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SCATTERING BY A NARROW GAP IN AN IMPEDANCE PLANE

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ABSTRACT

For a plane wave incident on a cavity-backed gap in an impedance plane, the coupled integral equations for the induced currents have been solved numerically and the far field scattering computed. The results are compared with a quasi-analytical solution previously derived and modified for the impedance plane. For narrow gaps of widths less than 0.15λ, the agreement is within 12 percent for H-polarization and 14 percent for E-polarization for the cavity geometries considered, limited to small surface impedances of the plane. Excellent agreement is obtained when the material filling of the gap is lossy.

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1. INTRODUCTION

A case of interest in radar cross section studies is the scattering from gaps and cracks in planar surfaces. Results were recently derived [1,2] for the narrow gap in a perfectly conducting ground plane. Of equal concern is the scattering from a gap of similar geometry but in a ground plane coated with a material having arbitrary dielectric properties.

The development of the solution in this paper is for planar surfaces large in extent compared to the width of the gap. As in [2], a set of coupled integral equations for the electric and magnetic currents which exist on the walls of the gap cavity and in aperture of the gap are developed by employing the equivalence principle [3]. Since a thinly coated conducting surface can be described by the impedance boundary condition [4], the half-space Green's function for an impedance plane has been developed and is applied. For plane wave incidence the integral equations are derived for a cavity of arbitrary shape filled with a homogeneous material. The equations are solved by the moment method and data for several cavities are presented.

A quasi-analytical solution was derived in [5] for the far field scattering from a uniform resistive or impedance insert in a perfectly conducting plane. In [2], the solution was applied to a narrow gap in the conducting ground plane. Accurate results were obtained by defining the surface impedance of the gap as the input impedance of the gap modeled as a shorted transmission line. Modifications are made to the quasi-analytical solution of [5] such that it can predict the scattering from a gap in an impedance plane with a small surface impedance.

The modified quasi-analytical solution is applied to the gap in the impedance plane for gap widths which are electrically small, and the results are compared with those obtained using the coupled integral equations. The limitations are determined for which this quasi-analytical solution provides an accurate design tool.

2. THEORETICAL DEVELOPMENT OF THE INTEGRAL EQUATIONS

The gap geometry under consideration is the two-dimensional one shown in Fig. 1. The plane y = 0 is an impedance plane of infinite extent for $|x| > w/2$ with a surface impedance $\bar{\eta}$. The plane y = 0 for $|x| < w/2$ contains the aperture A of the gap cavity whose walls S are perfectly conducting. The cavity is filled with a homogeneous dielectric material of permittivity $\varepsilon_1 = \varepsilon_r \varepsilon$ and permeability $\mu_1 = \mu_r \mu$, where the quantities without subscripts refer to free space. The incident plane wave is

$$\overline{H}^i, \overline{E}^i = \hat{z}\, e^{-ik(x\cos\phi_o + y\sin\phi_o)} \tag{1}$$

for H- and E-polarizations respectively, where k is the propagation constant in the free space region above the surface. A time factor $e^{-i\omega t}$ is assumed and suppressed.

2.1 H-Polarization

The equivalence principle is applied to the gap for the region y > 0 by shorting the gap with a perfect electric conductor in A and placing a magnetic current $\overline{J}^* = -\hat{y} \times \overline{E}(x,0)$ over it. The plane y = 0 is then an impenetrable surface with mixed boundary conditions: that of a perfectly conducting surface for $|x| < w/2$ and that of an impedance surface for $|x| > w/2$. For the tangential magnetic field, the impedance boundary condition is given by [4]

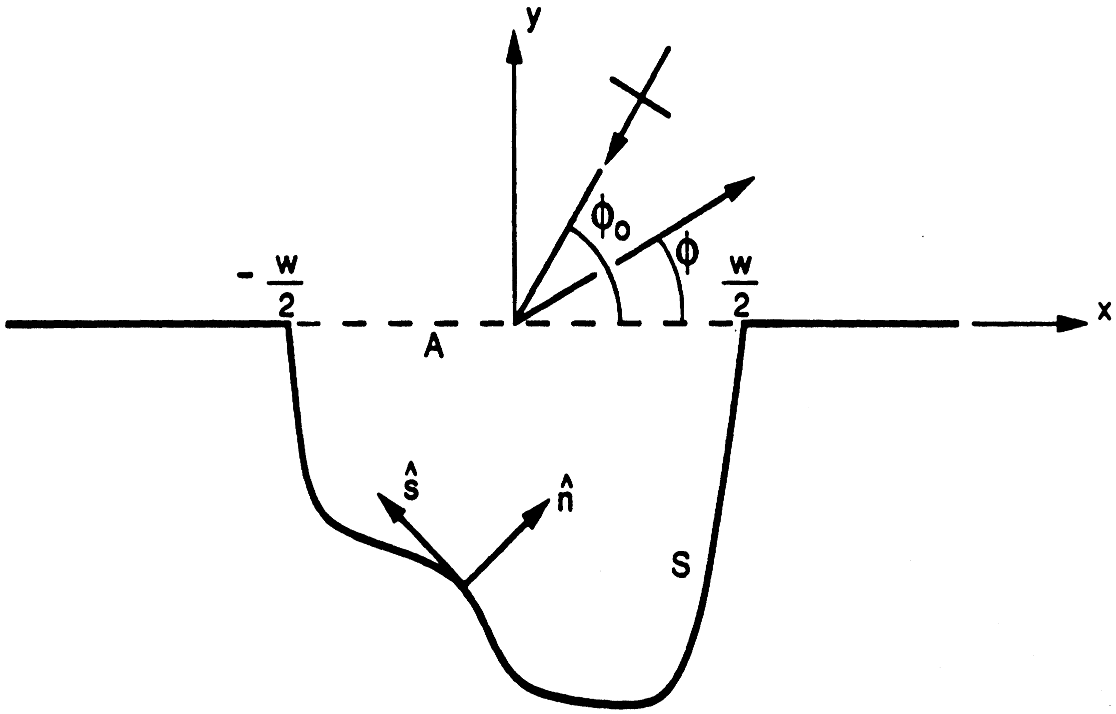

Figure 1. Narrow gap of arbitrary shape in an infinite ground plane.

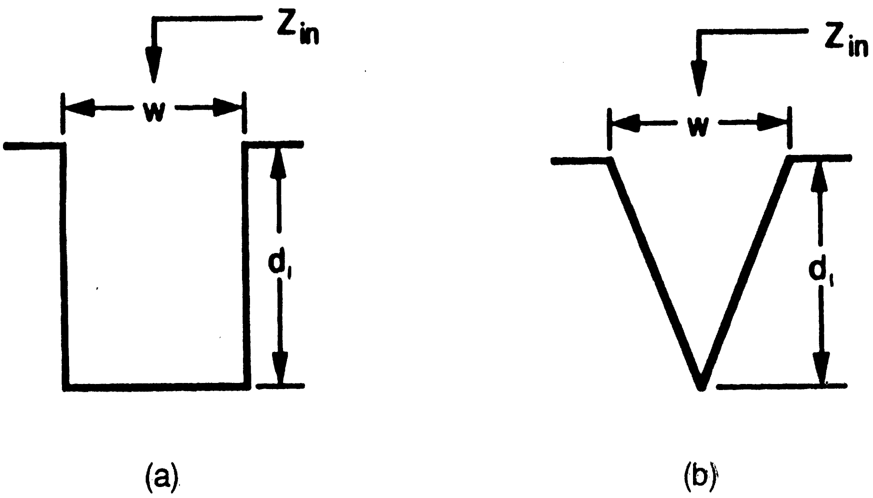

Figure 2. Rectangular and triangular gap geometries and dimensions.

$$\left(\frac{\partial}{\partial y} + \frac{ik\bar{\eta}}{Z}\right) H_z = 0 \tag{2}$$

where Z is the free space impedance. By applying Green's theorem, an integral equation for the total magnetic field can be written as

$$H_z(x,y) = H_z^i(x,y) + \int_A \left(J_x(x') \frac{\partial}{\partial y'} G(x,y;x',y') + ikYJ_z^*(x')\, G(x,y;x',0) \right)_{y' \to 0} dx'$$
$$+ \left[\int_{-\infty}^{-w/2} + \int_{w/2}^{\infty} \right] \left(H_z(x',0) \frac{\partial}{\partial y'} G(x,y;x',y') - G(x,y;x',0) \frac{\partial}{\partial y'} H_z(x',y') \right)_{y' \to 0} dx'$$

where G is the appropriate free space Green's function. This is certainly not a desirable integral equation to solve with the path of infinite extent over $|x| > w/2$. However, this path of integration over the y = 0 plane can be eliminated by applying the half space Green's function for an impedance plane, which also satisfies (2).

The half space Green's function for the impedance plane is

$$G = \frac{i}{4} \left\{ H_o^{(1)}\left(k\sqrt{(x-x')^2 + (y-y')^2} \right) - \frac{1}{\pi} \int_C \Gamma_H(\alpha)\, e^{ik[(x-x')\cos\alpha + |y+y'|\sin\alpha]} d\alpha \right\} \tag{3}$$

where $H_o^{(1)}$ is the zeroth order Hankel function of the first kind and

$$\Gamma_H(\alpha) = \frac{\bar{\eta}Y - \sin\alpha}{\bar{\eta}Y + \sin\alpha}$$

is the reflection coefficient for an H-polarized plane wave incident on an impedance boundary at the angle α. The path C is in the complex α plane, defined in Fig. 3. Note that as $\bar{\eta}$ approaches zero, (3) becomes the Green's function for a perfectly conducting plane for the H-polarization case. For the numerical solution of (3), a more quickly converging integral gives

$$G = \frac{i}{4}\left\{ H_o^{(1)}\left(k\sqrt{(x-x')^2 + (y-y')^2}\right) + H_o^{(1)}\left(k\sqrt{(x-x')^2 + (y+y')^2}\right) \right.$$

$$\left. - 2\int_0^\infty \tau_H \, e^{-\tau_H v} \, H_o^{(1)}\left(k\sqrt{(x-x')^2 + (y+y'+iv)^2}\right) dv \right\} \qquad (4)$$

where

$$\tau_H = \frac{k\bar{\eta}}{Z} \, .$$

Equation (4) was derived using a transform technique presented in [6].

For such a Green's function to be applied, the continuity of the impedance boundary condition over the entire y = 0 plane is necessary. This is accomplished by impressing a magnetic current source equivalent to $\bar{\eta}H_z$ over

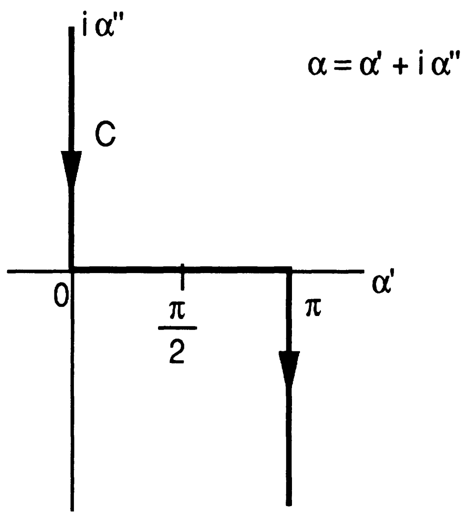

Figure 3. Path of integration in the complex α plane.

A. Applying then the half space Green's function of (4),

$$H_z(x,y) = H_z^i(x,y) - H_z^r(x,y) + ikY \int_A J_z^*(x')\, G(x,y;x',0)\, dx'$$
$$- \frac{ik}{Z} \int_A \bar{\eta} H_z(x',0)\, G(x,y;x',0)\, dx' \qquad (5)$$

where

$$H_z^r(x,y) = \frac{\bar{\eta}Y - \sin\phi_o}{\bar{\eta}Y + \sin\phi_o}\, e^{-ik(x\cos\phi_o - y\sin\phi_o)}$$

is the reflected plane wave, and $Y = 1/Z$. The second integral in (5) is the correction term for the additional impressed current , ensuring the original boundary condition for the total field at the shorted gap. That is, it removes the contribution of the scattered field due to the surface $\bar{\eta}$ over A which was not part of the original system. Observing the field in the aperture,

$$H_z(x,0) = \frac{2\sin\phi_o}{\bar{\eta}Y + \sin\phi_o}\, e^{-ikx\cos\phi_o} + ikY \int_A \left[J_z^*(x') - \bar{\eta}J_s(x')\right] G(x,0;x',0)\, dx' \qquad (6)$$

where $J_s(x') = [\hat{y} \times \bar{H}(x',0)] \cdot \hat{s}$ from (5). As shown in Fig. 1, $\hat{s}$ is tangential to the surface of the cavity wall, and in the aperture, $\hat{s} = \hat{x}$.

For the region $y < 0$ occupied by the cavity, a magnetic current $-\bar{J}^*$ is placed just below A to ensure the continuity of the tangential electric field in the open gap. The expression for the magnetic field in the shorted cavity is the same as that constructed in [2] since the region is independent of the impedance boundary. The integral equation for the currents on the cavity walls is [2,Eq. (7)],

$$J_s(s) = \frac{kY}{2}\varepsilon_r \int_A J_z^*(x')\, H_o^{(1)}\left(k_1\sqrt{(x-x')^2+y^2}\right) dx'$$

$$+\frac{ik_1}{2}\int_{S+A} J_s(s')\,\sin\beta'\, H_1^{(1)}\left(k_1\sqrt{(x-x')^2+(y-y')^2}\right) ds' \qquad (7)$$

where $k_1 = k\sqrt{\varepsilon_r\mu_r}$ and

$$\sin\beta' = \hat{z}\bullet\frac{(x-x')\,\hat{x}+(y-y')\,\hat{y}}{\sqrt{(x-x')^2+(y-y')^2}}\times\hat{s}'\ , \qquad (8)$$

valid at all points of S and A. To ensure the continuity of H_z through the aperture, the field expression for (7) [2,Eq. (9)] is matched to (6), resulting in

$$J_s(x) = \frac{2\sin\phi_o}{\overline{\eta}Y+\sin\phi_o}\, e^{-ikx\cos\phi_o} + ikY\int_A\left[J_z^*(x') - \overline{\eta}\, J_s(x')\right] G(x,0;x',0)\, dx' \qquad (9)$$

valid for x in A, and (7) and (9) constitute a pair of coupled integral equations for J_z^* and J_s.

In the far field, the large argument expression for the Green's function of (3) is found by saddle point integration, allowing C to become the path of steepest descent about $\alpha = \pi/2$. The scattered magnetic field is then given as

$$H_z^s = \sqrt{\frac{2}{\pi k\rho}}\, e^{i(k\rho-\pi/4)}\, P_H(\phi,\phi_o) \qquad (10)$$

where

$$P_H(\phi,\phi_o) = -\frac{kY}{2}\,\frac{1}{\overline{\eta}Y+1}\int_A\left[J_z^*(x') - \overline{\eta}J_s(x')\right] e^{-ikx'\cos\phi}\, dx' \qquad (11)$$

Thus the far field amplitude is given for the currents J_z^* and J_s over A determined from (7) and (9).

2.2 E-Polarization

The impedance boundary condition for the total tangential electric field is [4]

$$\left(\frac{\partial}{\partial y} + i\frac{kZ}{\bar{\eta}}\right) E_z = 0 \tag{12}$$

The half-space Green's function for an impedance plane with an E-polarized incident plane wave satisfying the same boundary condition is

$$G = \frac{i}{4}\left\{ H_o^{(1)}\left(k\sqrt{(x-x')^2 + (y-y')^2}\right) + \frac{1}{\pi}\int_C \Gamma_E(\alpha)\, e^{ik[(x-x')\cos\alpha + |y+y'|\sin\alpha]}\, d\alpha \right\} \tag{13}$$

where

$$\Gamma_E(\alpha) = \frac{\bar{\eta}Y - \csc\alpha}{\bar{\eta}Y + \csc\alpha} .$$

Note again that as $\bar{\eta}$ approaches zero, (13) becomes the Green's function for a perfectly conducting plane under E-polarized illumination. For a quickly convergent integral, the same expression is used as that given in (4), with τ_H replaced with

$$\tau_E = \frac{kZ}{\bar{\eta}} .$$

Using the same approach as for the H-polarization case, the total field for the region $y > 0$ is

$$E_z(x,y) = E_z^i(x,y) + E_z^r(x,y) - \int_A J_x^*(x') \frac{\partial}{\partial y'} G(x,y;x',y')\Big|_{y'\to 0} dx'$$

$$- \int_A E_z(x',0) \frac{\partial}{\partial y'} G(x,y;x',y')\Big|_{y'\to 0} dx' \tag{14}$$

where J_x^* is the assumed equivalent magnetic current on A and

$$E_z^r(x,y) = \frac{\bar{\eta}Y - \csc\phi_o}{\bar{\eta}Y + \csc\phi_o} e^{-ik(x\cos\phi_o - y\sin\phi_o)}$$

is the reflected plane wave. The second integral in (14) accounts for the scattering due to the added impressed current equivalent to $E_z(x',0)$ for the impedance plane. The dependence of this current on the surface impedance is shown more explicitly using [4] the boundary condition $E_z = -\bar{\eta}H_x$. From $H_x = -\frac{iY}{k}\frac{\partial E_z}{\partial y}$, the tangential component of the magnetic field in the aperture is

$$H_x(x,0) = -\frac{2Y}{\bar{\eta}Y + \csc\phi_o} e^{-ikx\cos\phi_o}$$

$$+ \frac{iY}{k} \lim_{y\to 0} \frac{\partial}{\partial y} \int_A \left[J_x^*(x') + \bar{\eta}J_z(x')\right] \frac{\partial}{\partial y'} G(x,y;x',y')\Big|_{y'\to 0} dx' \; . \qquad (15)$$

In the region y < 0 occupied by the cavity, the field source is the equivalent magnetic current J_x^* in the aperture and the contribution from the electric current J_z on the walls of the cavity. The integral equation constructed for the currents is [2,Eq. (15)]

$$J_z(s) = \frac{Y}{2k\mu_r}(\hat{n}\bullet\nabla)\frac{\partial}{\partial y}\int_A J_x^*(x')\, H_o^{(1)}\left(k_1\sqrt{(x-x')^2 + y^2}\right) dx'$$

$$+ \frac{ik_1}{2}\int_{S+A} J_z(s')\,\sin\beta\, H_1^{(1)}\left(k_1\sqrt{(x-x')^2 + (y-y')^2}\right) ds' \qquad (16)$$

where

$$\sin\beta = \hat{z}\bullet\frac{(x-x')\,\hat{x} + (y-y')\,\hat{y}}{\sqrt{(x-x')^2 + (y-y')^2}} \times\hat{s} \qquad (17)$$

valid at all points of S and A. When (15) is matched to the field expression for (16) with the observation point in the aperture,

$$J_z(x) = -\frac{2\,Y}{\bar{\eta}Y + \csc\phi_o}\, e^{-ikx\cos\phi_o}$$

$$+ \frac{iY}{k} \lim_{y\to 0} \frac{\partial}{\partial y} \int_A \left[J_x^*(x') + \bar{\eta}J_z(x')\right] \frac{\partial}{\partial y'} G(x,y;x',y')\Big|_{y'\to 0} dx' \qquad (18)$$

for x in A. Equations (16) and (18) are a pair of coupled integral equations for the currents J_x^* and J_z.

A third integral equation was developed in [2] for the electric field in the cavity of the gap. When the boundary condition on the perfectly conducting surface was applied, the expression for the currents on A and S came to be

$$J_x^*(x) = \frac{i}{2} \int_A J_x^*(x') \frac{\partial}{\partial y} H_o^{(1)}\left(k_1\sqrt{(x-x')^2 + y^2}\right) dx'$$

$$+ \frac{kZ\mu_r}{2} \int_{S+A} J_z(s')\, H_o^{(1)}\left(k_1\sqrt{(x-x')^2\ (y-y')^2}\right) ds' \qquad (19)$$

where $J_x^*(x)$ is non-zero only in A. Equations (18) and (19) are the pair of integral equations used to compute J_x^* and J_z. The scattering in the far field for E-polarization has a similar expression as that in (10), giving the far field amplitude

$$P_E(\phi,\phi_o) = -\frac{k}{2}\, \frac{\sin\phi}{\bar{\eta}Y + 1} \int_A \left[J_x^*(x') + \bar{\eta}J_z(x')\right] e^{-ikx'\cos\phi}\, dx' . \qquad (20)$$

3. APPLICATION OF THE QUASI-ANALYTICAL SOLUTION

Consider an impedance insert of width w and characterized by η in an impedance plane $\bar{\eta}$. For the H-polarization case, as in Section 2.1, the application of Green's theorem for the region y > 0 gives

$$H_z(x,y) = H_z^i(x,y) - H_z^r(x,y) + \frac{ik}{Z} \int_{-w/2}^{w/2} (\eta - \bar{\eta})\, H_z(x',0)\, G(x,y;x',0)\, dx' \tag{21}$$

where the incident and reflected fields are those described in (5) and G is the Green's function of (4), satisfying (2) in the plane y = 0. The integral of (21) is the scattering due to the impedance insert, and this clearly vanishes for $\eta = \bar{\eta}$.

The Green's function of (4) can be written in the form, for y' = 0,

$$G(x,y;x',0) = \frac{i}{2} H_o^{(1)}\left(k\sqrt{(x-x')^2 + y^2}\right) g(x,y;x',0|\bar{\eta}) \tag{22}$$

where the normalized half space impedance Green's function is

$$g(x,y;x',0|\bar{\eta}) = \left[1 - \frac{L_H(x,y;x',0|\bar{\eta})}{H_o^{(1)}\left(k\sqrt{(x-x')^2+y^2}\right)} \right]$$

for which the function L_H is the integral of (4). We note that as $\bar{\eta}$ approaches zero, L_H approaches zero, and as $\bar{\eta}$ approaches infinity, L_H approaches $H_o^{(1)}$. Thus |g| varies from one to zero for these limits. To remove the coordinate dependence, g is averaged over -w/2 < x < w/2, for x' and y set to zero. By curve fitting the numerically generated magnitude and phase, this average is found to be approximated analytically to within 5 percent as

$$g_H(w|\bar{\eta}) = e^{-c(w)\,\bar{\eta}/Z}\, e^{i\,\psi(w|\bar{\eta})} \tag{23}$$

where

$$c(w) = 0.245 + 1.267 w/\lambda$$

$$\psi(w|\bar{\eta}) = 1 - e^{-(0.098+1.760 w/\lambda)\,\bar{\eta}/Z}$$

for $0.02 < w/\lambda < 0.15$ and $\bar{\eta}/Z < 2$.

Now taking the observation point to be on the insert, (21) can be approximated by

$$H_z(x,0) = \frac{2\sin\phi_o}{\bar{\eta}Y + \sin\phi_o} e^{-ikx\cos\phi_o}$$

$$- \frac{k}{2Z}(\eta - \bar{\eta})\, g_H(w|\bar{\eta}) \int_{-w/2}^{w/2} H_z(x',0)\, H_o^{(1)}(k|x-x'|)\, dx' \qquad (24)$$

where the averaging of g has enabled us to remove the $\bar{\eta}$ dependence from the integral. The assumption that g is constant over w should hold for small w and small $\bar{\eta}$. The integral equation (24) can be written in the form [5]

$$-\frac{i}{2}\int_{-1}^{1} J_1(\zeta')\, H_o^{(1)}\left(\frac{kw}{2}|\zeta-\zeta'|\right) d\zeta' = e^{-i\frac{kw}{2}\zeta\cos\phi_o} + aJ_1(\zeta) \qquad (25)$$

where

$$J_1(\zeta) = \frac{ikw}{2}\,\frac{\eta - \bar{\eta}}{Z}\,\frac{\bar{\eta}Y + \sin\phi_o}{2\sin\phi_o}\, g_H(w|\bar{\eta})\, H_z(x,0)$$

and

$$a = \frac{2i}{kw}\,\frac{Z}{\eta - \bar{\eta}}\,\frac{1}{g_H(w|\bar{\eta})}\,. \qquad (26)$$

Referring to (11) and the integral of (21), the far field amplitude written in terms of J_1 is

$$P_H(\phi,\phi_o) = i\,\frac{1}{\bar{\eta}Y + 1}\,\frac{1}{g_H(w|\bar{\eta})}\,\frac{\sin\phi_o}{\bar{\eta}Y + \sin\phi_o}\int_{-1}^{1} J_1(\zeta')\, e^{-i\frac{kw}{2}\zeta'\cos\phi_o}\, d\zeta'\,. \qquad (27)$$

A simplification is made using [5]

$$J_1(\zeta) = \left[1 + \frac{iA\, g_H(w|\bar{\eta})}{\pi}(\bar{\eta}Y + 1)\,\frac{\bar{\eta}Y + \sin\phi_o}{\sin\phi_o}\, P_H(\phi,\phi_o)\right] J_2(\zeta)$$

where the integral equation that the modified current J_2 satisfies is [2,5]

$$\frac{1}{\pi}\int_{-1}^{1} J_2(\zeta') \, \ell n|\zeta-\zeta'| \, d\zeta' = 1 + aJ_2(\zeta)$$

for $-1 < \zeta < 1$ and

$$A = \ell n \frac{kw}{4} + \gamma - i\frac{\pi}{2}$$

where $\gamma = 0.5772157....$ is Euler's constant. The far field amplitude (27) becomes

$$P_H(\phi,\phi_o) = i\pi \frac{1}{\bar{\eta}Y + 1} \frac{1}{g_H(w|\bar{\eta})} \frac{\sin\phi_o}{\bar{\eta}Y + \sin\phi_o}\left[A + \frac{1}{K_H(a)}\right]^{-1} \tag{28}$$

with

$$K_H(a) = \frac{1}{\pi}\int_{-1}^{1} J_2(\zeta) \, d\zeta \ ,$$

for which an approximate expression is [2]

$$K_H(a) = -\frac{1}{\frac{\pi a}{2} + \ell n \, 2 + 0.1} \tag{29}$$

with a given in (26). Thus with the modifications to a and $P_{H,}$ the same quasi-analytical expressions are used to solve for the scattering of the impedance insert in an impedance plane.

Similarly, for E-polarization, the total field for the region $y > 0$ is

$$E_z(x,y) = E_z^i(x,y) + E_z^r(x,y)$$
$$+ \int_{-w/2}^{w/2} \left[E_z(x',0|\eta) - E_z(x',0|\bar{\eta})\right] \frac{\partial}{\partial y'} G(x,y;x',y')\Big|_{y'\to 0} dx' \ , \tag{30}$$

and the correlation of the field with the respective impedance surface is shown explicitly. The corresponding expression for the tangential magnetic field over the insert is

$$H_x(x,0) = -\frac{2\,Y}{\bar{\eta}Y + \csc\phi_o}\, e^{-ikx\cos\phi_o}$$

$$- \frac{i}{kZ} \lim_{y\to 0} \frac{\partial}{\partial y} \int_{-w/2}^{w/2} (\eta - \bar{\eta})\, H_x(x',0)\, \frac{\partial}{\partial y'} G(x,y;x',y')\Big|_{y'\to 0} dx' \qquad (31)$$

where G is (13). When (13) is expressed in the form of (4), the partial derivatives in (31) render the source and image Hankel functions zero for y' = 0, and G is given by (22) with

$$g(x,y;x',0|\bar{\eta}) = \frac{L_E\,(x,y;x',0|\bar{\eta})}{H_o^{(1)}\left(k\sqrt{(x-x')^2+y^2}\right)}$$

where L_E is the integral of (4) with τ_H replaced with τ_E . For this case we note that as $\bar{\eta}$ approaches zero, L_E approaches $H_o^{(1)}$, and as $\bar{\eta}$ approaches infinity, L_E approaches zero. Thus |g| varies from one to zero for these limits. The average over w of the normalized half space impedance Green's function is approximated analytically within 10 percent as

$$g_E(w|\bar{\eta}) = e^{-d(w)\,\bar{\eta}/Z}\, e^{i\,\vartheta(w|\bar{\eta})} \qquad (32)$$

where

$$d(w) = 1.558 - 4.226w/\lambda$$

$$\vartheta(w|\bar{\eta}) = -\,(0.380 - 0.80w/\lambda)\left(1 - e^{-2.586\sqrt{\bar{\eta}/Z}}\right)$$

for $0.025 < w/\lambda < 0.15$ and $\bar{\eta}/Z < 2$.

The tangential magnetic field on the insert is then approximated as

$$H_x(x,0) = -\frac{2Y}{\bar{\eta}Y + \csc\phi_o} e^{-ikx\cos\phi_o}$$

$$-\frac{i}{kZ}(\eta - \bar{\eta})\, g_E(w|\bar{\eta}) \left(k^2 + \frac{\partial^2}{\partial x^2}\right) \int_{-w/2}^{w/2} H_x(x',0)\, H_o^{(1)}(k\,|x-x'|)\, dx' \,. \quad (33)$$

Given this approximation, (33) can be equated to [5]

$$\left[\frac{\partial^2}{\partial\zeta^2} + \left(\frac{kw}{2}\right)^2\right] \frac{1}{2i} \int_{-1}^{1} J_3(\zeta')\, H_o^{(1)}\left(\frac{kw}{2}|\zeta-\zeta'|\right) d\zeta' = e^{-i\frac{kw}{2}\zeta\cos\phi_o} - bJ_3(\zeta)$$

with

$$J_3(\zeta) = -\frac{2i}{kw}\,\frac{\eta - \bar{\eta}}{Z}\,\frac{\bar{\eta}Y + \csc\phi_o}{2Y}\, g_E(w|\bar{\eta})\, H_x(x,0) \quad (34)$$

and

$$b = -\frac{ikw}{2}\,\frac{Z}{\eta - \bar{\eta}}\,\frac{1}{g_E(w|\bar{\eta})}\,. \quad (35)$$

Referring to (20) and using (32), the far field amplitude for the E-polarization case is

$$P_E(\phi,\phi_o) = -\frac{i\pi}{4}(kw)^2 \frac{\sin\phi}{\bar{\eta}Y + 1}\,\frac{1}{\bar{\eta}Y + \csc\phi_o}\,\frac{1}{g_E(w|\bar{\eta})}\, K_E(b) \quad (36)$$

with K_E approximated by [2,5]

$$K_E(b) = \frac{0.62}{b + 1.15}\,\frac{(b + 4.08)(b + 7.26)(b + 10.37)(b + 13.43)(b + 16.46)}{(b + 4.27)(b + 7.37)(b + 10.45)(b + 13.49)(b + 16.50)}\,. \quad (37)$$

As in [2], the analogy is drawn from the impedance insert to the narrow gap by equating the surface impedance η to the input impedance of the gap modeled as a transmission line. The expressions for various gap and cavity configurations are contained in [2], and the far field amplitude P_H and P_E can then be calculated accordingly by (28) and (36), respectively.

4. NUMERICAL RESULTS

The integral equation pairs (7), (9) and (18), (19) for H- and E-polarizations respectively were programmed for solution by the method of moments, using pulse basis and point matching functions. The half space impedance Green's function of (4) and its derivatives were evaluated analytically in handling the singularities of the integration and numerically otherwise as described in Appendix A. The application of the method of moments is described in Appendix B, which also contains the program listing used for generating the results. The quasi-analytical expressions of Section 3 were programmed for solution, as listed in Appendix C. The upper limit of $w/\lambda = 0.15$ was determined for the applicability of the quasi-analytical solution in [2], and this limit is maintained for the results listed here.

In Figs. 4 and 5 the magnitude and phase of the far field amplitude $P_H(\pi/2, \pi/2)$ are shown as a function of depth for a rectangular air-filled gap of width $w/\lambda = 0.15$, comparing the method of moments (MoM) and quasi-analytical (QA) solutions. For each method it was verified that as $\overline{\eta}$ approaches zero, the results approach that of the gap in a perfectly conducting plane in [2]. For $\overline{\eta}/Z = 0.1$ and 0.5 the difference between the peak amplitudes of each method are within 12 percent, and the phase curves show excellent agreement. As expected, $|P_H|$ is non-zero as d approaches zero, corresponding to the scattering from a perfectly conducting strip in an impedance plane. Consistent results were verified for $\overline{\eta}/Z = 0.5$ and w/λ as small as 0.025. As observed in [2], a cyclical behavior exists with increasing gap depth resulting from the periodicity of the impedance looking into the gap.

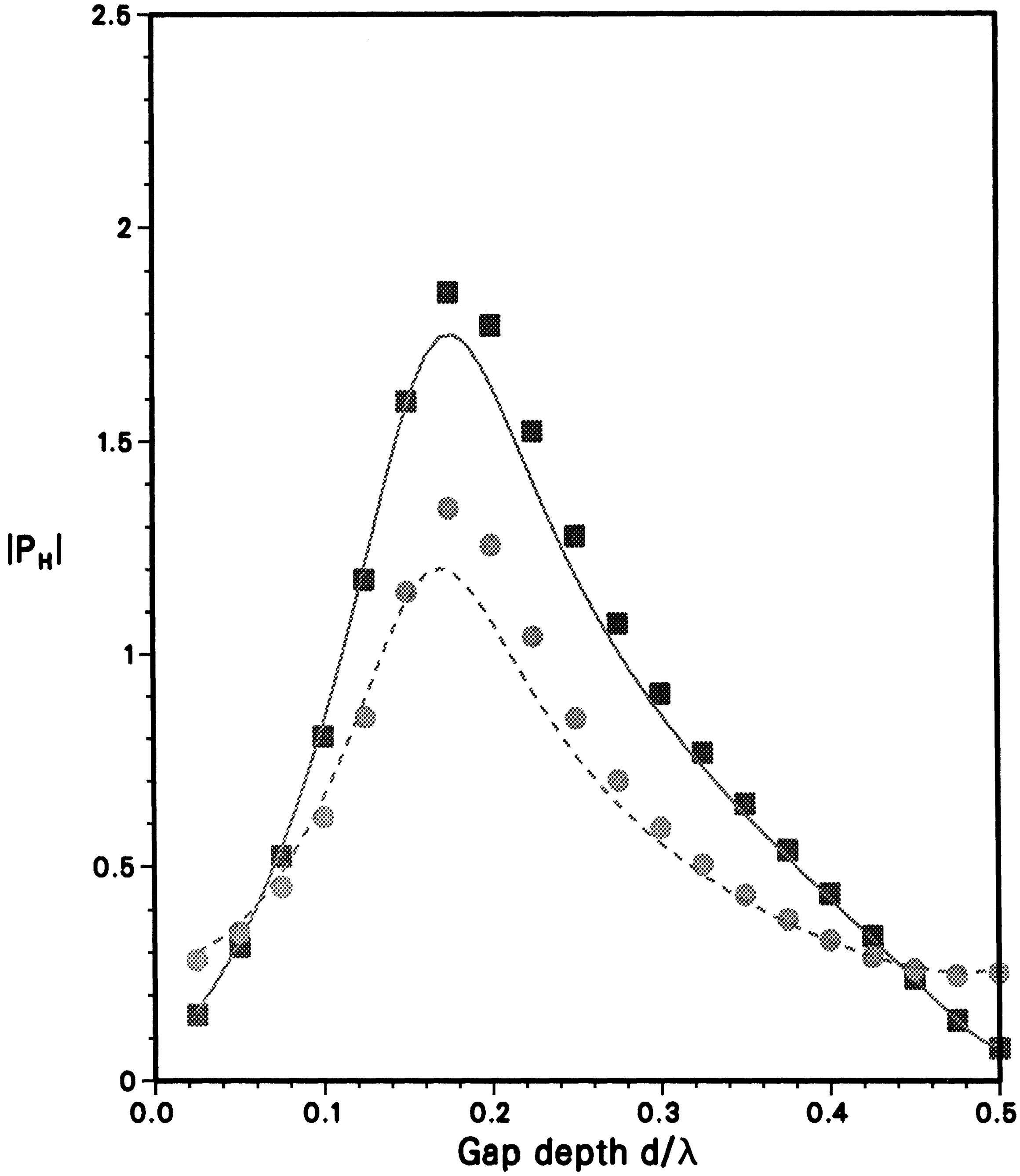

Figure 4. Modulus of the far field amplitude P_H for a rectangular gap of varying depth $d_1 = d$ with $\phi = \phi_o = \pi/2$ and $w/\lambda = 0.15$:

$\bar{\eta}/Z = 0.1$ ■ MoM, ——— QA
$\bar{\eta}/Z = 0.5$ ● MoM, - - - - QA

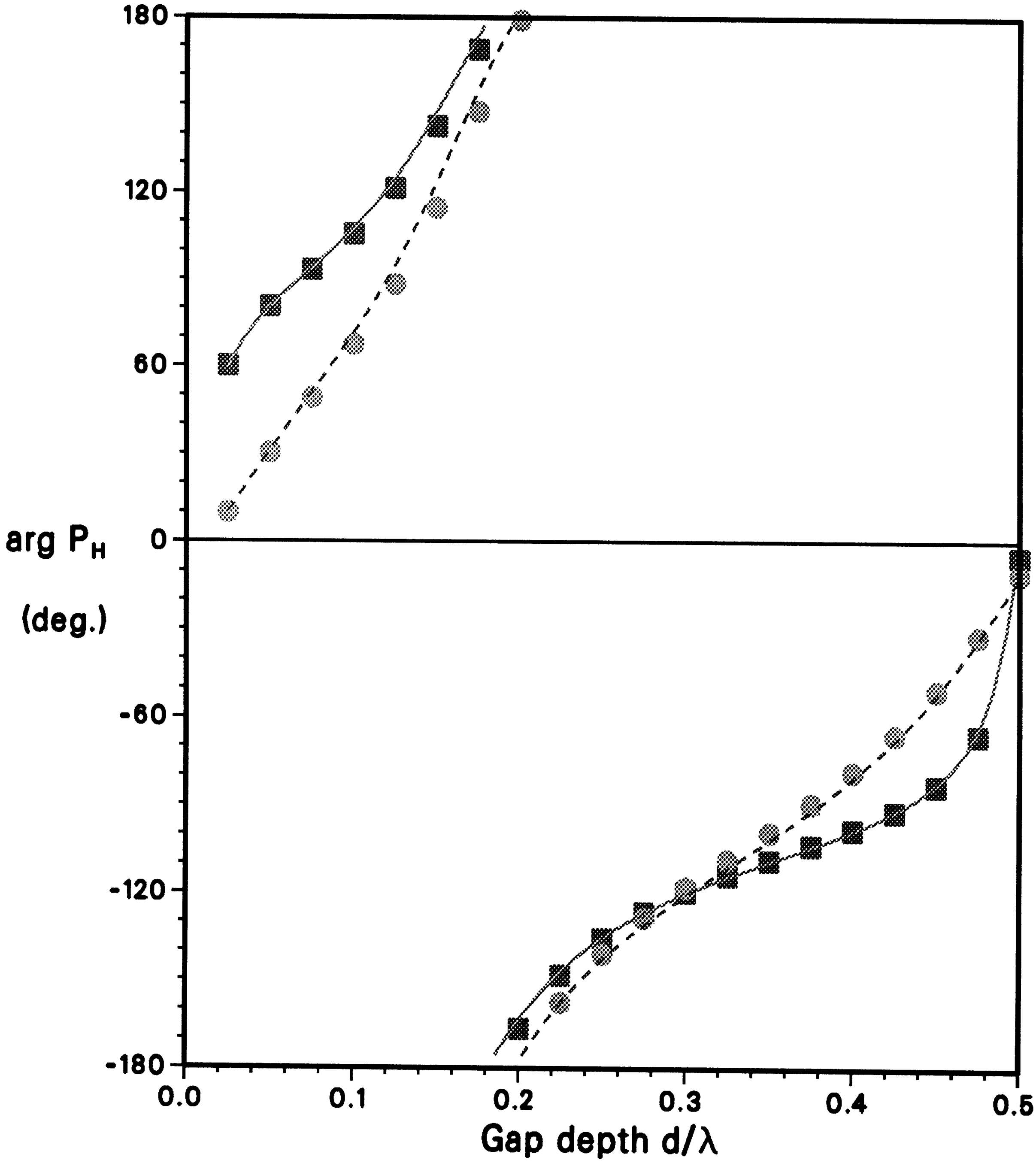

Figure 5. Argument of the far field amplitude P_H for a rectangular gap of varying depth $d_1 = d$ with $\phi = \phi_o = \pi/2$ and $w/\lambda = 0.15$:

$\overline{\eta}/Z = 0.1$ ■ MoM, ——— QA
$\overline{\eta}/Z = 0.5$ ● MoM, - - - - QA

The far field amplitude backscatter response to the rectangular gap with $d/\lambda = 0.15$ is contained in Fig. 6, showing excellent agreement between the MoM and QA solutions for all ϕ. Figure 7(a) shows the far field amplitude of the rectangular gap filled with a lossless dielectric having $\varepsilon_r = 2.5$ for $\overline{\eta}/Z = 0.5$ and 1.0. It was observed that as the relative permittivity of the gap filling was increased from 1, the prediction by the QA method improved, bringing the difference at the peaks within 14 percent for the $\overline{\eta}/Z = 1.0$ curve shown in the figure. The agreement is improved when some loss is introduced in the dielectric filling, as shown in Fig. 7(b) for $\varepsilon_r = 3 + i0.5$. For the V-shaped gap of Fig. 2(b), the far field amplitude is presented in Fig. 8 for varying depth. Similar results are expected for gaps of arbitrary geometries, given the appropriate surface impedance η of the gap necessary for the QA solution.

For the E-polarization case, the magnitude and phase are plotted in Figs. 9 and 10 for the rectangular air-filled gap for $\overline{\eta}/Z = 0.1$ and 0.3. The curves reveal the evanescent nature of the fields in the gap cavity, giving constant values for $d/\lambda > 0.1$. In Fig. 9, the greatest deviation occurs as d approaches zero, for which the MoM predicts a decrease in magnitude. For $\overline{\eta}/Z = 0.3$, the difference between the two methods is within 14 percent. The QA method breaks down then for $\overline{\eta}/Z > 0.4$ since the amplitude continues to increase for all d. Figure 10 shows that the phase information is lost in using the QA method. A plot of the backscatter from an air-filled rectangular gap with $d_1/\lambda = 0.2$ is contained in Fig. 11, and Fig. 12 shows the far field amplitude of the scattering from an air-filled V-shaped gap.

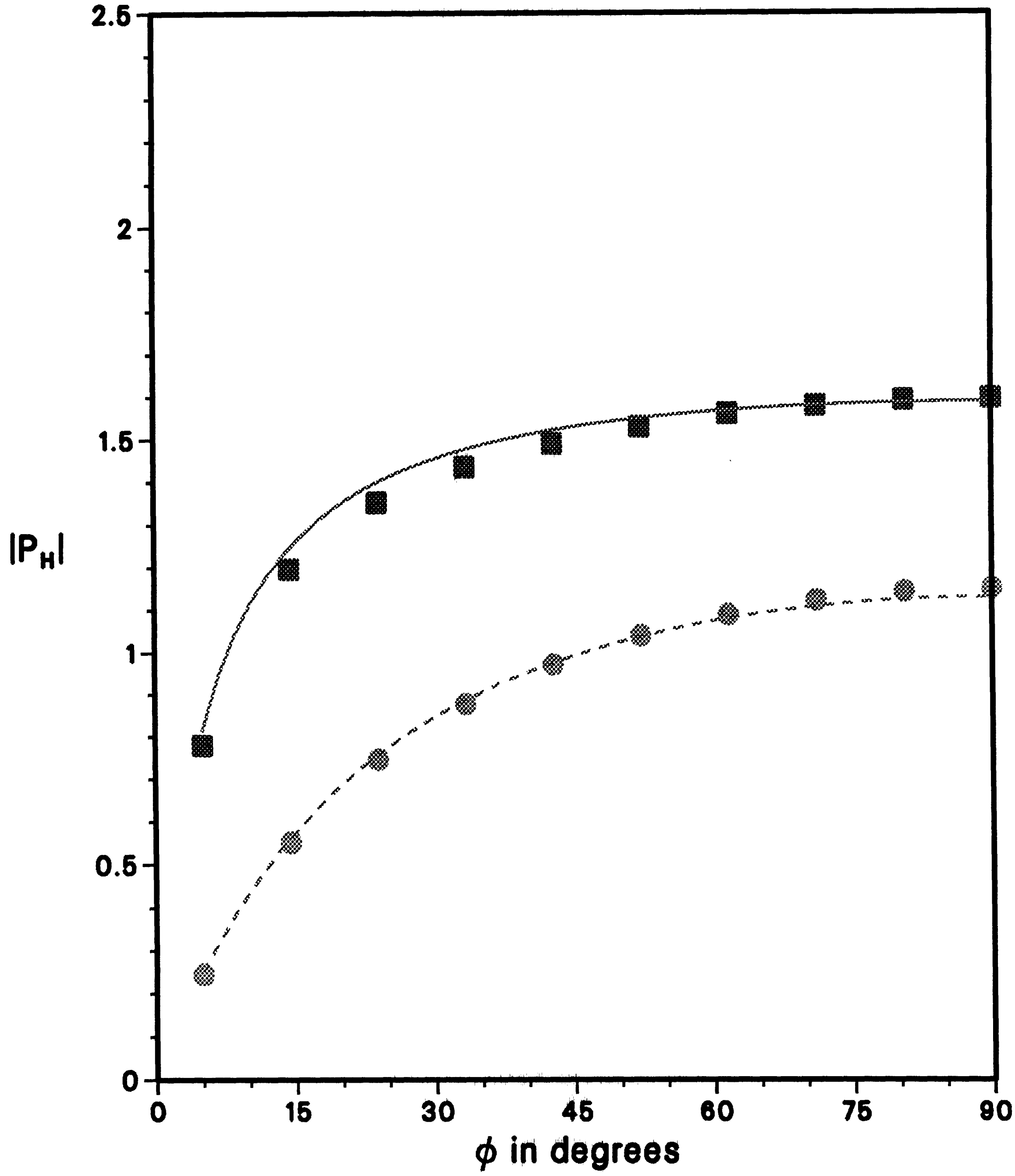

Figure 6. Modulus of the far field amplitude P_H for the backscatter from a rectangular gap of depth $d_1/\lambda = 0.15$ with $\phi_o = \phi$, $w/\lambda = 0.15$:

$\overline{\eta}/Z = 0.1$ ■ MoM, ——— QA
$\overline{\eta}/Z = 0.5$ ● MoM, - - - - QA

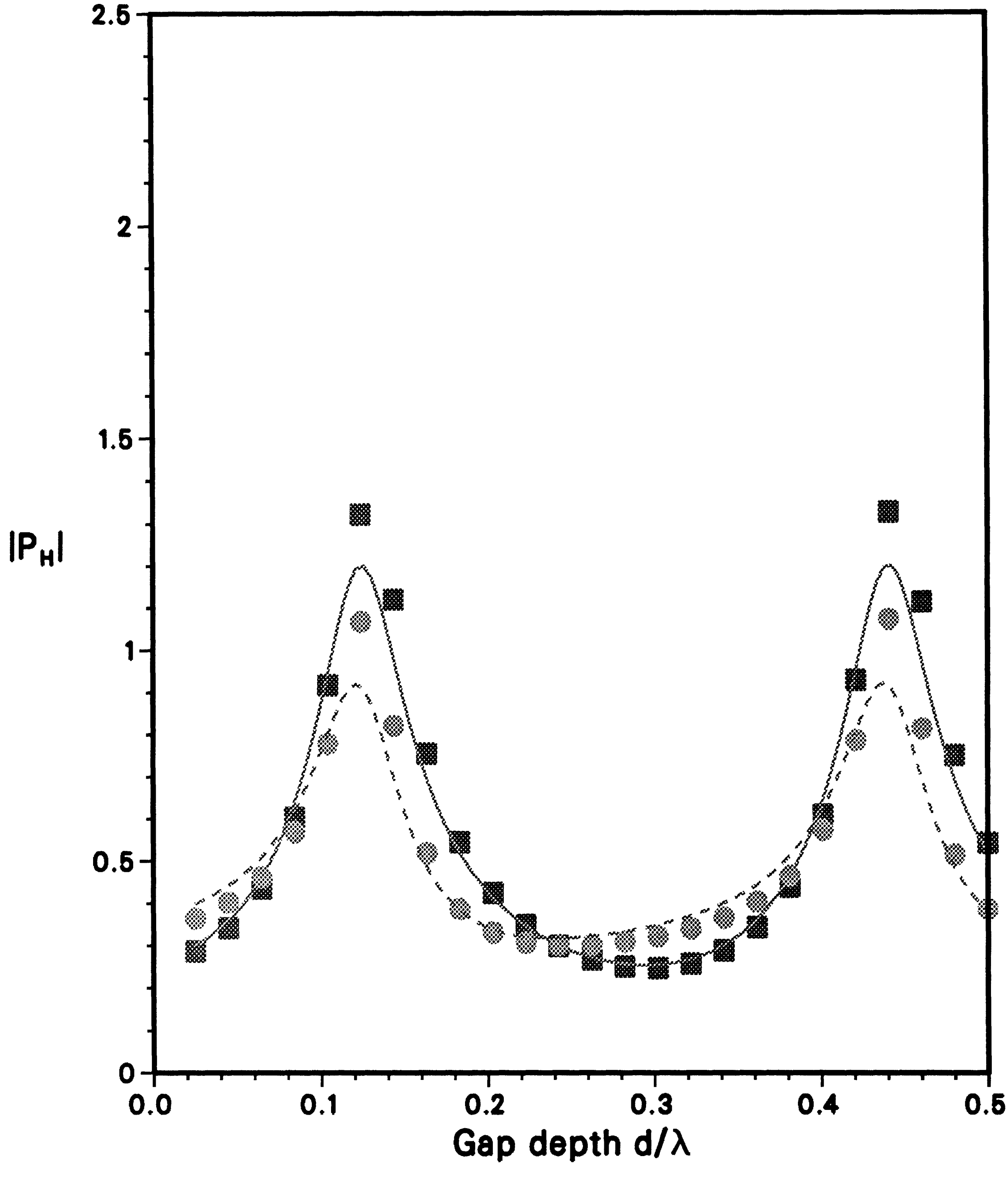

Figure 7(a). Molulus of the far field amplitude P_H for the material-filled rectangular gap of varying depth $d_1 = d$ with $\varepsilon_r = 2.5$, $\mu_r = 1$, $\phi = \phi_o = \pi/2$, and $w/\lambda = 0.15$:

$\bar{\eta}/Z = 0.5$	■	MoM, ——— QA
$\bar{\eta}/Z = 1.0$	●	MoM, - - - - QA

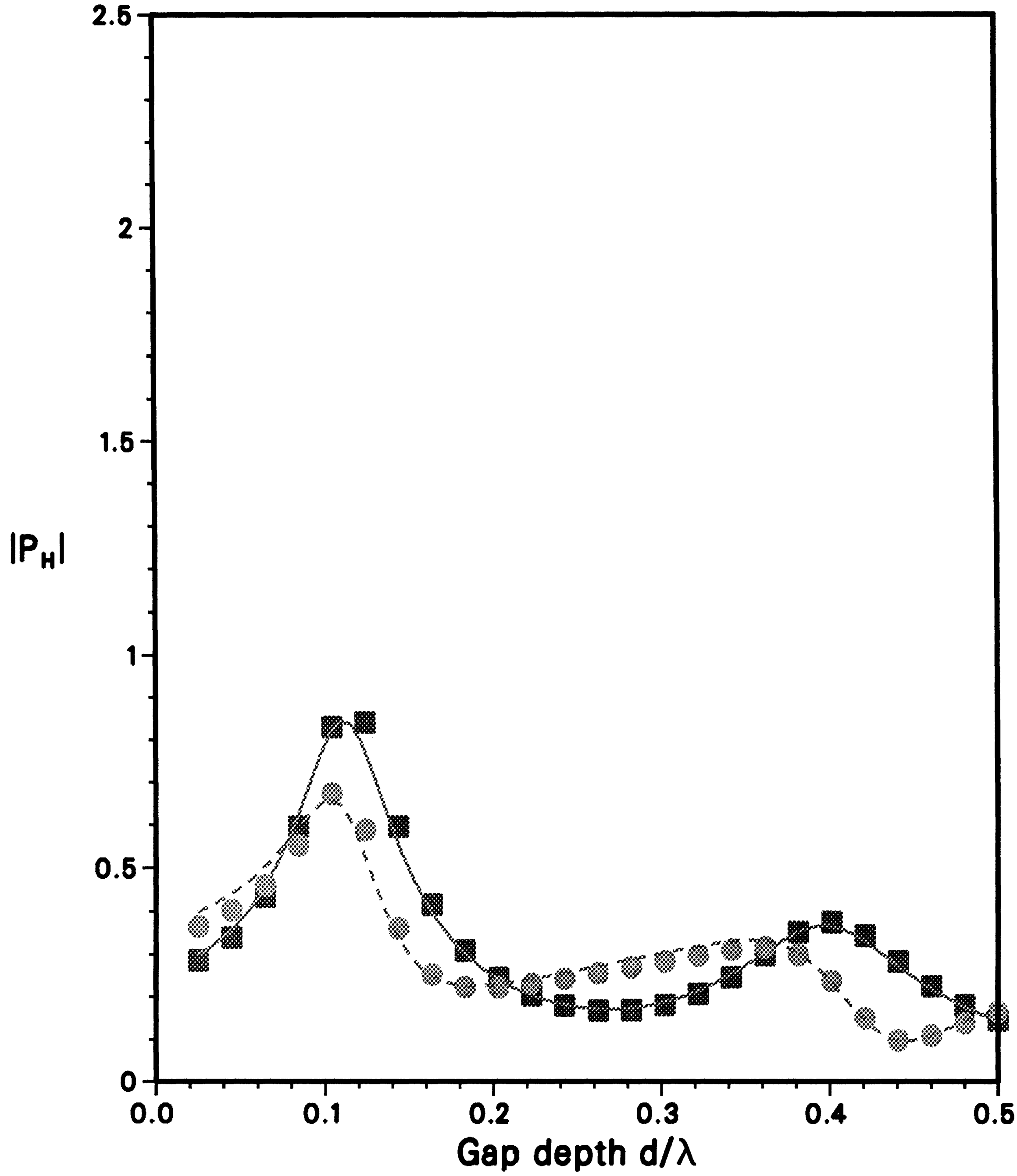

Figure 7(b). Modulus of the far field amplitude P_H for the material-filled rectangular gap of varying depth $d_1 = d$ with $\varepsilon_r = 3 + i0.5$, $\mu_r = 1$, $\phi = \phi_o = \pi/2$, and $w/\lambda = 0.15$:

$\bar{\eta}/Z = 0.5$ ■ MoM, ——— QA
$\bar{\eta}/Z = 1.0$ ● MoM, - - - - QA

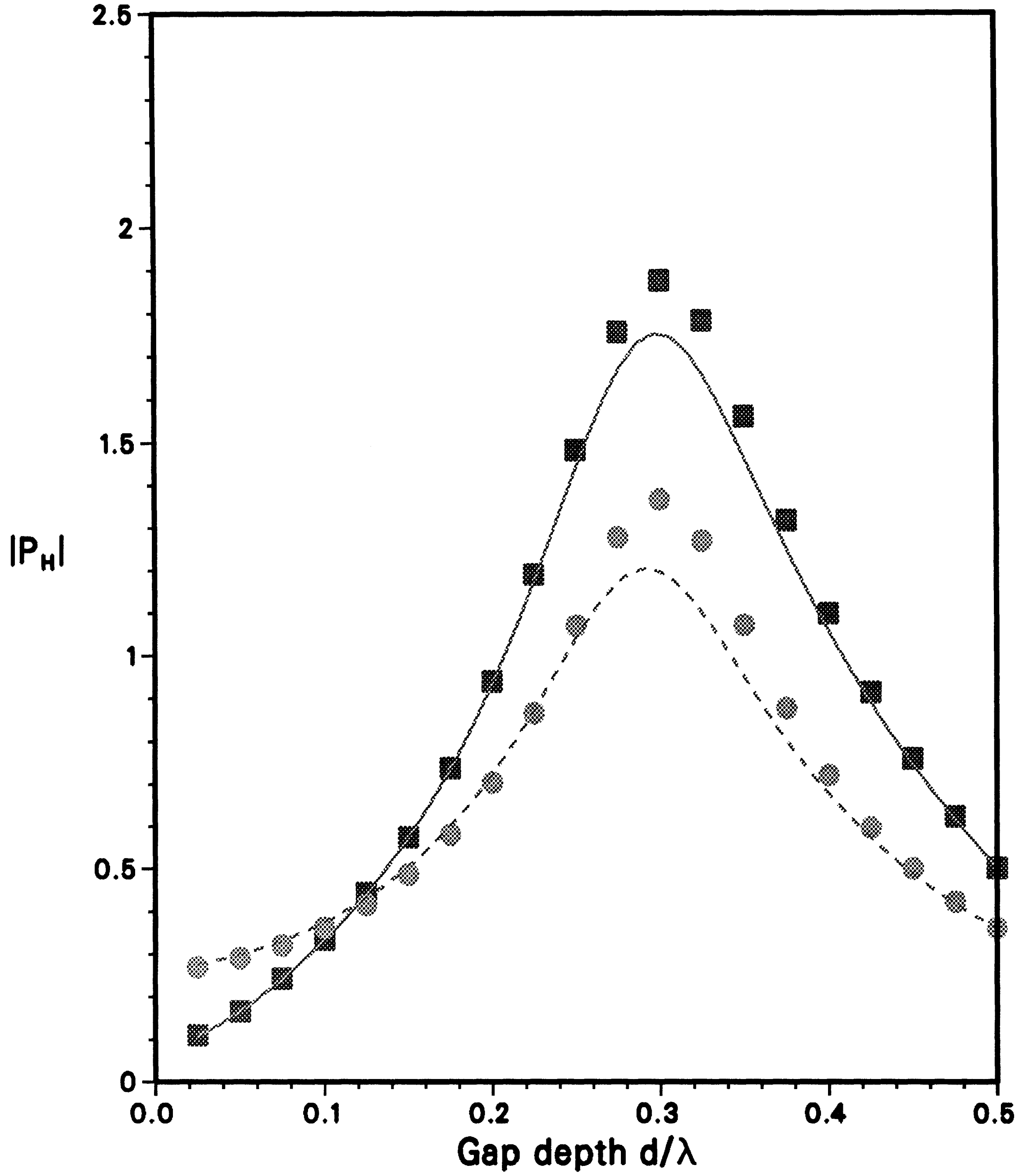

Figure 8. Modulus of the far field amplitude P_H for an air-filled V-shaped gap of varying depth $d_1 = d$ with $\phi = \phi_o = \pi/2$ and $w/\lambda = 0.15$:

$\bar{\eta}/Z = 0.1$ ■ MoM, ——— QA
$\bar{\eta}/Z = 0.5$ ● MoM, - - - - QA

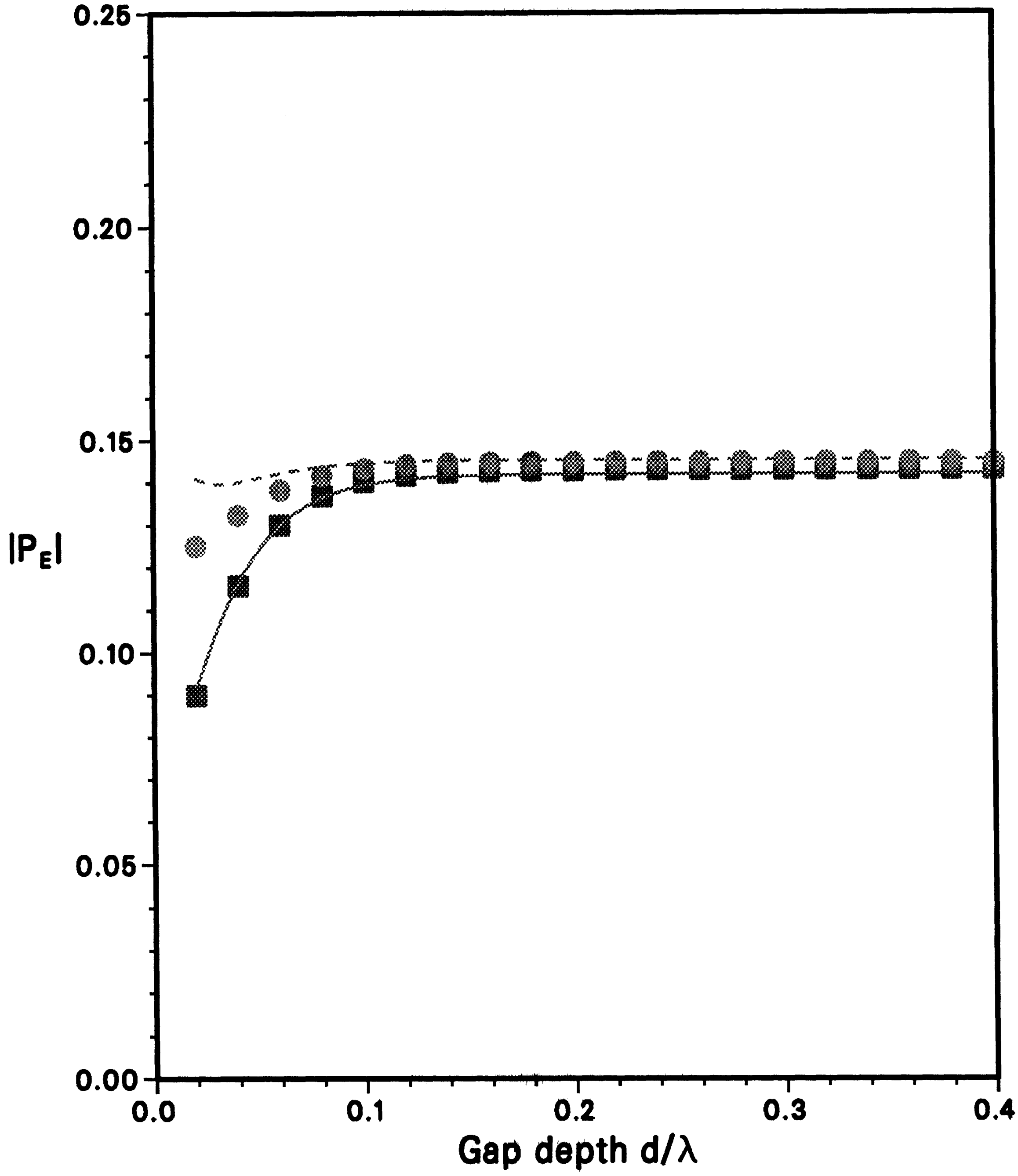

Figure 9. Modulus of the far field amplitude P_E for an air-filled rectangular gap of varying depth $d_1 = d$ with $\phi = \phi_o = \pi/2$ and $w/\lambda = 0.15$:

$\bar{\eta}/Z = 0.1$ ■ MoM, ——— QA
$\bar{\eta}/Z = 0.3$ ● MoM, - - - - QA

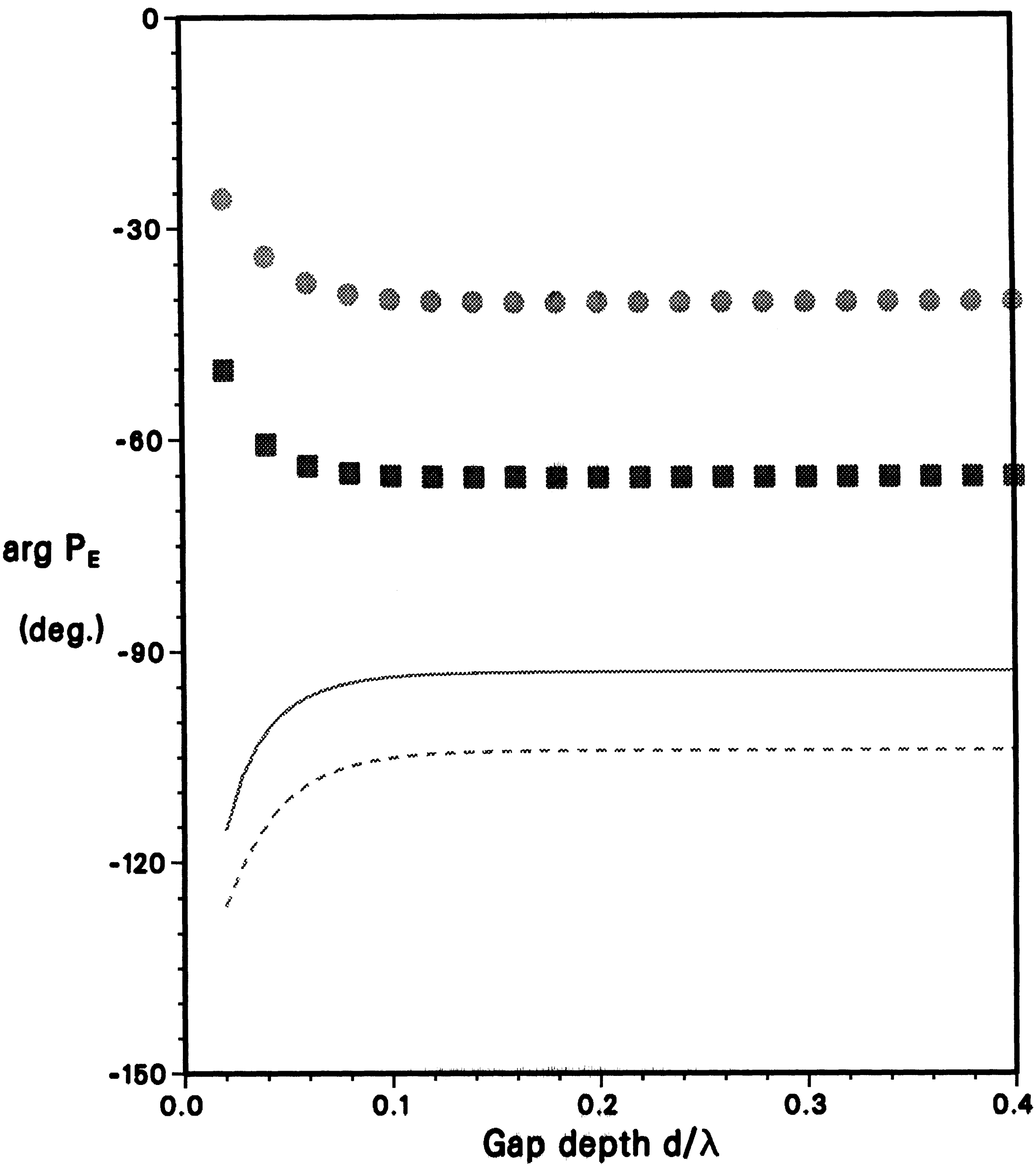

Figure 10. Argument of the far field amplitude P_E for an air-filled rectangular gap of varying depth $d_1 = d$ with $\phi = \phi_o = \pi/2$ and $w/\lambda = 0.15$:

$\overline{\eta}/Z = 0.1$ ■ MoM, ——— QA
$\overline{\eta}/Z = 0.3$ ● MoM, - - - - QA

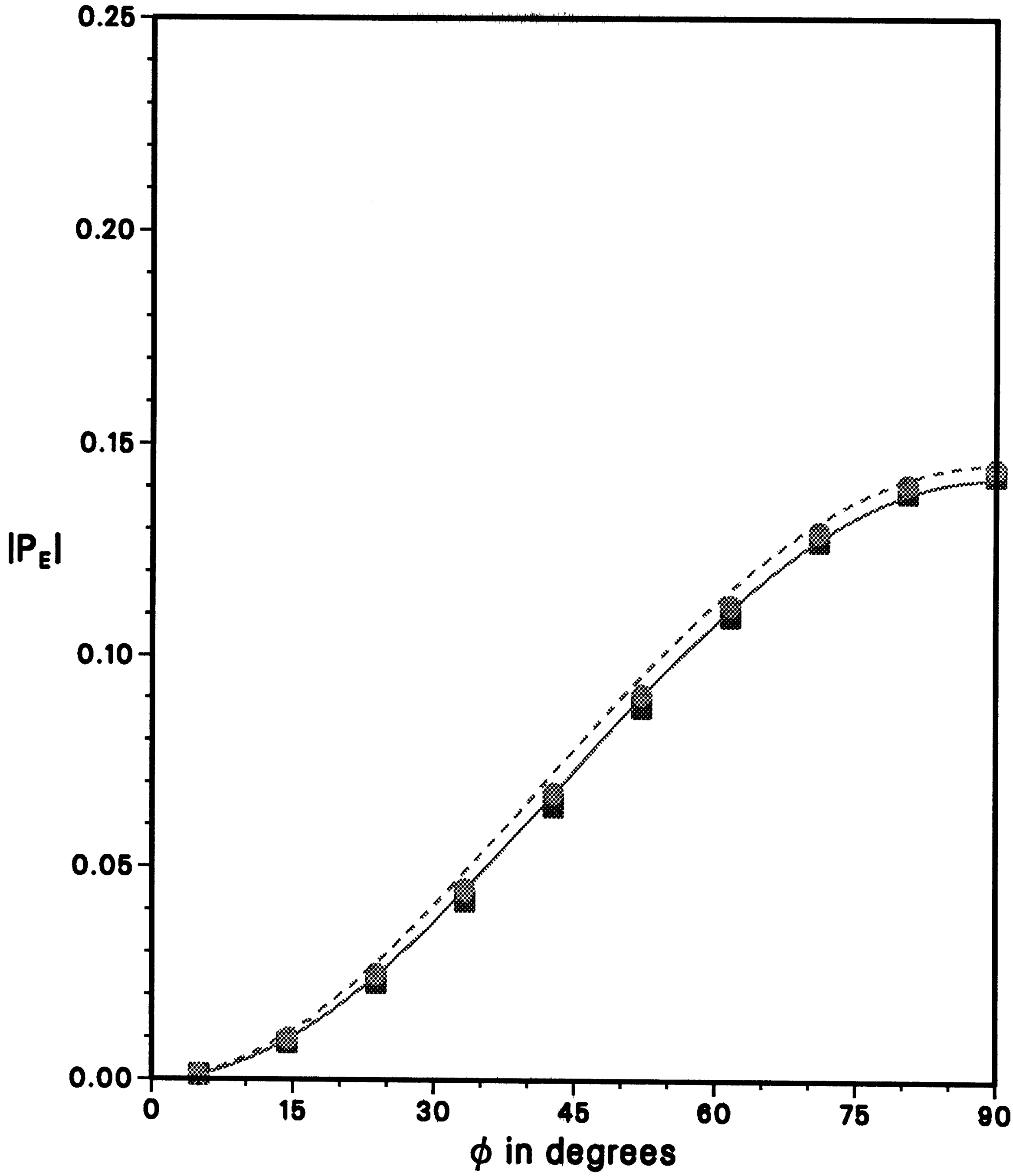

Figure 11. Modulus of the far field amplitude P_E for the backscatter from a rectangular gap of depth $d_1/\lambda = 0.2$ with $\phi_o = \phi$, $w/\lambda = 0.15$:

$\bar{\eta}/Z = 0.1$ ■ MoM, ——— QA
$\bar{\eta}/Z = 0.3$ ● MoM, - - - - QA

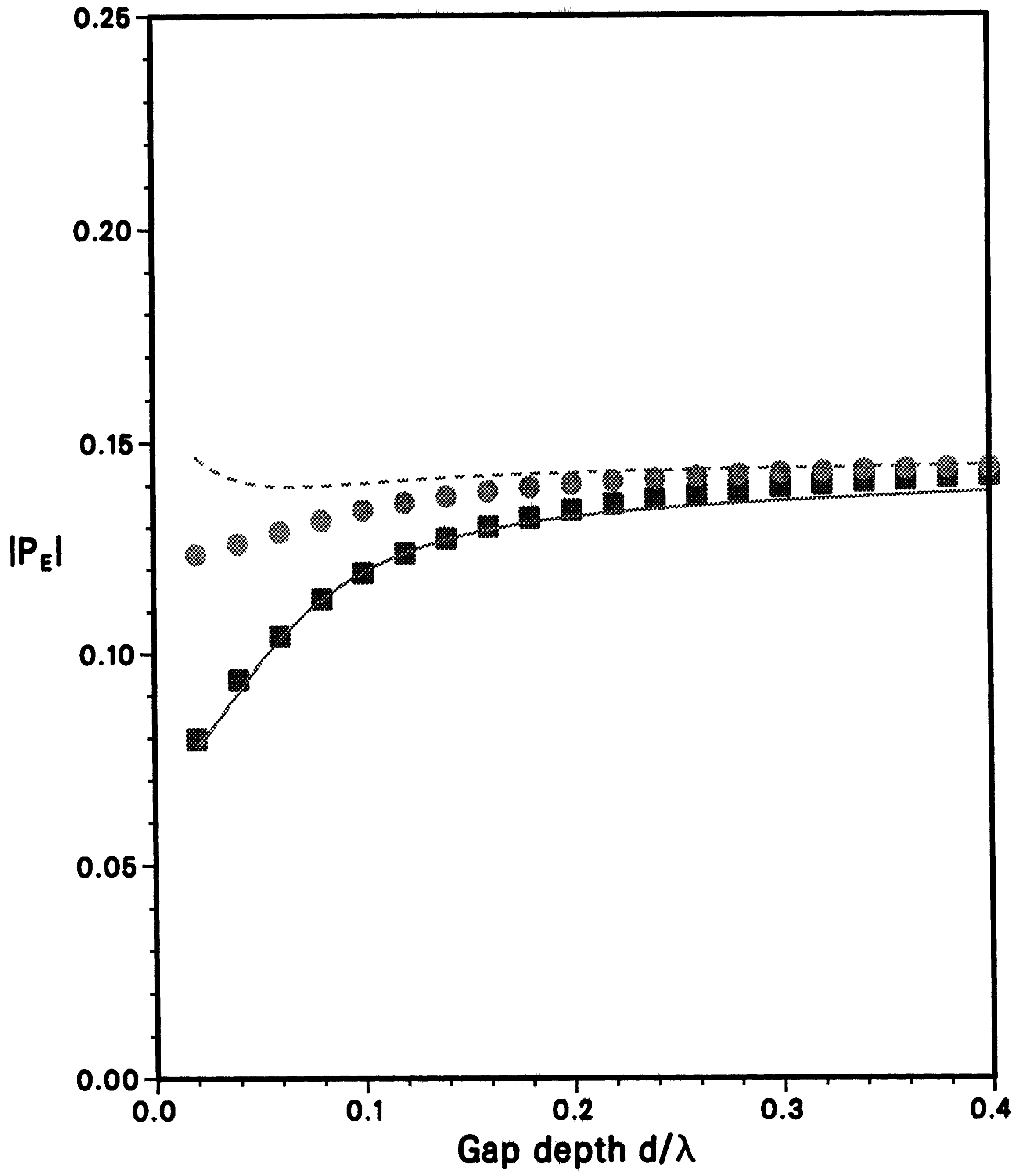

Figure 12. Modulus of the far field amplitude P_E for an air-filled V-shaped gap of varying depth $d_1 = d$ with $\phi = \phi_o = \pi/2$ and $w/\lambda = 0.15$:

$\bar{\eta}/Z = 0.1$ ■ MoM, ——— QA
$\bar{\eta}/Z = 0.3$ ● MoM, - - - - QA

5. CONCLUSIONS

The quasi-analytical solution of [2] was based on the low frequency approximation of the integral equations for a constant impedance insert in a perfectly conducting plane applied to a cavity-backed gap in the same such plane. This same solution has been applied to a gap in an impedance plane by modifying the expressions with coefficients dependent on the surface impedance of the plane and the width of the gap. For comparison, a solution was derived by the equivalence principle in conjunction with the half space impedance Green's function, and the exact integral equations were solved by the method of moments. For an air-filled gap, the quasi-analytical solution for the far-field amplitude is within 12 percent for an $\bar{\eta}/Z \leq 0.5$ for H-polarization and within 14 percent for an $\bar{\eta}/Z \leq 0.3$ for E-polarization, assuming $w/\lambda \leq 0.15$. The results are improved when the gap is material-filled with a relative permittivity greater than 2.5, within 14 percent for $\bar{\eta}/Z \leq 1$ for H-polarization. The quasi-analytical solution gives excellent agreement with the method of moments results when the gap is filled with a lossy dielectric material. Thus the quasi-analytical solution is a simple and accurate method for determining the scattering from a narrow gap in an impedance plane for surface impedances of small but significant values.

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APPENDIX A Evaluation of the Half Space Green's Function of an Impedance Plane

The half space Green's function for an impedance plane is given in (3) and, considering the integral, in a more quickly convergent form in (4). For the currents and observation in the gap, y' = y = 0, and (4) becomes

$$G(x,0;x',0) = \frac{i}{2}\left\{ H_o^{(1)}(k\,|x-x'|) - \lim_{y\to 0}\int_0^{\infty}\tau_H\, e^{-\tau_H v}\, H_o^{(1)}\left(k\sqrt{(x-x')^2 + (y+iv)^2}\right)dv \right\}. \quad (A.1)$$

The Hankel function is well defined, but the integral must be analyzed carefully, since its integrand contains a singularity at

$$v = v_o = |x-x'|$$

in the limit as y approaches zero. Using the small argument approximation of the Hankel function, the integral about this point is

$$L_H(x,0;x',0|\bar{\eta})\big|_{v_o} = \lim_{y\to 0}\int_{v_o-\Delta v}^{v_o+\Delta v} \tau_H\, e^{-\tau_H v}\left[A' + \frac{i2}{\pi}\ln\sqrt{(x-x')^2 + (y+iv)^2}\right]dv \quad (A.2)$$

where

$$A' = 1 + \frac{i2}{\pi}\left(\gamma + \ln\frac{k}{2}\right). \quad (A.3)$$

Assuming Δv is small such that the exponential term is nearly constant over $2\Delta v$, (A.2) is evaluated analytically in the limit as

$$L_H^o = \tau_H\, e^{-\tau_H v_o}\,(L_1 + L_2) \quad (A.4)$$

where

$$L_1 = 2A'\,\Delta v$$

and

$$L_2 = \frac{i}{\pi}\left\{ (v_o+\Delta v)\left[\ell n(2v_o\,\Delta v + \Delta v^2) + i\pi\right] - (v_o-\Delta v)\,\ell n(2v_o\,\Delta v - \Delta v^2)\right.$$
$$\left. -4\Delta v + v_o\left[\ell n\left(\frac{2v_o + \Delta v}{2v_o - \Delta v}\right) - i\pi\right]\right\}.$$

Over the other portions of the path of integration, L_H is evaluated numerically.

For the case x = x', we have $v_o = 0$, and the evaluation of (A.1) can be done as follows. In the application of the moment method, the path of the integral in (9) is segmented such that the currents are assumed to be constant over each segment Δx. The currents can then be removed from the integral, and with the self-cell, the integration of L_H over Δx is

$$\int_{x_o-\Delta x/2}^{x_o+\Delta x/2} \lim_{y\to 0}\left\{\int_0^{\Delta v} \tau_H\, e^{-\tau_H v}\left[A' + \frac{i2}{\pi}\ell n\sqrt{(x_o-x')^2 + (y+iv)^2}\right]dv \right.$$
$$\left. + \int_{\Delta v}^{\infty} \tau_H\, e^{-\tau_H v}\, H_o^{(1)}\left(k\sqrt{(x_o-x')^2 + (y+iv)^2}\right)dv\right\}dx',$$

where x_o is the midpoint of the self-cell. Evaluating the first integral within the brackets analytically, the self-cell expression of L_H becomes

$$S_H(\Delta x|\bar{\eta}) = L_H^{ox} + 2\int_{x_o}^{x_o+\Delta x/2} \lim_{y\to 0}\int_{\Delta v}^{\infty} \tau_H\, e^{-\tau_H v}\, H_o^{(1)}\left(k\sqrt{(x_o-x')^2 + (y+iv)^2}\right)dv\,dx' \quad \text{(A.5)}$$

where

$$L_H^{ox} = \tau_H\, e^{-\tau_H \frac{\Delta v}{2}}\,(L_3 + L_4) \qquad \text{(A.6)}$$

with

$$L_3 = \left(A' - \frac{i2}{\pi}\right)\Delta x\,\Delta v$$

and

$$L_4 = \frac{i}{\pi}\Delta x\left\{\Delta v\,\ln\left(\frac{\Delta x^2}{4} - \Delta v^2\right) - 2\Delta v + \frac{\Delta x}{2}\ln\left(\frac{\Delta x + 2\Delta v}{\Delta x - 2\Delta v}\right)\right.$$

$$\left. - \left(\frac{\Delta x}{4} - \frac{\Delta v^2}{\Delta x}\right)\left[\ln\left(\frac{\Delta x + 2\Delta v}{\Delta x - 2\Delta v}\right) + i\pi\right] + \frac{\Delta v}{2} + i\pi\frac{\Delta x}{4}\right\} .$$

The double integral in (A.5) is evaluated numerically. The self-cell expression of the Hankel function in (A.1) is well known and is given in Appendix B as needed.

For the E-polarization case, the derivatives of the Green's function must be considered. From (18), and using the form in (4) for (13),

$$\lim_{\substack{y'\to 0\\ y\to 0}} \frac{\partial^2}{\partial y\,\partial y'} G(x,y;x',y') = \frac{i}{2}\lim_{y\to 0}\left(k^2 + \frac{\partial^2}{\partial x^2}\right)\int_0^\infty \tau_E\, e^{-\tau_E v} H_0^{(1)}\left(k\sqrt{(x-x')^2 + (y+iv)^2}\right) dv \quad (A.7)$$

The order of the differential operator is reduced by applying the integral shown in (18) over the segment Δx. Given the endpoints x_1, x_2 of the segment, the integral L_E of (A.7) becomes

$$\int_{x_1}^{x_2}\int_0^\infty \tau_E\, e^{-\tau_E v} H_0^{(1)}\left(k\sqrt{(x-x')^2 + (y+iv)^2}\right) dv\, dx'$$

$$+ \frac{1}{k}\int_0^\infty \tau_E\, e^{-\tau_E v}\left[\frac{(x-x_2)}{\sqrt{(x-x_2)^2+(y+iv)^2}} H_1^{(1)}\left(k\sqrt{(x-x_2)^2+(y+iv)^2}\right)\right.$$

$$\left. - \frac{(x-x_1)}{\sqrt{(x-x_1)^2+(y+iv)^2}} H_1^{(1)}\left(k\sqrt{(x-x_1)^2+(y+iv)^2}\right)\right] dv , \quad (A.8)$$

in the limit as y approaches zero, where k^2 has been factored out. For v_o not equal to zero, the singularity in the first integral expression of (A.8) is handled in the same way as that for the H-polarization case, giving

$$L_E^o = \tau_E \, e^{-\tau_E v_o} (L_1 + L_2) \tag{A.9}$$

with L_1 and L_2 given above. The integration is done numerically over the remaining path, as well as with respect to x'. Let $L_E^{'}(x,\Delta x'|\overline{\eta})$ denote the second integral expression in (A.8). The contribution from the singularity is, using the small argument approximation of the first order Hankel function,

$$L_E^1 = \frac{\tau_E}{k} \left\{ e^{-\tau_E v_{o2}} \left[k(x-x_2)\,\Delta v + \frac{i}{\pi k}\left(\ln \frac{2v_{o2}-\Delta v}{2v_{o2}+\Delta v} + i\pi \right) \frac{(x-x_2)}{|x-x_2|} \right] \right.$$

$$\left. - \, e^{-\tau_E v_{o1}} \left[k(x-x_1)\,\Delta v + \frac{i}{\pi k}\left(\ln \frac{2v_{o1}-\Delta v}{2v_{o1}+\Delta v} + i\pi \right) \frac{(x-x_1)}{|x-x_1|} \right] \right\} \tag{A.10}$$

where

$$v_{o1} = |x-x_1| \, , \; v_{o2} = |x-x_2| \; .$$

Considering the integrations of (A.8) over the self-cell,

$$S_E(\Delta x|\overline{\eta}) = L_E^{ox} + 2 \int_{x_o}^{x_o+\Delta x/2} \lim_{y \to 0} \int_{\Delta v}^{\infty} \tau_E \, e^{-\tau_E v} \, H_o^{(1)}\left(k\sqrt{(x_o-x')^2 + (y+iv)^2} \right) dv \, dx'$$

$$+ L_E^{1x} - \frac{1}{k} \lim_{y \to 0} \int_{\Delta v}^{\infty} \tau_E \, e^{-\tau_E v} \frac{\Delta x}{\sqrt{(\Delta x/2)^2 + (y+iv)^2}} H_1^{(1)}\left(k\sqrt{(\Delta x/2)^2 + (y+iv)^2} \right) dv \tag{A.11}$$

where the analytical expressions are

$$L_E^{ox} = \tau_E \, e^{-\tau_E \frac{\Delta v}{2}} (L_3 + L_4) \tag{A.12}$$

and

$$L_E^{1x} = \tau_E \, e^{-\tau_E \frac{\Delta v}{2}} \frac{2}{k} \left[-k \frac{\Delta x}{4} \Delta v + \frac{i}{\pi k} \ln \left(\frac{\Delta x + 2\Delta v}{\Delta x - 2\Delta v} \right) \right] . \tag{A.13}$$

APPENDIX B Moment Method Solution of the Coupled Integral Equations

The integral equation pairs given by (7),(9) and (18),(19) are solved by the moment method [2,Appendix A]. Using pulse basis functions in the moment method, the aperture A and the cavity walls S shown in Fig. 1 are segmented into N cells of size Δs. The magnetic and electric currents are assumed to be constant over each of these segments. When the integrations of the coupled equations are taken over each segment, the current expressions can be removed as constants from the integrals. With the contour of integration discretized, the (x',y') coordinates become (x_i,y_i), i = 1,...,N, which describe the location of each of the segments. The Green's functions can then be expressed in terms of rotated coordinates (s,n) for the observation position and (s_i,n_i) for each segment or source position since the integration is with respect to the tangential vector $\hat{s}$ as shown in Fig. 1.

The expressions for the numerical solution of the coupled equations are developed in the following sections for the H- and E-polarization cases. Applying point matching, the magnetic and electric currents in the aperture and on the cavity walls are determined, and the far field amplitude is calculated.

A.1 H-Polarization

For the discretized contour of integration, (7) and (9) become

$$J_s(s,n) = \frac{kY}{2}\varepsilon_r \sum_{i=1}^{M} J_z^*(s_i) \int_{\Delta s_i} H_o^{(1)}\left(k_1\sqrt{(s-s_i)^2 + n^2}\right) ds_i$$

$$+\frac{ik_1}{2} \sum_{i=1}^{N} J_s(s_i,n_i) \int_{\Delta s_i} \sin\beta_i \, H_1^{(1)}\left(k_1\sqrt{(s-s_i)^2 + (n-n_i)^2}\right) ds_i \qquad (B.1)$$

$$J_s(s) = \frac{2\sin\phi_o}{\overline{\eta}Y + \sin\phi_o} e^{-iks\cos\phi_o}$$

$$-\frac{kY}{2}\sum_{i=1}^{M}\left[J_z^*(s_i) - \overline{\eta}J_s(s_i)\right]\int_{\Delta s_i}\left[H_o^{(1)}(k\,|s-s_i|) - L_H(s,0;s_i,0)\right]ds_i \qquad \text{(B.2)}$$

where M are the number of segments across the aperture and

$$\sin\beta_i = \frac{(n-n_i)}{\sqrt{(s-s_i)^2 + (n-n_i)^2}} . \qquad \text{(B.3)}$$

Applying point matching over the N segments of the aperture and cavity walls,

$$\sum_{i=1}^{N} I_i\left[-1\,\Big|_{j=i} + \frac{ik_1}{2}\int_{\Delta s_i}\sin\beta_{j,i}\,H_1^{(1)}(k_1\,R_{j,i})\,ds_i\right]$$

$$+\frac{kY}{2}\varepsilon_r\sum_{i=N+1}^{N+M} I_i\int_{\Delta s_i}H_o^{(1)}(k_1\,R_{j,i})\,ds_i = 0 \qquad \text{(B.4)}$$

$$\sum_{i=1}^{M} I_i\left[1\,\Big|_{j=i} - \frac{kY}{2}\overline{\eta}\int_{\Delta s_i}\left[H_o^{(1)}(k\,R_{j,i}) - L_H(s_j,0;s_i,0)\right]ds_i\right]$$

$$+\frac{kY}{2}\sum_{i=N+1}^{N+M} I_i\int_{\Delta s_i}\left[H_o^{(1)}(k\,R_{j,i}) - L_H(s_j,0;s_i,0)\right]ds_i = \frac{2\sin\phi_o}{\overline{\eta}Y + \sin\phi_o} e^{iks_j\cos\phi_o} \qquad \text{(B.5)}$$

where

$$R_{j,i} = \sqrt{(s_j-s_i)^2 + (n_j-n_i)^2} . \qquad \text{(B.6)}$$

The coordinate (s_j,n_j) is the observation position at the midpoint of the j^{th} segment. Hence, for i,j = 1,...,M,N+1,...,N+M, the segments are located in the aperture, and for i,j = M+1,...,N, the segments are located on the cavity walls. I_i in (B.4) and (B.5) are the electric currents, for i = 1,...,N, and the magnetic currents, i = N+1,...,N+M, to be determined.

In matrix form, (B.4) and (B.5) become

$$[Z_{j,i}][I_i] = [V_j] \quad \text{(B.7)}$$

The impedance matrix is given as

$$[Z_{j,i}] = \left[\begin{array}{c:c} Z_{e1} & Z_{m1} \\ \hdashline Z_{e2} & Z_{m2} \end{array} \right] \quad \text{(B.8)}$$

where the sets of elements are as follows:

$$Z_{e1} = \begin{cases} \dfrac{ik_1}{2} \displaystyle\int_{\Delta s_i} \sin\beta_{j,i}\, H_1^{(1)}(k_1 R_{j,i})\, ds_i & j \neq i \\ -1 & j = i \end{cases} \quad \text{(B.9)}$$

for i = 1,...,N and j = 1,...,N;

$$Z_{m1} = \begin{cases} \dfrac{kY}{2} \varepsilon_r \displaystyle\int_{\Delta s_i} H_o^{(1)}(k_1 R_{j,i})\, ds_i & j \neq i\text{-}N \\ \dfrac{kY}{2} \varepsilon_r\, 2(s_i - s_j) \left[\dfrac{i2}{\pi} \ln(R_{j,i}) - 1 + A_1' \right] & j = i\text{-}N \end{cases} \quad \text{(B.10)}$$

for i = N+1,...,N+M and j = 1,...,N;

$$Z_{e2} = \begin{cases} -\dfrac{kY}{2} \overline{\eta} \displaystyle\int_{\Delta s_i} \left[H_o^{(1)}(k R_{j,i}) - L_H(s_j,0;s_i,0) \right] ds_i & j \neq i\text{+}N \\ 1 - \dfrac{kY}{2} \overline{\eta} \left[2(s_i - s_j) \left(\dfrac{i2}{\pi} \ln(R_{j,i}) - 1 + A' \right) - S_H(\Delta s_i | \overline{\eta}) \right] & j = i\text{+}N \end{cases} \quad \text{(B.11)}$$

for i = 1,...,N and j = N+1,...,N+M;

$$Z_{m2} = \begin{cases} \frac{kY}{2} \int_{\Delta s_i} \left[H_o^{(1)}(k\, R_{j,i}) - L_H(s_j,0;s_i,0) \right] ds_i & j \neq i \\ \frac{kY}{2} \left[2(s_i - s_j)\left(\frac{i2}{\pi} \ln(R_{j,i}) - 1 + A' \right) - S_H(\Delta s_i | \bar{\eta}) \right] & j = i \end{cases} \qquad \text{(B.12)}$$

for i = N+1,...,N+M and j = N+1,...,N+M. In (B.10), the expression for A'_1 is

$$A'_1 = 1 + \frac{i2}{\pi}\left(\gamma + \ln \frac{k_1}{2} \right),$$

and in (B.11) and (B.12), S_H is given by (A.5). For the self-cells in (B.10) to (B.12), s_i is taken to be the endpoint of the i^{th} segment. The self-cell expressions were derived analytically, and a numerical integration is applied to the other segments. In the case of the V-shaped gap, the adjacent cells needed to be evaluated in the vicinity of y = -d, for $R_{j,i}$ less than one cell size, and these expressions can be found in Appendix A of [2].

The source matrix is given by

$$V_j = \begin{cases} 0 & j = 1,...,N \\ \frac{2 \sin\phi_o}{\bar{\eta} Y + \sin\phi_o} e^{-iks_j \cos\phi_o} & j = N+1,...,N+M \ , \end{cases} \qquad \text{(B.13)}$$

and the currents I_i are determined by solving (B.7), given that $[Z_{j,i}]$ is nonsingular. From (11), the far field amplitude at the angle ϕ is now

$$P_H(\phi,\phi_o) = -\frac{kY}{2}\,\frac{1}{\bar{\eta}Y + 1} \sum_{i=1}^{M} \left(I_{i+N} - \bar{\eta}\, I_i \right) \int_{\Delta x_i} e^{-ikx_i \cos\phi} dx_i \ . \qquad \text{(B.14)}$$

B.2 E-Polarization

The integral equation pair given by (18) and (19) was solved in the same manner as described for the H-polarization case. The elements of the impedance matrix defined in (B.8) for the E-polarization case are as follows:

$$Z_{e1} = \begin{cases} \frac{kZ}{2}\mu_r \int_{\Delta s_i} H_o^{(1)}(k_1 R_{j,i})\, ds_i & j \neq i \\ \frac{kZ}{2}\mu_r\, 2(s_i - s_j)\left[\frac{i2}{\pi}\ln(R_{j,i}) - 1 + A_1'\right] & j = i \end{cases} \tag{B.15}$$

for i = 1,...,N and j = 1,...,N;

$$Z_{m1} = \begin{cases} -\frac{ik_1}{2}\int_{\Delta s_i} \frac{y_j}{R_{j,i}} H_1^{(1)}(k_1 R_{j,i})\, ds_i & j \neq i\text{-}N \\ -1 & j = i\text{-}N \end{cases} \tag{B.16}$$

for i = N+1,...,N+M and j = 1,...,N;

$$Z_{e2} = \begin{cases} -\frac{kY}{2}\bar{\eta}\left[\int_{\Delta s_i} L_E(s_j,0;s_i,0|\bar{\eta})\, ds_i + L_E'(s_j,\Delta s_i|\bar{\eta})\right] & j \neq i\text{+}N \\ -1 - \frac{kY}{2}\bar{\eta}\, S_E(\Delta s_i|\bar{\eta}) & j = i\text{+}N \end{cases} \tag{B.17}$$

for i = 1,...,N and j = N+1,...,N+M;

$$Z_{m2} = \begin{cases} -\frac{kY}{2}\left[\int_{\Delta s_i} L_E(s_j,0;s_i,0|\bar{\eta})\, ds_i + L_E'(s_j,\Delta s_i|\bar{\eta})\right] & j \neq i \\ -\frac{kY}{2}\, S_E(\Delta s_i|\bar{\eta}) & j = i \end{cases} \tag{B.18}$$

for i = N+1,...,N+M and j = N+1,...,N+M. In the self-cell expression of (B.15), s_i is evaluated at the endpont of the i^{th} segment. In (B.17) and (B.18), S_E is given by (A.11).

The source matrix is

$$V_j = \begin{cases} 0 & j = 1,\dots,N \\ \dfrac{2Y}{\bar{\eta}Y + \csc\phi_o} e^{-iks_j \cos\phi_o} & j = N+1,\dots,N+M \ . \end{cases} \qquad \text{(B.19)}$$

Given that $[Z_{j,i}]$ is nonsingular, the currents I_i can be determined, and the far field amplitude is calculated from the expression

$$P_E(\phi,\phi_o) = -\frac{k}{2} \frac{\sin\phi}{\bar{\eta}Y + 1} \sum_{i=1}^{M} \left(I_{i+N} + \bar{\eta}\, I_i \right) \int_{\Delta x_i} e^{-ikx_i \cos\phi} dx_i \ . \qquad \text{(B.20)}$$

B.3 Program LIsting

The expressions for the impedance and source matrices and the far field amplitude were programmed for solution, as contained in the program listing of IMP.FTN below. The subroutines called by the program in addition to those listed are contained in the file GAPSUB.FTN of Appendix A of [2].

The numerical integration is done for the appropriate segments using Simpson's three-point composite integration over each segment. A segment size of $\Delta s_i/\lambda = 0.01$ was used for the results of Figs. 4 to 12. The numerical integration implemented for L_H and L_E is the Gauss-quadrature technique, and convergence was verified for $\bar{\eta}/Z \leq 2$. As listed, the program calculates the far field amplitude as a function of the gap depth using (B.14) for H-polarization and (B.20) for E-polarization.

```
c
c     IMP.FTN
c
c
c       This FORTRAN program computes the far field scattering due
c       to a narrow gap of arbitrary shape in an infinite impedance
c       plane.  The moment method is applied to solve the currents
c       of two coupled equations.
c
c       INPUT   The user is prompted from the subroutine
c               GAPROM for the polarization and angle of the
c               incident field, angle of far field observation,
c               relative permittivity of gap filling, shape and
c               dimensions of gap, segment size, number of
c               iterations with respect to gap depth, and
c               normalized surface impedance.
c
c       OUTPUT FILES
c               GAPDAT  Contains input data.
c               AMPDAT  Contains the magnitude of the far field.
c               PHADAT  Contains the phase of the far field.
c
c       SUBROUTINES
c               HANKZ1  Computes the Hankel functions of the first
c                       kind of orders zero and one.
c               CHANK   Computes the Hankel functions of the first
c                       kind of orders zero and one given a
c                       complex argument.
c               IMPGF   Calculates the value of the impedance
c                       plane Green's function.
c               SIMPGF  Calculates the value of the self-cell
c                       for the impedance plane Green's function.
c               CGECO   Factors a complex matrix and estimates the
c                       condition of the matrix.
c               CGESL   Solves the complex set of linear
c                       equations [A][x] = [b].
c
c
      integer pn
      parameter(pn=500)
      integer EorH,N,noS,gN,szN(50)

      real pi,k,phi,phio,w,d,maxC,etab,q(50,2)
      real dStp(50),wStp(50),p(pn,2),m(pn,2),szSd(50),psi(50)
      real posiX,posiY,spaceX,spaceY,stepX
      real ni,si,nj,sj,xj,yj,DelS,R,Rm,tanf

      complex czero,ci,er,ur,Ao,A,k1,cself0,cself1,ctemp0,ctemp1
      complex Z(pn,pn),LH0,LH1,LH0o,LH1o,aLH0,Ii(pn),Vj(pn)
      complex eta(pn),Hi(pn),Hs(pn),E(pn),Lsca,Psca
      complex GF0a(pn),GF0b(pn),GF1a(pn),GF1b(pn),GF0(pn),GF1(pn)

      integer ipvt(pn),iretrn
      real rc,krho
      complex wk(pn),H0,H1,H0o,H1o,ckrho,iH0o,iH1o
      logical Epol,Lossy,side,neg(50)
      common /prompt/ EorH,phio,phi,er,ur,igap,wStp,dStp,w,d,noS,
     &                q,maxC,noIter,etab

  1   format(i1)
  2   format(i5)
  3   format(g16.8)
  4   format(a4)
  6   format(2g16.8)

      open(1,file='gapdat')
      open(3,file='ampdat')
      open(4,file='phadat')

c...Declaring constant values
      czero=cmplx(0.0,0.0)
      ci=cmplx(0.0,1.)
      pi=4.0*atan(1.0)
      k=2*pi
      Zo=sqrt(4.e-07*pi/8.854e-12)
      Yo=1./Zo
      gam=0.5772157
      Ao=2*(log(k/2)+gam-ci*pi/2)
      iprg=1

c...Setting default values
      EorH=1
      phio=90.0
      phi=90.0
      er=cmplx(1.,0.0)
      ur=cmplx(1.,0.0)
      w=0.15
      d=0.2
      noS=3
      maxC=0.01
      noIter=30
      etab=1.
      adj=0.00001
      Epol=.false.
      Lossy=.false.
```

```
c...Prompting user for input data
      call gaprom(iprg)

      if(EorH .eq. 1) Epol=.true.
      phio=phio*pi/180.0
      phi=phi*pi/180.0
      drat=dStp(1)/d
      k1=k*csqrt(er*ur)
      A=2*(clog(k1/2)+gam-ci*pi/2)
      if(aImag(er) .ne. 0.0) Lossy=.true.

      dmin=.025
      if(Epol) dmin=0.02
      dmax=d
      if(noIter .ne. 1)then
        dstep=(dmax-dmin)/(noIter-1)
        d=dmin
      endif
      DO 700 iter=1,noIter

c...Determining coordinates of corner points given gap type
        if(igap .eq. 1)then
c    RECTANGULAR
           noS=3
           q(3,1)=w/2
           q(3,2)=-d
           q(4,1)=-w/2
           q(4,2)=-d
        else if(igap .eq. 2)then
c    L-SHAPED
           noS=7
           dStp(1)=d*drat
           dStp(2)=d*(1-drat)
           q(3,1)=w/2
           q(3,2)=-dStp(1)
           q(8,1)=-w/2
           q(8,2)=-dStp(1)
           q(4,1)=w/2+wStp(2)
           q(4,2)=-dStp(1)
           q(7,1)=-w/2-wStp(3)
           q(7,2)=q(4,2)
           q(5,1)=q(4,1)
           q(5,2)=q(4,2)-dStp(2)
           q(6,1)=q(7,1)
           q(6,2)=q(5,2)
        else if(igap .eq. 3)then
c    V-SHAPED
           noS=2
           q(3,1)=0.0
           q(3,2)=-d
           adj=0.75*maxC
        else if(igap .eq. 4)then
c    T-SHAPED
           noS=5
           dStp(1)=d*drat
           dStp(2)=d*(1-drat)
           q(3,1)=w/2
           q(3,2)=-dStp(1)
           q(4,1)=w/2+wStp(2)
           q(4,2)=-dStp(1)
           q(5,1)=q(4,1)
           q(5,2)=q(4,2)-dStp(2)
           q(6,1)=-w/2
           q(6,2)=q(5,2)
        endif
c...Corner points of gap at y=0
        q(1,1)=-w/2
        q(1,2)=0.0
        q(2,1)=w/2
        q(2,2)=0.0
        q(noS+2,1)=-w/2
        q(noS+2,2)=0.0

c******************************************************************
c*************   Current segment locations (xi,yi)   *************
c******************************************************************
      N=0
      do 175 l=1,noS+1
c...Size (length) of lth side of gap
        szSd(l)=sqrt((q(l+1,1)-q(l,1))**2
     &               +(q(l+1,2)-q(l,2))**2)
c..Angle of rotation for each side with respect to x axis
         if(q(l+1,1) .lt. q(l,1))then
            psi(l)=asin((q(l,2)-q(l+1,2))/szSd(l))
            neg(l)=.true.
         else
            psi(l)=asin((q(l+1,2)-q(l,2))/szSd(l))
            neg(l)=.false.
         endif

         szN(l)=int(szSd(l)/maxC)+1
         N=N+szN(l)
         spaceX=(q(l+1,1)-q(l,1))/szN(l)
         spaceY=(q(l+1,2)-q(l,2))/szN(l)
         posiX=q(l,1)
         posiY=q(l,2)
```

```
c...ENDPOINTS of each segment are p, MIDPOINTS are m in (x,y)
c   coordinates
         do 170 i=N-szN(l)+1,N
           p(i,1)=posiX
           p(i,2)=posiY
           m(i,1)=posiX+spaceX/2.0
           m(i,2)=posiY+spaceY/2.0
           posiX=posiX+spaceX
           posiY=posiY+spaceY
170      continue
175   continue
      p(N+1,1)=-w/2
      p(N+1,2)=0.0
c...Number of segments in the aperture
      gN=szN(1)
c...Number of current coefficients to be calculated
      NgN=N+gN
      print *, ' d = ',d,' N = ',N,' gN = ',gN
c...Initializing matrices to zero
      do 190 j=1,NgN
      do 180 i=1,NgN
         Z(j,i)=czero
180   continue
         Vj(j)=czero
190   continue

c*****************************************************************
c*****************     Impedance, Source,    *********************
c*****************   and Current Matrices   **********************
c*****************************************************************
c
c       VARIABLES:
c           HOo,HO  Hankel function of zero order in free
c                   space and in material er, respectively.
c           H1o,H1  Hankel function of first order in free
c                   space and in material er, respectively.
c           iH0o,   Evaluation of the integral expression of
c             iH1o  the half space Green's function.
c           Green's Function integrals:
c               LH0  Integral of H0.
c               LH1  Integral of H1 for H-pol, of dH1/dy for
c                    E-pol.
c               LH0o Integral of H0o and iH0o for H-pol and
c                    iH0o for E-pol.
c               LH1o Integral of iH1o for E-pol.
c               aLH0 Analytical integral of H0o for evaluation
c                    of adjacent cells for LH2, E-pol case.
c
c       The integration is done one side at a time, for j=1,...,N,
c       in the clockwise direction, starting at (x,y)=(-w/2,0).
c
      istart=1
      istop=szN(1)
c...Source point is i of the lth side, observation point is j
      do 230 l=1,noS+1
       do 220 i=istart,istop
       do 210 j=1,N
c      Coordinate rotation for observation point
         sj=m(j,1)*cos(psi(l))+m(j,2)*sin(psi(l))
         nj=m(j,1)*sin(psi(l))-m(j,2)*cos(psi(l))
         if(neg(l))then
            sj=-sj
            nj=-nj
         endif
c...Integration over ith segment
         LH0=czero
         LH1=czero
         LH0o=czero
         LH1o=czero
c   Magnitude between midpoints Rm=|r-r'|
         Rm=sqrt((m(j,1)-m(i,1))**2+(m(j,2)-m(i,2))**2)
       if(j .eq. i .or. Rm .le. adj)then
c   SMALL ARGUMENT APPROXIMATION integral for self-cell
c    and adjacent cells
         do 200 ip=i+1,i,-1
c        Coordinate rotation for source segment points
           si=p(ip,1)*cos(psi(l))+p(ip,2)*sin(psi(l))
           ni=p(ip,1)*sin(psi(l))-p(ip,2)*cos(psi(l))
           if(neg(l))then
              si=-si
              ni=-ni
           endif

           R=sqrt((sj-si)**2+(nj-ni)**2)
           if(j .eq. i .or. abs(nj-ni) .eq. 0.0)then
              tanf=pi/2
              absf=1.0
           else
              tanf=atan((si-sj)/abs(nj-ni))
              absf=abs(nj-ni)
           endif

           LH1=-k1**2/2*(nj-ni)*si
     &         +ci*2./pi*(nj-ni)/absf*tanf-LH1
           LH0=ci/pi*(2*(si-sj)*log(R)
     &           -(2-A)*si+2.0*abs(nj-ni)*tanf)-LH0
```

```fortran
            LH0o=ci/pi*(2*(si-sj)*log(R)
     &           -(2-Ao)*si+2.0*abs(nj-ni)*tanf)-LH0o
            if(j .eq. i) GOTO 202
200       continue
202       if(j .eq. i)then
            LH0=2*(LH0+ci/pi*(2-A)*sj)
            if(i .le. gN .and. j .le. gN)then
              if(i .eq. 1 .and. iter .eq. 1)then
                 iH1o=abs(sj-si)
                 call simpGF(0.,etab,Epol,iH0o,iH1o)
                 cself0=iH0o
                 cself1=iH1o
              endif
              if(Epol)then
                LH1o=cself0+cself1
              else
                LH0o=2*(LH0o+ci/pi*(2-Ao)*sj)-cself0
              endif
            endif
          endif
        else
c    SIMPSON'S THREE POINT COMPOSITE INTEGRATION
          do 204 ip=i+1,i,-1
c         Coordinate rotation for source segment endpoints
             si=p(ip,1)*cos(psi(l))+p(ip,2)*sin(psi(l))
             ni=p(ip,1)*sin(psi(l))-p(ip,2)*cos(psi(l))
             if(neg(l))then
                si=-si
                ni=-ni
             endif
             stepS=si
c         HANKEL FUNCTION evaluation at endpoints of segment
             R=sqrt((sj-si)**2+(nj-ni)**2)
             if(Lossy)then
               ckrho=k1*R
               call cHank(ckrho,2,H0,H1)
             else
               krho=Real(k1)*R
               call Hankzl(krho,2,H0,H1)
             endif
             if(i .le. gN .and. j .le. gN)then
               krho=k*R
               call Hankzl(krho,2,H0o,H1o)
               if(i .eq. 1 .and. iter .eq. 1)then
                 call impGF(krho,etab,Epol,ctemp0,ctemp1)
                 if(i .eq. ip)then
                    GF0a(j)=ctemp0
                    iH0o=GF0a(j)
                    GF1a(j)=ctemp1
                    iH1o=GF1a(j)
                 else
                    GF0b(j)=ctemp0
                    iH0o=GF0b(j)
                    GF1b(j)=ctemp1
                    iH1o=GF1b(j)
                 endif
               else
                 if(i .eq. ip)then
                    iH0o=GF0a(abs(j-i)+1)
                    iH1o=GF1a(abs(j-i)+1)
                 else
                    iH0o=GF0b(abs(j-i)+1)
                    iH1o=GF1b(abs(j-i)+1)
                 endif
               endif
             endif

             LH0=H0+LH0
             if(Epol)then
               LH1=k1*m(j,2)/R*H1+LH1
               if(i .le. gN .and. j .le. gN)then
                 LH1o=-LH1o-iH1o
                 LH0o=iH0o+LH0o
               endif
             else
               LH1=k1*(nj-ni)/R*H1+LH1
               LH0o=(H0o-iH0o)+LH0o
             endif
204       continue

c       Coordinate rotation for source segment midpoints
          si=m(i,1)*cos(psi(l))+m(i,2)*sin(psi(l))
          ni=m(i,1)*sin(psi(l))-m(i,2)*cos(psi(l))
          if(neg(l))then
             si=-si
             ni=-ni
          endif
          stepS=abs(stepS-si)
          DelS=2*stepS
c       HANKEL FUNCTION evaluation at midpoint of segment
          R=sqrt((sj-si)**2+(nj-ni)**2)
          if(Lossy)then
            ckrho=k1*R
            call cHank(ckrho,2,H0,H1)
          else
            krho=Real(k1)*R
```

```
            call Hankz1(krho,2,H0,H1)
          endif
          if(i .le. gN .and. j .le. gN)then
            krho=k*R
            call Hankz1(krho,2,H0o,H1o)
            if(i .eq. 1 .and. iter .eq. 1)then
              call impGF(krho,etab,Epol,ctemp0,ctemp1)
              GF0(j)=ctemp0
              iH0o=GF0(j)
              GF1(j)=ctemp1
              iH1o=GF1(j)
            else
              iH0o=GF0(abs(j-i)+1)
              iH1o=GF1(abs(j-i)+1)
            endif
          endif
c      GREEN'S FUNCTION INTEGRALS
          LH0=stepS/3*(4*H0+LH0)
          if(Epol)then
            LH1=stepS/3*(4*k1*m(j,2)/R*H1+LH1)
            if(i .le. gN .and. j .le. gN)then
              LH0o=stepS/3*(4*iH0o+LH0o)
              LH1o=LH0o+LH1o
            endif
          else
            LH1=stepS/3*(4*k1*(nj-ni)/R*H1+LH1)
            if(i .le. gN .and. j .le. gN)then
              LH0o=stepS/3*(4*(H0o-iH0o)+LH0o)
            endif
          endif
        endif
        if(i .ne. j .and. Rm .le. adj)then
           LH0=-LH0
        endif

        if(Epol)then
c    E-POL IMPEDANCE MATRIX
          Z(j,i)=k*Zo*ur/2*LH0
          if(i .le. gN .and. j .ne. i)then
             Z(j,N+i)=-ci/2*LH1
          else if(i .le. gN .and. j .eq. i)then
             Z(j,N+i)=-1.
          endif

          if(j .le. gN)then
            if(j .ne. i)then
               if(i .le. gN)then
                 Z(N+j,i)=-k/2*etab*LH1o
               else
                 Z(N+j,i)=czero
               endif
            else
               Z(N+j,i)=-1.-k/2*etab*LH1o
            endif
            if(i .le. gN)then
               Z(N+j,N+i)=-k*Yo/2*LH1o
            endif
          endif
        else
C    H-POL IMPEDANCE MATRIX
          if(j .ne. i)then
             Z(j,i)=ci/2*LH1
          else
             Z(j,i)=-1.
          endif
          if(i .le. gN)then
             Z(j,N+i)=k*Yo*er/2*LH0
          endif

          if(j .le. gN)then
            if(j .ne. i)then
               if(i .le. gN)then
                 Z(N+j,i)=-k*etab/2*LH0o
               else
                 Z(N+j,i)=czero
               endif
            else
               Z(N+j,i)=1.-k*etab/2*LH0o
            endif
            if(i .le. gN)then
               Z(N+j,N+i)=k*Yo/2*LH0o
            endif
          endif
        endif
210   continue

c...Incident Field (Source) matrix elements
       xj=m(i,1)
       Vj(i)=czero
       if(i .le. gN)then
         if(Epol)then
c    E-POL INCIDENT FIELD Hx
           Vj(N+i)=2*sin(phio)/(etab*sin(phio)+1)
     &              *Yo*cexp(-ci*k*xj*cos(phio))
         else
c    H-POL INCIDENT FIELD Hz
```

```
          Vj(N+i)=2*sin(phio)/(etab+sin(phio))
     &              *cexp(-ci*k*xj*cos(phio))
        endif
      endif
220   continue
        istart=istop+1
        istop=istop+szN(l+1)
230   continue


c...Calling subroutines to calculate the current matrix
      call CGECO(Z,pn,NgN,ipvt,rc,wk)
      call CGESL(Z,pn,NgN,ipvt,Vj,0)

      print *,'      The condition number is ',rc
      do 310 i=1,NgN
c   CURRENT MATRIX
         Ii(i)=Vj(i)
310   continue



C*****************************************************************
C*****************   Far Field Amplitude   **********************
C*****************************************************************
        Psca=czero
        DelX=w/gN/2
        do 600 i=1,gN
c...Simpson's three point composite integration over each
c   segment in the aperture
           Lsca=DelX/3*(cexp(-ci*k*p(i,1)*cos(phi))
     &           +4*cexp(-ci*k*m(i,1)*cos(phi))
     &           +cexp(-ci*k*p(i+1,1)*cos(phi)))
           if(Epol)then
              Psca=(Ii(N+i)+etab*Zo*Ii(i))*Lsca+Psca
            else
              Psca=(Yo*Ii(N+i)-etab*Ii(i))*Lsca+Psca
           endif
600     continue
        if(Epol)then
           Psca=-k/2*sin(phi)/(etab+1)*Psca
        else
           Psca=-k/2/(etab+1)*Psca
        endif

        write(3,*) d,cabs(Psca)
        write(4,*) d,180/pi*(atan2(aImag(Psca),Real(Psca)))
        print *,'      Exact: |Psca| = ',cabs(Psca),'  arg Psca = ',
     &  180/pi*(atan2(aImag(Psca),Real(Psca)))

        d=d+dstep
700     continue

800   call exit
      END
C*******************************************************************
        SUBROUTINE IMPGF(KRHO,ETAB,EPOL,LRE,LRM)
C*******************************************************************
        real pi,krho,k,nu,nuo,numax
        real*8 rts(64,64),coef(64,64)
        complex ci,carg,croot,Ao
        complex re,rm,eint,mint,Lre,Lrm,H0,H1
        logical Epol,self,second
        common /data/ rts,coef
        call gausq

        pi=3.141593
        ci=cmplx(0.,1.)
        k=2*pi
        gam=0.5772157
        Ao=1+ci*2/pi*(gam+alog(k/2))
        self=.false.
        second=.false.
        iroot=64
        ih=0

        alf=k/etab
        bet=k*etab
        nuo=krho/k
c...Determining size of del nu and max nu value necessary
        if(Epol)then
          dnu=alog(0.95)/(-alf)
          numax=alog(0.00001)/(-alf)
          ih=2
        else
          dnu=alog(0.95)/(-bet)
          numax=alog(0.00001)/(-bet)
        endif
        dnumax=sqrt((0.1/k)**2+nuo**2)-nuo
        if(dnu .gt. dnumax) dnu=dnumax
        rnumax=sqrt((12./k)**2+nuo**2)

        if(nuo .eq. 0.0)then
          self=.true.
          second=.true.
        endif
```

```
        if(dnu .ge. nuo .and. .not.(self))then
          dnu=nuo
          second=.true.
        endif
        if(numax .le. nuo+dnu) numax=2*(nuo+dnu)
        if(rnumax .lt. numax) numax=rnumax

c...Gauss-Quadrature
        Lre=0.0
        Lrm=0.0
       do 20 m=1,2
        if(.not.(m .eq. 1 .and. second))then
          if(m .eq. 1)then
            a=0.0
            b=nuo-dnu
          else if(m .eq. 2)then
            a=nuo+dnu
            b=numax
          else
            a=b
            b=numax
          endif

          mint=0.0
          eint=0.0
          do 70 i=1,iroot
            srts=rts(iroot,i)
            scoef=coef(iroot,i)
            nu=((b-a)*srts+b+a)/2
            croot=nuo**2-nu**2
            carg=k*csqrt(croot)
            if(m .eq. 1)then
              rarg=Real(carg)
              call Hankzl(rarg,ih,H0,H1)
            else
              call cHank(carg,ih,H0,H1)
            endif
            if(Epol)then
              rm=alf*exp(-alf*nu)*nuo*H1/carg
              re=alf*exp(-alf*nu)*H0
            else
              re=bet*exp(-bet*nu)*H0
            endif
            eint=scoef*re+eint
            mint=scoef*rm+mint
 70       continue

          Lre=(b-a)/2*eint+Lre
          Lrm=(b-a)/2*mint+Lrm
        endif
 20    continue

c...Singularity evaluation, nuo = |xj-xi|
        if(.not.(self))then
          if(Epol)then
            mint=alf*exp(-alf*nuo)
     &            *(nuo*dnu+ci/pi/k**2
     &            *(alog((2*nuo-dnu)/(2*nuo+dnu))+ci*pi))
            eint=alf*exp(-alf*nuo)
     &            *(2*Ao*dnu
     &              +ci/pi*((nuo+dnu)*(alog(2*dnu*nuo+dnu**2)
     &               +ci*pi)-(nuo-dnu)*alog(2*dnu*nuo-dnu**2)
     &               -4*dnu+nuo*(alog((2*nuo+dnu)/(2*nuo-dnu))
     &                -ci*pi)))
          else
            eint=bet*exp(-bet*nuo)
     &            *(2*Ao*dnu
     &              +ci/pi*((nuo+dnu)*(alog(2*dnu*nuo+dnu**2)
     &               +ci*pi)-(nuo-dnu)*alog(2*dnu*nuo-dnu**2)
     &               -4*dnu+nuo*(alog((2*nuo+dnu)/(2*nuo-dnu))
     &                -ci*pi)))
          endif
        endif
        Lre=Lre+eint
        Lrm=Lrm+mint
        return
        end
C*******************************************************************
        SUBROUTINE SIMPGF(KRHO,ETAB,EPOL,LRE,LRM)
C*******************************************************************
        real pi,k,nu,nuo,numax,numin,krho
        real*8 rts(64,64),coef(64,64)
        complex ci,carg,croot,Ao,H0,H1,H(64)
        complex re,rm,eint,mint,Lre,Lrm,ssLre,ssLrm
        logical Epol,self,second
        common /data/ rts,coef
        call gausq

        pi=3.141593
        ci=cmplx(0.,1.)
        k=2*pi
        gam=0.5772157
        Ao=1+ci*2/pi*(gam+alog(k/2))
        self=.false.
        second=.false.
        iroot=64
```

```
        numm=21
        ih=0

        alf=k/etab
        bet=k*etab
        nuo=krho/k
c...Determining size of del nu and max nu value necessary
        if(Epol)then
          dnu=alog(0.95)/(-alf)
          numax=alog(0.00001)/(-alf)
          ih=2
        else
          dnu=alog(0.95)/(-bet)
          numax=alog(0.00001)/(-bet)
        endif
        dnumax=sqrt((0.1/k)**2+nuo**2)-nuo
        if(dnu .gt. dnumax) dnu=dnumax
        rnumax=sqrt((12./k)**2+nuo**2)

        if(nuo .eq. 0.0)then
          self=.true.
          second=.true.
          dx=real(Lrm)
          if(dnu .ge. dx) dnu=1.05*dx/2
          numin=dnu
        endif
        if(dnu .ge. nuo .and. .not.(self))then
          dnu=nuo
          second=.true.
        endif
        if(numax .le. nuo+dnu) numax=2*(nuo+dnu)
        if(rnumax .lt. numax) numax=rnumax

c...Self-cell evaluation
        if(self)then
          if(Epol)then
            mint=alf*exp(-alf*dnu/2)*2*(-dx/2*dnu+ci/pi/k**2
     &            *alog((dx+dnu)/(dx-dnu)))
            eint=alf*exp(-alf*dnu/2)*2*((Ao-ci/pi*2)*dx*dnu
     &            +ci/pi*dx*(dnu*alog(dx**2-dnu**2)-2*dnu
     &             +dx*alog((dx+dnu)/(dx-dnu)))
     &            +ci/pi/2*((dx**2-dnu**2)*(alog((dx-dnu)/(dx+dnu))
     &              -ci*pi)+dx*dnu+ci*dx**2*pi))
          else
            eint=bet*exp(-bet*dnu/2)*2*((Ao-ci/pi*2)*dx*dnu
     &            +ci/pi*dx*(dnu*alog(dx**2-dnu**2)-2*dnu
     &             +dx*alog((dx+dnu)/(dx-dnu)))
     &            +ci/pi/2*((dx**2-dnu**2)*(alog((dx-dnu)/(dx+dnu))
     &              -ci*pi)+dx*dnu+ci*dx**2*pi))
          endif
          ssLre=eint
          ssLrm=mint
        endif

        rt=0.0
 13     if(rt .eq. 1.3) numm=numm+2

c...Integration over self-cell
       do 30 mm=1,numm
         if(mm .gt. 1)then
           nuo=nuo+dx/(numm-1)
           if(abs(numin-nuo) .le. 0.000001)then
              rt=1.3
              nuo=0
              print *,'ACK!!!!!!  Changing numm.'
              goto 13
           endif
           if(Epol)then
             dnu=alog(0.95)/(-alf)
           else
             dnu=alog(0.95)/(-bet)
           endif
           dnumax=sqrt((0.1/k)**2+nuo**2)-nuo
           if(dnu .gt. dnumax) dnu=dnumax
           if(nuo .lt. numin)then
              second=.true.
              self=.true.
           else
              if(dnu .ge. (nuo-numin))then
                dnu=nuo-numin
                second=.true.
              endif
           endif
           if(numax .le. nuo+dnu) numax=2*(nuo+dnu)
           if(rnumax .lt. numax) numax=rnumax
         endif

c    Gauss-Quadrature
        Lre=0.0
        Lrm=0.0
       do 20 m=1,2
        if(.not.(m .eq. 1 .and. second))then
          if(m .eq. 1)then
            a=numin
            b=nuo-dnu
          else if(m .eq. 2)then
```

```
            a=nuo+dnu
            if(nuo .lt. numin) a=numin
            b=numax
          else
            a=numax
            b=2*numax
          endif

          mint=0.0
          eint=0.0
          do 70 i=1,iroot
            srts=rts(iroot,i)
            scoef=coef(iroot,i)
            nu=((b-a)*srts+b+a)/2
            croot=((nuo)**2-nu**2)
            carg=k*csqrt(croot)
            if(m .eq. 1)then
              rarg=real(carg)
              call Hankz1(rarg,ih,H0,H1)
            else
              call cHank(carg,ih,H0,H1)
            endif
            if(Epol)then
              rm=alf*exp(-alf*nu)*2*nuo*H1/carg
              re=alf*exp(-alf*nu)*H0
            else
              re=bet*exp(-bet*nu)*H0
            endif
            eint=scoef*re+eint
            mint=scoef*rm+mint
 70       continue
          Lre=(b-a)/2*eint+Lre
          Lrm=(b-a)/2*mint+Lrm
        endif
 20     continue
c...Singularity evaluation, nuo = |x-xi|
        if(.not.(self))then
          if(Epol)then
            mint=alf*exp(-alf*nuo)
     &             *2*(nuo*dnu+ci/pi/k**2
     &             *(alog((2*nuo-dnu)/(2*nuo+dnu))+ci*pi))
            eint=alf*exp(-alf*nuo)
     &             *(2*Ao*dnu
     &               +ci/pi*((nuo+dnu)*(alog(2*dnu*nuo+dnu**2)
     &                +ci*pi)-(nuo-dnu)*alog(2*dnu*nuo-dnu**2)
     &                -4*dnu+nuo*(alog((2*nuo+dnu)/(2*nuo-dnu))
     &                 -ci*pi)))
          else
            eint=bet*exp(-bet*nuo)
     &             *(2*Ao*dnu
     &               +ci/pi*((nuo+dnu)*(alog(2*dnu*nuo+dnu**2)
     &                +ci*pi)-(nuo-dnu)*alog(2*dnu*nuo-dnu**2)
     &                -4*dnu+nuo*(alog((2*nuo+dnu)/(2*nuo-dnu))
     &                 -ci*pi)))
          endif
          Lre=Lre+eint
          Lrm=Lrm+mint
        endif

        H(mm)=Lre
        self=.false.
        second=.false.
 30     continue
        Lre=0.0
        do 40 i=1,numm-2,2
          Lre=H(i)+4*H(i+1)+H(i+2)+Lre
 40     continue
        Lre=ssLre+2*(dx/(numm-1)/3*Lre)
        Lrm=ssLrm-Lrm
        return
        end
C*******************************************************************
C SUBPROGRAM DATAINT CONTAINS INTEGRATION DATA
C*****************************************************************
C
      SUBROUTINE GAUSQ
      REAL*8 RTS(64,64),COEF(64,64)
C
      COMMON /DATA/ RTS,COEF
C

C
C+--------------------------------------------------------------+
C  FIXED POINTS FOR GAUSSIAN QUADRATURE                          |
C+--------------------------------------------------------------+
C
C---N=64
C
      RTS(64,1)=  .999305041735772D0
      RTS(64,2)=  .996340116771955D0
      RTS(64,3)=  .991013371476744D0
      RTS(64,4)=  .983336253884625D0
      RTS(64,5)=  .973326827789910D0
      RTS(64,6)=  .961008799652053D0
      RTS(64,7)=  .946411374858402D0
      RTS(64,8)=  .929569172131939D0
```

```
      RTS(64,9)=  .910522137078502D0
      RTS(64,10)= .889315445995114D0
      RTS(64,11)= .865999398154092D0
      RTS(64,12)= .840629296252580D0
      RTS(64,13)= .813265315122797D0
      RTS(64,14)= .783972358943341D0
      RTS(64,15)= .752819907260531D0
      RTS(64,16)= .719881850171610D0
      RTS(64,17)= .685236313054233D0
      RTS(64,18)= .648965471254657D0
      RTS(64,19)= .611155355172393D0
      RTS(64,20)= .571895646202634D0
      RTS(64,21)= .531279464019894D0
      RTS(64,22)= .489403145707052D0
      RTS(64,23)= .446366017253464D0
      RTS(64,24)= .402270157963991D0
      RTS(64,25)= .357220158337668D0
      RTS(64,26)= .311322871990210D0
      RTS(64,27)= .264687162208767D0
      RTS(64,28)= .217423643740007D0
      RTS(64,29)= .169644420423992D0
      RTS(64,30)= .121462819296120D0
      RTS(64,31)= .072993121787799D0
      RTS(64,32)= .024350292663424D0
      RTS(64,64)=-.999305041735772D0
      RTS(64,63)=-.996340116771955D0
      RTS(64,62)=-.991013371476744D0
      RTS(64,61)=-.983336253884625D0
      RTS(64,60)=-.973326827789910D0
      RTS(64,59)=-.961008799652053D0
      RTS(64,58)=-.946411374858402D0
      RTS(64,57)=-.929569172131939D0
      RTS(64,56)=-.910522137078502D0
      RTS(64,55)=-.889315445995114D0
      RTS(64,54)=-.865999398154092D0
      RTS(64,53)=-.840629296252580D0
      RTS(64,52)=-.813265315122797D0
      RTS(64,51)=-.783972358943341D0
      RTS(64,50)=-.752819907260531D0
      RTS(64,49)=-.719881850171610D0
      RTS(64,48)=-.685236313054233D0
      RTS(64,47)=-.648965471254657D0
      RTS(64,46)=-.611155355172393D0
      RTS(64,45)=-.571895646202634D0
      RTS(64,44)=-.531279464019894D0
      RTS(64,43)=-.489403145707052D0
      RTS(64,42)=-.446366017253464D0
      RTS(64,41)=-.402270157963991D0
      RTS(64,40)=-.357220158337668D0
      RTS(64,39)=-.311322871990210D0
      RTS(64,38)=-.264687162208767D0
      RTS(64,37)=-.217423643740007D0
      RTS(64,36)=-.169644420423992D0
      RTS(64,35)=-.121462819296120D0
      RTS(64,34)=-.072993121787799D0
      RTS(64,33)=-.024350292663424D0
C
C
C
      COEF(64,1)=  .001783280721696D0
      COEF(64,2)=  .004147033260562D0
      COEF(64,3)=  .006504457968978D0
      COEF(64,4)=  .008846759826363D0
      COEF(64,5)=  .011168139460131D0
      COEF(64,6)=  .013463047896718D0
      COEF(64,7)=  .015726030476024D0
      COEF(64,8)=  .017951715775697D0
      COEF(64,9)=  .020134823153530D0
      COEF(64,10)= .022270173808383D0
      COEF(64,11)= .024352702568710D0
      COEF(64,12)= .026377469715054D0
      COEF(64,13)= .028339672614259D0
      COEF(64,14)= .030234657072402D0
      COEF(64,15)= .032057928354851D0
      COEF(64,16)= .033805161837141D0
      COEF(64,17)= .035472213256882D0
      COEF(64,18)= .037055128540240D0
      COEF(64,19)= .038550153178615D0
      COEF(64,20)= .039953741132720D0
      COEF(64,21)= .041262563242623D0
      COEF(64,22)= .042473515123653D0
      COEF(64,23)= .043583724529323D0
      COEF(64,24)= .044590558163756D0
      COEF(64,25)= .045491627927418D0
      COEF(64,26)= .046284796581314D0
      COEF(64,27)= .046968182816210D0
      COEF(64,28)= .047540165714830D0
      COEF(64,29)= .047999388596458D0
      COEF(64,30)= .048344762234802D0
      COEF(64,31)= .048575467441503D0
      COEF(64,32)= .048690957009139D0
      COEF(64,64)= .001783280721696D0
      COEF(64,63)= .004147033260562D0
      COEF(64,62)= .006504457968978D0
      COEF(64,61)= .008846759826363D0
      COEF(64,60)= .011168139460131D0
```

```
      COEF(64,59)= .013463047896718D0
      COEF(64,58)= .015726030476024D0
      COEF(64,57)= .017951715775697D0
      COEF(64,56)= .020134823153530D0
      COEF(64,55)= .022270173808383D0
      COEF(64,54)= .024352702568710D0
      COEF(64,53)= .026377469715054D0
      COEF(64,52)= .028339672614259D0
      COEF(64,51)= .030234657072402D0
      COEF(64,50)= .032057928354851D0
      COEF(64,49)= .033805161837141D0
      COEF(64,48)= .035472213256882D0
      COEF(64,47)= .037055128540240D0
      COEF(64,46)= .038550153178615D0
      COEF(64,45)= .039953741132720D0
      COEF(64,44)= .041262563242623D0
      COEF(64,43)= .042473515123653D0
      COEF(64,42)= .043583724529323D0
      COEF(64,41)= .044590558163756D0
      COEF(64,40)= .045491627927418D0
      COEF(64,39)= .046284796581314D0
      COEF(64,38)= .046968182816210D0
      COEF(64,37)= .047540165714830D0
      COEF(64,36)= .047999388596458D0
      COEF(64,35)= .048344762234802D0
      COEF(64,34)= .048575467441503D0
      COEF(64,33)= .048690957009139D0
C
C---N=64
C
      RTS(64,1)=  .999305041735772D0
      RTS(64,2)=  .996340116771955D0
      RTS(64,3)=  .991013371476744D0
      RTS(64,4)=  .983336253884625D0
      RTS(64,5)=  .973326827789910D0
      RTS(64,6)=  .961008799652053D0
      RTS(64,7)=  .946411374858402D0
      RTS(64,8)=  .929569172131939D0
      RTS(64,9)=  .910522137078502D0
      RTS(64,10)= .889315445995114D0
      RTS(64,11)= .865999398154092D0
      RTS(64,12)= .840629296252580D0
      RTS(64,13)= .813265315122797D0
      RTS(64,14)= .783972358943341D0
      RTS(64,15)= .752819907260531D0
      RTS(64,16)= .719881850171610D0
      RTS(64,17)= .685236313054233D0
      RTS(64,18)= .648965471254657D0
      RTS(64,19)= .611155355172393D0
      RTS(64,20)= .571895646202634D0
      RTS(64,21)= .531279464019894D0
      RTS(64,22)= .489403145707052D0
      RTS(64,23)= .446366017253464D0
      RTS(64,24)= .402270157963991D0
      RTS(64,25)= .357220158337668D0
      RTS(64,26)= .311322871990210D0
      RTS(64,27)= .264687162208767D0
      RTS(64,28)= .217423643740007D0
      RTS(64,29)= .169644420423992D0
      RTS(64,30)= .121462819296120D0
      RTS(64,31)= .072993121787799D0
      RTS(64,32)= .024350292663424D0
      RTS(64,64)=-.999305041735772D0
      RTS(64,63)=-.996340116771955D0
      RTS(64,62)=-.991013371476744D0
      RTS(64,61)=-.983336253884625D0
      RTS(64,60)=-.973326827789910D0
      RTS(64,59)=-.961008799652053D0
      RTS(64,58)=-.946411374858402D0
      RTS(64,57)=-.929569172131939D0
      RTS(64,56)=-.910522137078502D0
      RTS(64,55)=-.889315445995114D0
      RTS(64,54)=-.865999398154092D0
      RTS(64,53)=-.840629296252580D0
      RTS(64,52)=-.813265315122797D0
      RTS(64,51)=-.783972358943341D0
      RTS(64,50)=-.752819907260531D0
      RTS(64,49)=-.719881850171610D0
      RTS(64,48)=-.685236313054233D0
      RTS(64,47)=-.648965471254657D0
      RTS(64,46)=-.611155355172393D0
      RTS(64,45)=-.571895646202634D0
      RTS(64,44)=-.531279464019894D0
      RTS(64,43)=-.489403145707052D0
      RTS(64,42)=-.446366017253464D0
      RTS(64,41)=-.402270157963991D0
      RTS(64,40)=-.357220158337668D0
      RTS(64,39)=-.311322871990210D0
      RTS(64,38)=-.264687162208767D0
      RTS(64,37)=-.217423643740007D0
      RTS(64,36)=-.169644420423992D0
      RTS(64,35)=-.121462819296120D0
      RTS(64,34)=-.072993121787799D0
      RTS(64,33)=-.024350292663424D0
C
C
```

```
C
      COEF(64,1)=  .001783280721696D0
      COEF(64,2)=  .004147033260562D0
      COEF(64,3)=  .006504457968978D0
      COEF(64,4)=  .008846759826363D0
      COEF(64,5)=  .011168139460131D0
      COEF(64,6)=  .013463047896718D0
      COEF(64,7)=  .015726030476024D0
      COEF(64,8)=  .017951715775697D0
      COEF(64,9)=  .020134823153530D0
      COEF(64,10)= .022270173808383D0
      COEF(64,11)= .024352702568710D0
      COEF(64,12)= .026377469715054D0
      COEF(64,13)= .028339672614259D0
      COEF(64,14)= .030234657072402D0
      COEF(64,15)= .032057928354851D0
      COEF(64,16)= .033805161837141D0
      COEF(64,17)= .035472213256882D0
      COEF(64,18)= .037055128540240D0
      COEF(64,19)= .038550153178615D0
      COEF(64,20)= .039953741132720D0
      COEF(64,21)= .041262563242623D0
      COEF(64,22)= .042473515123653D0
      COEF(64,23)= .043583724529323D0
      COEF(64,24)= .044590558163756D0
      COEF(64,25)= .045491627927418D0
      COEF(64,26)= .046284796581314D0
      COEF(64,27)= .046968182816210D0
      COEF(64,28)= .047540165714830D0
      COEF(64,29)= .047999388596458D0
      COEF(64,30)= .048344762234802D0
      COEF(64,31)= .048575467441503D0
      COEF(64,32)= .048690957009139D0
      COEF(64,64)= .001783280721696D0
      COEF(64,63)= .004147033260562D0
      COEF(64,62)= .006504457968978D0
      COEF(64,61)= .008846759826363D0
      COEF(64,60)= .011168139460131D0
      COEF(64,59)= .013463047896718D0
      COEF(64,58)= .015726030476024D0
      COEF(64,57)= .017951715775697D0
      COEF(64,56)= .020134823153530D0
      COEF(64,55)= .022270173808383D0
      COEF(64,54)= .024352702568710D0
      COEF(64,53)= .026377469715054D0
      COEF(64,52)= .028339672614259D0
      COEF(64,51)= .030234657072402D0
      COEF(64,50)= .032057928354851D0
      COEF(64,49)= .033805161837141D0
      COEF(64,48)= .035472213256882D0
      COEF(64,47)= .037055128540240D0
      COEF(64,46)= .038550153178615D0
      COEF(64,45)= .039953741132720D0
      COEF(64,44)= .041262563242623D0
      COEF(64,43)= .042473515123653D0
      COEF(64,42)= .043583724529323D0
      COEF(64,41)= .044590558163756D0
      COEF(64,40)= .045491627927418D0
      COEF(64,39)= .046284796581314D0
      COEF(64,38)= .046968182816210D0
      COEF(64,37)= .047540165714830D0
      COEF(64,36)= .047999388596458D0
      COEF(64,35)= .048344762234802D0
      COEF(64,34)= .048575467441503D0
      COEF(64,33)= .048690957009139D0
C
      RETURN
      END
```

APPENDIX C. Program Listing for the Quasi-Analytical Solution

The quasi-analytical solution presented in Section 3 was programmed for solution, as listed in the program IMPQA.FTN below. The subroutines used by this program are listed in GAPSUB.FTN in Appendix A of [2].

For H-polarization, the far field amplitude P_H is calculated from (28), and for E-polarization, P_E is calculated from (34). The parameters a and b in (26) and (33), respectively, are dependent on the effective surface impedance η of the gap, which is calculated from the formulas in [2] according to the specified shape of the cavity and the field polarization.

```
C
C     IMPQA.FTN
C
C
C       This FORTRAN program computes the far field scattering due
C       to a narrow gap of specified shape in an infinite impedance
C       plane.  The far field amplitude is calculated given the
C       input impedance of the gap, calculated from it's equivalent
C       transmission line model.
C
C       INPUT   The user is prompted from the subroutine
C               GAPROM for the polarization and angle of the
C               incident field, angle of far field observation,
C               relative permittivity of gap filling, shape and
C               dimensions of gap, segment size, number of
C               iterations with respect to gap depth, and
C               normalized input impedance.
C
C       OUTPUT FILES
C               GAPDAT  Contains input data.
C               IMPDAT  Effective surface impedance of the gap.
C               AMPDAT  Contains the magnitude of the far field.
C               PHADAT  Contains the phase of the far field.
C
C       SUBROUTINES
C               HANKZ1  Computes the Hankel functions of the first
C                       kind of orders zero and one.
C               CHANK   Computes the Hankel functions of the first
C                       kind of orders zero and one given a
C                       complex argument.
C               MODBES  Computes the modified Bessel functions of
C                       the first kind of orders zero and one.
C
C       FUNCTION
C               CTAN    Calculates the tangent of a complex
C                       argument.
C
C
      integer EorH,N,noS,gN,szN(50)
      real pi,k,phi,phio,w,d,maxC,q(50,2)
      real dStp(50),wStp(50),I0,I1,krho,psi
      complex czero,ci,ctemp,er,ur,k1,kc,carg,ctan,eta
      complex Zi,Z1,Zc,ZL,X,B1,B2,H0,H1,a,b,Ke,Kh,Ao,Psca,ge,gh
      logical Epol,Lossy
      common /prompt/ EorH,phio,phi,er,ur,igap,wStp,dStp,w,d,noS,
     &                 q,maxC,noIter,etab

    1 format(i1)
    2 format(i5)
    3 format(g16.8)
    4 format(a1)
    5 format(i3)
    6 format(2g16.8)

      open(1,file='gapdat')
      open(2,file='impdat')
      open(3,file='ampdat')
      open(4,file='phadat')

c...Declaring constant values
      czero=cmplx(0.0,0.0)
      ci=cmplx(0.0,1.)
      pi=4.0*atan(1.0)
      k=2*pi
      Eo=1.0
      Ho=1.0
      Zo=sqrt(4.e-07*pi/8.854e-12)
      Yo=1./Zo
      gam=0.5772157
      iprg=2

c...Setting default values
 10   EorH=1
      phio=90.0
      phi=90.0
      er=cmplx(1.,0.0)
      ur=cmplx(1.,0.0)
      w=0.15
      d=0.5
      noS=3
      maxC=0.01
      noIter=30
      etab=1.
      adj=0.00001
      Epol=.false.
      Lossy=.false.
      side=.true.

c...Prompting user for input data
 15   call gaprom(iprg)

      if(igap .eq. 5) GOTO 15
      if(EorH .eq. 1) Epol=.true.
      phio=phio*pi/180.0
      phi=phi*pi/180.0
      drat=dStp(1)/d
```

```
      if(aImag(er) .ne. 0.0) Lossy=.true.

      dmin=0.025
      if(Epol) dmin=0.02
      dmax=d
      if(noIter .ne. 1)then
        dstep=(dmax-dmin)/(noIter-1)
        d=dmin
      endif

      DO 700 iter=1,noIter
c*****************************************************************
c*********************    Gap Impedance    ***********************
c*****************************************************************

c...Complex propagation constant k1 and characteristic impedance
c    Z1 of the T-line model
        if(Epol)then
          k1=ci*k*csqrt((1./2/w)**2-er*ur)
          Z1=-ci*Zo*ur/csqrt((1./2/w)**2-er*ur)
        else
          Z1=Zo*csqrt(ur/er)
          k1=k*csqrt(ur*er)
        endif
          if(igap .eq. 1)then
c        RECTANGULAR
            ETA=-ci*Z1*ctan(k1*d)
          else if(igap .eq. 2 .or. igap .eq. 4)then
            w=wStp(1)
            w2=wStp(2)
            w3=wStp(3)
            d1=d*drat
            d2=d*(1-drat)
c         Propagation constant and characteristic impedance of
c         the arms of the T- or L-shaped gaps
            if(Epol)then
              kc=ci*k*csqrt((1./2/d2)**2-er*ur)
              Zc=-ci*Zo*d2*ur/csqrt((1./2/d2)**2-er*ur)
            else
              Zc=Zo*d2*csqrt(ur/er)
              kc=k*csqrt(ur*er)
            endif
            if(igap .eq. 4)then
c        L-SHAPED
              Zi=-ci*Zc*ctan(kc*w2)
              X=k1*Zc*w1
              B1=k1/Zc*d2*(d2/(d2+w1))*(1.-2./pi*log(2.))
              B2=k1/Zc*d2*(w1/(d2+w1))*(1.-2./pi*log(2.))
              ZL=(Zi-ci*X*(1-ci*B2*Zi))
     &           /((1-B1*X)*(1-ci*B2*Zi)-ci*B1*Zi)
            else
c        T-SHAPED
              Zi=-ci*Zc*(ctan(kc*w2)+ctan(kc*w3))
              X=k1*Zc*w1
              B1=k1/Zc*d2*(d2/(d2+w1))*0.7822
              ZL=(Zi-ci*X)/(1-ci*B1*(Zi-ci*X))
            endif
            ETA=Z1*(ZL-ci*Z1*w1*ctan(k1*d1))
     &           /(Z1*w1-ci*ZL*ctan(k1*d1))
          else if(igap .eq. 3)then
c        TRIANGULAR
            if(Lossy)then
               carg=k1*d
               call cHank(carg,3,H0,H1)
               ETA=-ci*Z1*H1/H0
            else
               if(Epol)then
                  rarg=Real(k1/ci)*d
                  call ModBes(rarg,I0,I1)
                  H0=I0
                  H1=I1
                  carg=ci
               else
                  rarg=Real(k1)*d
                  call Hankz1(rarg,2,H0,H1)
                  carg=1.
               endif
               ETA=-ci*Z1*Real(H1)/Real(H0)*carg
            endif
          endif
c          write(2,*) d,cabs(ETA)


c*****************************************************************
c******************    Far Field Amplitude    ********************
c*****************************************************************
        Rsdp=1./(etab+1)
        if(Epol)then
           theta=(-.380-0.8*w)*(1-exp(-2.586*sqrt(etab)))
           dw=1.558-4.266*w
           ge=exp(-dw*etab)*cexp(ci*theta)
           b=-ci*k*w/2*Zo/(ETA-etab*Zo)/ge
           Ke=0.62/(b+1.15)*(b+4.08)*(b+7.26)*(b+10.37)
     &        *(b+13.43)*(b+16.46)
     &        /((b+4.27)*(b+7.37)*(b+10.45)*(b+13.49)
     &        *(b+16.5))
```

```
            Ri=sin(phio)/(sin(phio)*etab+1)
            Psca=-ci*pi/4*(k*w)**2*sin(phi)*Rsdp/ge*Ri*Ke
         else
            psi=1-exp(-1*(0.098+1.76*w)*etab)
            cw=0.245+1.267*w
            gh=exp(-cw*etab)*cexp(ci*psi)
            a=ci*2/k/w*Zo/(ETA-etab*Zo)/gh
            Kh=-1./(pi/2*a+0.1+log(2.))
            Ao=log(k*w/4)+gam-ci*pi/2
            Ri=sin(phio)/(etab+sin(phio))
            Psca=ci*pi*Rsdp/gh*Ri*Kh/(1+Ao*Kh)
         endif

c...Outputting the far field magnitude and phase
         print *,' d = ',d
         print *,'      Analytical: |Psca| = ',cabs(Psca),
      &  '  arg Psca = ',180/pi*(atan2(aImag(Psca),Real(Psca)))
         write(3,*) d,cabs(Psca)
         write(4,*) d,180/pi*(atan2(aImag(Psca),Real(Psca)))

         d=d+dstep
700   continue

         print *,' Again (1=yes) ? '
         read(*,1)ians
         if(ians .eq. 1) GOTO 10

800   call exit
      END
```