theta(\mathbf{r})\mathbf{H}_1(\mathbf{r}) = \mathbf{H}^i - \Omega_1(\mathbf{J}_1 + \mathbf{J}_4) + -\frac{1}{\eta_1^2}\Lambda_1(\mathbf{M}_1 + \mathbf{M}_4) \qquad (52b)$$

$$\theta(\mathbf{r})\mathbf{E}_2(\mathbf{r}) = -\Lambda_2(\mathbf{J}_2 - \mathbf{J}_1) + \Omega_2(\mathbf{M}_2 - \mathbf{M}_1) \qquad (52c)$$

$$\theta(\mathbf{r})\mathbf{H}_2(\mathbf{r}) = -\Omega_2(\mathbf{J}_2 - \mathbf{J}_1) + -\frac{1}{\eta_2^2}\Lambda_2(\mathbf{M}_2 - \mathbf{M}_1) \qquad (52d)$$

$$\theta(\mathbf{r})\mathbf{E}_3(\mathbf{r}) = -\Lambda_3(\mathbf{J}_3 - \mathbf{J}_2) + -\Omega_3\mathbf{M}_2 \qquad (52e)$$

$$\theta(\mathbf{r})\mathbf{H}_3(\mathbf{r}) = -\Omega_3(\mathbf{J}_3 - \mathbf{J}_2) - -\frac{1}{\eta_3^2}\Lambda_3\mathbf{M}_2 \qquad (52f)$$

boundary conditions on the surfaces $S_1 - S_4$ can be given as

$$On S_1 : \mathbf{n} \times [\mathbf{E_1} - \mathbf{E_2}] = 0 \quad ; \quad \mathbf{n} \times [\mathbf{H_1} - \mathbf{H_2}] = 0 \qquad (53a)$$

$$On S_2 : \mathbf{n} \times [\mathbf{E_2} - \mathbf{E_3}] = 0 \quad ; \quad \mathbf{n} \times [\mathbf{H_2} - \mathbf{H_3}] = 0 \qquad (53b)$$

$$On S_3 : \mathbf{n} \times \mathbf{E_3} = 0 \ (EFIE) \quad ; \quad \mathbf{n} \times \mathbf{H_3} = \mathbf{J}_3 \ (MFIE) \qquad (54c)$$

$$On S_4 : \mathbf{n} \times \mathbf{E_1} = Z\mathbf{n} \times \mathbf{n} \times \mathbf{H_1} \qquad (54d)$$

Using $(54a - d)$ in $(53a - f)$, and after some straight forward manipulations we find the surface integral equation as

$$\mathbf{Z} = \begin{bmatrix} \Lambda_1 + \Lambda_2 & -\Omega_1 - \Omega_2 & -\Lambda_2 & \Omega_2 & 0 & \Lambda_1 & -\Omega_1 \\ \Omega_1 + \Omega_2 & \frac{1}{\eta_1^2}\Lambda_1 + \frac{1}{\eta_2^2}\Lambda_2 & -\Omega_2 & -\frac{1}{\eta_2^2}\Lambda_2 & 0 & \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 \\ -\Lambda_2 & \Omega_2 & \Lambda_2 + \Lambda_3 & -\Omega_2 - \Omega_3 & -\Lambda_3 & 0 & 0 \\ -\Omega_2 & -\frac{1}{\eta_2^2}\Lambda_2 & \Omega_2 + \Omega_3 & \frac{1}{\eta_2^2}\Lambda_1 + \frac{1}{\eta_3^2\Lambda_3} & -\Omega_3 & 0 & 0 \\ 0 & 0 & -\Lambda_3 & \Omega_3 & \Lambda_3 & 0 & 0 \\ \Lambda_1 & -\Omega_1 & 0 & 0 & 0 & \Lambda_1 + \frac{Z}{2} & -\Omega_1 \\ \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 & 0 & 0 & 0 & \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 + \frac{1}{2Z} \end{bmatrix} \qquad (55a)$$

$$\mathbf{I} = \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{M}_1 \\ \mathbf{J}_2 \\ \mathbf{M}_2 \\ \mathbf{J}_3 \\ \mathbf{J}_4 \\ \mathbf{M}_4 \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} \mathbf{E}^i(S_1) \\ \mathbf{H}^i(S_1) \\ 0 \\ 0 \\ 0 \\ \mathbf{E}^i(S_4) \\ \mathbf{H}^i(S_4) \end{bmatrix} \qquad (55b)$$

Problem 5

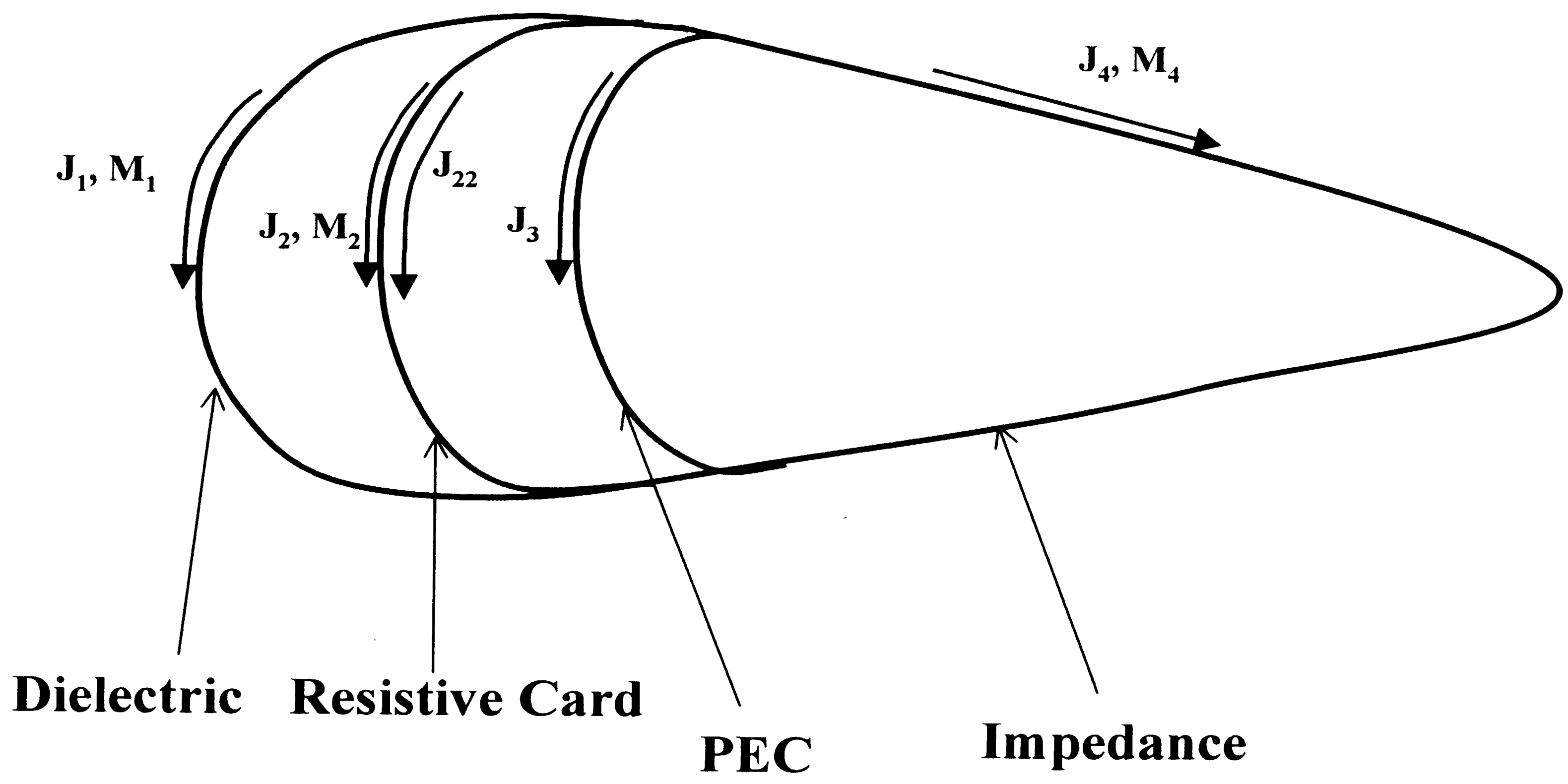


Fig.2.13 Geometry for Problem-5

In the fifth problem S_1 is a dielectric, S_2 is a resistive S_3 is a PEC and S_4 is an impedance surface. In this case fields can be as

$$\theta(\mathbf{r})\mathbf{E}_1(\mathbf{r}) = \mathbf{E}^i - \Lambda_1(\mathbf{J}_1 + \mathbf{J}_4) + \Omega_1(\mathbf{M}_1 + \mathbf{M}_4) \tag{56a}$$

$$\theta(\mathbf{r})\mathbf{H}_1(\mathbf{r}) = \mathbf{H}^i - \Omega_1(\mathbf{J}_1 + \mathbf{J}_4) + -\frac{1}{\eta_1^2}\Lambda_1(\mathbf{M}_1 + \mathbf{M}_4) \tag{56b}$$

$$\theta(\mathbf{r})\mathbf{E}_2(\mathbf{r}) = -\Lambda_2(\mathbf{J}_2 - \mathbf{J}_1) + \Omega_2(\mathbf{M}_2 - \mathbf{M}_1) \tag{56c}$$

$$\theta(\mathbf{r})\mathbf{H}_2(\mathbf{r}) = -\Omega_2(\mathbf{J}_2 - \mathbf{J}_1) + -\frac{1}{\eta_2^2}\Lambda_2(\mathbf{M}_2 - \mathbf{M}_1) \tag{56d}$$

$$\theta(\mathbf{r})\mathbf{E}_3(\mathbf{r}) = -\Lambda_3(\mathbf{J}_3 + \mathbf{J}_{22}) + -\Omega_3\mathbf{M}_2 \tag{56e}$$

$$\theta(\mathbf{r})\mathbf{H}_3(\mathbf{r}) = -\Omega_3(\mathbf{J}_3 + \mathbf{J}_{22}) - -\frac{1}{\eta_3^2}\Lambda_3\mathbf{M}_2 \tag{56f}$$

boundary conditions on the surfaces $S_1 - S_4$ can be given as

$$OnS_1 : \mathbf{n} \times [\mathbf{E_1} - \mathbf{E_2}] = 0 \quad ; \quad \mathbf{n} \times [\mathbf{H_1} - \mathbf{H_2}] = 0 \tag{57a}$$

$$OnS_2 : \mathbf{n} \times [\mathbf{E_2} - \mathbf{E_3}] = 0 \quad ; \quad \mathbf{n} \times [\mathbf{E_2} + \mathbf{E_3}] = 2\eta_1 R\mathbf{n} \times \mathbf{n} \times [\mathbf{H_2} - \mathbf{H_3}] \tag{57b}$$

$$OnS_3 : \mathbf{n} \times \mathbf{E_3} = 0(EFIE) \quad ; \quad \mathbf{n} \times \mathbf{H_3} = \mathbf{J_3}(MFIE) \tag{57c}$$

$$OnS_4 : \mathbf{n} \times \mathbf{E_1} = Z\mathbf{n} \times \mathbf{n} \times \mathbf{H_1} \tag{57d}$$

Using $(57a - d)$ in $(56a - f)$, and after some straight forward manipulations we find the surface integral equation as

$$\mathbf{Z} =$$

$$\begin{bmatrix}
\Lambda_1+\Lambda_2 & -\Omega_1-\Omega_2 & -\Lambda_2 & \Omega_2 & 0 & 0 & \Lambda_1 & -\Omega_1 \\
\Omega_1+\Omega_2 & \frac{1}{\eta_1^2}\Lambda_1+\frac{1}{\eta_2^2}\Lambda_2 & -\Omega_2 & -\frac{1}{\eta_2^2}\Lambda_2 & 0 & 0 & \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 \\
-\Lambda_2 & \Omega_2 & \Lambda_2+\frac{\eta_1 R}{2} & -\Omega_2 & \frac{-\eta_1 R}{2} & 0 & 0 & 0 \\
-\Omega_2 & -\frac{1}{\eta_2^2}\Lambda_2 & \Omega_2 & \frac{1}{\eta_2^2}\Lambda_2+\frac{1}{\eta_3^2\Lambda_3}+\frac{R}{2\eta_1} & -\Omega_3 & -\Omega_3 & 0 & 0 \\
0 & 0 & \frac{\eta_1 R}{2} & \Omega_3 & \Lambda_3+\frac{\eta_1 R}{2} & \Lambda_3 & 0 & 0 \\
0 & 0 & 0 & \Omega_3 & \Lambda_3 & \Lambda_3 & 0 & 0 \\
\Lambda_1 & -\Omega_1 & 0 & 0 & 0 & 0 & \Lambda_1+\frac{Z}{2} & -\Omega_1 \\
\Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 & 0 & 0 & 0 & 0 & \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1+\frac{1}{2Z}
\end{bmatrix} \tag{58a}$$

$$\mathbf{I} = \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{M}_1 \\ \mathbf{J}_2 \\ \mathbf{M}_2 \\ \mathbf{J}_{22} \\ \mathbf{J}_3 \\ \mathbf{J}_4 \\ \mathbf{M}_4 \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} \mathbf{E}^i(S_1) \\ \mathbf{H}^i(S_1) \\ 0 \\ 0 \\ 0 \\ 0 \\ \mathbf{E}^i(S_4) \\ \mathbf{H}^i(S_4) \end{bmatrix} \tag{58b}$$

Problem 6

In the sixth problem S_1 is a dielectric, S_2 is a CABC S_3 is a PEC and S_4 is an Impedance surface.

In this case fields can be as

$$\theta(\mathbf{r})\mathbf{E}_1(\mathbf{r}) = \mathbf{E}^i - \Lambda_1(\mathbf{J}_1 + \mathbf{J}_4) + \Omega_1(\mathbf{M}_1 + \mathbf{M}_4) \tag{59a}$$

$$\theta(\mathbf{r})\mathbf{H}_1(\mathbf{r}) = \mathbf{H}^i - \Omega_1(\mathbf{J}_1 + \mathbf{J}_4) + -\frac{1}{\eta_1^2}\Lambda_1(\mathbf{M}_1 + \mathbf{M}_4) \tag{59b}$$

$$\theta(\mathbf{r})\mathbf{E}_2(\mathbf{r}) = -\Lambda_2(\mathbf{J}_2 - \mathbf{J}_1) + \Omega_2(\mathbf{M}_2 - \mathbf{M}_1) \tag{59c}$$

$$\theta(\mathbf{r})\mathbf{H}_2(\mathbf{r}) = -\Omega_2(\mathbf{J}_2 - \mathbf{J}_1) + -\frac{1}{\eta_2^2}\Lambda_2(\mathbf{M}_2 - \mathbf{M}_1) \tag{59d}$$

$$\theta(\mathbf{r})\mathbf{E}_3(\mathbf{r}) = -\Lambda_3(\mathbf{J}_3 + \mathbf{J}_{22}) + \Omega_3\mathbf{M}_{22} \tag{59e}$$

$$\theta(\mathbf{r})\mathbf{H}_3(\mathbf{r}) = -\Omega_3(\mathbf{J}_3 + \mathbf{J}_{22}) - \frac{1}{\eta_3^2}\Lambda_3\mathbf{M}_{22} \tag{59f}$$

boundary conditions on the surfaces $S_1 - S_4$ can be given as

$$On S_1 : \mathbf{n} \times [\mathbf{E_1} - \mathbf{E_2}] = 0 \quad ; \quad \mathbf{n} \times [\mathbf{H_1} - \mathbf{H_2}] = 0 \tag{60a}$$

$$\begin{bmatrix} \mathbf{E}^- \\ \eta\mathbf{n} \times \mathbf{H}^- \end{bmatrix} = \begin{bmatrix} \chi_{11} & \chi_{12} \\ \chi_{21} & \chi_{22} \end{bmatrix} \begin{bmatrix} \mathbf{E}^+ \\ \eta\mathbf{n} \times \mathbf{H}^+ \end{bmatrix} \tag{60b}$$

$$On S_3 : \mathbf{n} \times \mathbf{E_3} = 0(EFIE) \quad ; \quad \mathbf{n} \times \mathbf{H_3} = \mathbf{J_3}(MFIE) \tag{60c}$$

$$On S_4 : \mathbf{n} \times \mathbf{E_1} = Z\mathbf{n} \times \mathbf{n} \times \mathbf{H_1} \tag{60d}$$

Using $(60a-d)$ in $(59a-f)$, and after some straight forward manipulations we find the surface integral equation as

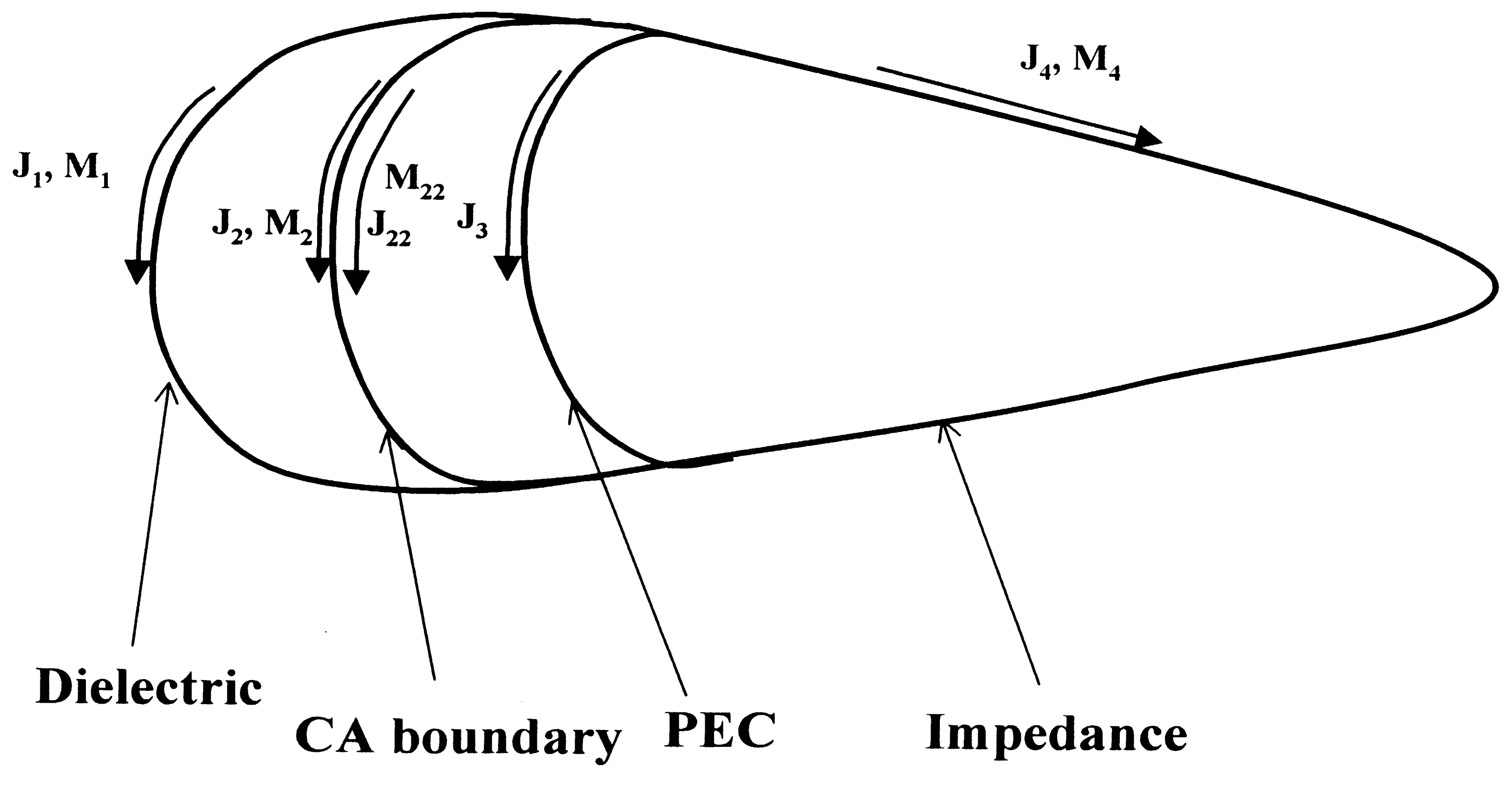


Fig.2.14 Geometry for Problem-6

$$\mathbf{Z} = \begin{bmatrix} \Lambda_1+\Lambda_2 & -\Omega_1-\Omega_2 & -\Lambda_2 & \Omega_2 & 0 & 0 & 0 & \Lambda_1 & -\Omega_1 \\ \Omega_1+\Omega_2 & \frac{1}{\eta_1^2}\Lambda_1+\frac{1}{\eta_2^2}\Lambda_2 & -\Omega_2 & -\frac{1}{\eta_2^2}\Lambda_2 & 0 & 0 & 0 & \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 \\ -\Lambda_2 & \Omega_2 & \Lambda_2-\frac{\eta\chi_{22}}{2\chi_{21}} & -\Omega_2 & -\frac{\eta}{2\chi_{21}} & 0 & 0 & 0 & 0 \\ -\Omega_2 & -\frac{1}{\eta_2^2}\Lambda_2 & \Omega_1 & \frac{1}{\eta_1^2}\Lambda_2+\frac{\chi_{11}}{2\eta\chi_{12}} & 0 & -\frac{1}{2\eta\chi_{12}} & 0 & 0 & 0 \\ 0 & 0 & \frac{\eta}{2}(\chi_{12}-\frac{\chi_{11}\chi_{22}}{\chi_{21}}) & 0 & \Lambda_2-\frac{\eta\chi_{11}}{2\chi_{21}} & -\Omega_2 & \Lambda_3 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2\eta}(\chi_{21}-\frac{\chi_{11}\chi_{22}}{\chi_{12}}) & \Omega_3 & \frac{1}{\eta_2^2}\Lambda_3+\frac{\chi_{22}}{2\eta\chi_{12}} & \Omega_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Lambda_3 & -\Omega_3 & \Lambda_3 & 0 & 0 \\ \Lambda_1 & -\Omega_1 & 0 & 0 & 0 & 0 & 0 & \Lambda_1+\frac{Z}{2} & -\Omega_1 \\ \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 & 0 & 0 & 0 & 0 & 0 & \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1+\frac{1}{2Z} \end{bmatrix} \quad (61a)$$

$$\mathbf{I} = \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{M}_1 \\ \mathbf{J}_2 \\ \mathbf{M}_2 \\ \mathbf{J}_{22} \\ \mathbf{M}_{22} \\ \mathbf{J}_3 \\ \mathbf{J}_4 \\ \mathbf{M}_4 \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} \mathbf{E}^i(S_1) \\ \mathbf{H}^i(S_1) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \mathbf{E}^i(S_4) \\ \mathbf{H}^i(S_4) \end{bmatrix} \quad (61b)$$

Problem 7

In the second problem S_1 is a dielectric, S_3 and S_4 are PEC surfaces.

In this case fields can be as

$$\theta(\mathbf{r})\mathbf{E}_1(\mathbf{r}) = \mathbf{E}^i - \Lambda_1(\mathbf{J}_1 + \mathbf{J}_4) + \Omega_1\mathbf{M}_1 \quad (62a)$$

$$\theta(\mathbf{r})\mathbf{H}_1(\mathbf{r}) = \mathbf{H}^i - \Omega_1(\mathbf{J}_1 + \mathbf{J}_4) + -\frac{1}{\eta_1^2}\Lambda_1\mathbf{M}_1 \quad (62b)$$

$$\theta(\mathbf{r})\mathbf{E}_2(\mathbf{r}) = -\Lambda_2(\mathbf{J}_3 - \mathbf{J}_1) - \Omega_2\mathbf{M}_1 \quad (62c)$$

$$\theta(\mathbf{r})\mathbf{H}_2(\mathbf{r}) = -\Omega_2(\mathbf{J}_3 - \mathbf{J}_1) + \frac{1}{\eta_2^2}\Lambda_2\mathbf{M}_1 \quad (62d)$$

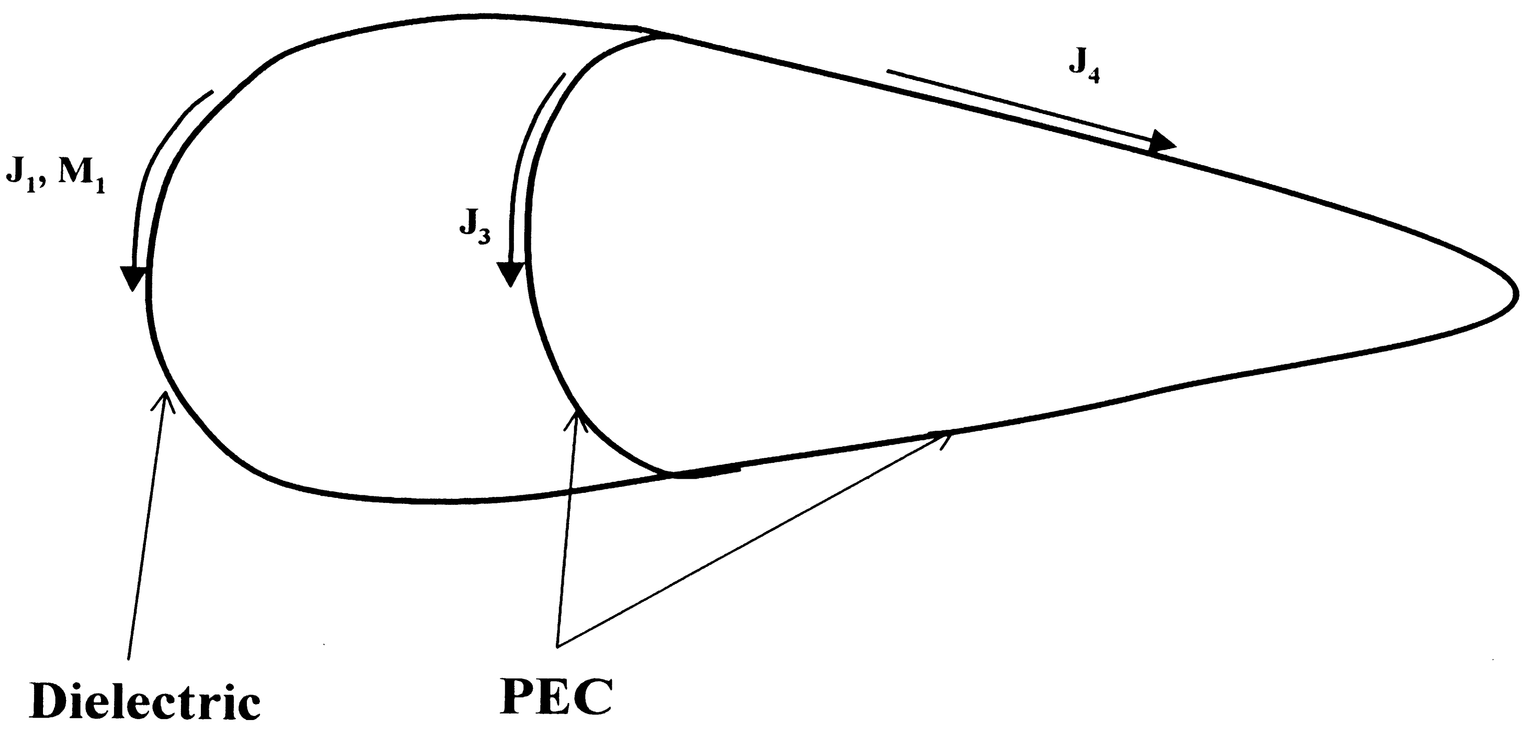


Fig.2.14 Geometry for Problem-7

boundary conditions on the surfaces $S_1 - S_4$ can be given as

$$On S_1 : \mathbf{n} \times [\mathbf{E_1} - \mathbf{E_2}] = 0 \quad ; \quad \mathbf{n} \times [\mathbf{H_1} - \mathbf{H_2}] = 0 \tag{63a}$$

$$On S_3 : \mathbf{n} \times \mathbf{E_3} = 0 (EFIE) \quad ; \quad \mathbf{n} \times \mathbf{H_3} = \mathbf{J}_3 (MFIE) \tag{63b}$$

$$On S_4 : \mathbf{n} \times \mathbf{E_1} = 0 (EFIE) \quad ; \quad \mathbf{n} \times \mathbf{H_1} = \mathbf{J}_4 (MFIE) \tag{63c}$$

Using $(63a - c)$ in $(62a - d)$, and after some straight forward manipulations we find the surface integral equation as

$$\mathbf{Z} = \begin{bmatrix} \Lambda_1 + \Lambda_2 & -\Omega_1 - \Omega_2 & -\Lambda_2 & \Lambda_1 \\ \Omega_1 + \Omega_2 & \frac{1}{\eta_1^2}\Lambda_1 + \frac{1}{\eta_2^2}\Lambda_2 & -\Omega_2 & \Omega_1 \\ -\Lambda_2 & \Omega_2 & \Lambda_2 & 0 \\ \Lambda_1 & -\Omega_1 & 0 & \Lambda_1 \end{bmatrix} \tag{64a}$$

$$\mathbf{I} = \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{M}_1 \\ \mathbf{J}_3 \\ \mathbf{J}_4 \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} \mathbf{E}^i(S_1) \\ \mathbf{H}^i(S_1) \\ 0 \\ \mathbf{E}^i(S_4) \end{bmatrix} \tag{64b}$$

Problem 8

In the eight problem S_1 is a dielectric, S_3 is a PEC and S_4 is an impedance surface.

In this case fields can be as

$$\theta(\mathbf{r})\mathbf{E}_1(\mathbf{r}) = \mathbf{E}^i - \Lambda_1(\mathbf{J}_1 + \mathbf{J}_4) + \Omega_1(\mathbf{M}_1 + \mathbf{M}_4) \tag{65a}$$

$$\theta(\mathbf{r})\mathbf{H}_1(\mathbf{r}) = \mathbf{H}^i - \Omega_1(\mathbf{J}_1 + \mathbf{J}_4) + -\frac{1}{\eta_1^2}\Lambda_1\mathbf{M}_1 \tag{65b}$$

$$\theta(\mathbf{r})\mathbf{E}_2(\mathbf{r}) = -\Lambda_2(\mathbf{J}_3 - \mathbf{J}_1) - \Omega_2\mathbf{M}_1 \tag{65c}$$

$$\theta(\mathbf{r})\mathbf{H}_2(\mathbf{r}) = -\Omega_2(\mathbf{J}_3 - \mathbf{J}_1) + \frac{1}{\eta_2^2}\Lambda_2\mathbf{M}_1 \tag{65d}$$

boundary conditions on the surfaces $S_1 - S_4$ can be given as

$$On S_1 : \mathbf{n} \times [\mathbf{E_1} - \mathbf{E_2}] = 0 \quad ; \quad \mathbf{n} \times [\mathbf{H_1} - \mathbf{H_2}] = 0 \tag{66a}$$

$$On S_3 : \mathbf{n} \times \mathbf{E_3} = 0 (EFIE) \quad ; \quad \mathbf{n} \times \mathbf{H_3} = \mathbf{J}_3 (MFIE) \tag{66b}$$

$$On S_4 : \mathbf{n} \times \mathbf{E_1} = Z\mathbf{n} \times \mathbf{n} \times \mathbf{H_1} \tag{66c}$$

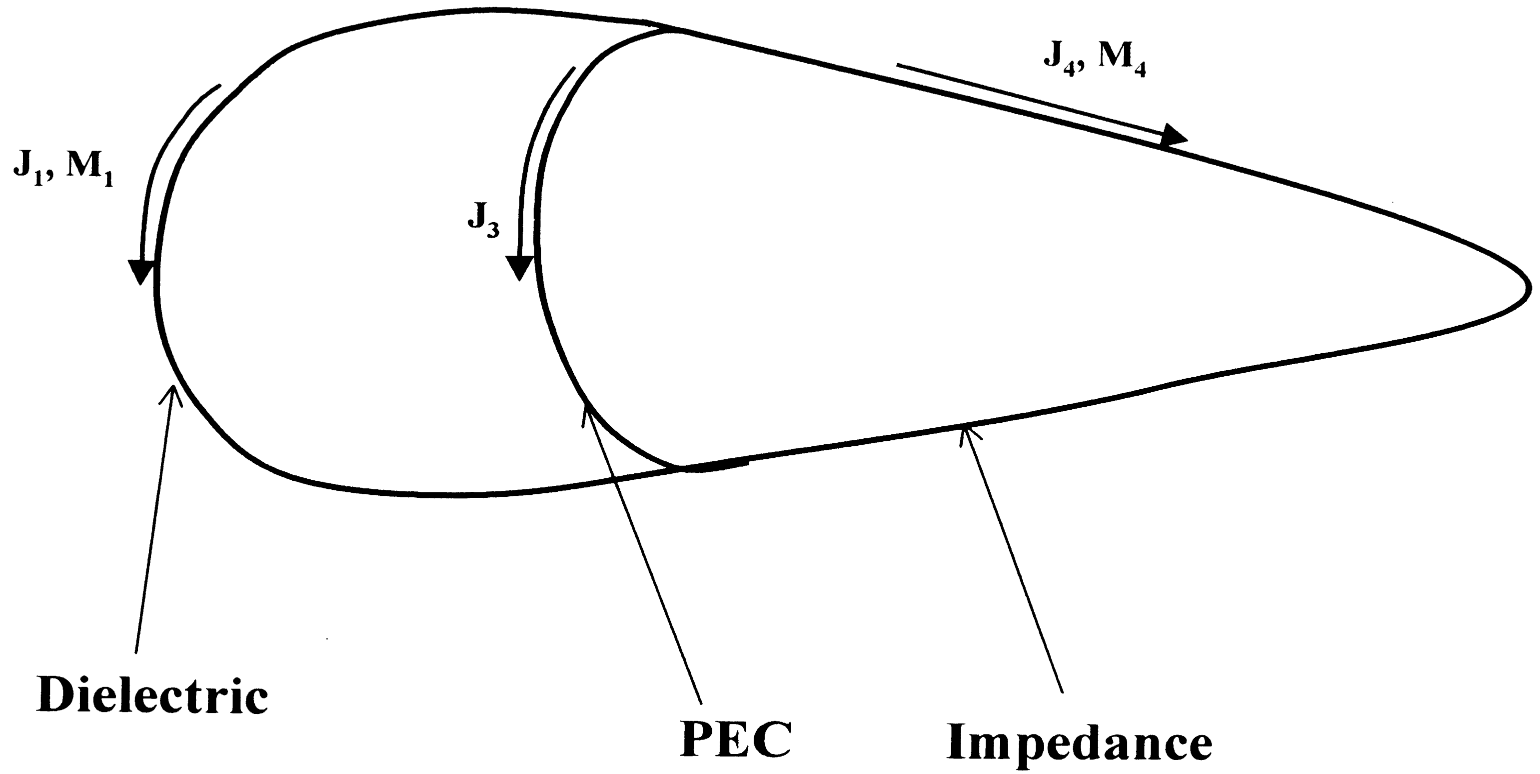


Fig.2.14 Geometry for Problem-8

Using $(66a-c)$ in $(65a-d)$, and after some straight forward manipulations we find the surface integral equation as

$$\mathbf{Z} = \begin{bmatrix} \Lambda_1+\Lambda_2 & -\Omega_1-\Omega_2 & -\Lambda_2 & \Lambda_1 & -\Omega_1 \\ \Omega_1+\Omega_2 & \frac{1}{\eta_1^2}\Lambda_1+\frac{1}{\eta_2^2}\Lambda_2 & -\Omega_2 & \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 \\ -\Lambda_2 & \Omega_2 & \Lambda_2 & 0 & 0 \\ \Lambda_1 & -\Omega_1 & 0 & \Lambda_1+\frac{Z}{2} & -\Omega_1 \\ \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 & 0 & \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1+\frac{1}{2Z} \end{bmatrix} \tag{67a}$$

$$\mathbf{I} = \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{M}_1 \\ \mathbf{J}_3 \\ \mathbf{J}_4 \\ \mathbf{M}_4 \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} \mathbf{E}^i(S_1) \\ \mathbf{H}^i(S_1) \\ 0 \\ \mathbf{E}^i(S_4) \\ \mathbf{H}^i(S_4) \end{bmatrix} \tag{67b}$$

Problem 9

In the eight problem S_2 is a resistive, S_3 and S_4 are PEC surfaces.

In this case fields can be as

$$\theta(\mathbf{r})\mathbf{E}_1(\mathbf{r}) = \mathbf{E}^i - \Lambda_1(\mathbf{J}_2+\mathbf{J}_4) + \Omega_1\mathbf{M}_2 \tag{68a}$$

$$\theta(\mathbf{r})\mathbf{H}_1(\mathbf{r}) = \mathbf{H}^i - \Omega_1(\mathbf{J}_2+\mathbf{J}_4) + -\frac{1}{\eta_1^2}\Lambda_1\mathbf{M}_2 \tag{68b}$$

$$\theta(\mathbf{r})\mathbf{E}_2(\mathbf{r}) = -\Lambda_2(\mathbf{J}_3+\mathbf{J}_{22}) - \Omega_2\mathbf{M}_2 \tag{68c}$$

$$\theta(\mathbf{r})\mathbf{H}_2(\mathbf{r}) = -\Omega_2(\mathbf{J}_3+\mathbf{J}_{22}) + \frac{1}{\eta_2^2}\Omega_2\mathbf{M}_2 \tag{68d}$$

boundary conditions on the surfaces $S_1 - S_4$ can be given as

$$OnS_2 : \mathbf{n}\times[\mathbf{E_2}-\mathbf{E_3}] = 0 \quad ; \quad \mathbf{n}\times[\mathbf{E_2}+\mathbf{E_3}] = 2\eta_1 R\mathbf{n}\times\mathbf{n}\times[\mathbf{H_2}-\mathbf{H_3}] \tag{69a}$$

$$OnS_3 : \mathbf{n}\times\mathbf{E_3} = 0(EFIE) \quad ; \quad \mathbf{n}\times\mathbf{H_3} = \mathbf{J}_3(MFIE) \tag{69b}$$

$$OnS_4 : \mathbf{n}\times\mathbf{E_1} = 0\ (EFIE) \quad ; \quad \mathbf{n}\times\mathbf{H_1} = \mathbf{J}_4\ (MFIE) \tag{69c}$$

Using $(69a-c)$ in $(68a-d)$, and after some straight forward manipulations we find the surface integral equation as

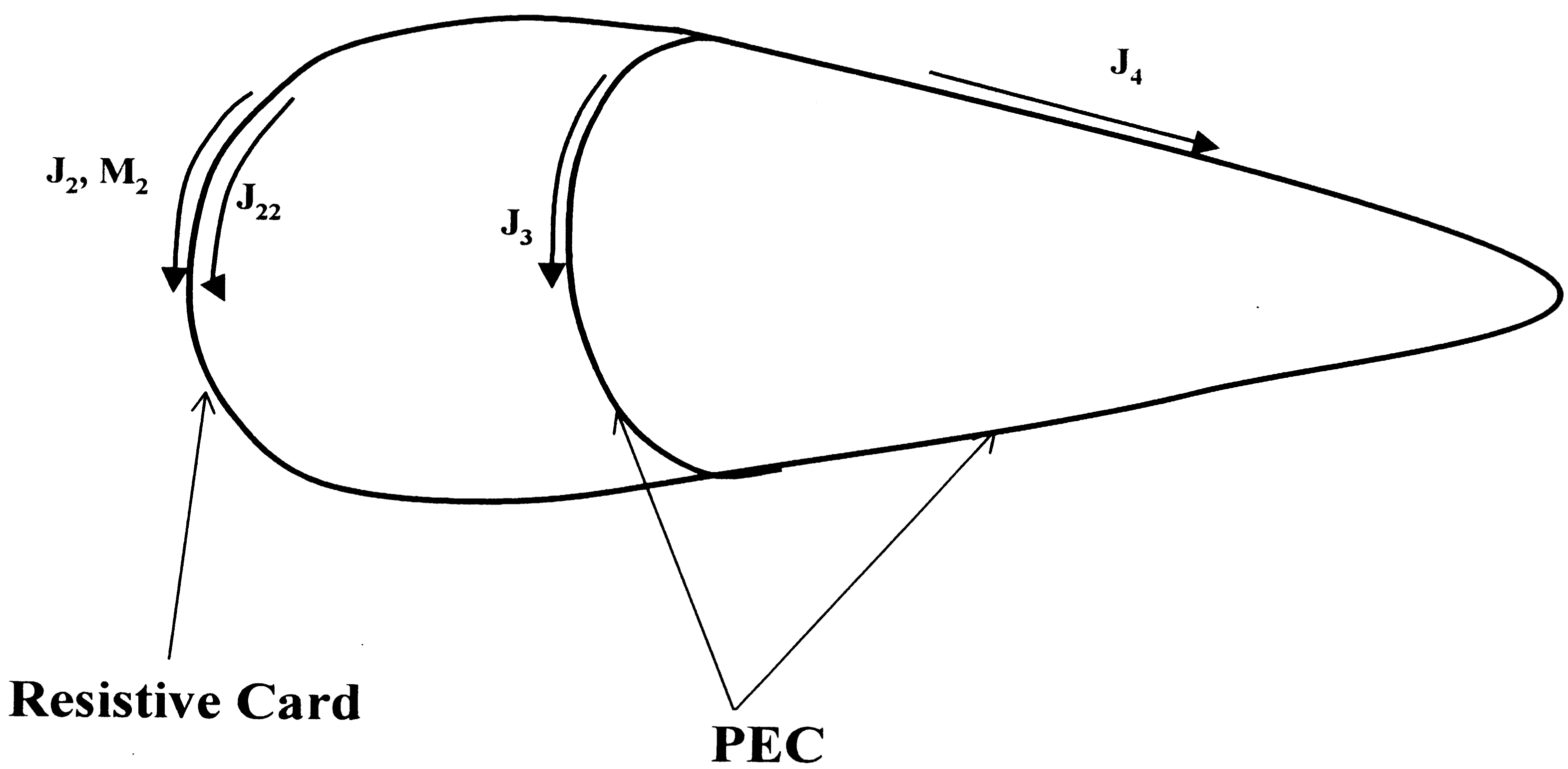


Fig.2.14 Geometry for Problem-9

$$\mathbf{Z} = \begin{bmatrix} \Lambda_1 + \frac{\eta_1 R}{2} & -\Omega_1 & \frac{-\eta_1 R}{2} & 0 & \Lambda_1 \\ \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 + \frac{1}{\eta_2^2\Lambda_2} + \frac{R}{2\eta_1} & -\Omega_2 & -\Omega_2 & \Omega_1 \\ \frac{\eta_1 R}{2} & \Omega_2 & \Lambda_2 + \frac{\eta_1 R}{2} & \Lambda_2 & 0 \\ 0 & \Omega_2 & \Lambda_2 & \Lambda_2 & 0 \\ \Lambda_1 & -\Omega_1 & 0 & 0 & \Lambda_1 \end{bmatrix} \tag{70a}$$

$$\mathbf{I} = \begin{bmatrix} \mathbf{J}_2 \\ \mathbf{M}_2 \\ \mathbf{J}_{22} \\ \mathbf{J}_3 \\ \mathbf{J}_4 \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} \mathbf{E}^i(S_1) \\ \mathbf{H}^i(S_1) \\ 0 \\ 0 \\ \mathbf{E}^i(S_4) \end{bmatrix} \tag{70b}$$

Problem 10

In the eight problem S_2 is a resistive, S_3 is a PEC and S_4 is an impedance surface.

In this case fields can be as

$$\theta(\mathbf{r})\mathbf{E}_1(\mathbf{r}) = \mathbf{E}^i - \Lambda_1(\mathbf{J}_2 + \mathbf{J}_4) + \Omega_1(\mathbf{M}_2 + \mathbf{M}_4) \tag{71a}$$

$$\theta(\mathbf{r})\mathbf{H}_1(\mathbf{r}) = \mathbf{H}^i - \Omega_1(\mathbf{J}_2 + \mathbf{J}_4) + -\frac{1}{\eta_1^2}\Lambda_1(\mathbf{M}_2 + \mathbf{M}_4) \tag{71b}$$

$$\theta(\mathbf{r})\mathbf{E}_2(\mathbf{r}) = -\Lambda_2(\mathbf{J}_3 + \mathbf{J}_{22}) - \Omega_2\mathbf{M}_2 \tag{71c}$$

$$\theta(\mathbf{r})\mathbf{H}_2(\mathbf{r}) = -\Omega_2(\mathbf{J}_3 + \mathbf{J}_{22}) + \frac{1}{\eta_2^2}\Omega_2\mathbf{M}_2 \tag{72d}$$

boundary conditions on the surfaces $S_1 - S_4$ can be given as

$$OnS_2 : \mathbf{n} \times [\mathbf{E_2} - \mathbf{E_3}] = 0 \quad ; \quad \mathbf{n} \times [\mathbf{E_2} + \mathbf{E_3}] = 2\eta_1 R\mathbf{n} \times \mathbf{n} \times [\mathbf{H_2} - \mathbf{H_3}] \tag{73a}$$

$$OnS_3 : \mathbf{n} \times \mathbf{E_3} = 0(EFIE) \quad ; \quad \mathbf{n} \times \mathbf{H_3} = \mathbf{J}_3(MFIE) \tag{73b}$$

$$OnS_4 : \mathbf{n} \times \mathbf{E_1} = Z\mathbf{n} \times \mathbf{n} \times \mathbf{H_1} \tag{73c}$$

Using $(73a - c)$ in $(72a - d)$, and after some straight forward manipulations we find the surface integral equation as

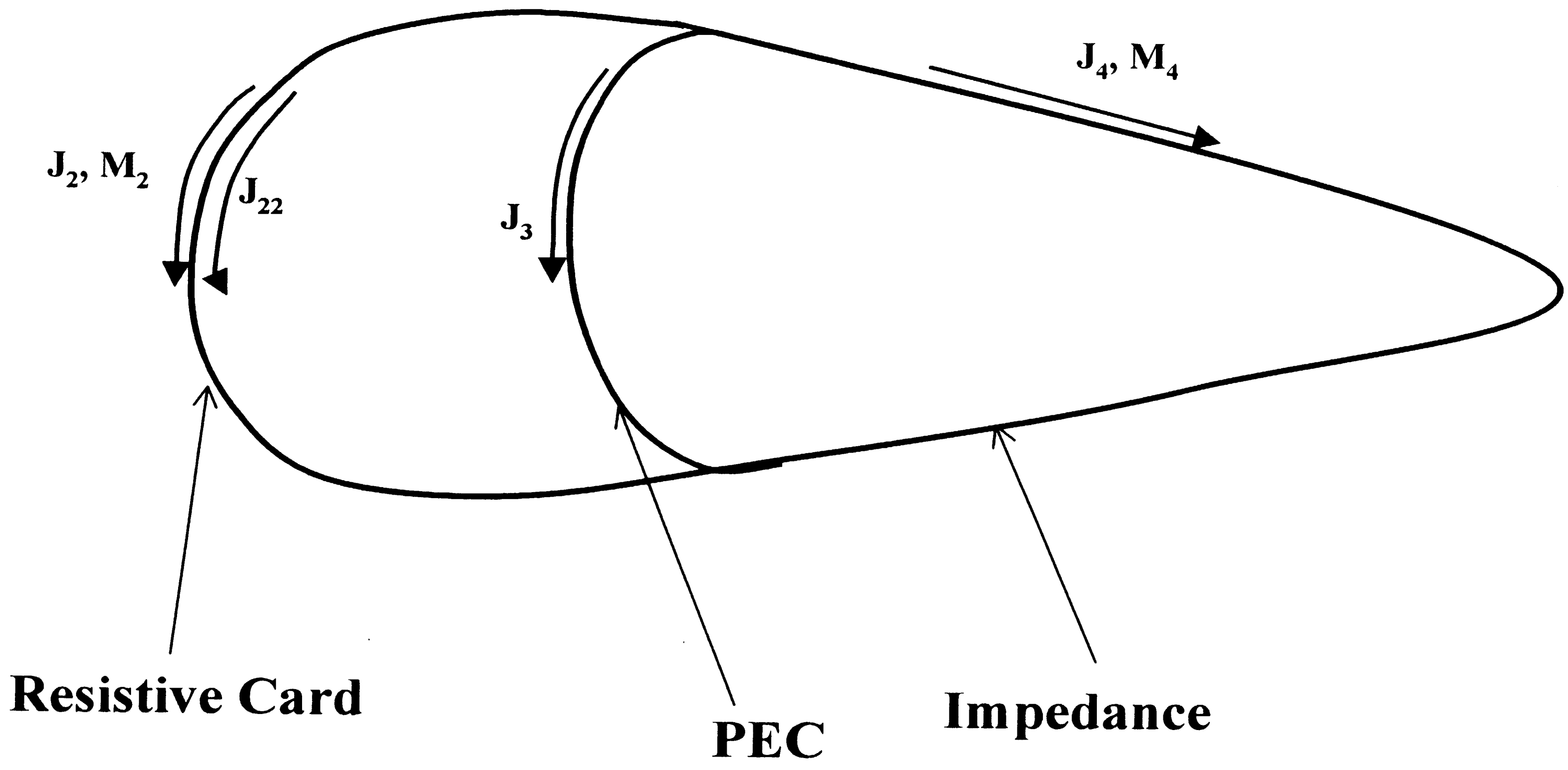


Fig.2.15 Geometry for Problem-10

$$
\mathbf{Z} = \begin{bmatrix}
\Lambda_1 + \frac{\eta_1 R}{2} & -\Omega_1 & \frac{-\eta_1 R}{2} & 0 & \Lambda_1 & -\Omega_1 \\
\Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 + \frac{1}{\eta_2^2\Lambda_2} + \frac{R}{2\eta_1} & -\Omega_2 & -\Omega_2 & \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 \\
\frac{\eta_1 R}{2} & \Omega_2 & \Lambda_2 + \frac{\eta_1 R}{2} & \Lambda_2 & 0 & 0 \\
0 & \Omega_2 & \Lambda_2 & \Lambda_2 & 0 & 0 \\
\Lambda_1 & -\Omega_1 & 0 & 0 & \Lambda_1 + \frac{Z}{2} & -\Omega_1 \\
\Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 & 0 & 0 & \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 + \frac{1}{2Z}
\end{bmatrix} \tag{74a}
$$

$$
\mathbf{I} = \begin{bmatrix} \mathbf{J}_2 \\ \mathbf{M}_2 \\ \mathbf{J}_{22} \\ \mathbf{J}_3 \\ \mathbf{J}_4 \\ \mathbf{M}_4 \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} \mathbf{E}^i(S_1) \\ \mathbf{H}^i(S_1) \\ 0 \\ 0 \\ \mathbf{E}^i(S_4) \\ \mathbf{H}^i(S_4) \end{bmatrix} \tag{74b}
$$

Problem 11

In the eight problem S_2 is a CA boundary, S_3 and S_4 are PEC surfaces.

In this case fields can be as

$$\theta(\mathbf{r})\mathbf{E}_1(\mathbf{r}) = \mathbf{E}^i - \Lambda_1(\mathbf{J}_2 + \mathbf{J}_4) + \Omega_1\mathbf{M}_2 \tag{75a}$$

$$\theta(\mathbf{r})\mathbf{H}_1(\mathbf{r}) = \mathbf{H}^i - \Omega_1(\mathbf{J}_2 + \mathbf{J}_4) + -\frac{1}{\eta_1^2}\Lambda_1\mathbf{M}_2 \tag{75b}$$

$$\theta(\mathbf{r})\mathbf{E}_2(\mathbf{r}) = -\Lambda_2(\mathbf{J}_3 + \mathbf{J}_{22}) - \Omega_2\mathbf{M}_2 \tag{75c}$$

$$\theta(\mathbf{r})\mathbf{H}_2(\mathbf{r}) = -\Omega_2(\mathbf{J}_3 + \mathbf{J}_{22}) - \frac{1}{\eta_2^2}\Omega_2\mathbf{M}_{22} \tag{75d}$$

boundary conditions on the surfaces $S_1 - S_4$ can be given as

$$On S_2 : \begin{bmatrix} \mathbf{E}^- \\ \eta\mathbf{n}\times\mathbf{H}^- \end{bmatrix} = \begin{bmatrix} \chi_{11} & \chi_{12} \\ \chi_{21} & \chi_{22} \end{bmatrix} \begin{bmatrix} \mathbf{E}^+ \\ \eta\mathbf{n}\times\mathbf{H}^+ \end{bmatrix} \tag{76a}$$

$$On S_3 : \mathbf{n}\times\mathbf{E_3} = 0(EFIE) \quad ; \quad \mathbf{n}\times\mathbf{H_3} = \mathbf{J}_3(MFIE) \tag{76b}$$

$$On S_4 : \mathbf{n}\times\mathbf{E_1} = 0(EFIE) \quad ; \quad \mathbf{n}\times\mathbf{H_1} = \mathbf{J}_4(MFIE) \tag{76c}$$

Using $(76a - c)$ in $(75a - d)$, and after some straight forward manipulations we find the surface integral equation as

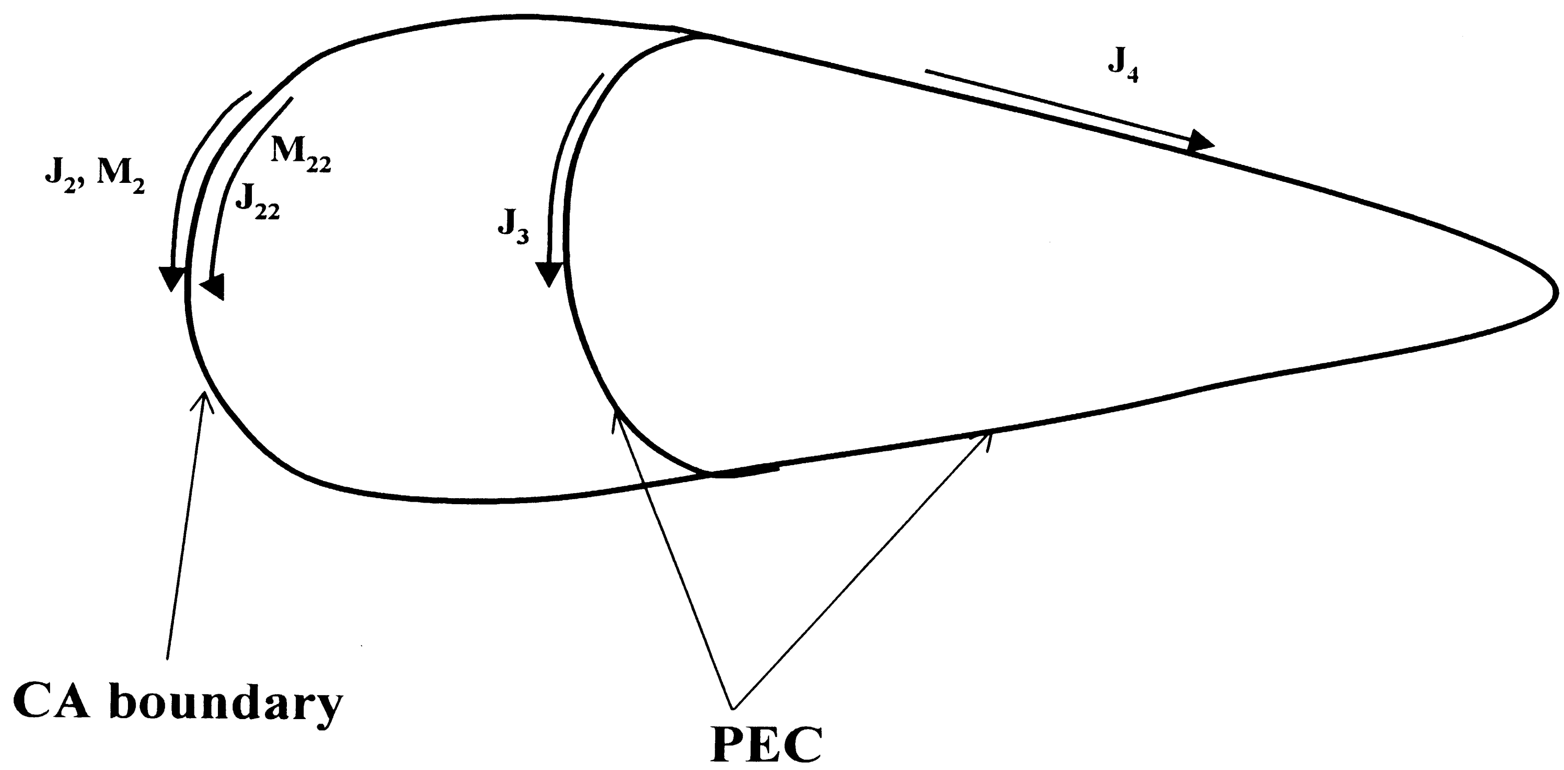


Fig.2.16 Geometry for Problem-11

$$\mathbf{Z}=\begin{bmatrix} \Lambda_1-\frac{\eta\chi_{22}}{2\chi_{21}} & -\Omega_1 & -\frac{\eta}{2\chi_{21}} & 0 & 0 & \Lambda_1 \\ \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1+\frac{\chi_{11}}{2\eta\chi_{12}} & 0 & -\frac{1}{2\eta\chi_{12}} & 0 & \Omega_1 \\ \frac{\eta}{2}(\chi_{12}-\frac{\chi_{11}\chi_{22}}{\chi_{21}}) & 0 & \Lambda_2-\frac{\eta\chi_{11}}{2\chi_{21}} & -\Omega_2 & \Lambda_2 & 0 \\ 0 & \frac{1}{2\eta}(\chi_{21}-\frac{\chi_{11}\chi_{22}}{\chi_{12}}) & \Omega_2 & \frac{1}{\eta_2^2}\Lambda_2+\frac{\chi_{22}}{2\eta\chi_{12}} & \Omega_2 & 0 \\ 0 & 0 & \Lambda_2 & -\Omega_2 & \Lambda_2 & 0 \\ \Lambda_1 & -\Omega_1 & 0 & 0 & 0 & \Lambda_1 \end{bmatrix} \quad (77a)$$

$$\mathbf{I}=\begin{bmatrix} \mathbf{J}_2 \\ \mathbf{M}_2 \\ \mathbf{J}_{22} \\ \mathbf{M}_{22} \\ \mathbf{J}_3 \\ \mathbf{J}_4 \end{bmatrix} \qquad \mathbf{V}=\begin{bmatrix} \mathbf{E}^i(S_1) \\ \mathbf{H}^i(S_1) \\ 0 \\ 0 \\ 0 \\ \mathbf{E}^i(S_4) \end{bmatrix} \quad (77b)$$

Problem 12

In the eight problem S_2 is a CA boundary, S_3 is a PEC and S_4 is an Impedance surface.

In this case fields can be as

$$\theta(\mathbf{r})\mathbf{E}_1(\mathbf{r})=\mathbf{E}^i-\Lambda_1(\mathbf{J}_2+\mathbf{J}_4)+\Omega_1(\mathbf{M}_2+\mathbf{M}_4) \quad (78a)$$

$$\theta(\mathbf{r})\mathbf{H}_1(\mathbf{r})=\mathbf{H}^i-\Omega_1(\mathbf{J}_2+\mathbf{J}_4)+-\frac{1}{\eta_1^2}\Lambda_1(\mathbf{M}_2+\mathbf{M}_4 \quad (78b)$$

$$\theta(\mathbf{r})\mathbf{E}_2(\mathbf{r})=-\Lambda_2(\mathbf{J}_3+\mathbf{J}_{22})-\Omega_2\mathbf{M}_{22} \quad (78c)$$

$$\theta(\mathbf{r})\mathbf{H}_2(\mathbf{r})=-\Omega_2(\mathbf{J}_3+\mathbf{J}_{22})-\frac{1}{\eta_2^2}\Omega_2\mathbf{M}_{22} \quad (78d)$$

boundary conditions on the surfaces S_1-S_4 can be given as

$$OnS_2:\begin{bmatrix}\mathbf{E}^-\\ \eta\mathbf{n}\times\mathbf{H}^-\end{bmatrix}=\begin{bmatrix}\chi_{11} & \chi_{12}\\ \chi_{21} & \chi_{22}\end{bmatrix}\begin{bmatrix}\mathbf{E}^+\\ \eta\mathbf{n}\times\mathbf{H}^+\end{bmatrix} \quad (79a)$$

$$OnS_3:\mathbf{n}\times\mathbf{E_3}=0(EFIE)\quad;\quad \mathbf{n}\times\mathbf{H_3}=\mathbf{J}_3(MFIE) \quad (79b)$$

$$OnS_4:\mathbf{n}\times\mathbf{E_1}=Z\mathbf{n}\times\mathbf{n}\times\mathbf{H_1} \quad (79c)$$

Using $(79a-c)$ in $(78a-d)$. and after some straight forward manipulations we find the surface integral equation as

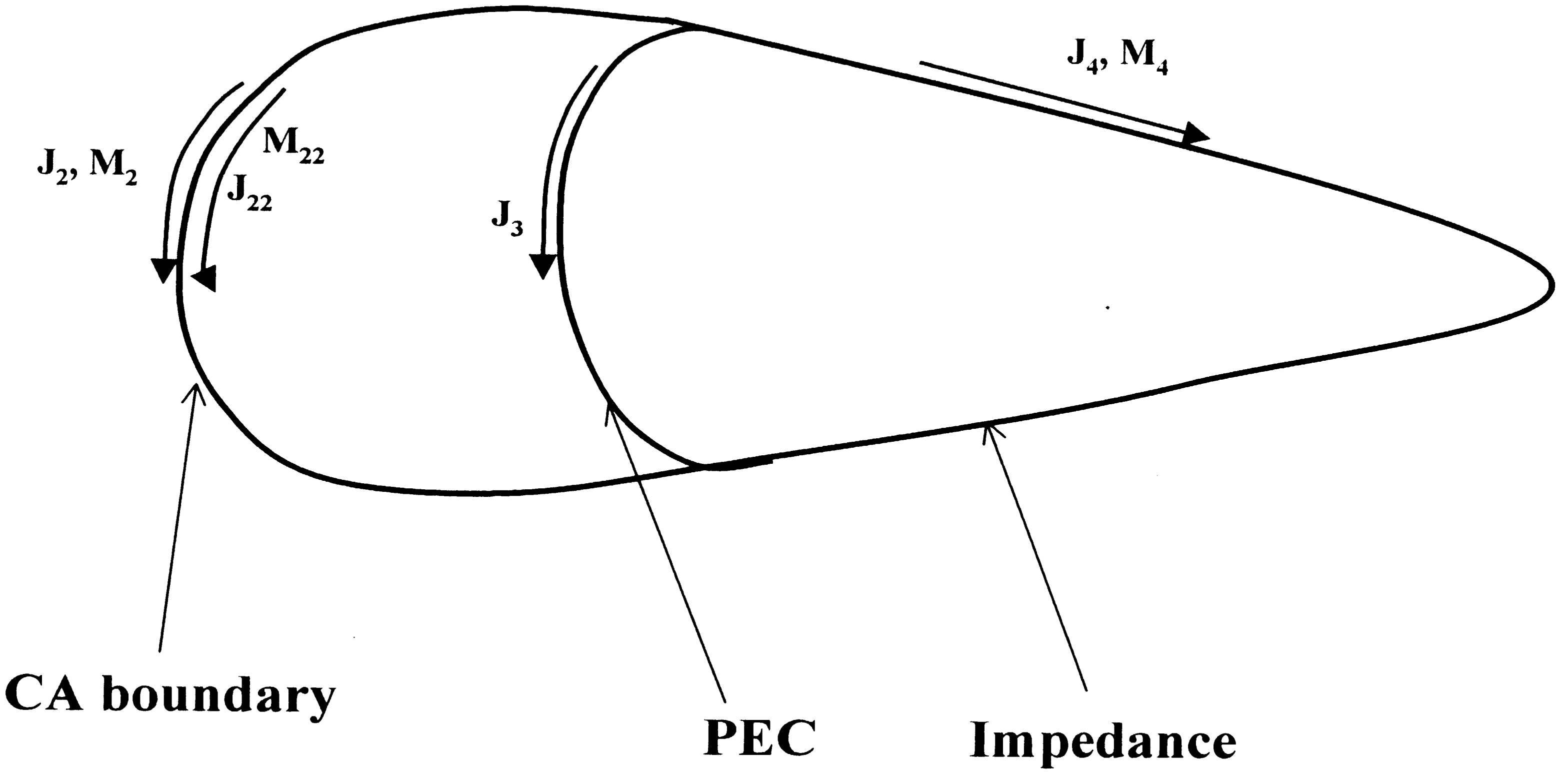


Fig.2.17 Geometry for Problem-12

$$\mathbf{Z} = \begin{bmatrix} \Lambda_1 - \frac{\eta\chi_{22}}{2\chi_{21}} & -\Omega_1 & -\frac{\eta}{2\chi_{21}} & 0 & 0 & \Lambda_1 & -\Omega_1 \\ \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 + \frac{\chi_{11}}{2\eta\chi_{12}} & 0 & -\frac{1}{2\eta\chi_{12}} & 0 & \Omega_1 & -\frac{1}{\eta_1^2}\Lambda_1 \\ \frac{\eta}{2}(\chi_{12} - \frac{\chi_{11}\chi_{22}}{\chi_{21}}) & 0 & \Lambda_2 - \frac{\eta\chi_{11}}{2\chi_{21}} & -\Omega_2 & \Lambda_2 & 0 & 0 \\ 0 & \frac{1}{2\eta}(\chi_{21} - \frac{\chi_{11}\chi_{22}}{\chi_{12}}) & \Omega_2 & \frac{1}{\eta_2^2}\Lambda_2 + \frac{\chi_{22}}{2\eta\chi_{12}} & \Omega_2 & 0 & 0 \\ 0 & 0 & \Lambda_2 & -\Omega_2 & \Lambda_2 & 0 & 0 \\ \Lambda_1 & -\Omega_1 & 0 & 0 & 0 & \Lambda_1 + \frac{Z}{2} & -\Omega_1 \\ \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 & 0 & 0 & 0 & \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 + \frac{1}{2Z} \end{bmatrix} \tag{80a}$$

$$\mathbf{I} = \begin{bmatrix} \mathbf{J}_2 \\ \mathbf{M}_2 \\ \mathbf{J}_{22} \\ \mathbf{M}_{22} \\ \mathbf{J}_3 \\ \mathbf{J}_4 \\ \mathbf{M}_4 \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} \mathbf{E}^i(S_1) \\ \mathbf{H}^i(S_1) \\ 0 \\ 0 \\ 0 \\ \mathbf{E}^i(S_4) \\ \mathbf{H}^i(S_4) \end{bmatrix} \tag{80b}$$

Problem 13

In the thirteenth problem S_3 is a PEC and S_4 is an Impedance surface.

In this case fields can be as

$$\theta(\mathbf{r})\mathbf{E}_1(\mathbf{r}) = \mathbf{E}^i - \Lambda_1(\mathbf{J}_3 + \mathbf{J}_4) + \Omega_1\mathbf{M}_4 \tag{81a}$$

$$\theta(\mathbf{r})\mathbf{H}_1(\mathbf{r}) = \mathbf{H}^i - \Omega_1(\mathbf{J}_3 + \mathbf{J}_4) + -\frac{1}{\eta_1^2}\Lambda_1\mathbf{M}_4 \tag{81b}$$

boundary conditions on the surfaces $S_1 - S_4$ can be given as

$$On S_3 : \mathbf{n} \times \mathbf{E_3} = 0(EFIE) \quad ; \quad \mathbf{n} \times \mathbf{H_3} = \mathbf{J}_3(MFIE) \tag{82a}$$

$$On S_4 : \mathbf{n} \times \mathbf{E_1} = Z\mathbf{n} \times \mathbf{n} \times \mathbf{H_1} \tag{82b}$$

Using $(82a, b)$ in $(81a, b)$, and after some straight forward manipulations we find the surface integral equation as

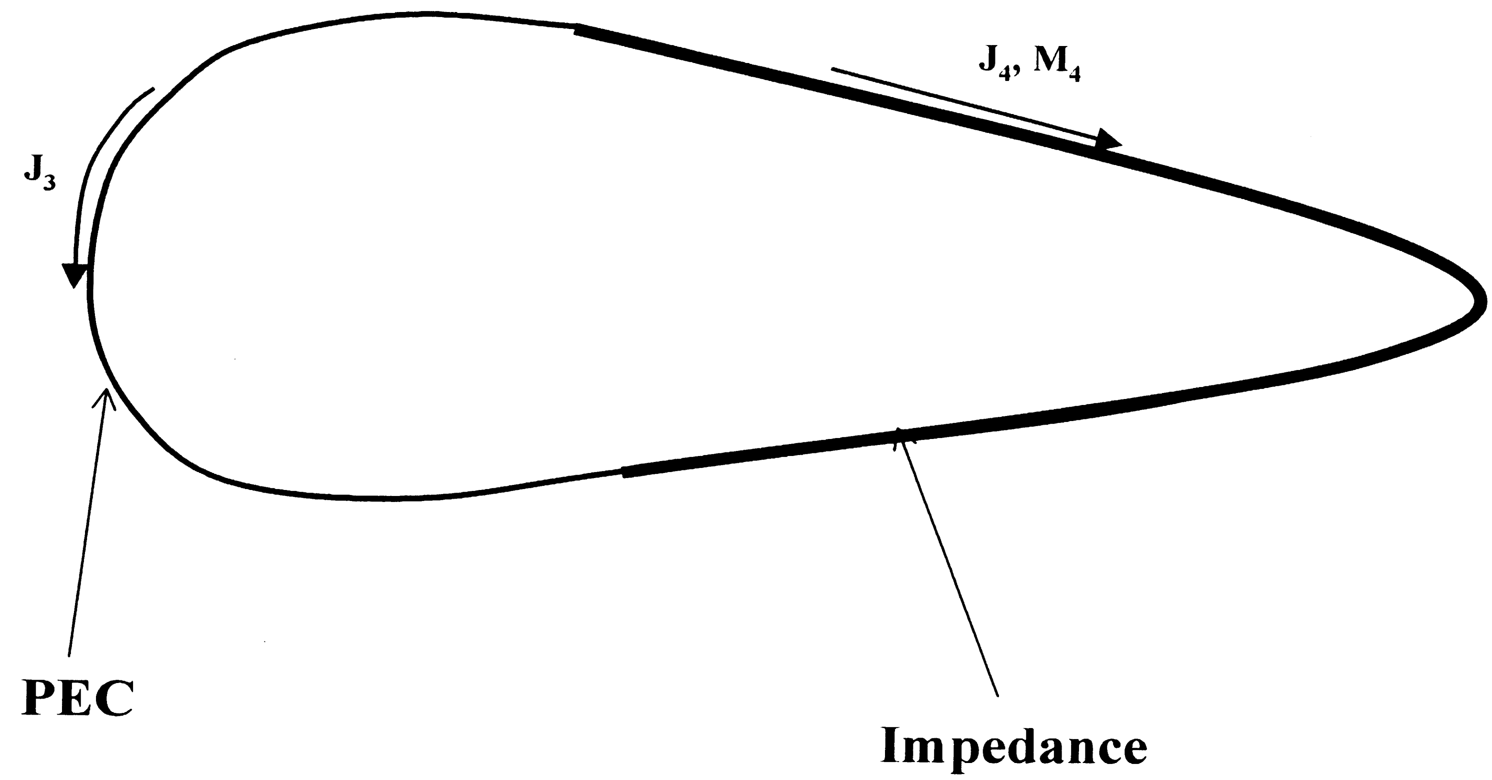


Fig.2.18 Geometry for Problem-13

$$\mathbf{Z} = \begin{bmatrix} \Lambda_1 & \Lambda_1 & -\Omega_1 \\ \Lambda_1 & \Lambda_1 + \frac{Z}{2} & -\Omega_1 \\ \Omega_1 & \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 + \frac{1}{2Z} \end{bmatrix} \tag{83a}$$

$$\mathbf{I} = \begin{bmatrix} \mathbf{J}_3 \\ \mathbf{J}_4 \\ \mathbf{M}_4 \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} \mathbf{E}^i(S_3) \\ \mathbf{E}^i(S_4) \\ \mathbf{H}^i(S_4) \end{bmatrix} \tag{83b}$$

Problem 14

In the thirteenth problem S_3 is an impedance and S_4 is a PEC surface.

In this case fields can be as

$$\theta(\mathbf{r})\mathbf{E}_1(\mathbf{r}) = \mathbf{E}^i - \Lambda_1(\mathbf{J}_3 + \mathbf{J}_4) + \Omega_1\mathbf{M}_3 \tag{84a}$$

$$\theta(\mathbf{r})\mathbf{H}_1(\mathbf{r}) = \mathbf{H}^i - \Omega_1(\mathbf{J}_3 + \mathbf{J}_4) + -\frac{1}{\eta_1^2}\Lambda_1\mathbf{M}_3 \tag{84b}$$

boundary conditions on the surfaces $S_1 - S_4$ can be given as

$$OnS_3 : \mathbf{n} \times \mathbf{E_1} = Z\mathbf{n} \times \mathbf{n} \times \mathbf{H_1} \tag{85a}$$

$$OnS_4 : \mathbf{n} \times \mathbf{E_1} = 0(EFIE) \quad ; \quad \mathbf{n} \times \mathbf{H_1} = \mathbf{J}_4(MFIE) \tag{85b}$$

Using $(85a, b)$ in $(84a, b)$, and after some straight forward manipulations we find the surface integral equation as

$$\mathbf{Z} = \begin{bmatrix} \Lambda_1 + \frac{Z}{2} & -\Omega_1 & \Lambda_1 \\ \Omega_1 & \frac{1}{\eta_1^2}\Lambda_1 + \frac{1}{2Z}\Omega_1 & \\ \Lambda_1 & -\Omega_1 & \Lambda_1 \end{bmatrix} \tag{86a}$$

$$\mathbf{I} = \begin{bmatrix} \mathbf{J}_3 \\ \mathbf{M}_3 \\ \mathbf{J}_4 \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} \mathbf{E}^i(S_3) \\ \mathbf{H}^i(S_3) \\ \mathbf{E}^i(S_4) \end{bmatrix} \tag{86b}$$

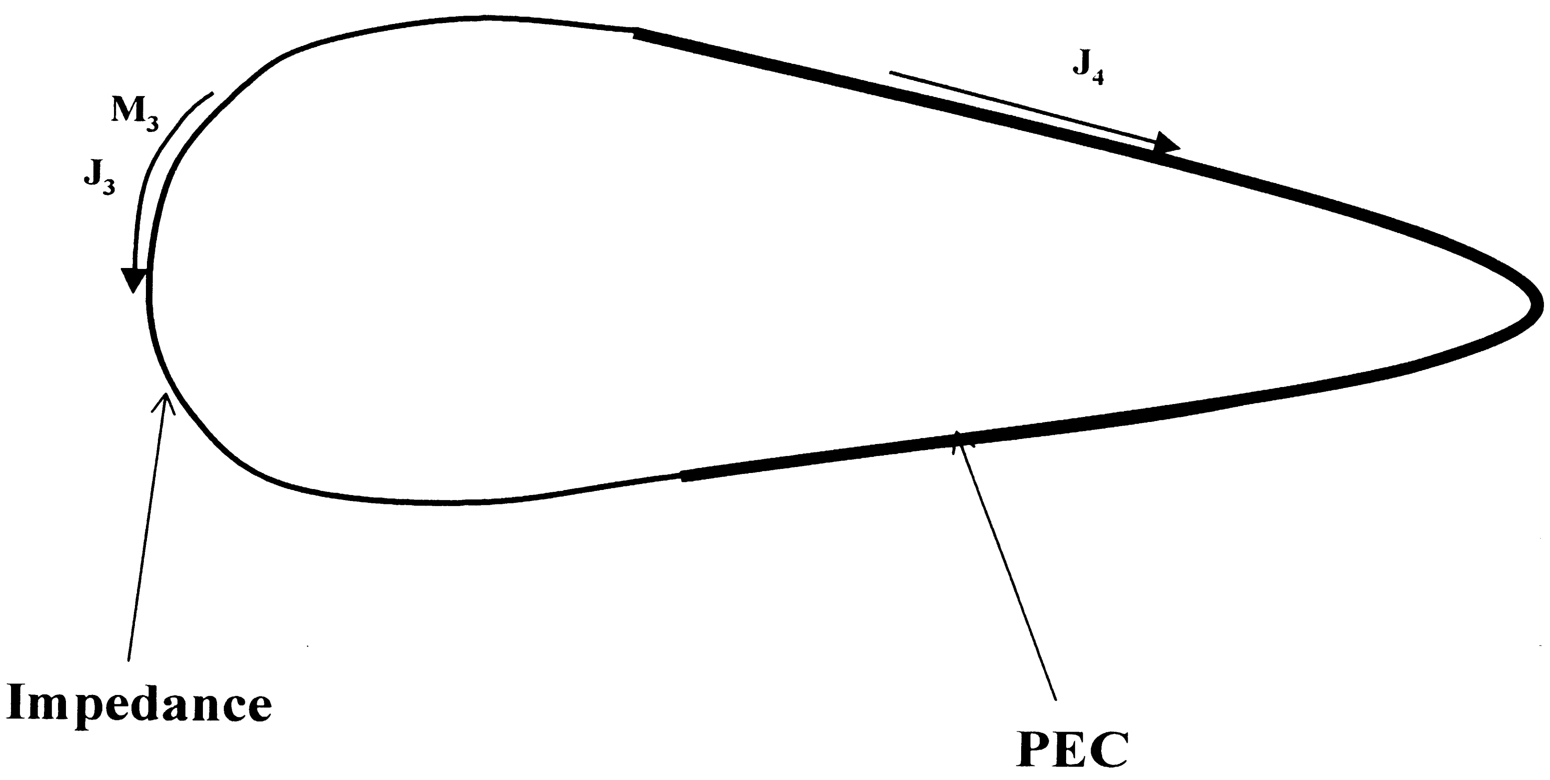


Fig.2.19 Geometry for Problem-14

TEST STRUCTURES (SINGLE SURFACES)

TEST STRUCTURES-I(PEC Box-750 Unknowns)

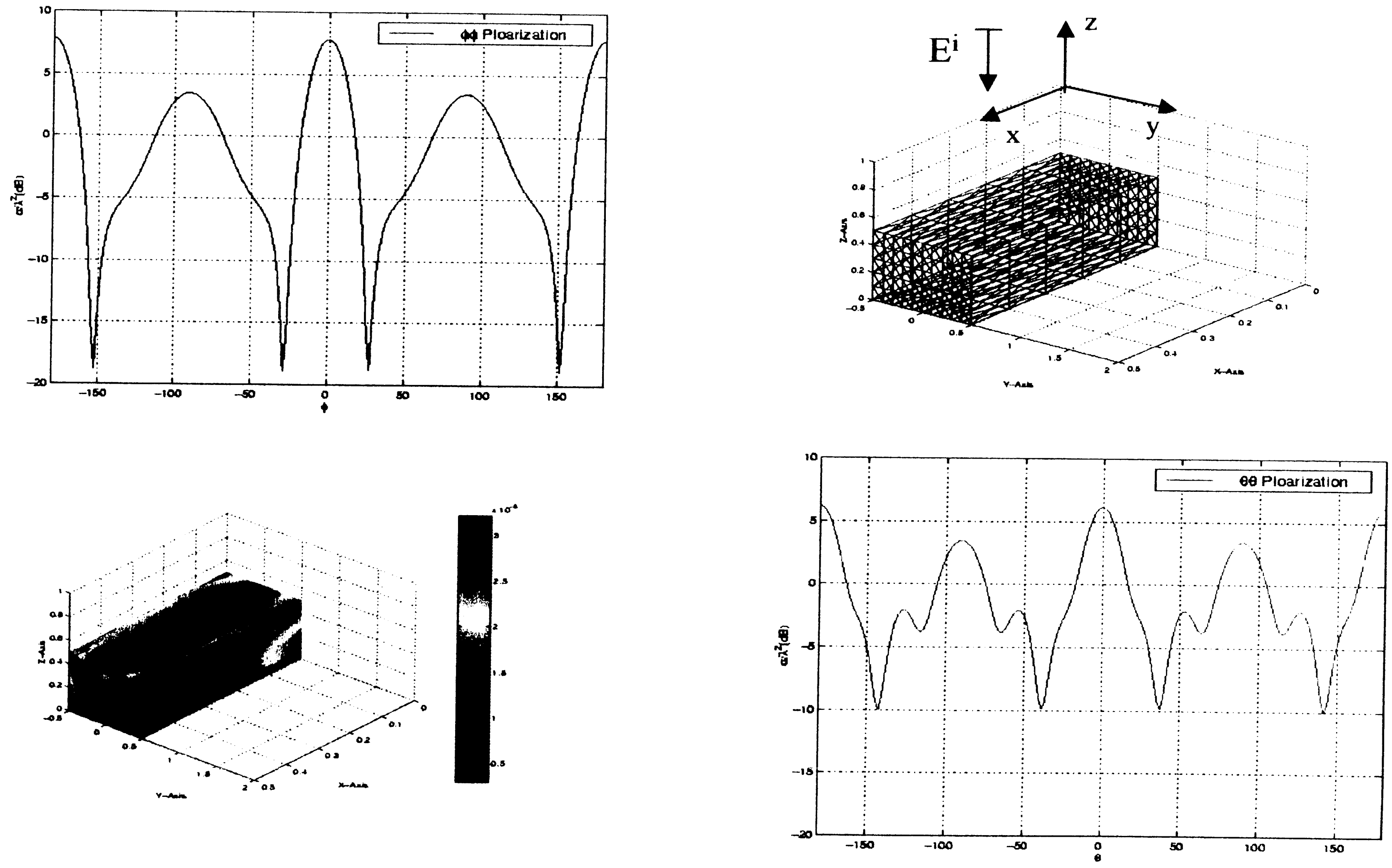


Fig.3.1 Monostatic Radar Cross Section from a Box and Induced Electric Currents

TEST STRUCTURES-II(PEC Cube-450 Unknowns)

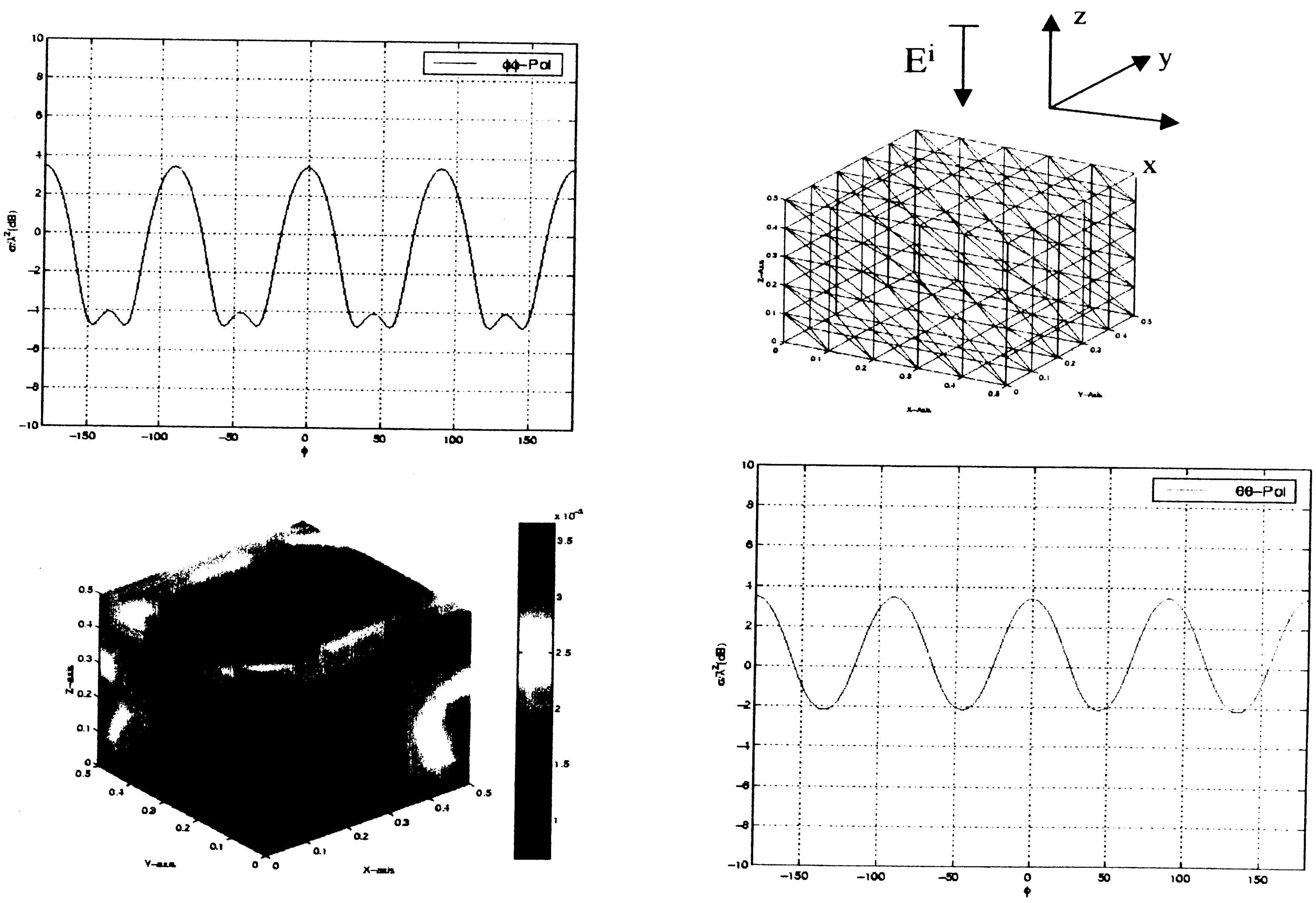


Fig.3.2 Monostatic Radar Cross Section from a Cube and Induced Electric Currents

TEST STRUCTURES-III(PEC Sphere- 540 Unknowns)

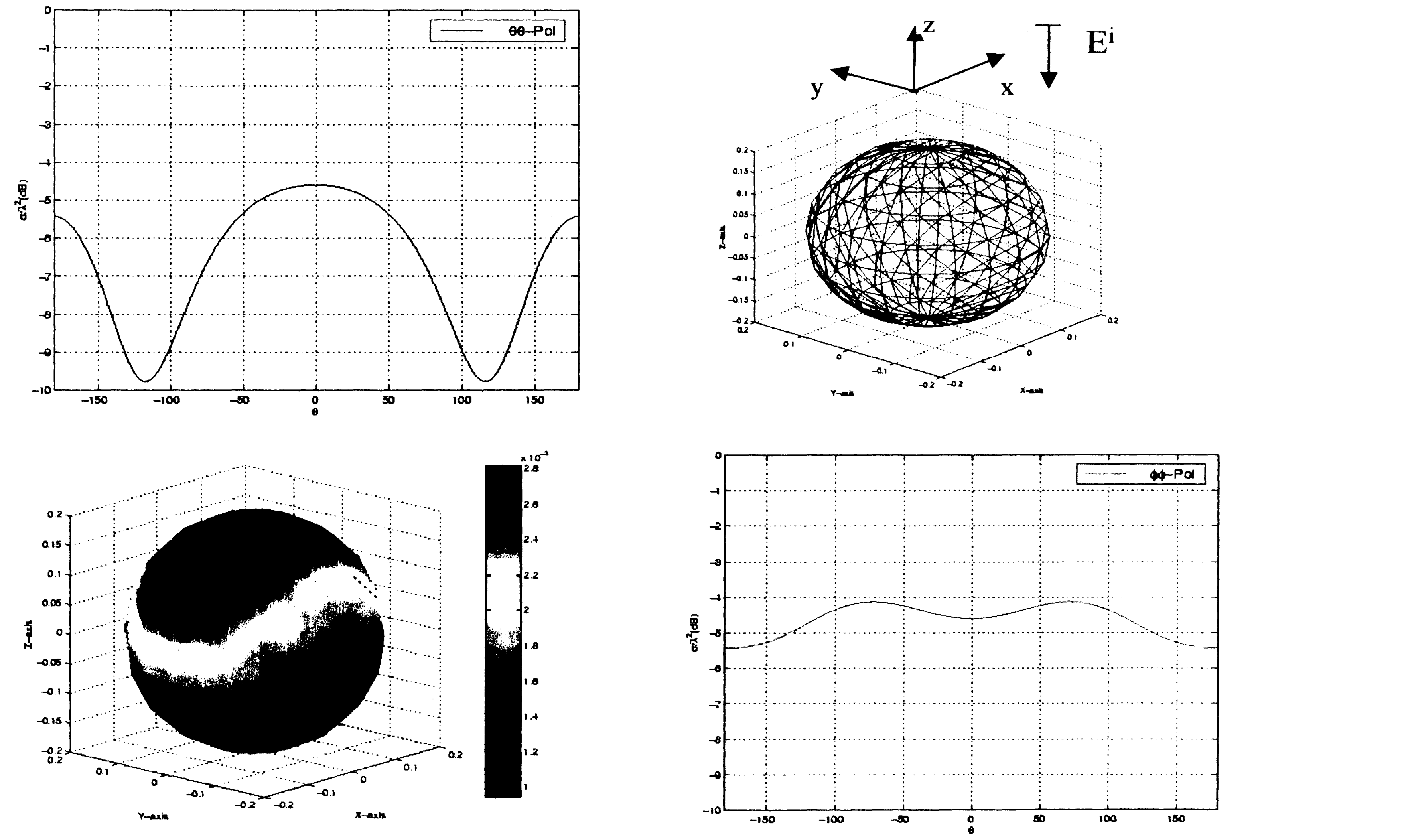


Fig.3.3 Monostatic Radar Cross Section from a Sphere and Induced Electric Currents

TEST STRUCTURES-IV(PEC Plate- 1240 Unknowns)

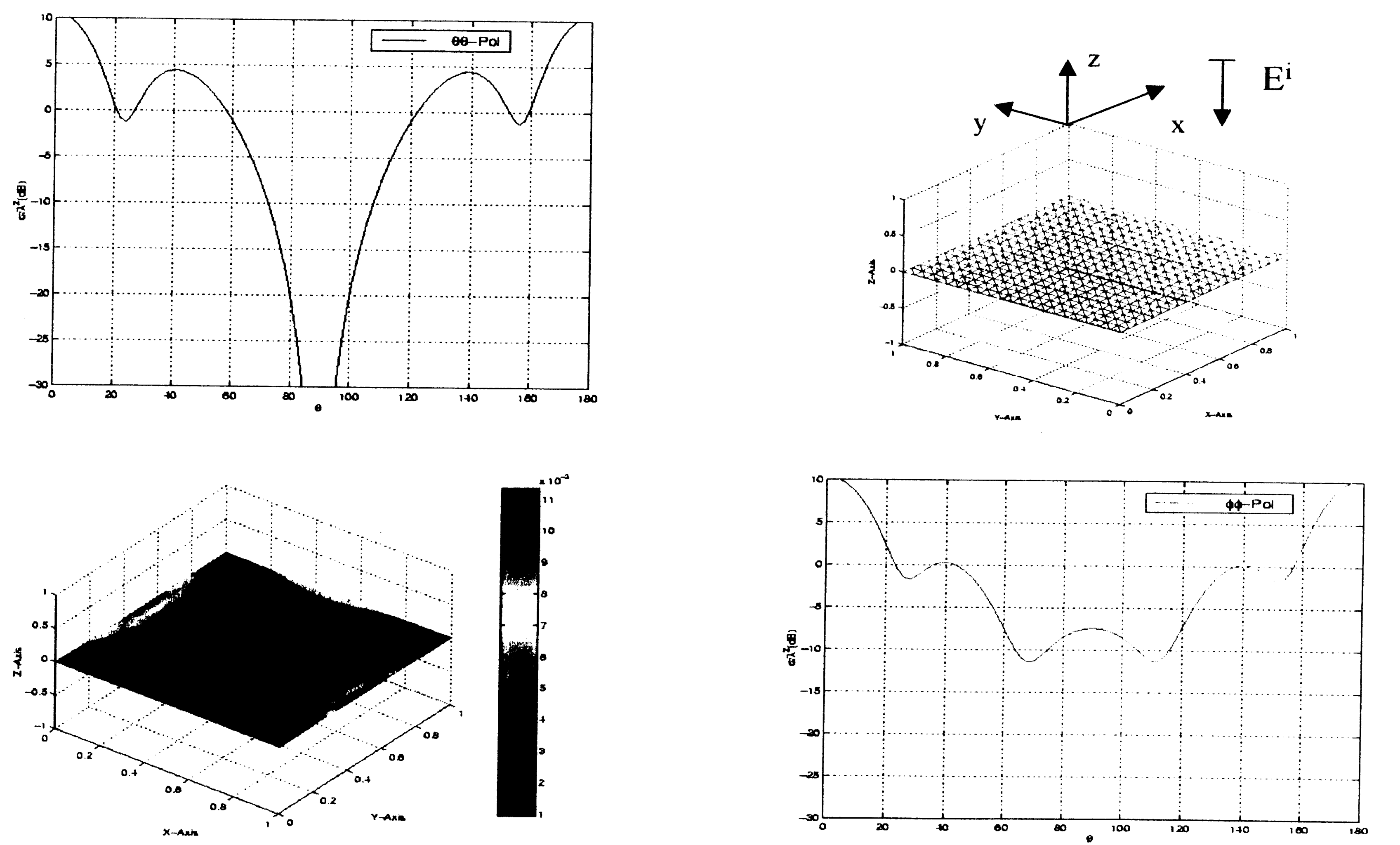


Fig.3.4 Monostatic Radar Cross Section from a plate and Induced Electric Currents

TEST STRUCTURES-V(NASA Almond-2GHz-2130 Unknowns)

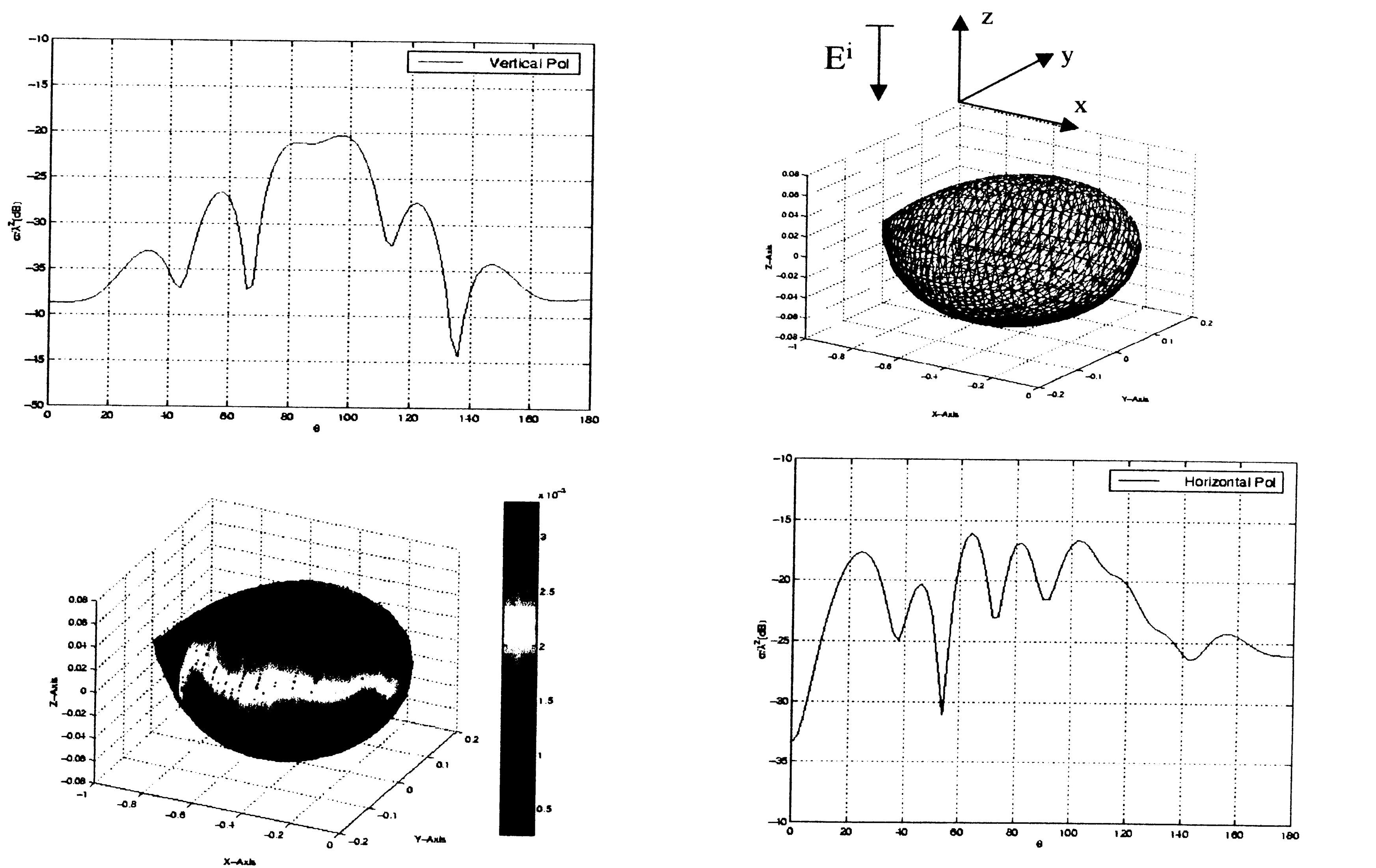


Fig.3.5 Monostatic Radar Cross Section from 14” Nasa Almond and Induced Electric Currents

TEST STRUCTURES-V(Dielectric Sphere(ε_r=1.75-j0.3)-360 Unknowns)

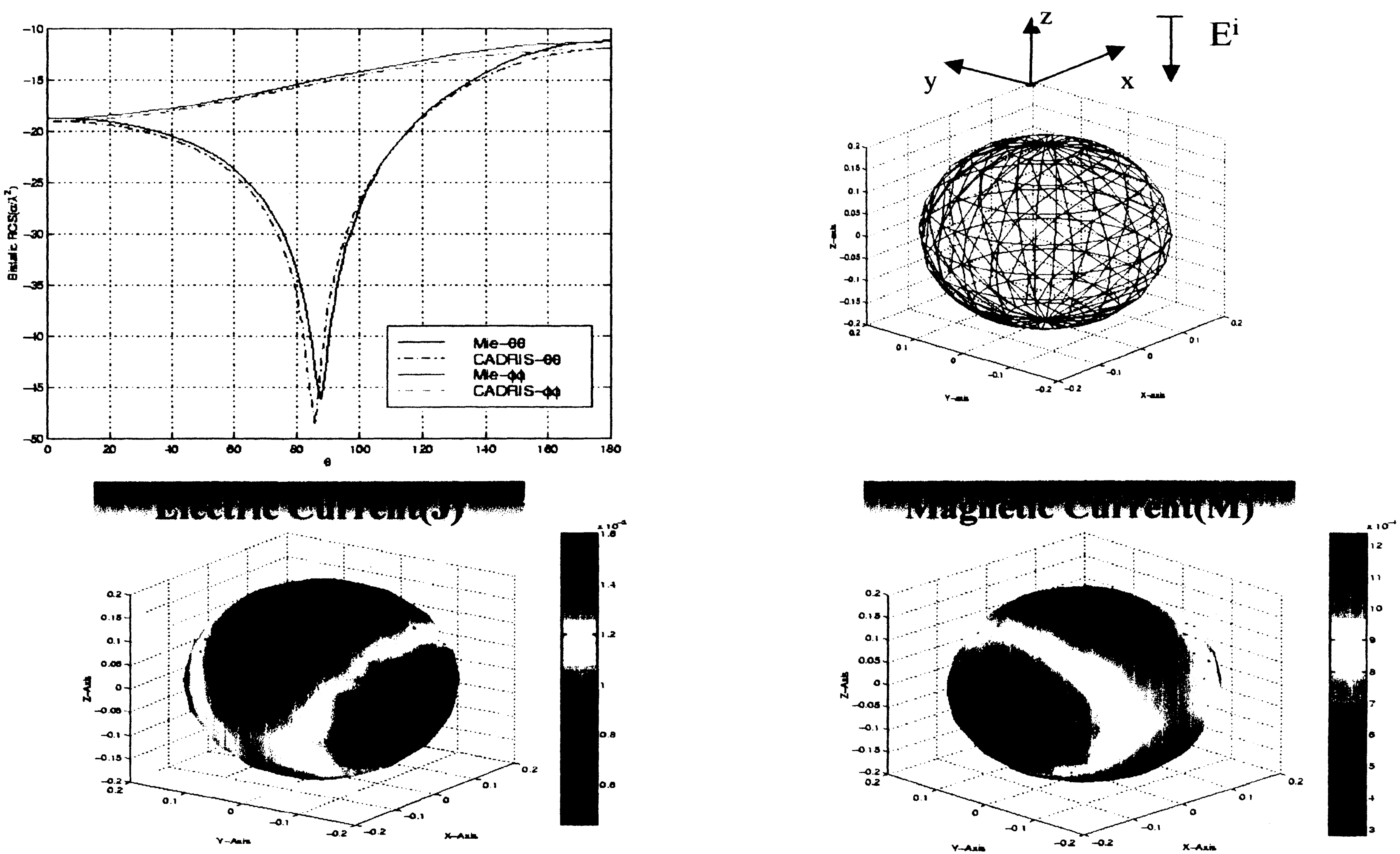


Fig.3.6 Bistatic Radar Cross Section from a dielectric Sphere
and Induced Electric and Magnetic Currents

TEST STRUCTURES-V(Resistive Sheet-1λX1λ-320 Unknowns)

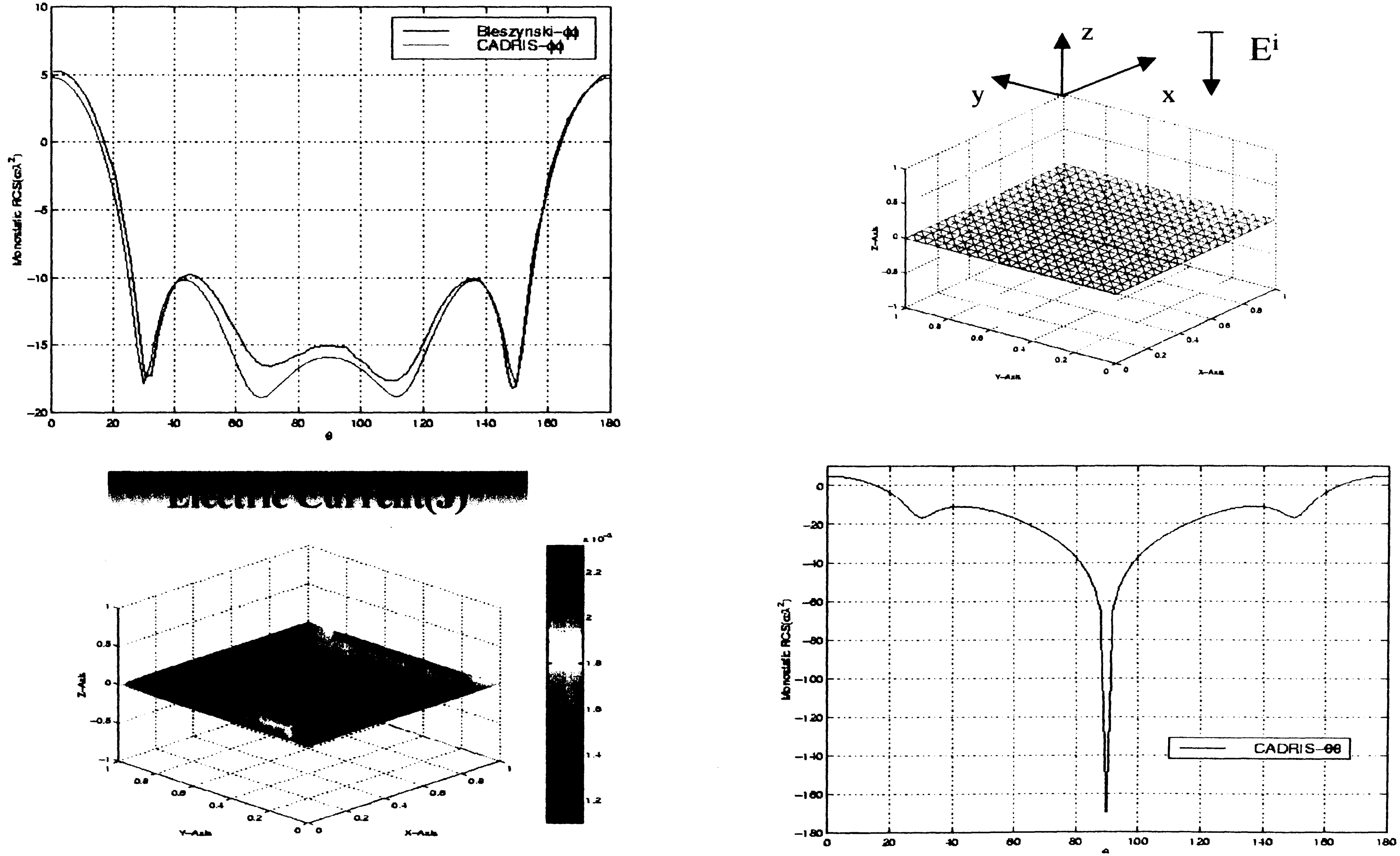


Fig.3.7 Monostatic Radar Cross Section from a Resistive Plate and Induced Electric Currents

TEST STRUCTURES-V(Resistive Sheet-1240 Unknowns)

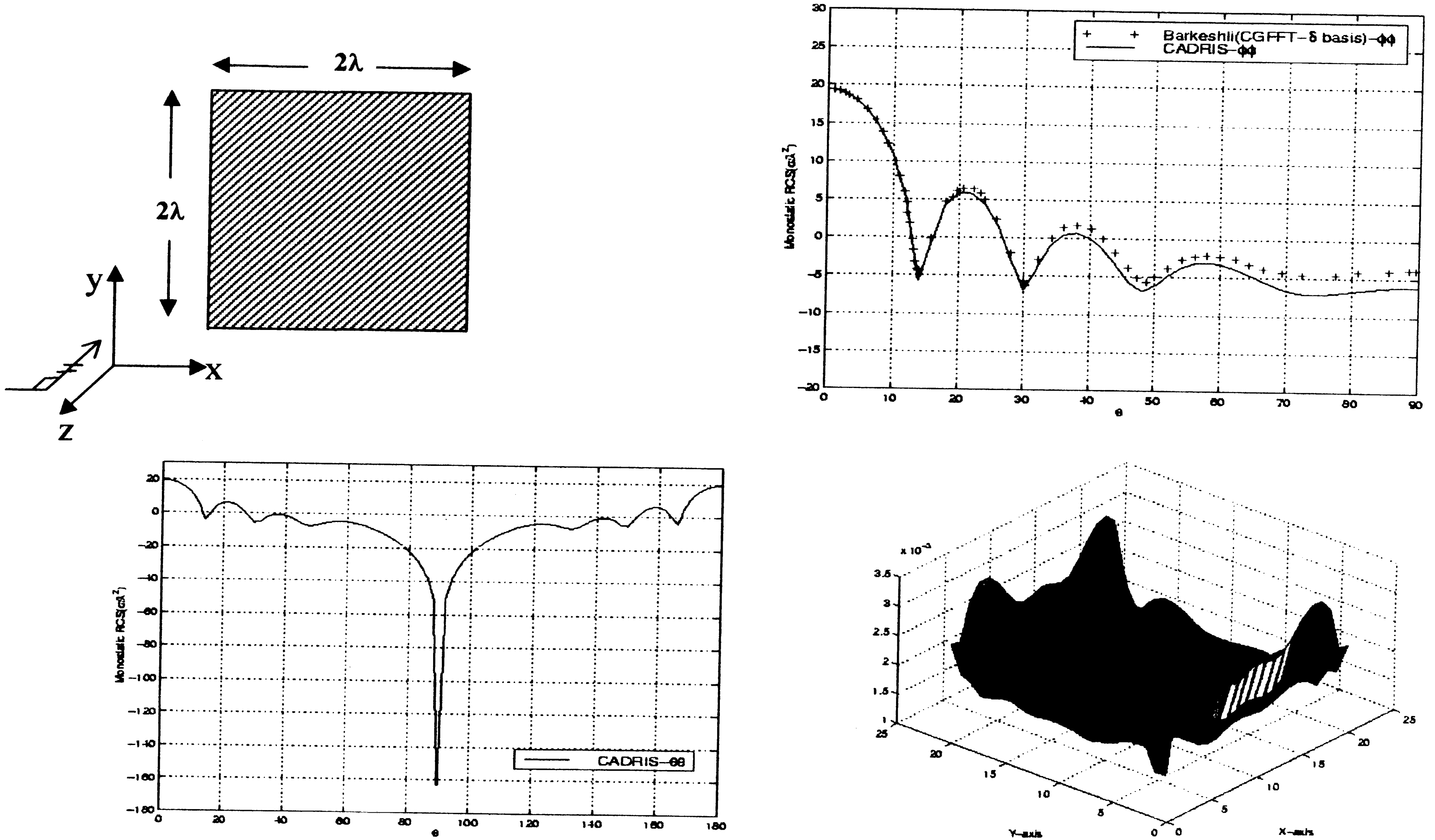
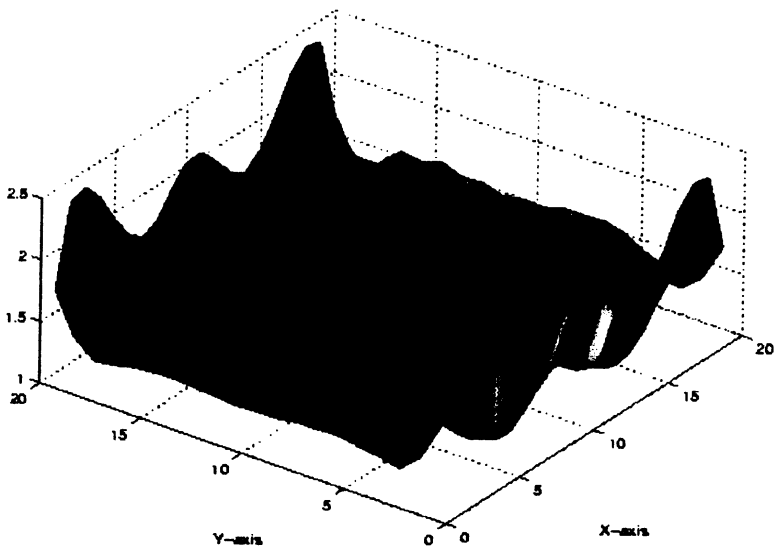
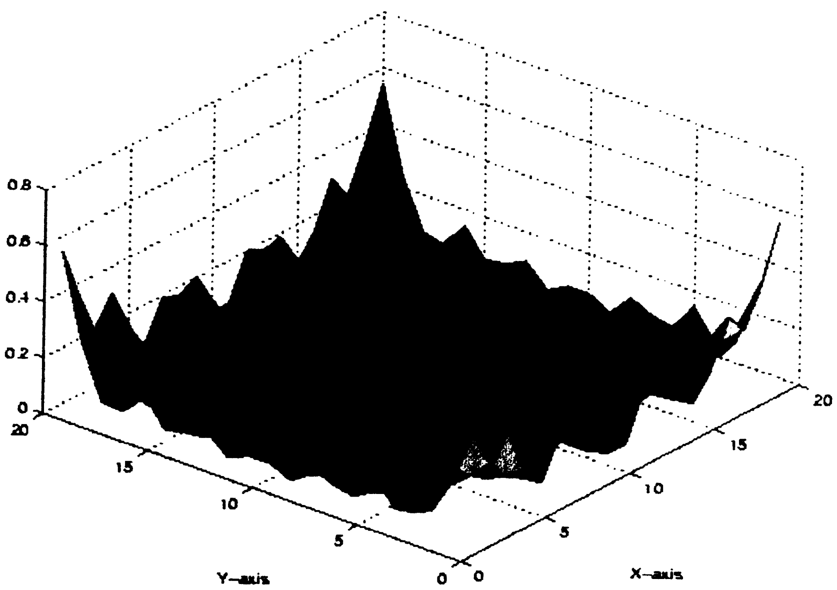


Fig.3.8 Monostatic Radar Cross Section from a Resistive Plate and Induced Electric Currents

TEST STRUCTURES-V(Resistive Sheet-1240 Unknowns)



a) Co-polar(Z_p=Z_0/4)



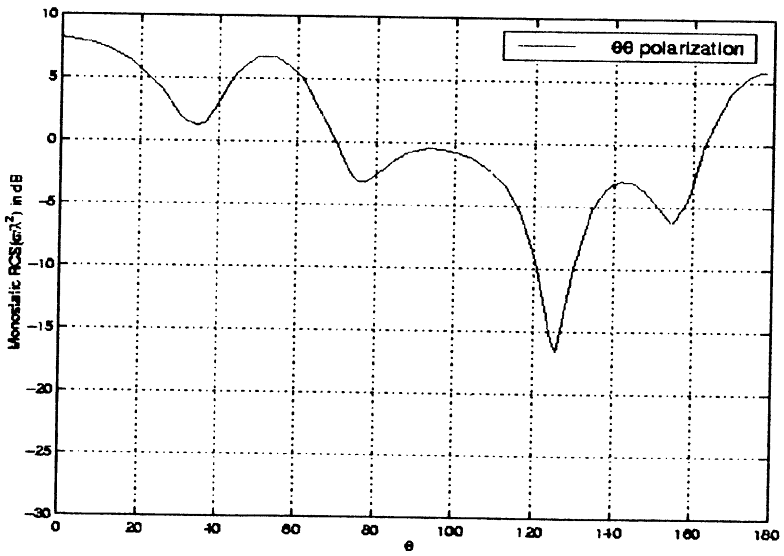
b) Cross-polar(Z_p=Z_0/4)

Fig.3.9 Induced Electric Currents a) Co-polar and b) Cross- polar

TEST STRUCTURES (COMPOSITE SURFACES)

Example (Validation Geometry-II)

Dielectric(ε_r=2.6)
S3: PEC
S4: PEC



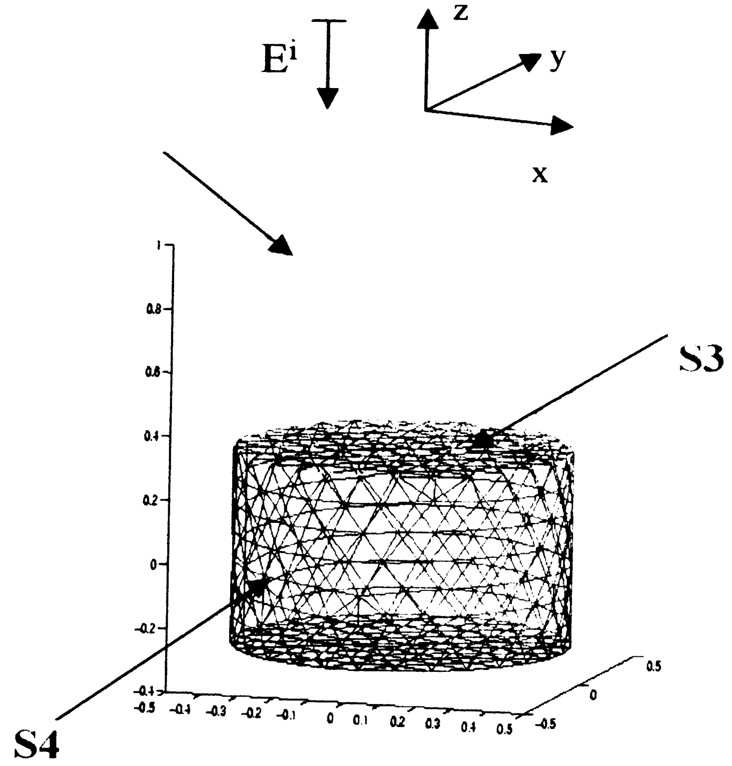


Fig.3.10 Bistatic Radar Cross Section from a metal backed dielectric cylinder

Actual Blade (2λX0.5λ) 6350 Unknowns

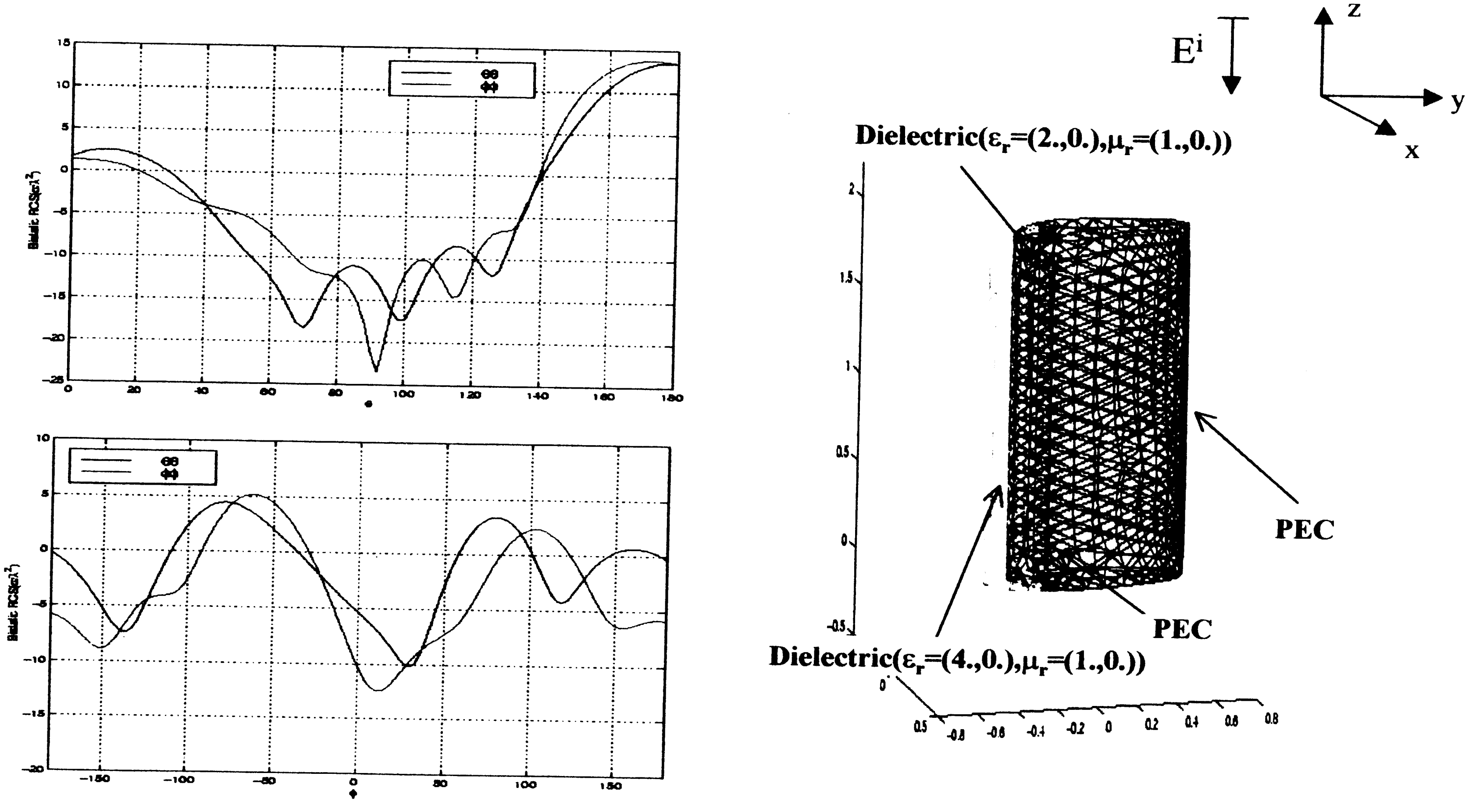


Fig.3.11 Bistatic Radar Cross Section from a blade(2 dielectric and 2 PEC surfaces)

Actual Blade (2λX0.5λ) 7049 Unknowns

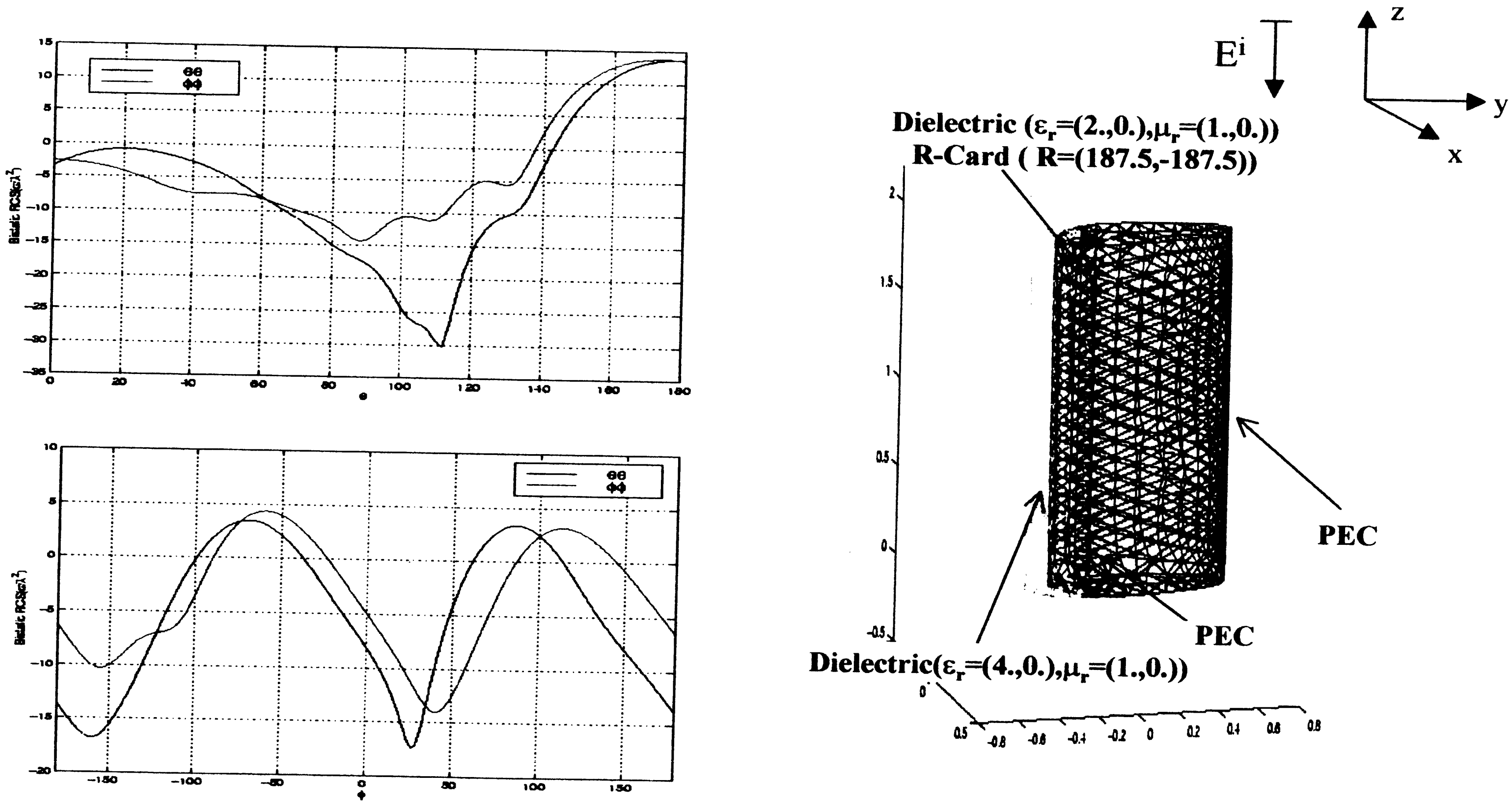
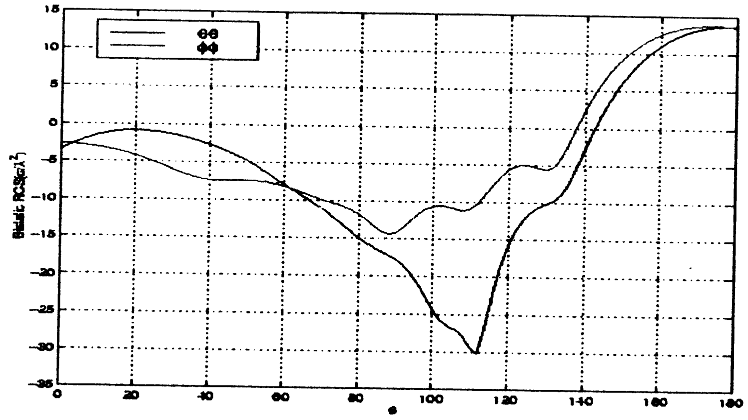
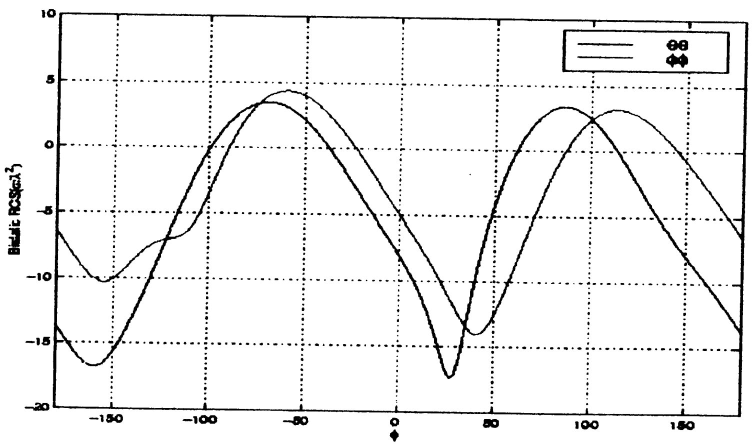
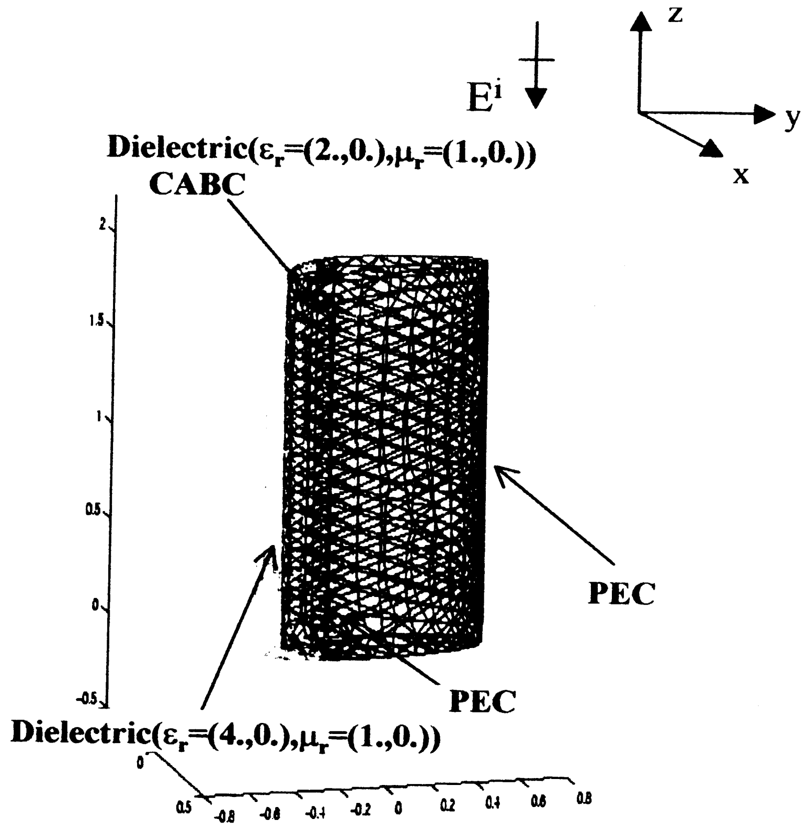


Fig.3.12 Bistatic Radar Cross Section from a blade (R-coated case)

Actual Blade (2λX0.5λ) 7748 Unknowns







$$\begin{bmatrix} \xi_{11} & \xi_{12} \\ \xi_{21} & \xi_{22} \end{bmatrix} = \begin{bmatrix} (1.,0.) & (0.0001,0.) \\ (1,-1) & (1.,0.) \end{bmatrix}$$

Fig.3.13 Bistatic Radar Cross Section from a blade (CA-boundary)