A HEURISTIC SOLUTION FOR THE DIFFRACTION BY A LOSSY MATERIAL WEDGE ON A GROUND PLANE

by

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ABSTRACT

A heuristic diffraction coefficient is presented for the diffraction by a lossy material wedge on a ground plane. Its derivation is such that it accounts for all reflection boundaries associated with the reflected and refracted ray mechanisms. The final form of the diffraction coefficient consists of GTD and physical optics terms.

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I. Introduction

The growing application of different materials on real-world geometries prompts the investigation of the scattering properties from these structures. One such geometry of interest is the junction of a tapered lossy dielectric/magnetic material over a perfectly conducting surface. However, if an analytical investigation is to be carried out, one must choose a canonical geometry which models the above junction. A mathematical model suitable for this application is an infinite lossy material wedge over a ground plane as shown in Fig. 1.

The problem of diffraction by a dielectric/magnetic wedge over a ground plane is essentially that of diffraction by a dielectric wedge once image theory is invoked. Unfortunately, there is no existing asymptotic (high frequency) solution for general wedge geometry and arbitrary material parameters. For example, solutions by Rawlins [1] and Berntsen [2] are only valid for a wedge with small dielectric constants. A recent solution by Joo, Ra and Shin [3] which is valid for large dielectric constant values involves the numerical evaluation of expansion coefficients used for calculation of the diffracted field. Their solution is confined to a right angle wedge and is not applicable for any extensive use. In addition, we note that none of the above solutions have seriously considered any magnetic (lossy) properties of the wedge.

Our assumption that the material wedge is associated with some loss simplifies the analysis considerably and allows one to obtain a simple solution in the context of the Uniform Geometrical Theory of Diffraction UTD [4] and Physical Optics. One method for analysis could follow the heuristic formulation of Burnside etc [5,6]. This requires

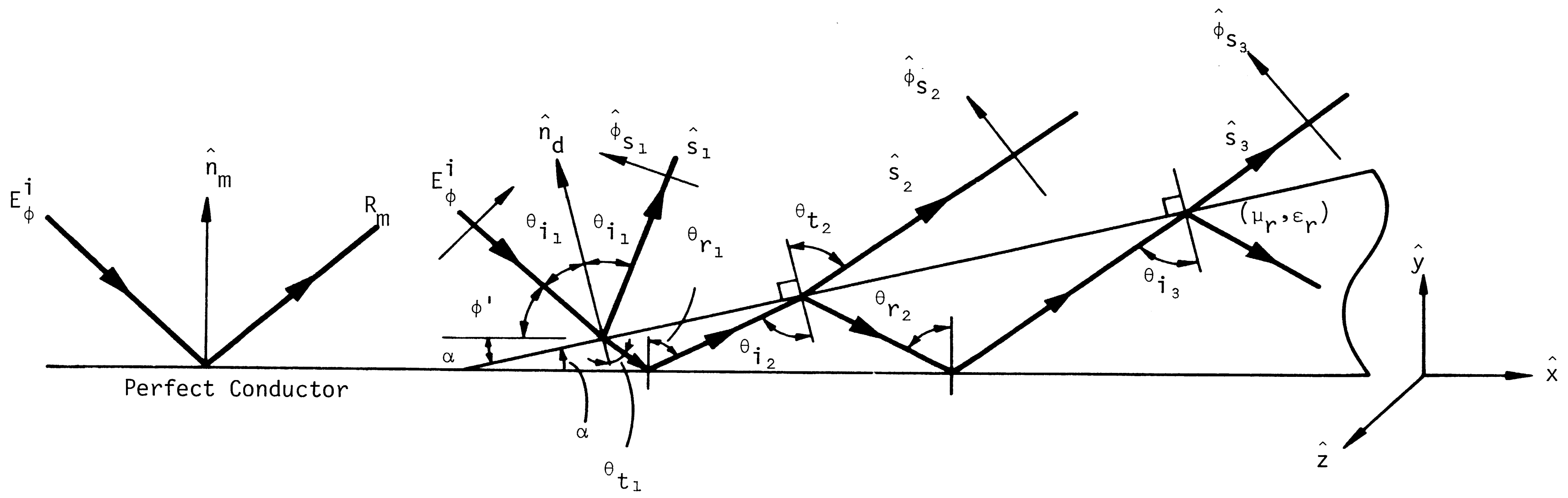

Fig. 1: Incident, transmitted, and reflected rays and their angles for a lossy material wedge on a ground plane.

that the original diffraction coefficient for the perfectly conducting wedge be modified such that the uniformity and continuity of the diffraction coefficient be maintained at the reflection boundaries when the faces of the wedge are other than perfectly conducting. Burnside and Burgener [5] obtained such heuristic solutions for a dielectric or impedance edge, and their solution was later found [7,8] to be fairly accurate.

A similar heuristic solution can be obtained for an impedance wedge (non-penetrable) and since the same formulation is being applied, it is expected that this solution would be of sufficient accuracy. However, in the case of a penetrable material wedge or the configuration in Fig. 1, the application of the above heuristic formulation becomes more involved because of the greater number of reflection boundaries caused by the internally transmitted/reflected ray mechanisms as shown in Fig. 2. Also, for the case of a lossy wedge, the reflected rays are no longer phase congruent. This fact prohibits at this time the calculation of a compact reflection coefficient to be used in a pure UTD heuristic solution. By considering each of the reflection boundaries separately we can arrive at a physical optics solution for the diffraction by the geometry in Fig. 1. The edge diffraction effects associated with multiply reflected rays will be modelled using a physical optics approach. Meanwhile the diffraction associated with the reflection boundaries from the ground plane and the principle reflected wave of the wedge will be accounted for via a UTD equivalent wedge formulation. Our final diffraction coefficent will then be a combination of UTD and physical optics terms.

The varying thickness (being linear) of the material medium as a function of x causes a complication for finding the required

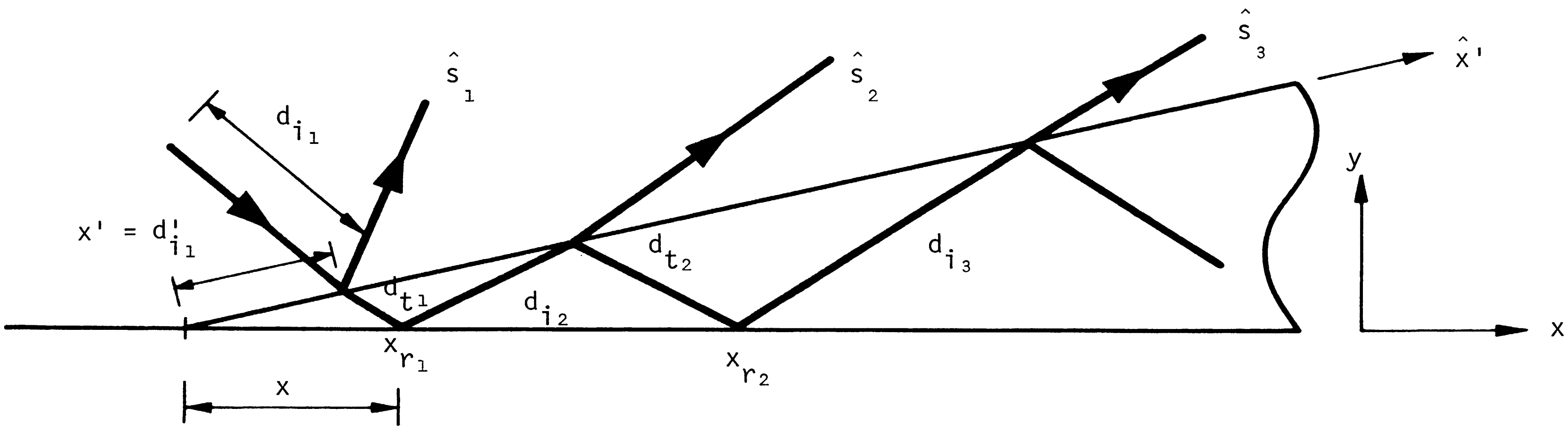

Fig. 2: Incident and reflected rays and their path lengths for a lossy material wedge on a ground plane.

compact diffraction coefficient for each particular species of rays to be included in the appropriate term of the diffraction coefficient. In our analysis these compact diffraction coefficients are found in an indirect manner via the use of equivalent surface currents at the boundary of the material which then is integrated to find the total physical optics field in the far zone. Subsequently, a compact physical optics diffraction coefficient is easily extracted.

This report is divided into four sections. The first is the ray analysis of the wedge geometry (Section II). In this Section (II) the generalized formulas for the refraction/reflection angles of the ray mechanisms within the dielectric wedge are found. In addition, the associated relative path lengths are computed. With this information, reflection boundaries can be defined which are important in the field calculation and formulation of the diffraction coefficients. The third section of this report uses an equivalent current formulation to solve for the far zone physical optics field. The initial portion of this section (III) derives the complex propagation constant of the lossy material wedge. With the characteristics of the ray paths within the wedge known, we assume that a parallel (hard) polarized plane wave is incident upon the lossy material wedge over a ground plane. Using ray tracing techniques the field of the multiply reflected waves within the wedge are found at the material wedge surface. Knowing the surface fields, the equivalent magnetic and electric surface currents are calculated. Appropriate magnetic and electric potential functions are then defined whose linear combination will yield the far zone field. The end result of this analysis is the derivation of the physical optics field corresponding to the multiply reflected/refracted ray mechanisms.

The analysis up to this point will yield all of the multiply refracted/reflected fields associated with the material wedge geometry. However, our interest is the total evaluation of the diffracted field from the junction (tip) the material forms with the perfectly conducting plane. In Section IV of this report we construct a heuristic diffraction coefficient for the diffracted field which when applied in its uniform form maintains total field continuity at the primary reflection boundaries. The approach here employs the use of the uniform diffraction coefficient derived by Kouyoumjian and Pathak for a perfectly conducting wedge to evaluate the diffraction effects corresponding to the prime reflection boundaries. In addition, physical optics (PO) diffraction coefficients will be derived for modeling the diffraction effects associated with the higher order mechanisms.

The last section of this report is a summary.

Before beginning the ray analysis of the wedge, it is important to state the basic assumptions and required specifications made throughout this report.

In this report we are concerned only with plane waves at normal incidence. An $e^{j\omega t}$ time dependence is always assumed and suppressed.

Two types of electric field polarizations may be considered. The normal (or soft) polarization case in which the $\bar{E}$ field is parallel to the wedge edge ($\hat{z}$ axis), and parallel (or hard) polarization case in which the $\bar{E}$ field lies in the plane defined by the incident ray and the vector normal to the surface.

Further, our interest is mainly in backscattering and for ϕ angles in the range of $2\pi/3 < \phi < \pi$. In view of this we note that the backscattered field for the perpendicular (soft) polarization of incidence (E_z^i) will be of nominal value and thus our analysis will be confined to the hard case only. In addition, no surface or lateral waves will be considered in this analysis. These are highly attenuated because of the lossy material assumed here.

II. Ray Analysis of the Wedge Geometry

In this section we develop the required ray analysis which will be used later for the evaluation of the equivalent surface currents on the material face of the wedge.

Figures 1 and 2 show the internal reflections within the lossy material wedge placed upon a perfectly conducting plane. The wedge angle is α (measured with respect to the positive x axis). In Fig. 1, θ_{ij}, θ_{rj}, θ_{tj} represent incident, reflected and transmitted angles respectively. R_s is the primary reflection coefficient of the wave reflected from the material wedge ($\hat{s}_1$). R_m is the reflection coefficient of the reflected wave from the perfectly conducting plane. The incident wave is multiply reflected within the wedge and portions are reflected back into the incident medium via equivalent reflected rays $\hat{s}_2$, $\hat{s}_3$ etc.

Using Fig. 1 as a guide, the general equations for the reflected and incident angles at various stages of the bouncing rays are easily derived and the results are given below. The incident angle (θ_{i_1}) with respect to the normal of the wedge is a known value. The transmitted angle θ_{t_1} and all others subsequent transmitted angles are calculated using Snell's law and the propagation factor k_d.

$$\alpha \;=\; \text{wedge angle}$$

$$\theta_{i_1} \;:\; \text{known incident angle}$$

$$\sin \theta_{t_1} = \mathrm{Re}\left(\frac{k_o}{k_d}\right) \sin \theta_{i_1}, \text{ air to material} \tag{1a}$$

$$\sin \theta_{tj} = \mathrm{Re}\left(\frac{k_d}{k_o}\right) \sin \theta_{ij} \quad j > 1, \text{ material to air} \tag{1b}$$

$$\theta_{r_1} = \theta_{t_1} + \alpha \tag{2}$$

$$\theta_{i_2} = 2\theta_{r_1} - \theta_{t_1} \tag{3}$$

$$\theta_{rj} = 2\theta_{ij} - \theta_{r(j-1)} \quad j \geq 2 \tag{4}$$

$$\theta_{ij} = 2\theta_{r(j-1)} - \theta_{i(j-1)} \quad j > 2 \tag{5}$$

The distance traveled by the multiply reflected rays is depicted in Fig. 2. It is convenient to use $x = 0$, $y = 0$ as our reference point. Also we define another coordinate system rotated about the $\hat{z}$ axis and which has its new x' axis coinciding with the surface of the wedge. From geometrical considerations

$$x' = \frac{x \cos (\theta_{t_1} + \alpha)}{\cos \theta_{t_1}} \quad . \tag{6}$$

This relationship allows one to easily relate the position on the x axis to the position on the wedge surface. The separation distance between the incident wavefront on the face of the wedge and at $x = 0$ is d_{i_1}. This distance, along with the travel distances of the multiple reflections d_{ij} and d_{tj} shown in Fig. 2 are given by

$$d_{i_1} = x' \cos(\phi' + \alpha) \tag{7}$$

$$d_{t_1} = x \frac{\sin \alpha}{\cos \theta_{t_1}} \tag{8}$$

$$d_{i_2} = d_{t_1} \frac{\cos \theta_{t_1}}{\cos \theta_{i_2}} \tag{9}$$

$$d_{tj} = d_{ij} \frac{\cos \theta_{r(j - 1)}}{\cos \theta_{rj}} ; j \geq 2 \tag{10}$$

$$d_{ij} = d_{t(j - 1)} \frac{\cos \theta_{i(j - 1)}}{\cos \theta_{ij}} ; j > 2 \tag{11}$$

by substituting (11) into (10) we obtain

$$d_{tj} = \frac{d_{t(j - 1)} \cos \theta_{i(j - 1)} \cos \theta_{r(j - 1)}}{\cos \theta_{ij} \cos \theta_{rj}} ; j > 2 \tag{12}$$

Conversely, by reversing the prior substitution we have:

$$d_{ij} = d_{i(j - 1)} \frac{\cos \theta_{r(j - 2)} \cos \theta_{i(j - 1)}}{\cos \theta_{r(j - 1)} \cos \theta_{ij}} j > 2 \tag{13}$$

Using the generalized ray geometry and Snell's law along with the knowledge of the wedge angle, the angle of incidence, and the material parameters of the wedge we can immediately calculate the reflection boundary angles associated with the multiply reflected rays transversed within the material wedge.

III. Equivalent Current Formulation of the Far Zone Field

Let us set up the problem from the beginning. We have a perfectly conducting ground plane lying on the y = 0 plane. At x = 0 we place the edge of a lossy material wedge which goes to infinity on the $\hat{x}$ axis (see Fig. 1). The material wedge has complex constitutive parameters ε_r and μ_r and has an angular opening of α degrees. A plane wave with parallel (hard) polarization is normally incident upon the defined surface at an angle ϕ', where $0 < \phi' < \pi/2 - \alpha$. ϕ' is defined with respect to the negative $\hat{x}$ axis.

Qualitatively, the plane wave incident upon the perfectly conducting surface will be reflected in accordance with the law of reflection. However, the tracing of the multiple reflected rays within the material wedge requires a detailed analysis. The initial incident wave on the material face of the wedge is first reflected according to the law of reflection and partially transmitted into the wedge. The transmitted wave then travels a path length d_{t_1} (see Fig. 2) before it reflects off the ground plane. Subsequently, it propagates back to the material-free space interface through a path length of d_{i_2}. At this boundary, part of the wave energy is again transmitted to the receiver and the rest reflects back into the wedge. The angle at which the transmitted wave emerges is governed by Snell's law and the corresponding ray is referred to as $\hat{s}_2$. The reflected wave continues on a similar path as the initial transmitted wave, propagating through a distance d_{t_2}, d_{i_3} (see Fig. 2) at which point it is back at another material-free space boundary. Using Snell's law and Fresnel transmission coefficients again, we can again find the direction of the emerging reflected ray (in free space), $\hat{s}_3$. The procedure may be repeated an infinite number of times.

However, since the path lengths are increasing, the multiply reflected waves will eventually attenuate to negligible values due to the material loss in the wedge. Therefore, we can obtain reasonable results by accounting for a limited number of multiply reflected rays (of which $\hat{s}_2$, $\hat{s}_3$, etc are examples). We note again that the surface or totally reflected rays are not included. This condition is reached when

$$\sin \theta_{tj} = 1 = \mathrm{Re}\left(\frac{k_d}{k_o}\right) \sin \theta_{ij} \quad . \tag{14}$$

The path lengths within the material and the associated angles of incidence and reflection are given in Section II of this report.

The constitutive parameters of the lossy material wedge are given by

$$\mu_r = \mu_r' - j\mu_r'' \tag{15}$$

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r'' \tag{16}$$

where μ_r is the complex permeability and ε_r is the complex permittivity.

Using the basic definition of the propagation constant k_d and the complex parameters μ_r and ε_r, we can express the complex propagation constant as:

$$k_d = \omega\sqrt{\mu_r \varepsilon_r} \tag{17}$$

$$k_d = k' - jk'' \tag{18}$$

where

$$k' = k_o k_r' \tag{19}$$

and

$$k'' = k_o k''_r \quad . \tag{20}$$

In order to define k'_r and k''_r, let

$$K_{mr} = [(\mu'_r\varepsilon'_r - \varepsilon''_r\mu''_r)^2 + (\mu''_r\varepsilon'_r + \varepsilon''_r\mu'_r)^2]^{1/4} \tag{21}$$

and

$$\phi_d = \tan^{-1}\left(\frac{\mu''_r\varepsilon'_r + \varepsilon''_r\mu'_r}{\mu'_r\varepsilon'_r - \varepsilon''_r\mu''_r}\right) . \tag{22}$$

Using K_{mr} and ϕ_d we define k'_r and k''_r as:

$$k'_r = K_{mr} \cos \frac{\phi_d}{2} \quad , \tag{23}$$

$$k''_r = K_{mr} \sin \frac{\phi_d}{2} \; . \tag{24}$$

In order to ensure that the wave decays, k''_r must be positive. The propagation factor can now be written as

$$e^{-jk_o s} = e^{-k_o k''_r s} \, e^{-jk_o k'_r s} \tag{25}$$

where $e^{-k_o k''_r s}$ is the decay factor, $e^{-jk_o k'_r s}$ is the phase factor, and s is the path length traveled within the wedge itself.

Having an expression for the complex propagation constant (k_d) we can derive Snell's law. The equations using Snell's law for incidence from free space into material and vice versa are given by equations (1a) and (1b). Finally, for reference purposes we state below the Fresnel reflection and transmission coefficients at the wedge free space interface. For parallel polarization at normal incidence the reflection (Γ) and transmission (T) coefficients are

$$\Gamma = \frac{\eta_1 \cos\theta_1 - \eta_2 \cos\theta_2}{\eta_1 \cos\theta_1 + \eta_2 \cos\theta_2} \tag{26}$$

and

$$T = \frac{2\eta_1 \cos\theta_1}{\eta_1 \cos\theta_1 + \eta_2 \cos\theta_2}\ , \tag{27}$$

where

$$\eta_j = \sqrt{\frac{\mu_{rj}}{\varepsilon_{rj}}}\ . \tag{28}$$

The subscripts with 1 denote the medium of the incident wave and 2 denotes the transmitted medium.

Using x = y = 0 as our reference point, the transmitted wave (parallel polarization) within the material of the wedge after the first interaction of the incident plane wave with the material face is given by

$$E_{\phi_{t_1}}(x_{r_1}) = E_\phi^i(x = 0)\, T_1\, e^{-jk_d d_{t_1}}\, e^{-jk_o d_{i_1}}\ . \tag{29}$$

In the above expression T_1 is the complex Fresnel transmission coefficient (see Eq. 27) from free space into the lossy material, d_{i_1} and d_{t_1} are given in Eqs. (7) and (8). Since k_d is complex, we are already accounting for the attenuation factor of the plane wave within the material wedge.

The wave in (29) is then incident upon the perfect conductor and is reflected to give a reflected wave

$$E_{\phi_{i_2}}(x_{i_2}) = E_{\phi_{t_1}} e^{-jk_d d_{i_2}} \tag{30}$$

propagating toward the material-free space interface.

The field transmitted into free space as a result of $E_{\phi_{i_2}}$ is given by

$$E_{\phi_{s_2}} = E_{\phi_{i_2}} T_2 e^{-jk_o s} \tag{31}$$

where T_2 is the transmission coefficient from the material to free space and s is the distance from the wedge to the point of observation.

Substituting (29) and (30) into (31) we can write the expression for the first multiply reflected field as

$$E_{\phi_{s_2}} = E_{\phi}^{i}(x = 0)\, T_1 T_2\, e^{-jk_o d_{i_1}}\, e^{-jk_d(d_t + d_{i_2})} , \tag{32}$$

where

$$d_{i_1} = x' f_{i_1} \tag{33}$$

$$d_{t_1} = x' f_{t_1} \tag{34}$$

$$d_{i_2} = x' f_{i_2} \tag{35}$$

In general, additional multiply reflected waves (R_{sj} with $j \geq 2$) can be found according to

$$E_{\phi s_w} = E_\phi^i(x = 0)T_1 \; e^{-jk_o d_{i_1}} \exp\left[-jk_d \sum_{m=2}^{w} (d_t(n - 1) + d_{im})\right] T_w$$

$$\cdot \prod_{m=2}^{w-1} R_m \; , \qquad (36)$$

where

$$d_{ij} = x' f_{ij} \; , \qquad (37)$$

and

$$d_{tj} = x' f_{tj} \; , \qquad (38)$$

and the T_j's with R_j's are transmission and reflection coefficients which are defined in Eqs. (27) and (26), respectively.

For the case of reflection directly off the material wedge, the reflection coefficient called R_s (for ray $\hat{s}_1$) is given simply by Eq. (26).

It will be useful to generalize f_{ij} and f_{tj} for future calculations in an attempt to formulate a general diffraction coefficient. Since d_{ij} and d_{tj} general forms are known, we find from Eqs. (7) through (13) that

$$f_{i_1} = \cos(\phi' + \alpha) \; , \qquad (39)$$

$$f_{i_2} = f_{t_1} \frac{\cos \theta_{t_1}}{\cos \theta_{i_2}} \; , \qquad (40)$$

$$f_{ij} = \frac{\cos \theta_{i(j - 1)}}{\cos \theta_{ij}} f_{t(j - 1)} \; , \; j > 2 \qquad (41)$$

$$f_{t_1} = \frac{\sin \alpha}{\cos(\theta_{t_1} + \alpha)} \; , \qquad (42)$$

and

$$f_{tj} = \frac{\cos \theta_{r(j-1)}}{\cos \theta_{rj}} f_{ij} \quad \cdot \quad j > 1 \quad . \tag{43}$$

To solve for the reflected magnetic field we define $\hat{s}_j$ as the unit vector in the direction of propagation of the transmitted wave and $\hat{\phi}_{sj}$ as the unit vector in the direction of the parallel polarized reflected electric field (see Fig. 1).

$$\bar{H}_j = \frac{1}{z_o} \hat{s}_j \times \hat{\phi}_{sj} E_{\phi_{sj}} \quad ; \quad z_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} \tag{44}$$

or

$$H_{zj} = \frac{1}{z_o} E_{\phi_{sj}} \quad . \tag{45}$$

Presently, we have the reflected field components at the surface of the wedge. To obtain the physical optics far field quantities we will define equivalent magnetic and electric currents ($\bar{J}_{s_2}$ and $\bar{M}_{s_2}$) and vector potentials components $A_{\phi_{s_2}}$ and F_z. With the vector potentials known it will be easy to obtain the physical optics field quantities. In the next section we will extract from these results a diffraction coefficient for each of the multiply reflected rays ($\hat{s}_2$, $\hat{s}_3$ etc.).

Once the surface fields of the wedge are known for a multiply reflected wave we can define (using $\hat{s}_2$ as an example) a set of equivalent electric and magnetic currents as

$$\bar{J}_{s_2} = \hat{n}_d \times \bar{H}_{s_2} \quad , \tag{46}$$

$$\bar{M}_{s_2} = \bar{E}_{s_2} \times \hat{n}_d \tag{47}$$

At this time it is convenient to rotate the $\hat{x}$ and $\hat{y}$ axis around the $\hat{z}$ axis such that the $\hat{x}'$ axis is parallel to the wedge as shown in Fig. 3.

From Fig. 3 we see that $\hat{n}_d$ is parallel to $\hat{y}'$ and $\hat{\phi}_{s_2} \cdot \hat{x}' = -\cos(\theta_{t_2})$. These relations are utilized in subsequent calculations.

Evaluating Eq. (47) in the prime coordinate system and solving for the $\hat{\phi}_{s_2}$ component results in

$$J_{\phi_{s_2}} = \hat{\phi}_{s_2} \cdot \bar{J}_{s_2} = \hat{x}' \frac{E_{\phi_{s_2}}}{Z_0} \cdot \hat{\phi}_{s_2} = -\cos\theta_{t_2} \frac{E_{\phi_{s_2}}}{Z_0} . \tag{48}$$

Also, for the magnetic current in Eq. (47) we have

$$\bar{M}_{s_2} = -\hat{z}' \cos\theta_{t_2} E_{\phi_{s_2}} . \tag{49}$$

The far zone electric field can now be calculated by [9]

$$E^s_{\phi_{s_2}} = -j\omega\mu A_{\phi_{s_2}} - jkF_z , \tag{50}$$

where $A_{\phi_{s_2}}$ and F_z are the electric and magnetic vector potentials. The vector potentials are calculated using a two-dimensional approach and a large argument approximation for the Hankel function $H_0^{(2)}$.

For the electric potential we have that

$$A_{\phi_{s_2}} = \frac{1}{\sqrt{8jks\pi}} \int_0^\infty J_{\phi_{s_2}} e^{-jk_0|\rho-\rho'|} dx' \tag{51}$$

with

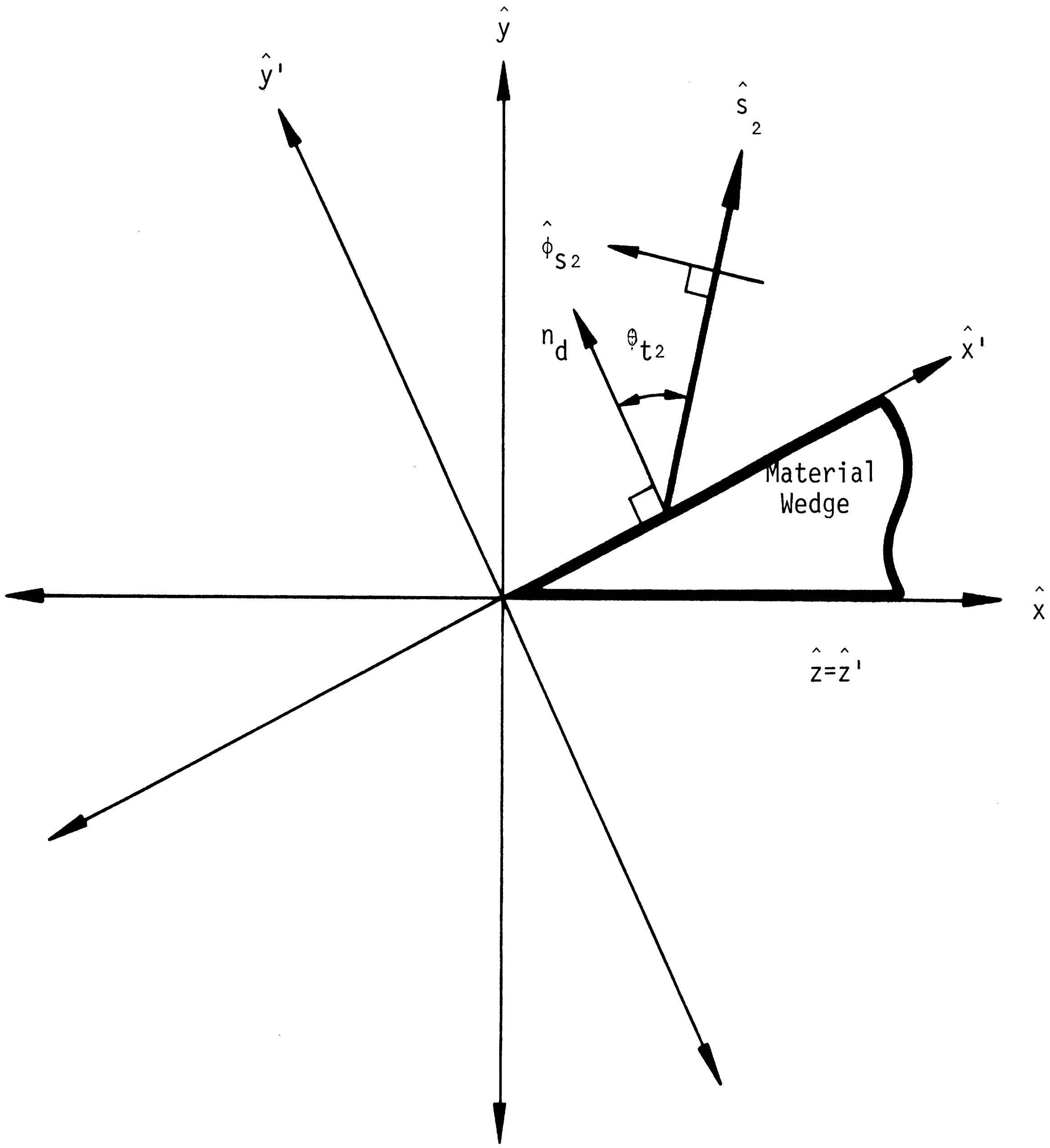

Fig. 3: Rotation of the $\hat{x}$-$\hat{y}$ axis about the $\hat{z}$ axis such that $\hat{x}'$ coincides with the wedge surface.

$$|\rho - \rho'| = |s - s'| \sim s - s' \cos(\phi - \phi') \quad . \tag{52}$$

According to Fig. 4 it is also found that

$$s - s' \cos(\phi - \phi') = s - x' \sin \theta_{t_2} \quad . \tag{53}$$

Thus

$$A_{\phi_{s_2}} = \frac{e^{-jk_o s}}{\sqrt{8jk_o s\pi}} \int_0^\infty J_{\phi_{s_2}} e^{jk_o x' \sin \theta_{t_2}} dx' \quad . \tag{54}$$

Likewise,

$$F_z = \frac{e^{-jk_o s}}{\sqrt{8\pi jk_o s}} \int_0^\infty M_z e^{jk_o x' \sin \theta_{t_2}} dx' \quad . \tag{55}$$

Substituting Eqs. (48) and (49) into (54) and (55) respectively, and finally inserting (54) and (55) into (50) we have

$$E^s_{\phi_{s_2}} = 2\sqrt{\frac{k_o}{8\pi}} e^{j\pi/4} \frac{e^{-jk_o s}}{\sqrt{s}} \int_0^\infty E_{\phi_{s_2}} \cos \theta_{t_2} e^{jk_o x' \sin \theta_{t_2}} dx' \quad . \tag{56}$$

Using the representation for $E_{\phi_{s_2}}$ (Eq.(32)) into (56) and evaluating the integrals results in

$$E^s_{\phi_{s_2}} = \sqrt{\frac{k_o}{2\pi}} e^{-j\pi/4} \frac{T_1 T_2 \cos \theta_{t_2} E^i_\phi}{k_o f_{i_1} + k_d(f_{t_1} + f_{i_2}) - k_o \sin \theta_{t_2}} \frac{e^{-jk_o s}}{\sqrt{s}} \quad . \tag{57}$$

One can now extrapolate the general physical optics fields for any higher order multiply reflected ray. In doing this it should be noted that we are assuming that the ray is not totally reflected (trapped) in the material wedge. Subject to this condition, it is found that

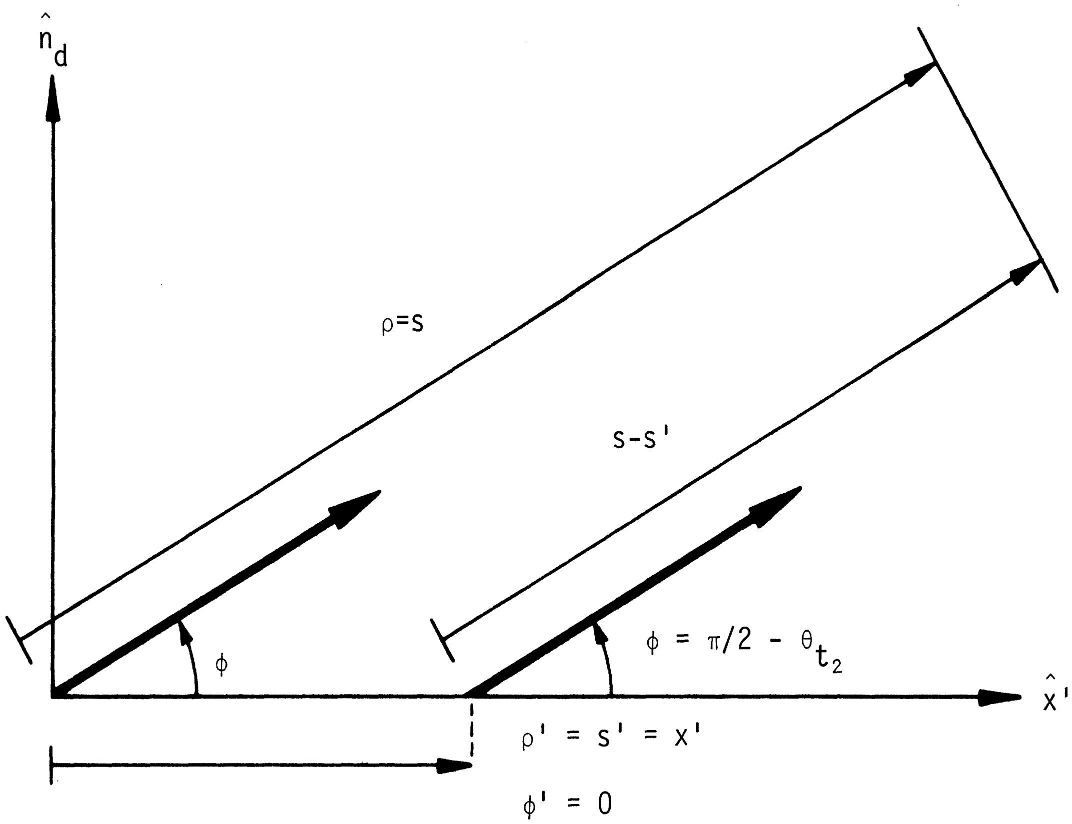

Fig. 4: From the wedge surface to a far field point of observation for a multiply reflected wave.

$$E^s_{\phi_{sm}} = \sqrt{\frac{k_o}{2\pi}}\, e^{-j\pi/4} \frac{\cos\theta_{TM}\, T_1 T_M \prod_{n=2}^{M-1} R_n}{k_o f_{i_1} + k_d\left[\sum_{n=1}^{m-1} f_{tn} + \sum_{n=2}^{m} f_{in}\right] - k_o \sin\theta_{tm}}$$

$$\cdot\; E^i_\phi \frac{e^{-jk_o s}}{\sqrt{s}}\;, \qquad (58)$$

where $E_{\phi_{sm}}$ is the physical optics field for the ray, along the $\hat{s}_m$ direction. Further, the coefficients T_j and R_j are defined by Eqs. (27) and (26) respectively, and s is defined from the point of transmission on the wedge to the point of observation.

IV. Construction of Diffraction Coefficients

Figure 5 shows the reflection boundaries for the multiply reflected waves.

For a constant direction of incidence, as the observation angle varies, pure geometrical optics would predict abrupt transitions in the total field due to the discontinuities in the multiply reflected fields. In the case of a perfectly conducting wedge such discontinuities are eliminated by the existence of the diffracted field. When the diffracted field is calculated in a uniform sense the transition regions of the two reflection boundaries are continuous. According to GTD [4,10] the total field is given by

$$E^{total} = E^i u^i + E^r u^r + E^d\;, \qquad (59)$$

where E^i is the incident field, E^r is the reflected field and E^d is the diffracted field by the edge of the wedge. In addition u^i and u^r are unit step functions which are zero in the region beyond the shadow and reflection boundaries.

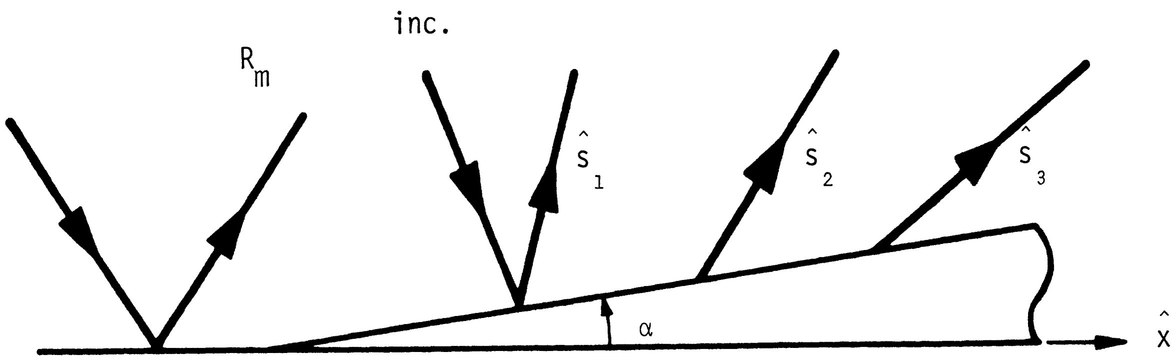

Fig. 5: Single and multiple reflection boundaries.

The diffracted field for an impedance wedge (non-penetrable) can be expressed in a heuristic manner as

$$E^d_\phi = -E^i_{\phi'} D_h \frac{e^{-jks}}{\sqrt{s}} \tag{60}$$

with

$$D_h(\phi,\phi') = -\frac{e^{-j\pi/4}}{2n\sqrt{2\pi k}} \left\{ \frac{2 \sin\frac{\pi}{n} F\left(2kL \cos^2\frac{\beta^-}{2}\right)}{\cos\frac{\pi}{n} - \cos\frac{\beta^-}{n}} \right.$$

$$+ R_2 \cot\left(\frac{\pi + \beta^+}{2n}\right) F\left[2kL \cos^2\left(\frac{2\pi n - \beta^+}{2}\right)\right]$$

$$\left. + R_1 \cot\left(\frac{\pi - \beta^+}{2n}\right) F\left[2kL \cos^2\frac{\beta^+}{2}\right] \right\} \tag{61}$$

where $R_{1,2}$ are the reflection coefficients on $\phi = 0$ and $\phi = n\pi$ faces of the wedge given by

$$R_{\substack{1\\2}} = \begin{cases} \dfrac{\sin\phi' - \eta_1}{\sin\phi' + \eta_1} \\[2ex] \dfrac{\sin(n\pi - \phi') - \eta_2}{\sin(n\pi + \phi') + \eta_2} \end{cases}, \tag{62}$$

$$\beta_\pm = \phi \pm \phi' , \tag{63}$$

$$L = S \tag{64}$$

assuming a plane wave incidence. Note that ϕ (scattering angle) and ϕ' (incident angle) are measured with respect to one of the wedge faces. For our case this is the negative x axis (the position on which the ground plane is located). The transition function F(x) is defined in [4] and $\eta_{1,2}$ are the normalized surface impedances on the appropriate faces of the wedge. The above heuristic diffraction coefficient in (61) was derived on the basis that continuity of the total field maintained at the reflection boundaries. For the case of the impedance half-plane where the exact solution is known it has been found [7,8] to give excellent results. Note that when $\eta_{1,2} \to 0$ then $R_{\frac{1}{2}} = 1$, and the above diffraction coefficient reduces to the rigorous one for the perfectly conducting case.

The form of the last two terms in (61) suggests a method for constructing a diffraction coefficient to model the diffraction terms associated with the ground plane and the principle reflected ray $\hat{s}_1$ for the geometry under consideration.

Such a UTD diffraction coefficient (which does not account for the scattering associated with multiply reflected rays) has the form

$$D_h(\phi,\phi') = -\frac{e^{-j\pi/4}}{2n\sqrt{2\pi k_o}} \left\{ \left(\cot\left(\frac{\pi - \beta^-}{2n}\right) + \cot\left(\frac{\pi + \beta^-}{2n}\right) \right) \cdot F\left[2k_o L \cos^2 \frac{\beta^-}{2}\right] + \cot\left(\frac{\pi - \beta^+}{2n}\right) F\left[2k_o S \cos^2 \frac{\beta^+}{2}\right] + R_s \cot\left(\frac{\pi + \beta^+}{2n}\right) F\left[2k_o S \cos\left(\frac{2\pi n_o - \beta^+}{2}\right)\right] \right\}, \tag{65}$$

where

$$n = 1 - \frac{\alpha}{\pi} \qquad (\alpha = \text{wedge angle}) \tag{66}$$

ϕ' = angle of incidence W.R.T. negative x axis

ϕ = angle of observation W.R.T. negative x axis

R_s is defined in Eq. (26).

We note that the first term of the above diffraction coefficient is associated with the shadow boundaries and is therefore very small for our geometry. Assuming also that $s \to \infty$, a simpler form for the diffraction coefficient is given by

$$D_h(\phi,\phi') = -\frac{e^{-j\pi/4}}{2n\sqrt{2\pi k_o}}\left\{\cot\left(\frac{\pi - \beta^+}{2n}\right) + R_s \cot\left(\frac{\pi + \beta^+}{2n}\right)\right\} \quad . \tag{67}$$

We note that the above diffraction coefficient is no longer uniform. However, in the region of interest, where

$$0 < \phi \ , \quad \phi' < \pi/2 - \alpha$$

equation (67) is still valid.

To account for the diffraction due to the boundaries of mulitply reflected waves, we turn to the physical optics far field expressions derived in Eq. (58). Equation (58) is written as

$$E^s_{\phi_{sm}} = D^{P.O.}_{h_{sm}}(\phi,\phi')\, E^i_\phi \frac{e^{jk_o s}}{\sqrt{s}} \quad . \tag{68}$$

It is now recognized that $D^{P.O.}_{h_{sm}}$ is the physical optics diffraction coefficient for the multiply reflected ray $\hat{s}_m$

$$D^{P.O.}_{h_{sm}} = \sqrt{\frac{1}{2\pi k_o}}\, e^{-j\pi/4} \frac{\sin(\phi + \alpha) T_1 T_m \prod_{n=2}^{m-1} R_n}{f_{i_1} + k_r\left[\sum_{n=1}^{m-1} f_{tn} + \sum_{n=2}^{m} f_{in}\right] + \cos(\phi + \alpha)} \tag{69}$$

Clearly, (69) is valid only for the "visible" (transmitted into free space from the wedge) ray mechanisms. One may also question the accuracy of the physical optics diffraction coefficient. However, preliminary calculations indicate that for small ϕ', there are no visible higher order terms provided ε_r is relatively large. Therefore for this case, the total diffracted field would be dominated by the GTD terms in Eq. (67).

The "visible" higher order mechanisms will more likely appear when ϕ' is closer toward the normal of the wedge face. In that region and provided backscattering is of interest, the physical optics calculations are expected to give good results.

We can now write a total diffraction coefficient for a lossy wedge on a ground plane by combining (67) and (69) to yield

$$D_h(\phi,\phi') = -\frac{e^{-j\pi/4}}{\sqrt{2\pi k_o}}\left\{ \frac{1}{2n}\left[\cot\left(\frac{\pi - \beta^+}{2n}\right) + R_s \cot\left(\frac{\pi + \beta^+}{2n}\right)\right]\right.$$

$$\left. -\sum_{M=2}^{M_T} \frac{\sin(\phi+\alpha)T\, T_M \prod_{n=2}^{M-1} R_n}{f_{i_1} + k_r\left[\sum_{n=1}^{M-1} f_{tn} + \sum_{n=2}^{M} f_{in}\right] + \cos(\phi+\alpha)} \right\}. \tag{70}$$

$M_T \geq 2$ is the number of multiply reflected rays (minus 1)

V. Summary

The plane wave diffraction by a lossy material wedge on a perfectly conducting plane was studied. A ray analysis of the wedge geometry was performed to find the location of the reflected and multiply reflected boundaries. With this information combined with ray tracing techniques, the multiply reflected field at the wedge surface were found. Equivalent surface currents were then derived. Finally, the far zone physical optics field was formulated using potential functions which were derived from the equivalent surface currents. The major result of this procedure yielded a physical optics diffraction coefficient associated with the higher order terms.

A uniform diffraction coefficient was subsequently derived on the basis of UTD, by imposing continuity of the total field at the reflection boundaries associated with the ground plane and principle reflected ray from the wedge. Finally, the UTD and physical optics diffraction coefficients were combined and conditions of validity were discussed.

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