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# COLLISIONAL EFFECTS ON THE DRIFT-CYCLOTRON INSTABILITY

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The drift-cyclotron instability in a weakly collisional plasma is considered including temperature perturbations. Collisions are described by a model Fokker–Planck equation. The growth rate of the instability is obtained analytically.

## 1. Introduction

It is well known (see for example, Ichimaru [1]) that if the drift-wave frequency is near the harmonics of the ion-cyclotron frequency, coupling between drift waves and ion-Bernstein waves can occur. Because both types of waves propagate predominantly perpendicular to the external magnetic field, the coupling can be very efficient. Here, the free energy in the spatial inhomogeneity is fed into the ion-Bernstein waves, thus driving the instability.

The effect of collisions on instabilities has often been investigated by means of the BGK collision model. However, this model is not really suitable for describing charged particle collisions [2], and is also questionable for describing short wavelength phenomena [3]. It is therefore necessary to use either the full Fokker–Planck collision operator, or some better model operator. Due to its complexity, the full Fokker–Planck operator leads to approximate solutions which are often divergent [4, 5]. On the other hand, the model Fokker–Planck operator of Lenard and Bernstein [6] has proven [2, 7–10] to be both sufficiently realistic and tractable. This practical model incorporates diffusion and friction in velocity space while neglecting the velocity dependence of the collision frequency, as well as the anisotropy of the dynamic friction force. It nevertheless conserves number density, momentum, and energy in collisions [2, 8, 9].

Yu [11] considered the effect of collisions on the drift-cyclotron instability using the model Fokker–Planck operator but neglected temperature perturbations. The latter assumption corresponds to the so-called isothermal approximation, which has been shown to be inadequate in certain problems [12].

In this paper, we therefore reconsider the drift-cyclotron instability by including temperature perturbations consistently. Analytical results for the growth rates are presented for small collision frequencies and compared with previous calculations.

## 2. Formulation

Consider a low-$\beta$ plasma situated in a constant magnetic field $B_0\hat{z}$. The plasma density is assumed to have a gradient along the $x$ axis. The temperature gradient has been neglected. A temperature difference between the electrons and ions is maintained, say, by an external heat source for the electrons and an external heat sink for the ions [12].

The kinetic equation with the model Fokker–Planck term is [12]

$$\frac{\partial f_j}{\partial t} + v \cdot \frac{\partial f_j}{\partial r} + \frac{e_j}{m_j}(E + v \times B) \cdot \frac{\partial f_j}{\partial v} = C_{jj}(f_j) + C_{jl}(f_j), \tag{1}$$

where

$$C_{jj}(f_j) = \nu_{jj} \frac{\partial}{\partial v} \cdot \left[ \frac{T_j}{m_j} \frac{\partial}{\partial v} + (v - u_j) \right] f_j$$

describes collisions among like particles, and

$$C_{jl}(f_j) = \nu_{jl} \frac{\partial}{\partial v} \cdot \left[ \frac{T_j}{m_j} \frac{\partial}{\partial v} + v - \frac{\nu_{lj} m_e}{\nu_{jl} m_j} u_j \right] f_j$$

describes collisions among unlike particles. Here, $j$, $l$ stand for either the electrons or the ions. The collision operators $C_{jj}$ and $C_{jl}$ conserve number density, momentum, and energy in collisions, provided that there is no external current.

We shall also use the following macroscopic conservation and Poisson equations,

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial r} \cdot (n_j u_j) = 0, \tag{2}$$

$$m_j n_j \left( \frac{\partial u_j}{\partial t} + u_j \cdot \frac{\partial u_j}{\partial r} \right) = -\frac{\partial}{\partial r} n_j T_j + \frac{\partial}{\partial r} \cdot \pi_j + n_j e_j \left( E + \frac{u_j \times B}{C} \right), \tag{3}$$

$$\frac{\partial}{\partial r} \cdot E = 4\pi \sum_j e_j n_j, \tag{4}$$

where $\pi_j$ is the traceless stress tensor, which can be neglected if the collision frequency is small.

Assuming that the density gradient is weak, to lowest order we obtain from eq. (1) the steady state

$$f_{0j}(r, v) = \left[ 1 + \left( x + \frac{v_y}{\Omega_j} \right) \epsilon_j \right] f_{Mj}(v), \tag{5}$$

where

$$\epsilon_j = \frac{1}{n_j} \frac{\mathrm{d}n_j}{\mathrm{d}x}, \quad \Omega_j = \frac{e_j B_0}{m_j c},$$

and

$$f_{Mj}(v) = n_j \left( \frac{m_j}{2\pi T_j} \right)^{\frac{3}{2}} \exp\left( -\frac{m_j v^2}{2T_j} \right).$$

The like and unlike particle collision terms in eq. (1) can be combined. Letting $f_j = f_{j0} + f_{j1} \exp(-\mathrm{i}\omega t + \mathrm{i}k \cdot r)$ where $f_{j1} \ll f_{j0}$, we obtain to first order

$$(3\nu_j + \mathrm{i}\omega - \mathrm{i}k \cdot v) f_{j1} - \left[ \frac{e_j}{m_j c}(v \times B_0) - \nu_j v \right] \frac{\partial f_{j0}}{\partial v} + \frac{\nu_j T_{j0}}{m_j} \frac{\partial^2 f_{j1}}{\partial v^2}$$
$$= \left( \frac{e_j}{m_j} E_1 + \nu_j u_{j1}^* \right) \cdot \frac{\partial f_{j0}}{\partial v} + \frac{\nu_j T_{j1}}{m_j} \frac{\partial^2 f_{j0}}{\partial v^2}, \tag{6}$$

where

$$u_{j1}^* = \frac{1}{\nu_j} \left( \nu_{jj} u_j + \nu_{jl} \frac{m_e n_e}{m_j n_j} u_l \right)$$

and

$$\nu_j = \nu_{jj} + \nu_{jl}.$$

In the above, magnetic perturbations are neglected, since all of the modes involved are electrostatic. In the following we shall assume that collisions are weak, such that the collision frequencies are smaller than any of the wave frequencies.

## 3. The dispersion relation

Eq. (6) can be solved exactly by means of a Fourier transformation in the velocity space together with the method of characteristics. The general solution can be obtained [12] after eliminating $u_j$, $T_{j1}$, $\phi_1$ *via* the moment and Poisson equations (2), (3), and (4). For our purpose, in the limit $k_z v_{T_e} < \omega \approx n\Omega_i$, $b_j \equiv \frac{1}{2} k_\perp^2 v_{T_j}^2 / \Omega_j^2 > 1$, $\nu_j < \Omega_i$, we obtain the following dispersion relation

$$k^2 \lambda_i^2 = 1 + \frac{\omega_i^* - \omega - \mathrm{i} b \nu_i}{\omega + \mathrm{i} b_i - n\Omega_i} \Delta + \frac{T_i}{T_e} \left( b_e + \frac{\omega_e^*}{\omega} \right) + S_1 + S_2 + \frac{T_i}{T_e} S_3 + S_4, \tag{7}$$

where $\omega_j^* = ck_\perp T_j \epsilon / e_j B_0$, $\Delta = (2\pi b_i)^{-\frac{1}{2}}$, $\lambda_i = v_{Ti}/\omega_{pi}$, and $n$ is the harmonic number. The functions $S_1$, $S_2$ and $S_3$ are rather complicated and are given in ref. 12. We have also defined

$$S_4 = -\frac{\mathrm{i}\nu_i}{\omega - n\Omega_\mathrm{i}} \frac{\epsilon}{k_\perp} n\Delta \left[ 1 + 2\left(1 + \frac{\omega_j^* - \omega - \mathrm{i}b_i\nu_i}{\omega + \mathrm{i}b\nu_i - n\Omega_\mathrm{i}}\Delta\right)\left(\frac{1}{2} - \frac{\omega^2 - \omega\omega_i^*}{k^2 v_{Ti}^2}\right)\right].$$

## 4. Results

We briefly derive the collisionless limit [13], which is the first order solution in our approximation. Accordingly, we set $\nu_\mathrm{e} = \nu_\mathrm{i} = 0$, the dispersion relation (7) then becomes

$$\delta - \frac{\omega_\mathrm{e}^*}{\omega} = \frac{(\omega - \omega^*)\Delta}{\omega - n\Omega_\mathrm{i}}, \tag{8}$$

where

$$\delta = 1 + k^2\lambda_i^2(1 + \omega_\mathrm{pe}^2/\Omega_\mathrm{e}^2).$$

Eq. (8) contains two branches. One is the electron drift-wave branch, namely

$$\omega_1 \approx \omega_\mathrm{e}^*/\delta. \tag{9}$$

The other branch describes the ion Bernstein waves, namely

$$\omega_2 \approx n\Omega_\mathrm{i}(1 + \Delta/\delta). \tag{10}$$

If these two branches intersect each other, then one expects unstable complex roots near the intersections. The condition for the instability to occur is [1, 11, 13]

$$2(\delta - 1)\,\omega_1(\omega_1 + \omega_2)\Delta > (\omega_1 - \omega_2)^2(\delta - 2\Delta). \tag{11}$$

The corresponding growth rate is

$$\gamma_0 = n\Omega_\mathrm{i}\,\Delta^{\frac{1}{2}}(1 - 1/\delta)^{\frac{1}{2}}. \tag{12}$$

Since $\Delta \ll 1$, in order for the instability to occur, the coupling condition $\omega_1 \approx \omega_2$ must be satisfied. It should be mentioned that the growing solution here does not originate from Landau type wave-particle resonant interaction.

We now investigate the effect of collisions and finite $k_z$, including temperature perturbations. It is clear that the real part of the frequency is unaffected by collisions. Letting $\omega = \omega_1 + \mathrm{i}\gamma_0 + \mathrm{i}\gamma_1$, $\gamma_1 < \gamma_0$, we obtain from eq. (7) the first order growth rate

$$\begin{aligned}\gamma_1 = {} & \frac{-2k_z^2 v_{Te}^2\,\Delta^{\frac{1}{2}}(\delta - 1)^{\frac{1}{2}}}{n\Omega_\mathrm{i}\,\delta^{\frac{3}{2}}} - \frac{4\nu_i\Delta}{\delta}\,[n^2(2+\delta) + 2b] \\ & - \frac{\nu_i}{2}\frac{k_z^2 v_{Ti}^2}{n^2\Omega_\mathrm{i}^2}\left\{\frac{1}{\delta - 1}\left[2(b - 4n^2) + \frac{3b\delta(1+\delta)}{(\delta - 1)\Delta}\right] + \frac{T_\mathrm{e}}{T_\mathrm{i}} + \frac{1}{\delta}\right\} + \nu_\mathrm{e}\frac{k_z^2}{k^2} - \frac{\nu_i \epsilon n\,\Delta^{\frac{3}{2}}}{2k_\perp(\delta - 1)^{\frac{1}{2}}\delta^{\frac{1}{2}}} \\ & \times \left\{1 + [1 + \Delta\delta(\delta - 1)^{\frac{1}{2}}]\left(1 + \frac{2n^2\mu\Omega_\mathrm{i}^2}{\omega_\mathrm{pi}^2}\right)\right\},\end{aligned} \tag{13}$$

where $\mu = 1 + \omega_\mathrm{pe}^2/\Omega_\mathrm{e}^2$. The last term can be shown to originate from temperature perturbation effects.

## 5. Conclusion

We have shown that electron–ion collisions increase the growth rate of the drift-cyclotron instability provided that the waves have a component in the direction of the magnetic field ($k_z \neq 0$). This effect may be related to the drift dissipative instability [14]. On the other hand, ion–ion collisions tend to decrease the growth rate. The effect of temperature perturbations is to increase the ion–ion collisional damping rate, while leaving the electron collisions unaffected. In view of this and previous investigations [5, 12, 15], it is clear that temperature perturbations should be included when model collision operators are used.

In this paper we have not investigated the nonlinear development of the instability. However, if collisions are sufficiently strong, the instability can be quenched either by strong ion–ion collisional damping or by destruction of the coupling condition $\omega_1 \approx \omega_2$ due to frequency shift. On the other hand, if the collisions are weak, a final low-frequency turbulent state might be expected, leading to heating of ions.

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