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ON IONOSPHERIC AERODYNAMICS*

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Summary—This paper presents theoretical methods to determine the gas dynamic and the electrostatic effects due to the interaction caused by a rapidly moving body in the ionosphere. The principles of the methods are derived from the kinetic theory of collision-free plasma. It is shown that the collective behavior of the collision-free plasma makes it possible to use the fluid approach to treat the problems of ionospheric aerodynamics. Various solutions to the system of fluid and field equations that have direct bearing on the ionospheric aerodynamics are presented and discussed. Physical significances of the mathematical results are stressed. Some outstanding unsolved problems in ionospheric aerodynamics are elaborated and discussed.

LIST OF SYMBOLS AND UNITS (EM)

All symbols are defined in the text. Only those that are repeatedly used are listed here. In the description of electromagnetic fields Gaussian cgs units are adopted throughout. These symbols are compiled in a separate group with their corresponding quantities in rationalized mks units with conversion factors listed in tandem.

GROUP A (electromagnetic)

Gaussian	*Quantity*	*mks*
$\mathbf{B}$	magnetic induction	$(4\pi/\mu_0)^{1/2}\mathbf{B}$
$\mathbf{H}$	magnetic field	$(4\pi\mu_0)^{1/2}\mathbf{H}$
c	speed of light	$(\epsilon_0\mu_0)^{-1/2}$
$\mathbf{E}$	electric field	$(4\pi\epsilon_0)^{1/2}\mathbf{E}$
ϕ	electric potential	$(4\pi\epsilon_0)^{1/2}\phi$
e	charge on proton	$e/(4\pi\epsilon_0)^{1/2}$
ρ_c	charge density	$\rho_c/(4\pi\epsilon_0)^{1/2}$
$\mathbf{J}$	current density	$\mathbf{J}/(4\pi\epsilon_0)^{1/2}$
ϵ	dielectric constant (ϵ_0, in free space)	ϵ/ϵ_0
μ	permeability (μ_0, in free space)	μ/μ_0
$\mathbf{D}$	displacement	$(4\pi/\epsilon_0)^{1/2}\mathbf{D}$
$\overleftrightarrow{\sigma}$	electrical conductivity	$\overleftrightarrow{\sigma}/(4\pi\epsilon_0)$
ω_p	plasma frequency	
Ω_α	Larmor frequency for particle of type α	
λ_D	Debye shielding length	
e_α	charge on particle of type α	

GROUP B (mechanical)

$\mathbf{x}$	position
$\mathbf{v}$	particle velocity
v_α	thermal speed of particles of type α
$f_\alpha(\mathbf{x}, \mathbf{v}, t)$	distribution function of particles of type α
T	temperature
n_α	number density of particles of type α
m_α	mass of particle of type α
l	mean free path
ν	collision frequency
κ	Boltzmann constant
h	Planck constant
p	scalar pressure
$\overleftrightarrow{\mathbf{P}}$	pressure tensor
$\mathbf{V}$	flow velocity
$\mathbf{q}$	thermal flux
ρ	mass density

1. INTRODUCTION

The advent of space flight which entails the interaction between a rapidly moving, electrically charged body, e.g. a spacecraft, and a tenuous ionized gas, e.g. the terrestrial ionosphere or the interplanetary gas, brings the aerodynamicists to bear with a new aspect of aerodynamics—the ionospheric aerodynamics.† This interdisciplinary science which combines gas dynamics with electromagnetic fields poses some distinctive traits of its own even though it shares the basic principles of plasma physics with studies on electrical discharges and nuclear fusion. The research on electrical discharges in gases dated back several decades to the works of J. J. Thomson and then I. Langmuir and many other prominent physicists. It was Langmuir who coined the word "plasma" to mean that part of a gaseous discharge which contains almost equal charge densities of free electrons and positive ions. Intensive studies on plasma began with the realization of possible controlled release of nuclear fusion energy in the earlier 1950s.[1] Important contributions to the basic understanding of plasma phenomena have also been made by astronomers and geophysicists in their studies of the solar activities[2] and of the atmospheric electricity.[3] A new incentive to the study of space plasma was added in the late 1950s when the solar wind in the interplanetary space was discovered.[4]

The speed of a spacecraft, of course, varies but its typical value is mesothermal which means it lies intermediate between the thermal speeds of ambient ions and electrons; specifically, it satisfies the following conditions: $(\kappa T_i/m_i)^{1/2} \ll V_\infty \ll (\kappa T_e/m_e)^{1/2}$. The typical values of these thermal speeds in the upper ionosphere of the earth are 1 and 100 km/sec respectively while a representative speed of an artificial earth satellite is of the order of 10 km/sec. The fact that the motion is hyperthermal with respect to the ambient ions and almost stationary to the swiftly moving thermal electrons tends to cause separation in the flow field. This, however, is re-

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† Also known as electrogasdynamics or simply abbreviated as EGD.

strained by the natural tendency of the plasma to preserve electrical neutrality, i.e. to have a minimal if not zero charge separation. These contrasting influences on a mesothermally moving plasma force it to develop local charge separation that confines to a distance of the order of a Debye shielding length (λ_D) in the absence of an external magnetic field.[5]

It is of interest to investigate further the nature of the disturbances of particles and fields caused by a charged body in rapid motion. It is commonly known by an aerodynamicist that there is fluid disturbance in the vicinity of a moving body. If the disturbed fluid is of the charged species, electric current can be induced whenever a differential motion of the oppositely charged particles prevails; in other words, there will be induced field of charge separation present. An electric field will influence the motions, hence the distributions of the charged particles which, in turn, further affect the field. This inherent coupling effect of particles and field almost invariably plays a primary role in the problems of ionospheric aerodynamics. We shall see that this particle-field interaction is governed by a system of coupled nonlinear equations. It is in the unravelling of this nonlinear coupling effect that various schemes of mathematical approximations have been introduced in the contemporary studies of ionospheric aerodynamics. Unfortunately, some of those approximations are not justifiable for the problems at hand and the consequences are particularly serious for those cases where the nonlinear coupling of particle and field is inherent with the problem, such as the near wake.[5] A contemporary belief that a bad approximation can be compensated by a numerical iteration of a nonlinear system is not necessarily valid because the iteration process in question may not always converge. Thus a self-consistent solution of particle and field distributions may not be obtained as expected.[5]

Another aspect of the ionospheric aerodynamics that could be of eminent interest to the aerodynamicists concerns with the method of formulation of the problems, to wit, the particle versus the fluid view of the problems. It is to be reminded that the flow medium, namely the ionosphere, is very tenuous; the mean free paths of the ambient particles, based on some binary collision cross sections, are much larger than the size of the body in motion. Had the particles of flow been neutral species, the particle approach, i.e. to use the Boltzmann equation of kinetics, will be naturally adopted.[6] With charged particles, the long-range nature of the inter-particle forces leads to their collective behavior in a collision-free plasma hence makes it much more amenable to the fluid approach* which is comparatively simpler. It must, however, be noted that with the use of a fluid approach, those plasma phenomena which are associated with the microscopic velocity distribution of the particles will be lost through the preliminary averaging process over the velocity space from which the macroscopic equations of flow and field have been formulated. A case in point here is the Landau damping of collison-free plasma oscillations which disappears when the fluid approach is used. The justification for the use of a fluid approach can be strengthened by citing the successful continuum theory of bow shock resulting from the impinging of solar wind on the terrestrial magnetosphere. On the other hand, the elucidation of the collision-free shock transition process which is obviously a problem of kinetics, is beyond the reach of the fluid approach.

It is the intent of the present review to discuss both methods of approach particularly the connection between them. Several applications of these methods to the ionospheric problems of interest will be presented. The topic of plasma diagnostic probes which is a very important subject of the field is excluded here because there have been several recent reviews on plasma probes.[5,7]

The presentation includes primarily theoretical studies of the problems of interest. Efforts are made to elucidate the physical essence of the phenomena using the simplest mathematical tools we are acquainted with. The description of experimental results is kept to a minimum† partly because of the scarcity of laboratory experiments that really meet the requirement of a collision-free plasma. The *in situ* observations from space probes are indeed plentiful. Unfortunately, these measurements of aerodynamically related quantities for purposes other than the interest of aerodynamics are inadequate to be used for elucidating the physics of fluids in question. In fact, the exposition of the need to appreciate the aerodynamic significances in the space research and teaching is the primary motivation of the present paper.

2. NATURE OF THE EARTH'S IONOSPHERE

The atmosphere of a planet is characterized by several parameters: the temperature, the density, the chemical composition and the degree of ionization. These properties vary with increasing altitude above the planetary surface. The density of an atmosphere decreases continuously with increasing altitude and is related to the temperature of the atmosphere by the hydrostatic equation and the equation of state of the atmospheric gas in question. It is well known from statistical mechanics of gases that a mixture of gases with different molecular masses, such as the atmosphere when left undisturbed to itself for a sufficient length of time, would diffuse through each other and attain an isothermal

* Also known as the continuum approach.

† Additional references on the contemporary studies of plasma, both theoretical and experimental, are available in ref. 5.

equilibrium in which each constituent gas is distributed exponentially according to Dalton's law, whereby the lighter gases will predominate in the higher altitude and the heavier ones in the lower.[8] The atmosphere near the ground level is constantly disturbed by the atmospheric circulation and turbulent mixing. This keeps the atmosphere homogeneous in composition until about 105 km above the Earth, according to the sounding rocket observations, where the diffusive separation of the constituent gases commences. It has been found that at this altitude, called the turbopause, where the molecular diffusion begins to dominate over the turbulent diffusion, the atmosphere starts to distribute according to Dalton's law. At an altitude of about 200 km atomic oxygen, produced by photochemical dissociation of the molecular oxygen, takes over from the molecular nitrogen as the main constituent, only to be superseded by helium and then hydrogen. The Earth indeed has an outer hydrogen atmosphere, but not below 1000 km or so.[9]

From experiments on radio transmission, it was found that ionized layers, conducting electric currents and reflecting radio waves, must exist in the upper atmosphere of the Earth. These layers are called the ionosphere which contains free electrons and ions. Accurate soundings of these layers has been made possible by means of radio-echo techniques. In fact, the structure of the Earth's lower ionosphere had been indirectly measured long before the space age. None the less, extensive direct measurements of the ionospheric parameters, the density and temperature of electrons and ions, from the use of sounding rockets and spacecrafts consolidate our knowledge of the earth's ionosphere.[9]

The ionospheric layers are designated by letters D, E and F, subdivided when necessary. The boundary between the D and E regions is generally taken to be at 90 km altitude and that between the E and F regions at 150 km. The upper ionosphere merges without clear-cut distinction into the magnetosphere, a tenuous region extending out to distances of several earth radii. The magnetosphere contains ionized gases and energetic charged particles trapped in the earth's magnetic field: besides the MeV particles of the "Van Allen" radiation belts, there are the lower energetic particles that cause auroral displays when they are dumped into the upper atmosphere near the poles.[10]

It has long been established that the solar ionizing radiation is primarily responsible for the production of the ionosphere. Measurements by rocket-borne detectors have convincingly shown that the ionizing UV- and X-radiations from the sun are sufficiently strong and are absorbed at the appropriate altitudes to produce the D, E and F_1 layers. At times, and particularly in high latitudes, the energetic particles bombard the atmosphere from above and produce ionization in the ionosphere. Various processes exist to annihilate the ionization by recombination of positively and negatively charged species in the ionosphere. The electron concentrations in the ionospheric layers are essentially controlled by the balance in production and loss of ionization. As an example, it is known that F_1 and F_2 layers are produced by the same photon radiation; the electron density in the F_2 layer is, however, greater even though the radiation is relatively weakly absorbed. The compensating factor is that the recombination of electrons and ions in the F_2 layer is much slower. This steady state theory holds except at night. In the lower ionosphere the balance between production and loss is complicated, particularly below the turbopause where the small-scale eddies and wave motions keep the atmosphere well mixed. On the other hand, at altitudes higher than 250 km, transport processes including effects of wind and electric field become important. At much higher altitude, the Earth's magnetic field plays a conspicuous role in determining the structure of the ionosphere. Electrons and protons enter the atmosphere from above spiralling down magnetic lines of force and give rise to auroral emissions and ionization. The same lines of force extend outwards to form part of the boundary of the magnetosphere.[11,12]

Typical electron density vs. altitude at midlatitude is shown in Fig. 1 for the purpose of illustration. It is based on the midlatitude ionospheric measurements[10] and intended to demonstrate that the D-layer almost disappears at night, and the F_1 and F_2 layers come close to merging. Curve "I" corresponds to minimum of sunspot cycles while "II", maximum of sunspot cycles.

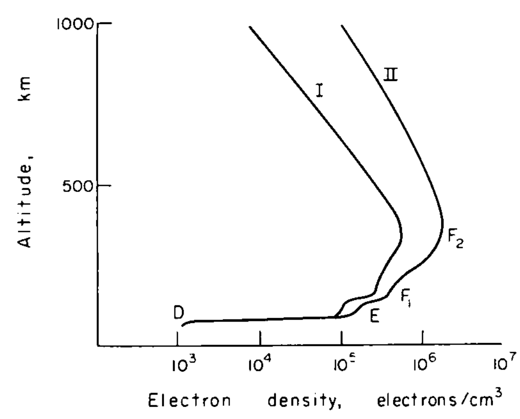

FIG. 1. Ionospheric profile of the Earth at mid-latitude.
'I'—minimum of sunspot cycle.
'II'—maximum of sunspot cycle.

3. KINETIC PROPERTIES OF IONIZED GASES

Although an ionized gas is a gaseous mixture, it behaves kinetically very different from that of the neutral molecular species. Many of these properties peculiar to the ionized gases have important bearings on the understanding of ionospheric aerodynamics. We shall discuss them in the following.

3.1. *Electrical Neutrality and Shielding Effect*

An ionized gas may be composed of several components: positively and negatively charged ions, free electrons and also neutral atoms and molecules. Each of these component species can be characterized by its concentration and temperature. In the simplest case all the ions are atomic species with single positive charges, and neutral molecules are fully dissociated into neutral atoms. The ionized gas thus contains only three components: free electróns, ions and neutral atoms; it contains no surplus charge of either kind, that is to say, the total amount of positive charge contributed by the ions will be closely equal to the total amount of negative charges contributed by the electrons, hence the state of the gas is electrically neutral. To preserve this electrical neutrality-status has been a unique characteristic of an ionized gas. It is of interest to see how the charged particles react when a local deviation from charge neutrality is forced upon the gas.

For simplicity, consider a fully ionized gas containing free electrons and singly charged ions only. Let an isolated positive point charge (Ze) be introduced to the gas. In equilibrium, the spatial distribution of the electrons is given by the Boltzmann distribution:* $n_e = n_\infty \exp(e\phi/\kappa T)$ where $\phi(\mathbf{x})$ is the field potential due to charge imbalance. Ignoring ion motion† and letting the site of the charge particle (Ze) be the origin of a spherical coordinate, the Poisson equation becomes

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) = -4\pi\left[en_\infty - en_\infty \exp\left(\frac{e\phi}{\kappa T}\right) - Ze\delta(\mathbf{x})\right] \simeq \frac{4\pi n_\infty e^2}{\kappa T} + 4\pi eZ\delta(\mathbf{x}) \quad (3.1)$$

at distances such that $e\phi(r)/\kappa T \ll 1$ where $\delta(\mathbf{x})$ is the Dirc delta function. The solution to eqn. (3.1) is

$$\phi(r) = -\frac{Ze}{r}\exp\left(-\frac{r}{\lambda_D}\right) \quad (3.2)$$

where $\lambda_D = (\kappa T/4\pi n_\infty e^2)^{1/2}$ is known as the Debye shielding length (or simply Debye length), being first derived by Debye in his electrolyte theory.

Notice that the potential (3.2) is a shielded Coulomb potential field where λ_D is the effective cut-off distance; it gives the magnitude of the sheath thickness within which charge neutrality may not be valid. The above analysis shows that the response of the charged particles to the extraneous charge (Ze) is to create a potential field to shield the influence of its charge imbalance. The physical interpretation of the shielding effect is that the electric field tends to polarize the gas while the thermal motion of the charged particles tends to weaken the polarization. As a consequence, shielding of the field occurs at some finite-distance (λ_D) which increases with temperature (T) and decreases as the number density (n_∞) increases. If we imagine that all the positively charged ions in the gas behave as the above-mentioned point charge, each of them must have a shielded field of its own. A quiescent ionized gas in equilibrium thus has a lumpy (grainy) structure where λ_D defines the size of the grainedness. The above disclosure has an important bearing on the dynamics of ionized gases: the fact that the potential between charged particles is the shielded Coulomb rather than Coulomb field means that the effective range for the binary collisions is significantly shorter, hence the Boltzmann collision integral for the charged particles is no longer divergent as with Coulomb interactions.[13]

The parameter $N_D \equiv n_\infty\lambda_D^3$ is a measure of the maximum number of charges that bunch together. It is basic to the concept of plasma that the concentration of the charged particles in an ionized gas is sufficiently large for their motions to be dominated by their cooperative electromagnetic interactions. To fulfill the above-mentioned definition of a plasma we may specify the conditions as follows:

$$n_\infty\lambda_D^3 \gg 1, \quad \lambda_D \ll L \quad (3.3)$$

where L is a length characterizing the dimensions of the plasma. These conditions are also supplemented by that of charge neutrality in the definition of a plasma herein discussed.

3.2. *Multi-mode Oscillations and Waves*

With the induced electrostatic and magnetic fields acting as restoring forces, various types of oscillations and waves may be initiated by disturbances in a plasma. Furthermore, on account of the particle bunching discussed in Section 3.1, the induced fields can provide a means of coupling between the particles so that they can participate in, and influence, coherent wave propagation even in a collision-free plasma. The nature of the restoring forces depends on the types of disturbances, e.g. the effect of charge separations is restored by electrostatic forces and the displacement of plasma as a whole can be restored by the "elasticity" of the magnetic lines of force which are displaced with the conducting plasma. In addition, a plasma, just like any other gas, exhibits elasticity towards pressure changes. Owing to the presence of these possible restoring actions, the initial disturbance of whatever cause may lead to the appearance of various oscillations and waves. We shall draw attention here only to some of the basic modes of plasma oscillations.

It is convenient to treat the response of the plasma to an electromagnetic field by the use of a dielectric tensor ($\overleftrightarrow{\epsilon}$). In so doing, we can separate the dynamical part of the problem from the part concerned with the electromagnetic field equations.

* Also called the Maxwell–Boltzmann distribution.

† If ion distribution is also taken to be Boltzmann, the Debye shielding length becomes $1/\sqrt{2}$ times smaller.

The Maxwell equations for a disturbed electromagnetic field can be written as follows:

$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + 4\pi \mathbf{J} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}, \quad (3.4)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}. \quad (3.5)$$

The oscillatory motions of the plasma particles depend on the fields to which the particles are subjected; the only fields that need consideration in eqns. (3.4) and (3.5) are the small perturbations. Any static, external fields present will only affect the dielectric properties of the plasma through the equation of motion. To complete the set of the above equations we must have Ohm's law to connect the current with the electric field:

$$\mathbf{J} = \overleftrightarrow{\boldsymbol{\sigma}} \cdot \mathbf{E} \quad (3.6)$$

where $\overleftrightarrow{\boldsymbol{\sigma}}$ is the conductivity tensor, or equivalently we may introduce the dielectric tensor to connect the displacement vector (**D**) with the electric field:

$$\mathbf{D} = \overleftrightarrow{\boldsymbol{\epsilon}} \cdot \mathbf{E}. \quad (3.7)$$

If the system in question is homogeneous spatially and all perturbed quantities have time dependence proportional to $\exp(-i\omega t)$, we may introduce Fourier transforms of variables in the above equations and considering variation of the form $\exp[i(\mathbf{k}\cdot\mathbf{x} - \omega t)]$, we may replace the differential operators $(\partial/\partial t)$ by $-i\omega$ and ∇ by $-i\mathbf{k}$ to obtain the Fourier transformed system of the electromagnetic field equations which, after the elimination of variable **B**, becomes

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) + \frac{\omega^2}{c^2}\overleftrightarrow{\boldsymbol{\epsilon}} \cdot \mathbf{E} = 0. \quad (3.8)$$

Equation (3.8) describes the basic behavior of plane waves in a plasma in the $(\mathbf{k}, \omega)$-space.

3.2.1. *Collision-free plasma high-frequency oscillations in a weak magnetic field*

If the range of oscillation frequencies is sufficiently high, we can discuss the electrostatic and electromagnetic waves with the assumption that the ions play no essential role other than to provide a background which assures the existence of an equilibrium, quasi-neutral state. We also ignore the particle collisions and external magnetic field in the present cold plasma approximation.

The electric current is

$$\mathbf{J} = -\frac{ne}{c}\mathbf{v} \quad (3.9)$$

where the electron velocity can be determined from its equation of motion in a homogeneous medium

$$m_e\frac{\partial \mathbf{v}}{\partial t} = -e\mathbf{E}. \quad (3.10)$$

Equations (3.9) and (3.10), after Fourier transform, become

$$\mathbf{J}(\omega) = \frac{ine^2}{m_e c\omega}\mathbf{E}(\omega), \quad (3.11)$$

$$\mathbf{v}(\omega) = -\frac{ie}{m_e\omega}\mathbf{E}(\omega). \quad (3.12)$$

From eqn. (3.6), after Fourier transform and substitutions of eqns. (3.11) and (3.12), we obtain the conductivity tensor in $(\mathbf{k}, \omega)$-space.

$$\overleftrightarrow{\boldsymbol{\sigma}} = \frac{ine^2}{m_e c\omega}\overleftrightarrow{\mathbf{I}} \quad (3.13)$$

where $\overleftrightarrow{\mathbf{I}}$ is the unit dyad. From (3.7), after Fourier transform and some algebraic manipulations, we obtain

$$\overleftrightarrow{\boldsymbol{\epsilon}} = \left(1 - \frac{\omega_p^2}{\omega^2}\right)\overleftrightarrow{\mathbf{I}} \quad (3.14)$$

where

$$\omega_p = \left(\frac{4\pi ne^2}{m_e}\right)^{1/2} \quad (3.15)$$

denotes the electron plasma frequency.

The wave equation (3.8), after substitution of (3.14), becomes

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) + \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right)\mathbf{E} = 0 \quad (3.16)$$

which can be rewritten after the field **E** is separated into longitudinal and transverse components, with respect to the wave vector **k**, i.e. $\mathbf{E} = \mathbf{E}_\parallel + \mathbf{E}_\perp$ in the following form:

$$\mathbf{k} \times [\mathbf{k} \times (\mathbf{E}_\parallel + \mathbf{E}_\perp)] + \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right)(\mathbf{E}_\parallel + \mathbf{E}_\perp) = 0. \quad (3.17)$$

Notice that $\mathbf{k} \times \mathbf{E}_\parallel = 0$ and $\mathbf{k} \times \mathbf{E}_\perp = 0$ by definition of $\mathbf{E}_\parallel$ and $\mathbf{E}_\perp$ respectively, so that

$$(\omega^2 - \omega_p^2)\mathbf{E}_\parallel + (\omega^2 - \omega_p^2 - c^2k^2)\mathbf{E}_\perp = 0. \quad (3.18)$$

In the case of the longitudinal mode of oscillation, we have $\mathbf{E}_\perp = 0$ and $\mathbf{E}_\parallel \neq 0$ and the dispersion relation leads to

$$\omega^2 = \omega_p^2 \quad (3.19)$$

which denotes the plasma electron oscillations; on the other hand, when the oscillation mode is transverse, we have $\mathbf{E}_\parallel = 0$ and $\mathbf{E}_\perp \neq 0$. Thus the dispersion relation becomes

$$\omega^2 = \omega_p^2 + c^2k^2 \quad (3.20)$$

which refers to the electromagnetic waves.

In the electrostatic mode of wave propagation in a plasma, negligible magnetic field is generated because $\mathbf{E}_\parallel$ is parallel to the wave vector implying that the curl of the electric field vanishes. It is further noted that $\mathbf{E}_\parallel$ is nonzero only if $\omega^2 = \omega_p^2$ for any **k**; hence all waves in this mode have the same frequency. Another noteworthy aspect of the present electrostatic waves is the consequence of the cold plasma approximation herein used. It is valid if the phase velocity of the waves $\omega/|\mathbf{k}| \gg v_e$, the electron thermal velocity. If an electron moves with a velocity comparable to the phase velocity, it sees a d.c. field and hence can resonate with the wave, either gaining or losing energy, even for a weak field. Consequently, electron thermal effects modify the wave dispersion giving rise to a change of phase velocity and to a damping known as Landau damping in a collision-free plasma.[11]

The inclusion of particle collision effect in the wave analysis will show that collisions tend to damp out the plasma electron oscillations in the time of the order $2/\nu$ where ν is the collision frequency. The electromagnetic waves are less sensitive to the collision effect. The reason is that its wave energy is carried partly by the plasma particles and partly by free space. At high frequencies, it is carried mostly by the space and thus particle collisions are unimportant.

The effect of an external magnetic field on the electrostatic and the electromagnetic waves is very complicated. Interested readers should consult special treatise.[14]

3.2.2. *Low-frequency plasma oscillations*

When oscillations of low frequencies are excited in a plasma, both ions and electrons will take part in action. If the frequency of oscillation is sufficiently low, the thermal motions of the electrons are able to set themselves in equilibrium in the electric field created by the motion of the ions. A similar procedure as in the high-frequency case discussed earlier can be set up with, however, much heavier algebra to determine the dispersion relation for the low-frequency plasma waves.[15] We shall not consider in detail the mechanism of ion oscillations except to note that their propagation through a plasma is analogous to the propagation of acoustic waves through a gas of neutral particles. If there is thermal equilibrium between the electrons and ions in the plasma, i.e. if $T_e = T_i$, the ion acoustic wave propagates with a velocity of the order of thermal velocity of the ions. In a bi-thermal plasma where $T_e \gg T_i$, the velocity of propagation of the ion acoustic wave is found to be of the order of $(\kappa T_e/m_i)^{1/2}$; in other words, the ion wave travels with the velocity which the ions would have if their temperature were equal to the electron temperature. It has been shown in theoretical analysis that ion acoustic waves can propagate freely in a plasma which has ions much colder than electrons, and are rapidly attenuated in an isothermal plasma. So far, our discussion on the plasma low-frequency oscillations have been limited to the longitudinal modes.

Among the transverse modes of plasma oscillations that have frequency much lower than the Larmor frequencies of the ions the Alfven wave which is an electromagnetic wave propagating along the magnetic lines of force in a plasma is of special interest in the ionosphere studies. In the idealized, dissipationless plasma, the nature of Alfven waves can be elucidated with the concept of frozen fields which means the magnetic field lines stick to the plasma fluid and move with the local fluid.

It should be noted that the magnetic field should have no appreciable effect on the electromagnetic wave field in a plasma if the frequency of oscillation is much greater than the Larmor frequency of the plasma electrons. On the other hand, if it is much lower than the Larmor frequency of the plasma ions, assuming for simplicity that the direction of propagation is parallel to the direction of the constant external magnetic field, the lines of force, frozen into the medium, assume a wave-like form and bring the plasma into oscillatory motion with them. A large proportion of the electromagnetic energy associated with the wave is thus converted into the kinetic energy of oscillation of the medium. The situation is somewhat similar to the propagation of elastic waves along a string. If the mass per unit length is higher at some point of the string than elsewhere, the velocity of the wave is reduced accordingly. In the propagation of electromagnetic waves in the direction of a constant magnetic field in plasma, the lines of force behave like elastic strings along which oscillations are transmitted. These strings are loaded by the plasma, and therefore the velocity of the wave is reduced.[16]

Our discussion on the plasma wave properties has been formal in the sense that some of the dispersion relations of the waves were formulated from the basic governing equations of particles and field without questioning the physical basis for the existence of wave motions, which are fluid properties, in a collisionless plasma. This subtle implication is based on the long-range nature of Coulomb forces between charged particles whose interactions is not so much in the nature of a collision as it is a reflection of the bunching effect of the so-called self-consistent field due to the encounterings at large distances. This collective behavior of charged particles is one of the fundamental properties of a plasma medium and will be more thoroughly discussed in the following.

3.3. *Collective Behavior of Collision-free Plasma*

In the earlier studies on rarefied plasma, it had been found that there were many anomalous phenomena which could not be explained on the basis of the classical Boltzmann kinetics, e.g. thickness of a plasma shock front is appreciably smaller than the mean free path of the medium; the plasma diffusion flux across magnetic field is abnormally high—a phenomenon known as the Bohm diffusion after its discoverer.

Following our earlier discussion of the Debye shielding effect on the Coulomb field of a charged particle, we note that the statistical effects of charged-particle interactions can be conveniently classified into two groups: (i) with particle separation $r < \lambda_D$ their random interactions can be treated in the manner of the Brownian motions, i.e. expressed in the form of the Fokker–Planck "collision" in the theory of Brownian motion;[17] (ii) with $r > \lambda_D$

the particles have finite correlations, i.e. behave collectively, the plasma hence having a configuration somewhat like a lattice that has a spacing equal to a wavelength of propagating plasma oscillations. Thus λ_D can be identified with the shortest possible wavelength of a longitudinal plasma wave. The collective behavior of plasma particles due to distant interactions is reflected in the self-consistent field effect. This finite correlation of particle motions accounts for the quasi-continuum nature of a collision-free plasma that supports the wave motions and even shock fronts.[18]

The resonant interactions between propagating plasma waves and those particles which have velocities close to that of the wave could provide the mechanism of damping or growth of the wave depending on the particle velocity distribution as first discussed by Landau. The case of wave-growth which is the cause of a micro-instability of the system can lead to the Bohm diffusion phenomenon mentioned earlier and the plasma turbulence, to cite a few. Since plasma in a magnetic field contains a large number of natural resonant frequencies, considering the collective oscillations of both ions and electrons, their Larmor frequencies and several combinations of these, it is one of the most challenging jobs in plasma research to determine various modes of micro-instabilities, the wave-particle interactions, particularly the non-linear ones if the wave amplitude grows sufficiently large. One particular mode worth noticing is the drift wave which can occur spontaneously by an instability driven by density and temperature gradients.[19] It could be primarily responsible for the collision-free shock transition process, a problem yet to be solved.

4. PARTICLE-SURFACE INTERACTION

When a charged particle impinges on a solid surface, one of several alternatives may happen, e.g. it may be neutralized electrically and reflected as a neutral particle, thus lost to the population of the charged species; it may reflect specularly and retains its charge, etc. An incident particle of high energy may cause secondary emission from the surface. This charge accretion by the surface is one of the inputs that determine the resultant surface potential of the body in question.[5]

The study of particle-surface interaction has been made difficult by the uncertainties of the actual surface condition which is one of the factors influencing the nature of particle accommodation. An ideal surface may be defined as one with which the bulk properties of the material persists to the geometric surface. On the other hand, with a real surface which is generally finished by mechanical grinding, polishing, etc. there exists a surface layer, about 10 μm thick, which has usually been violently distorted. Hence the use of ideal solid surface models for particle accommodation study must be done with caution. The adsorption of gas by a real surface is another factor that adds to the uncertainty of the surface condition. Physically adsorbed gas layers attached to a solid surface may be removed by moderate heating. Adsorption behavior, from the kinetic point of view, may be conveniently described by the sticking probability for molecules of the gas striking the surface. Sticking probability lies in the range 0.1 to 1 for the first monolayers and falls sharply as more layers are adsorbed. The amount of adsorption quickly reaches an equilibrium value which depends upon the partial pressure of the adsorbate and the surface temperature, e.g. with a partial pressure of 10^{-7} mm Hg of oxygen a clean metallic surface will acquire an oxide monolayer in about a minute.

Fortunately, in an ionospheric aerodynamic problem where the surface potential is never larger than a few volts, it is a close approximation to consider all incident ions neutralized by picking up a negative charge of equal amount at impact; electrons, absorbed. This assumption is used throughout the present study.

4.1. *Equilibrium Surface Potential without Photoemission*[20,21]

We shall illustrate the attainment of equilibrium surface potential (ϕ_s) by using a spherical model of radius R in a quiescent plasma in equilibrium at a temperature T. To a stationary sphere of zero initial surface potential, the incident flux of thermal electrons is larger than that of thermal ions, assuming singly charged. The negative surface potential thus acquired on account of the net amount of negative charges accreted by the sphere tends to repel the electrons and attract the ions. An equilibrium surface potential (ϕ_s) will be reached finally when the thermal electron influx balances the thermal ion influx. We have

$$A_e\left(\frac{8\kappa T}{\pi m_e}\right)^{1/2} = A_i\left(\frac{8\kappa T}{\pi m_i}\right)^{1/2} \tag{4.1}$$

where A_e, A_i denote the effective collection areas of the ambient thermal electrons and ions respectively.[20] An estimation of the equilibrium surface potential of a spherical body moving at mesothermal speeds, namely $(\kappa T_i/m_i)^{1/2} \ll V_\infty \ll (\kappa T_e/m_e)^{1/2}$ was made.[21] It goes without saying that other sources of charge inputs such as the photoemission of electrons from the surface due to solar radiation, the intensive charge flux input if the body is in the radiation belt of the earth, etc., would change the equilibrium surface potential.

4.2. *Equilibrium Surface Potential with Photoemission*[20,21]

The photoemission effect upon the surface potential of a spacecraft in deep space can be strong enough that the equilibrium potential takes a positive value ($\phi_s > 0$) relative to the ambient plasma. Under such conditions, the thermal ion flux becomes negligible. We consider now the extreme case with a sphere of radius R that has the thermal electron flux in balance with the photoemission current $I_{Ph}(\phi_s, \phi_w)$ which depends on the surface potential (ϕ_s) and the photoelectric work function (ϕ_w)

$$en_e\left(\frac{8\kappa T}{\pi m}\right)^{1/2}\left(1-\frac{e\phi_s}{\kappa T}\right)=\frac{I_{Ph}}{\pi R^2}. \tag{4.2}$$

Equation (4.2) can be solved by a graphical method provided the photoelectric work function (ϕ_w) of the body material is given. It has been estimated that a positive surface potential larger than a few volts is unlikely unless the spacecraft is in the radiation belt where extremely intensive radiation flux is expected.

5. KINETIC AND CONTINUUM THEORIES OF IONOSPHERIC FLOWS

The mathematical representation of a gaseous motion can be made at various levels of approximations. Considering a gaseous motion as a collected motion of a large number of particles each of which may interact with the others by means of the inter-particle forces, a microscopic approach to the problem starts with the application of the conservation laws of mechanics to the particle motions and the use of mathematics of statistics to determine mean values of the particle motions that are macroscopic observables of the flow field. This is known as the kinetic approach to the problems of flows. On the other hand, a macroscopic approach starts with the application of the laws of mechanics to the motion of a representative fluid element whose macroscopic behavior can be construed as giving the average behavior of particles it contains. The latter approach is called the continuum theory of flow or fluid theory because it focuses attention on a fluid element as the starting point. It appears that the formal difference between the two-mentioned microscopic and macroscopic approaches is merely a matter of procedure. There is, however, much subtlety in the choice between them for the solution of a flow problem.

If our interest in the upper atmospheric flows is restricted to the neutral particle species for which the mean free paths are many order of magnitude larger than a characteristic length of the moving body, a free molecular flow would be expected; the kinetic approach is the logical choice because there is no meaningful representative fluid element in such a collision-free flow. On the other hand, with an ionospheric flow where the charged species dominate, a quasi-continuum flow prevails even for a collision-free plasma because of their collective behavior discussed in Section 3.3. Consequently, fluid approach to an ionospheric flow becomes possible provided certain conditions are met.[18] It is intended here to formulate the governing equations of the system from both the kinetic and the continuum points of view emphasizing particularly the connection between them. It is expected that in so doing we can appreciate how certain plasma behavior peculiar to the microscopic velocity distribution have been averaged out and lost to the system at the level of macroscopic description of the flow. Nevertheless, the asset of simplicity and directness of the continuum approach is too familiar to the aerodynamicists to be enumerated here.

5.1. *Kinetic Equations*

The ionospheric plasma is considered to be composed only of singly charged and neutral particles which interact according to the laws of classical mechanics. Inelastic collisions, e.g. charge-exchange collision which is possibly important in comet gas dynamics, are excluded herein.

5.1.1. *Distribution function in the phase space* (x, v)

The statistical mechanical state of a plasma can be described by separate distribution functions each of which denotes a particular species of the mixture, e.g. $f_\alpha(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{x}\, d\mathbf{v}$ represents the probable number of particle species α in the six-dimensional phase space $d\mathbf{x}\, d\mathbf{v}$ at time t where α is used to designate ions ($\alpha = i$), electrons ($\alpha = e$) or neutral ($\alpha = n$). The kinetic equation known as the Boltzmann equation for the distribution function $f_\alpha(\mathbf{x}, \mathbf{v}, t)$ governing the statistical distribution of species α that stream in the phase space $(\mathbf{x}, \mathbf{v})$ can be written[13]

$$\frac{\partial f_\alpha}{\partial t}+\mathbf{v}\cdot\frac{\partial f_\alpha}{\partial \mathbf{x}}+\mathbf{a}\cdot\frac{\partial f_\alpha}{\partial \mathbf{v}}=\left(\frac{\delta f_\alpha}{\delta t}\right)_c \tag{5.1}$$

where $\mathbf{a}(\mathbf{x}, \mathbf{v}, t)$ is the particle acceleration due to the Lorentz force $e_\alpha(\mathbf{E}+\mathbf{v}\times\mathbf{B}/c)$ on a particle charge e_α and possibly the gravitational force if it is significant. The electrostatic field $\mathbf{E}$ is the self-consistent electrostatic field which takes account of the distant (collective) part of the Coulomb interaction while the collision term $(\delta f_\alpha/\delta t)_c$ takes into account the near-neighbor interaction of the particles. This *ad hoc* division of the many-body interaction effect into the long-ranged collective interaction and the short-ranged discrete collisions and the subsequent assumption of the dominance of the former contribution must be considered as one of the fundamental assumptions of the contemporary plasma physics.

The self-consistency of the electromagnetic field is built into the equation when the field is required to satisfy the Maxwell equation of the electromagnetic fields (3.4), (3.5) supplemented by

$$\frac{\partial}{\partial \mathbf{x}}\cdot\mathbf{E}=4\pi\rho_c \tag{5.2}$$

and

$$\frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{B} = 0 \tag{5.3}$$

where ρ_c is the net charge density. The current **J** in eqn. (3.4) is

$$\mathbf{J} = \mathbf{J}_{\text{ext.}} + \sum_\alpha e_\alpha \int \mathbf{v} f_\alpha(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v} \tag{5.4}$$

where $\mathbf{J}_{\text{ext.}}$ is an externally produced current while the second term on the right-hand side of eqn. (5.4) represents the current produced by the plasma itself. The summation is over various species that compose the plasma. A similar equation relates the net charge density to the distribution functions:

$$\rho_c = \sum_\alpha e_\alpha \int f_\alpha\, d\mathbf{v}. \tag{5.5}$$

The determination of the term $(\delta f_\alpha/\partial t)_c$ in eqn. (5.1) that represents the effect of nearby collisions has been of considerable interest in the contemporary plasma studies. It is noted that in the classical kinetic equation (5.1) of Boltzmann, the collision term has been formulated on the basis of the binary assumption which says that encounters in which more than two molecules take part are negligible in frequency and effect compared with binary encounters. The binary assumption has worked out well for the dilute gases of neutral particles having small fields.* In the analysis of collisions between charged particles difficulties arise that stem from the long-range nature of the Coulomb force. A given particle is subject to the influence of a large number of surrounding particles, and all of these particle interactions must be taken into account. Thus, instead of dealing with the binary collisions, we must cope with a many-body problem, for which an effective rigorous analysis is not yet available. A working hypothesis has been introduced to approximate this many-body interaction.[13] Other than the collective effect of distant interactions and cumulative effect of interactions between a given particle and its neighbors within a distance of Debye length (λ_D) can be effectively treated as equivalent to diffusion in velocity space (Fokker–Planck Process[13]).

Consider the plasma particles undergoing random motions, akin to the Brownian motions,[17] as a result of a large number of small deflections. It is appropriate to describe such a stochastic process by the transition probability $\psi(\mathbf{v}, \Delta\mathbf{v})$ that a particle with velocity **v** acquires a velocity increment $\Delta\mathbf{v}$ in time Δt. For the Markov Process[17] ψ does not depend explicity on time and the distribution then evolves according to the relation:

$$f_\alpha(\mathbf{x}, \mathbf{v}, t) = \int f_\alpha(\mathbf{x}, \mathbf{v} - \Delta\mathbf{v}, t - \Delta t)\psi(\mathbf{v} - \Delta\mathbf{v}, \Delta\mathbf{v})\, d(\Delta\mathbf{v}) \tag{5.6}$$

* Referring to the short-range inter-molecular force fields.

assuming that a time interval Δt exists, which is long enough for a particle to suffer a large number of collisions, but short enough so that its velocity does not change much. The discrete collision effect is

$$\left(\frac{\delta f_\alpha}{\delta t}\right)_c = \lim_{\Delta t \to 0} \frac{[f_\alpha(\mathbf{x}, \mathbf{v}, t) - f_\alpha(\mathbf{x}, \mathbf{v}, t - \Delta t)]}{\Delta t}. \tag{5.7}$$

To evaluate $(\delta f_\alpha/\delta t)_c$ (5.7), we expand the functions under the integral in eqn. (5.6) into Taylor series,

$$f_\alpha(\mathbf{x}, \mathbf{v}, t) = \int d(\Delta\mathbf{v}) \times \left\{ f_\alpha(\mathbf{x}, \mathbf{v}, t)\psi(\mathbf{v}, \Delta\mathbf{v}) - \Delta t \psi \frac{\partial f_\alpha}{\partial t} - \Delta\mathbf{v} \cdot \left[\frac{\partial f_\alpha}{\partial \mathbf{v}} + f_\alpha \frac{\partial \psi}{\partial \mathbf{v}}\right] + \frac{1}{2}\Delta\mathbf{v}\Delta\mathbf{v} : \left[\psi \frac{\partial^2 f_\alpha}{\partial \mathbf{v}\partial \mathbf{v}} + 2\frac{\partial f_\alpha}{\partial \mathbf{v}}\frac{\partial \psi}{\partial \mathbf{v}} + f_\alpha \frac{\partial^2 \psi}{\partial \mathbf{v}\partial \mathbf{v}}\right] + \cdots \right\}$$

which, after the use of the normalization condition for ψ,

$$\int \psi\, d(\Delta\mathbf{v}) = 1 \tag{5.8}$$

becomes

$$\Delta t \left(\frac{\delta f_\alpha}{\delta t}\right)_c = -\frac{\partial}{\partial \mathbf{v}} \cdot (\langle \Delta\mathbf{v} \rangle f_\alpha) + \frac{1}{2}\frac{\partial^2}{\partial \mathbf{v}\partial \mathbf{v}} : (\langle \Delta\mathbf{v}\Delta\mathbf{v} \rangle f_\alpha) \tag{5.9}$$

where

$$\langle \Delta\mathbf{v} \rangle = \int \psi(\mathbf{v}, \Delta\mathbf{v})\Delta\mathbf{v}\, d(\Delta\mathbf{v}),$$
$$\langle \Delta\mathbf{v}\Delta\mathbf{v} \rangle = \int \psi(\mathbf{v}, \Delta\mathbf{v})\Delta\mathbf{v}\Delta\mathbf{v}\, d(\Delta\mathbf{v}).$$

The expression (5.9) can be used after ψ is prescribed for an assumed model of the scattering process; its first term gives rise to a slowing-down effect and is called the coefficient of dynamic friction; its second term has the effect of spreading out an initial particle beam thus named the coefficient of diffusion.

A result essentially equivalent to expression (5.9) was obtained by Landau[12] who started with the Boltzmann binary collision integral and postulated that owing to the long-range nature of electrostatic forces, the number of collisions resulting in a small deflection θ are much more numerous than those of large θ, hence change of particle velocity $\Delta\mathbf{v}$ will be small for most collisions so that it is reasonable to expand the integrand of Boltzmann's collision integral in powers of $\Delta\mathbf{v}$ and keep the first and second powers. A justification for Landau's procedure, which caused some earlier controversies, can be made on the ground that it makes no difference to the final result whether the interactions are simultaneous (as a many-body problem) or successive (as the summed binary effects).

The collision effect between charged and neutral particles can be adequately represented by the classical Boltzmann binary collision integral with the Maxwellian molecular model (interaction force law $\propto r^{-5}$) as, for example, was done by Allis.[23]

In the upper ionosphere where the density is very low and the atmosphere is strongly ionized, not only the effect of the neutral atoms but also the complete discrete collision $(\delta f_\alpha/\delta t)_c$ are negligible compared with the collective effect of the distant charged particles, eqn. (5.1) can be simplified to the collision-free Boltzmann equation with self-consistent field, often called the Vlasov equation:

$$\frac{\partial f_\alpha}{\partial t}+\mathbf{v}\cdot\frac{\partial f_\alpha}{\partial \mathbf{x}}+\mathbf{a}\cdot\frac{\partial f_\alpha}{\partial \mathbf{v}}=0. \tag{5.10}$$

5.1.2. *Formal solution of Vlasov equation*

The distribution function $f(\mathbf{x},\mathbf{v},t)$ satisfying eqn. (5.10) and a given initial state $f_0(\mathbf{x},\mathbf{v},0)$ can be determined formally by the method of characteristics. Notice that the subsidiary (characteristics) equations to eqn. (5.10) in Cartesian coordinates can be written

$$\frac{dt}{1}=\frac{dx_1}{v_1}=\frac{dx_2}{v_2}=\frac{dx_3}{v_3}=\frac{dv_1}{a_1}=\frac{dv_2}{a_2}=\frac{dv_3}{a_3} \tag{5.11}$$

which define the particle trajectories of the collision-free plasma. The general solution to eqn. (5.11) can be prescribed as follows:

$$x_\beta = x_\beta(I_1, I_2, \ldots, I_6, t);\quad v_\beta = v_\beta(I_1, I_2, \ldots, I_6, t)\quad (\beta = 1,2,3) \tag{5.12}$$

where $I_1, I_2, \ldots, I_6$ are the six constants of integration. The integrals of motion are

$$I_\beta = I_\beta(\mathbf{x},\mathbf{v},t)\quad (\beta = 1,2,\ldots,6). \tag{5.13}$$

According to the theory of partial differential equations, the general solution to eqn. (5.10) is an arbitrary function of the integrals of motion (5.13), namely

$$f = F(I_1, I_2, \ldots, I_6). \tag{5.14}$$

The particular solution to the given initial value problem is obtainable by requiring that $F(I_1, I_2, \ldots, I_6)$ satisfies the given initial distribution $f_0(\mathbf{x},\mathbf{v},0)$.

An application of the above result to the study of stellar dynamics was made by Jeans. The functional relation (5.14) is known as Jeans theorem in stellar dynamics. Extensive applications of the theorem (5.14) to specify ion trajectories in an electrostatic field of sheath and wakes in ionospheric gas dynamics were also made.[5] The advantage of using the theorem (5.14) to the study of collision-free plasma over the calculation of assorted ion trajectories with assumed parameters of field and initial conditions is computational. A judicious choice of $F(I_1, I_2, \ldots, I_6)$ with sufficient number of integrals (I_β) not only saves much computations for individual trajectories but makes the numerical iterations* much illuminating to gain physical insight because of its relative simplicity in formulation.

* Which is almost always needed when the system is nonlinear such as the near-wake problem.[5]

5.1.3. *Particle orbit theory of collision-free plasma*

In view of the equivalence between the Jeans theorem approach and the consideration of the particle trajectories to treat the collision-free plasma as shown in Section 5.1.2, it should be possible to replace the Vlasov equation by the equations of motion (5.11). As to how effective the latter approach is depends on the method used to handle the results of particle trajectories thus obtained in order to formulate the space charge density (ρ_c) needed in eqn. (5.2). The job can become unwieldy considering the number of essential parameters for the determination of the assorted trajectories. It is for the purpose of efficient mathematical classification of the particle trajectories that an alternative approach has been introduced[24] which uses the Hamilton–Jacobian equation in classical mechanics to describe particle motion in lieu of the Newtonian equations (5.11). However, since the wave function of the Schroedinger equation which is the counterpart of the Hamilton–Jacobian equation is more accessible for describing particle density, it is initially used even though the quantum mechanical effect is negligible. The transformation of the Schroedinger equation to the Hamilton–Jacobian equation can be made by the introduction of the W.K.B.J. approximation.[25] This approach is found particularly effective to treat the interaction problems of mesothermal collision-free plasmas in the absence of magnetic fields[5] where the electrostatic approximation is appropriate.

The Schroedinger equation for an ion of mass m_i and charge in steady motion is

$$h^2\nabla^2\Psi(\mathbf{x})+8\pi^2 m_i[E_i-e\phi(\mathbf{x})]\Psi=0 \tag{5.15}$$

where h denotes the Planck constant; $\phi(\mathbf{x})$ the field potential; E_i the total energy of the particle in question. The wave function $\Psi(\mathbf{x})$ has meaning such that $|\Psi|^2\,d\mathbf{x}$ is the probability of finding a particle in volume element $d\mathbf{x}$. Thus, if there are n_∞ noninteracting identical particles per unit volume, $n_\infty|\Psi|^2$ will represent the particle density at $\mathbf{x}$. Equation (5.15) can be used to describe the distribution of monoenergetic ions in the problems of mesothermal flows.[24]

A short-wavelength approximation, known as the W.K.B.J. method,[25] can be introduced to eqn. (5.15), we let

$$\Psi = A\exp[iW(\mathbf{x})] \tag{5.16}$$

and substitute it in eqn. (5.15)

$$h^2[(\nabla W)^2-i\nabla^2 W]=8\pi^2 m_i(E_i-e\phi) \tag{5.17}$$

which, under short-wavelength approximation, reduces to

$$h^2(\nabla W)^2=8\pi^2 m_i(E_i-e\phi). \tag{5.18}$$

Equation (5.18) is the Hamilton–Jacobian equation in classical mechanics. W is proportional to the action. It can be determined as the formal solution to eqn. (5.18)

$$W(\mathbf{x}) = \int \frac{2\pi(2m_i)^{1/2}}{h}\{E_i - e\phi[\mathbf{x}(S)]\}^{1/2}\, dS \qquad (5.19)$$

where S is a parameter in terms of which the ion orbits are described.

It is noted that the mathematical advantage of using the Schroedinger equation for the present purpose stems from the fact that it is linear for a given $\phi(\mathbf{x})$. This makes it very convenient to unravel the nonlinearly coupled system of the particle-field interactions under electrostatic approximation which will be shown later.

5.2. *Continuum Equations*

The kinetic flow analysis (Section 5.1) which involves the use of distribution functions in a six-dimensional phase space $(\mathbf{x}, \mathbf{v})$ is not only saddled with considerable mathematical difficulties but also provides too much detailed information on plasma particle behaviors which are not needed in determining the flow observables:

$$\left.\begin{array}{ll} \text{number density:} & n_\alpha(\mathbf{x}, t) = \int f_\alpha\, d\mathbf{v}, \\ \text{flow velocity:} & \mathbf{V}_\alpha(\mathbf{x}, t) = \frac{1}{n_\alpha}\int \mathbf{v} f_\alpha\, d\mathbf{v}, \\ \text{pressure tensor:} & \overleftrightarrow{\mathbf{P}}_\alpha(\mathbf{x}, t) = m_\alpha \int (\mathbf{v} - \mathbf{V}_\alpha) \\ & \qquad \times (\mathbf{v} - \mathbf{V}_\alpha) f_\alpha\, d\mathbf{v}, \\ \text{thermal flux:} & \mathbf{q}(\mathbf{x}, t) = \frac{m_\alpha}{2}\int (\mathbf{v} - \mathbf{V}_\alpha) \\ & \qquad \times (\mathbf{v} - \mathbf{V}_\alpha)^2 f_\alpha\, d\mathbf{v}, \end{array}\right\} \qquad (5.20)$$

etc.

It would be desirable to develop governing equations of flows that have the flow quantities (5.20) as their unknown functions. This is the primary motivation for introducing the moment form of the kinetic equation (5.1). Notice that the flow quantities (5.20) are the zeroth, the first, the second and the contracted third moment respectively of the distribution function.

5.2.1. *Transport equation*

In spite of the importance of the distribution function which serves as a universal joint to all the flow quantities (5.20) of interest, it is sometimes found necessary to circumvent the mathematical difficulties by forming moments of the terms of its governing eqn. (5.1). Physically speaking, it is intended to transform eqn. (5.1) in such a way that it may be possible to calculate the average value of any quantity $Q(\mathbf{v})$ without knowing much about the distribution function itself. Notice that $Q(\mathbf{v})$ designates

$$1, \mathbf{v}, m(\mathbf{v} - \mathbf{V}_\alpha)(\mathbf{v} - \mathbf{V}_\alpha) \text{ or } \tfrac{1}{2}m_\alpha(\mathbf{v} - \mathbf{V}_\alpha)(\mathbf{v} - \mathbf{V}_\alpha)^2 \ldots,$$

for number density, flow velocity, pressure tensor or thermal flux ..., respectively, as given in flow quantities (5.20).

Multiplying both sides of eqn. (5.1) by $Q(\mathbf{v})$ and integrating over the velocities of the particles of type α, we obtain[13]

$$\frac{\partial(n_\alpha\langle Q\rangle)}{\partial t} + \frac{\partial}{\partial \mathbf{x}} \cdot (n_\alpha\langle vQ\rangle) - n_\alpha \mathbf{a} \cdot \left\langle \frac{\partial Q}{\partial \mathbf{v}} \right\rangle = \left\langle Q\left(\frac{\delta f_\alpha}{\delta t}\right)_c \right\rangle \qquad (5.21)$$

where an angular parenthesis denotes the average over velocity space of the quantity enclosed. The term on the right-hand side of eqn. (5.21) needs further clarification. It is recalled that $(\delta f_\alpha/\delta t)_c$ represents the rate of change of f_α due to close clllisions, a comparison of eqns. (5.1) and (5.21) suggests the moment of the collision term might represent the rate of change of $n_\alpha\langle Q\rangle$ due to collisions.

It is of interest to observe the analogous relationship between eqns. (5.1) and (5.21) to that between the equations of boundary layers, in fluid dynamics,[26] due to Prandtl and von Karman.* Recall that Prandtl's equation satisfies pointwise the boundary-layer flow field while von Karman's equation, being a moment of Prandtl's equation, satisfies the momentum conservation only on the average across the boundary layer. The relaxed condition imposed in the latter approach to the boundary layers makes the solution untenable for a unique pointwise description of the flow field. This loss of accuracy in flow-field description is compensated by its mathematical simplicity and versatility in applications. The comparison of the cited alternative approaches to the boundary-layer problems in fluid dynamics is very much in parallel with that of the two alternative approaches (5.1) and (5.21) to gas-kinetic problems. While the kinetics eqn. (5.1) provides a pointwise description in the phase-space $(\mathbf{x}, \mathbf{v})$, the continuum eqn. (5.21) obtained as the velocity moments of eqn. (5.1) satisfies the transport of $Q(\mathbf{v})$ on the average in the velocity-space. The physical basis for the use of eqn. (5.21) is that the time characteristic of the randomizing collision effect on the relaxation of an arbitrary velocity distribution to its locally near-equilibrium state is very small compared with the time constant of the hydrodynamic flow. Under such a condition the flow quantities (5.20) would be relatively insensitive to the microscopic velocity distributions. It thus points out the important role played by the molecular collision effects.

There is another thorny question pertaining to the moment approach (5.21), namely the closure problem of the moment equations. The formal development of the moment equations by assigning

* More properly, known as the von Karman's momentum integral.

successive order of velocity moments leads to a set of equations which contains more unknowns than the number of available equations. For instance, an $(n+1)$th moment is, in general, generated in an nth moment equation. Various means of closure on the basis of physical considerations are introduced to obtain a closed set of moment equations as will be shown later.

5.2.2. *Conservation equations of a two-component plasma*

A two-component plasma is regarded as two interacting fluids, the electron gas and ion gas. They interact through collisions as well as the electromagnetic field to which both fluids contribute and are attached

Let $Q = 1$; $m_\alpha \mathbf{v}$ in eqn. (5.21) we obtain the continuity equation

$$\frac{\partial n_\alpha}{\partial t}+\frac{\partial}{\partial \mathbf{x}}\cdot(n_\alpha \mathbf{V}_\alpha)=0 \quad (\alpha = i, e) \qquad (5.22)$$

and the momentum equation, after some algebraic simplification,[13]

$$m_\alpha n_\alpha\left(\frac{\partial \mathbf{V}_\alpha}{\partial t}+\mathbf{V}_\alpha\cdot\frac{\partial \mathbf{V}_\alpha}{\partial \mathbf{x}}\right)-n_\alpha e_\alpha(\mathbf{E}+\mathbf{V}_\alpha\times\mathbf{B}/c)+\frac{\partial}{\partial \mathbf{x}}\cdot\overleftrightarrow{\mathbf{P}}=\mathscr{F}_\alpha \qquad (5.23)$$

where $\mathscr{F}_\alpha$ represents the momentum transferred by collisions from the other component to component α. $\mathscr{F}_\alpha$ can be evaluated only after the collision model has been specified.[13]

The eqns. (5.22) and (5.23) for n_α and $\mathbf{V}_\alpha$ are rigorous consequences of eqn. (5.1) but they do not constitute a closed set of equations from which n_α, $\mathbf{V}_\alpha$ can be computed as functions of position ($\mathbf{x}$) and time (t). In comparison to the continuity-equation and the Navier–Stokes equation of the classical fluid dynamics,[26] which are the counterpart to eqns. (5.22) and (5.23), except for the electromagnetic (external) force and a closure condition. For a gas of neutral molecules, the corresponding moment eqns. (5.22) and (5.23) can be made a closed set by introducing a particular constitutive relation between the pressure tensor and the rate of strain. Should the constitutive relation be linear and the fluid isotropic, the Navier–Stokes equation will be recovered from the momentum conservation eqn. (5.23) for neutral species provided the Stokes viscosity relation* between shear and volume viscosities is used.[27]

We shall now introduce several closure approximations in order to obtain closed sets of equations for the flow quantities n_α, $\mathbf{V}_\alpha$, etc.

(a) *Cold plasma hypothesis.* In the limit of zero plasma temperature, the spread in particle velocities vanishes. Hence

$$\frac{\partial}{\partial \mathbf{x}}\cdot\overleftrightarrow{\mathbf{P}}_\alpha \simeq 0 \quad \text{and} \quad \mathscr{F}_\alpha \simeq 0.$$

Equations (5.22) and (5.23), together with Maxwell equations of the electromagnetic field, form a closed set which has been used extensively particularly in the study of wave propagation in the ionosphere.[28] When the cold plasma equations are used, it is important that the speed of plasma disturbances of interest must be large compared with the thermal velocities of the particles.

(b) *Locally Maxwellian hypothesis.* If the collisions are so frequent that the microscopic velocity distribution is sufficiently isotropic such that to the first approximation a local Maxwellian distribution is maintained

$$f_\alpha(\mathbf{x},\mathbf{v},t)=n_\alpha(\mathbf{x},t)\left[\frac{m_0}{2\pi\kappa T_\alpha(\mathbf{x},t)}\right]^{3/2}\times\exp\left\{-\frac{m_\alpha[\mathbf{v}-\mathbf{V}_\alpha(\mathbf{x},t)]}{2\kappa T_\alpha(\mathbf{x},t)}\right] \qquad (5.24)$$

under the condition (5.24), the contracted third moment $\mathbf{q}$, i.e. the thermal flux and the off-diagonal components of the pressure tensor $\overleftrightarrow{\mathbf{P}}_\alpha$ vanish. The pressure tensor, in fact, becomes $\overleftrightarrow{\mathbf{P}}_\alpha = p_\alpha\overleftrightarrow{\mathbf{I}}$ where $\overleftrightarrow{\mathbf{I}}$ is the unit dyad ($\overleftrightarrow{\mathbf{I}}=\mathbf{e}_\gamma\delta_{\gamma\nu}\mathbf{e}_\nu$) where the scalar pressure $p_\alpha = n_\alpha\kappa T_\alpha$.

Letting $Q = m_\alpha\mathbf{v}\mathbf{v}$ in eqn. (5.21) and making use of eqns. (5.22) and (5.23) and also condition (5.24), we obtained finally an extremely simplified second moment equation in the following form:†

$$\left(\frac{\partial}{\partial t}+\mathbf{V}_\alpha\cdot\frac{\partial}{\partial \mathbf{x}}\right)p_\alpha+\frac{5}{3}p_\alpha\frac{\partial}{\partial \mathbf{x}}\cdot\mathbf{V}_\alpha=0. \qquad (5.25)$$

Replacing $\partial/\partial\mathbf{x}\cdot\mathbf{V}_\alpha$ in eqn. (5.25) by means of the continuity equation, we obtain

$$\left(\frac{\partial}{\partial t}+\mathbf{V}_\alpha\cdot\frac{\partial}{\partial \mathbf{x}}\right)\left(\frac{p_\alpha}{n_\alpha^{5/3}}\right)=0 \qquad (5.26)$$

which is an adiabatic relation with $\gamma = \frac{5}{3}$ for monotonic gases.

Equations (5.22) and (5.23) after simplification together with Maxwell equations of the electromagnetic fields constitute the basis of a popular fluid model of a plasma. They bear a strong resemblance to the Euler equations of ideal gas dynamics.

Now suppose there is a strong external magnetic field $\mathbf{H}$ in the direction of the z-axis. The Larmor gyrations of the plasma particles will cause the velocity distribution to have cylindrical symmetry about the z-axis if the Larmor frequencies are higher than the collision frequencies. This is sometimes taken into account by assuming a diagonal, but anisotropic pressure tensor.[29]

(c) *Electostatic approximation.* If the electric field curl-free condition, $\partial/\partial\mathbf{x}\times\mathbf{E}\simeq 0$ is valid, there is no time-varying magnetic field and a scalar field potential $\phi(\mathbf{x},t)$ exists such that $\mathbf{E}=-\partial\phi/\partial\mathbf{x}$. The system of the electromagnetic field equations col-

* Tantamount to zero bulk viscosity.

† Note that all the collision terms vanish.

lapses into Poisson equation

$$\nabla^2\phi = -4\pi\rho_c. \tag{5.27}$$

Equation (5.27) together with Vlasov equation complete a mathematical description of a collision-free plasma with only electrostatic interparticle forces included. This is an important set of equations governing many of the problems of interest in ionospheric aerodynamics.

6. SIMILARITY PARAMETERS OF IONOSPHERIC MESOTHERMAL FLOWS

In the studies of many mesothermal aerodynamic problems of the ionosphere where the plasma interaction near the moving body is of interest, the electrostatic approximation of the equations of flows (Section 5.2.2(c)) is valid. The magnetic field effect is small because the Larmor radii of ions are usually much larger than the characteristic size of the body. Under the mesothermal flow condition, the thermal electrons maintain a quasi-equilibrium state in spite of the moving body, hence the magnetic-field effect on the plasma interaction through the Larmor electron gyrations is not important. The physical picture of the steady state plasma interaction due to a mesothermally moving and negatively charged body of characteristic size R in the ionosphere can be viewed as follows. The ions, which are significantly disturbed from their equilibrium distribution, move among electrons which are essentially in a Maxwell–Boltzmann distribution: $n_e = n_\infty \exp[e\phi/\kappa T]$ where the field potential $\phi(\mathbf{x})$ is due to charges both on the body and of the space. It is of interest to determine the particle and field distributions in the disturbed zone of particle-field interaction.

6.1. *Dimensionless Transformations of the Governing Equations*

The equations governing the above-mentioned plasma interaction consist, as usual, of two groups: the particle equations are the kinetic equation (5.1) for the ions and the Maxwell–Boltzmann distribution for the electrons; the field equations of Maxwell is reduced to a single equation (Poisson equation) (5.27) under the present electrostatic approximation.

In order to gain some physical insight into this coupled particle-field set of equations, we rewrite them in terms of dimensionless quantities. In defining these quantities, the following reference magnitudes as standard of comparison are introduced: the ion thermal speed $v_i = (\kappa T_i/m_i)^{1/2}$ as characteristic velocity; the body size R as the macroscopic characteristic length; effective range of particle collisions d (defined as a transverse distance from a scattering center that a scattered particle is deflected by an angle $\theta \gtrsim 90°$) as the microscopic characteristic length; ambient ion density, n_∞. The dimensionless representations (with superscript*) are as follows:

$$t = (R/v_i)t^*, \quad \mathbf{x} = R\mathbf{x}^*, \quad \phi = (\kappa T/e)\phi^*,$$
$$\mathbf{v} = v_i\mathbf{v}^*, \quad f_i = (n_\infty/v_i^3)f_i^*, \quad \Delta t = (d/v_i)(\Delta t)^*.$$

In terms of the above dimensionless quantities, eqns. (5.1), (5.7) become

$$\frac{\partial f_i^*}{\partial t^*} + \mathbf{v}^*\cdot\frac{\partial f_i^*}{\partial \mathbf{x}^*} - \frac{\partial \phi^*}{\partial \mathbf{x}^*}\cdot\frac{\partial f_i^*}{\partial \mathbf{v}^*} = \frac{R}{\sqrt{2}\times\pi l}\left\{-\frac{\partial}{\partial \mathbf{v}^*}\cdot\left[\frac{\langle\Delta\mathbf{v}^*\rangle}{(\Delta t)^*}f_i\right] + \frac{1}{2}\frac{\partial^2}{\partial\mathbf{v}^*\partial\mathbf{v}^*}\cdot\left[\frac{\langle\Delta\mathbf{v}^*\Delta\mathbf{v}^*\rangle}{(\Delta t)^*}f_i^*\right]\right\} \tag{6.1}$$

where the characteristic mean free path $l = (\sqrt{2}\,\pi d^2 n_\infty)^{-1/2}$ eqn. (5.27) becomes

$$\left(\frac{\lambda_D}{R}\right)^2\nabla^{*2}\phi^* = e^{\phi^*} - n_i/n_\infty \tag{6.2}$$

where λ_D is the Debye shielding length.

It is observed from eqns. (6.1) and (6.2) that the gas dynamics and the electrostatic similarity between different model systems, depends on two similarity parameters: Knudsen number (l/R) and a shielding parameter (λ_D/R). Let us investigate further the physical significance of the dimensionless equations (6.1) and (6.2). From eqn. (6.1) we observe that when the Knudsen number is very small, say $l/R \ll 1$, the collision term dominates over the streaming-term. This implies that the flow field in question is collision-controlled, hence the particle velocity distribution is not expected to deviate much from the locally Maxwellian distribution on which corresponds to continuum flow of an inviscid fluid (see Section 5.2.2(b)). On the other hand, when the Knudsen number (l/R) is extremely large, we have the collision-free plasma flows with self-consistent fields due to particle distant encounters, known as their collective behavior (Section 3.3). This is of primary interest to the study of ionospheric aerodynamics.

It is observed that eqn. (6.2) has several distinctive features: (i) its coefficient of the most high differentiated term $(\lambda_D/R)^2 \ll 1$ for aerodynamics of finite bodies; (ii) the other coefficients are of order unity. The electrostatic field potential $\phi(\mathbf{x})$ must thus behave akin to the boundary layers in applied mechanics.[30] The field potential $\phi(\mathbf{x})$ would have small gradient* except in a narrow region very close to the body, known as the "boundary layer" in aerodynamics. It will be called the plasma sheath in the ionospheric flows where the stream impinges on the body. The field configuration in the near wake behind a moving body is much more complex.[5] In any event for a region far away from the body, the term on the left-hand side of eqn. (6.2)

*The gradient can be moderately large in the near wake, see the discussion that follows.

plays an insignificant role, we have a quasi-neutral flow for which $n_i/n_\infty \simeq n_e/n_\infty = \exp\phi^*$.

6.2. *Asymptotic Solutions*

Following the discussion of Section 6.1, we have, under the condition $\lambda_D/R \ll 1$, the quasi-neutral flows i.e. with $n_e(\mathbf{x}) \simeq n_i(\mathbf{x})$ outside of the sheath and the near wake. This corresponds to the zeroth order approximation to eqn. (6.2). It designates flows both upstream beyond the frontal sheath and the far wake downstream. We have from eqn. (6.2)

$$n_i(\mathbf{x}) \simeq n_e(\mathbf{x}) = n_\infty \exp(e\phi/\kappa T) \tag{6.3}$$

the field potential

$$\phi(\mathbf{x}) = -\frac{\kappa T}{e} \ln\left[\frac{n_\infty}{n_i(\mathbf{x})}\right]. \tag{6.4}$$

Gurevich[31] suggested that the wake behind a spherical satellite be divided into two regions in one of which the ion density is not very small such that $(R/\lambda_D)^2 n_i(\mathbf{x})/n_\infty > 1$; in the other the ion density is so small that $(R/\lambda_D)^2 n_i(\mathbf{x})/n_\infty < 1$. These refer to the regions far from and close to the body respectively. The boundary separating them is so defined by the condition

$$\left(\frac{R}{\lambda_D}\right)^2 \frac{n_i(\mathbf{x})}{n_\infty} = 1. \tag{6.5}$$

The field potential in the near-region can be approximated by the use of

$$\nabla^2\phi = 0 \tag{6.6}$$

which is equivalent to assuming that the effect of space charge density is negligible compared with that of the surface charge.

7. PLASMA SHEATH AND NEAR WAKE

In view of the discussion on the nature of eqns. (6.1) and (6.2) which govern the distributions of particles and field near a moving body (R), nonlinear analysis for the sheath region in front of the body and the near wake behind it are mandatory unless the body size is small ($R \ll \lambda_D$). This stems from the fact that the nonlinear relation of the particle-field coupling is inherent with the problem. We shall treat these aspects of the ionospheric aerodynamics first in order to gain a glimpse of the nature of plasma interaction. Our contemporary apprehensions about these problems are not at par. While the solution to the sheath problem has almost been satisfactorily resolved, the theory of near wake is very primitive, nowhere near to being a satisfactory solution.

7.1. *Plasma Sheath*

On account of the boundary-layer nature of the field equation (6.2), it is expected that adjacent to the surface where the plasma stream impinges on the body, a thin sheath exists in which the variation of $\phi(\mathbf{x})$ across the sheath is rapid. In view of the extreme disparity in field potential gradient across and along the sheath, we shall use a simplified quasi- one-dimensional sheath analysis.[24]

Consider a collision-free mesothermal plasma stream, which is composed of fully ionized and singly charged particles, impinging on a negatively charged plate at a small angle β (Fig. 2). It is assumed that the incident electrons are absorbed; ions neutralized and re-emitted as neutrals. The plate is fixed to the yz-plane as shown in Fig. 2 and is assumed to be infinitely large, hence no edge effect will be considered, and the field potential depends on x only, $\phi = \phi(x)$. Furthermore, it is assumed that the absolute value is large enough that only the electrons at the rare high energy tail of its distribution can reach the plate and be lost. Thus the Maxwell–Boltzmann equilibrium distribution for electrons is approximately valid

$$n_e(\mathbf{x}) = n_\infty \exp\left[\frac{e\phi(\mathbf{x})}{\kappa T_e}\right] \tag{7.1}$$

where n_∞ denotes the electron density at the outer edge of the sheath where $n_{i\infty} = n_{e\infty} = n_\infty$.

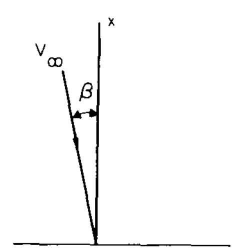

FIG. 2. Plasma sheath at an inclined plane ($\phi_s < 0$).

The field potential obeys the Poisson equation

$$\frac{d^2\phi}{dx^2} = -4\pi e(n_i - n_e) = -4\pi e\left[n_i - n_\infty \exp\left(\frac{e\phi}{\kappa T}\right)\right] \tag{7.2}$$

and boundary conditions:

$$\begin{aligned} \phi &= \phi_s, \quad x = 0, \\ \phi &= 0, \quad x = \infty, \end{aligned} \tag{7.3}$$

which implies that the sheath potential decays asymptotically to zero at ambient. The ion density $n_i(\mathbf{x})$ in eqn. (7.2) can be determined by means of either the Vlasov equation (5.8) or the Schroedinger equation (5.13). The latter is chosen for mathematical simplicity.

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{8\pi^2 m_i}{h^2}[E_i - e\phi(x)]\Psi = 0 \tag{7.4}$$

where E_i denotes the energy of an impinging ion relative to the reference frame fixed to the plate. The stream is approximately monoenergetic (E_i) since $V_\infty \gg v_i$.

Let

$$\Psi(x, y) = \Psi_1(x)\Psi_2(y) \tag{7.5}$$

be a trial for the separation of variables in $\Psi(x, y)$. Equation (7.4) is resolvable into two component equations:

$$\frac{d\Psi_1}{dx^2} + \frac{8\pi^2 m_i}{h^2}[E_i - e\phi(x)]\Psi_1 = A^2\Psi_1,$$

$$\frac{d^2\Psi_2}{dy^2} = A^2\Psi_2$$

where A^2 is a constant. Through the W.K.B.J. transformation (5.16), the solutions to the above equations can be readily obtained which, after the introduction of the upstream boundary conditions (7.3), gives

$$|\Psi|^2 = \frac{1}{\left(1 - \dfrac{e\phi}{E_i \cos^2\beta}\right)^{1/2}} = n_i/n_\infty. \tag{7.6}$$

It is of interest to compare the present result (7.6) with that obtained by Ginzburg[32] who studied a special case ($\beta = 0$) of the problem using hydrodynamic equations suitable for continuum flows. It is found that if the contribution due to pressure-term in reference 32 is neglected for a hyperthermal flow as with the present case, the result thus obtained would agree with eqn. (7.6) when β is put to zero.

The solution to eqn. (7.2) after the substitution of n_i (7.6) can be written:

$$x = \int_{\phi_s}^{\phi} (8\pi n_\infty)^{-1/2}\{2E_i \cos^2\beta(\sqrt{(1 - e\phi/E_i \cos^2\beta)} - 1) - \kappa T_e[1 - \exp(e\phi/\kappa T_e)]\}^{-1/2}. \tag{7.7}$$

The sheath potential $\phi(x)$, given in solution (7.7) is shown in Fig. 3. It is observed from Fig. 3, along with the number densities n_i (7.6) and n_e (7.1), that they all show the rapid variation nearest to the plate, in fact more than 95% of the drops in their

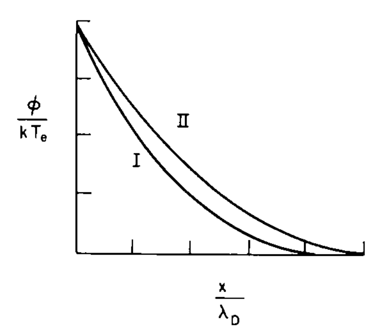

FIG. 3. Field potential of sheath.

'I': $\frac{V_\infty \cos\beta}{v_i} = 2$. 'II': $\frac{V_\infty \cos\beta}{v_i} = 4$.

values occur in a minute distance from the plate which may be defined as the thickness of the sheath. This unique "boundary-layer" behavior of the sheath makes it possible for the use of singular perturbation analysis.[33]

The fact that the variations of field variables, ϕ, n_i and n_e are infinitely stronger across than along the sheath makes it feasible to use the present inclined-plate solution to approximate the frontal sheath of a three-dimensional body which can be sliced into convex ring sections facing a mesothermal stream at different inclination angles (β). This approach is analogous to the modified Newtonian theory of sphere drag at hypersonic speeds using the oblique shock relation.[34]

7.2. *Near Wake*

When a negatively charged circular disk of radius R moves at a mesothermal speed along the direction of its axis in a collision-free plasma, a momentary void develops immediately behind the disk. Both the ambient electrons and ions rush to populate the void region. Since the electrons have a greater mobility they leave the ions behind giving rise to an electric field which decelerates them very rapidly while slightly accelerating the heavy ions. As a result, the velocities of electrons and ions filling the void tend to become equal. The front of the plasma motion thus has a local negative field potential because of the excess electrons therein. This apparently is responsible for the presence of a V-shaped potential valley in the near wake.[5] For a moving blunt body having a width R across the stream and with the above depicted nonlinear coupling between particles and field is an inherent trait of the near wake.

Inasmuch as a satisfactory theory of near wake is not yet available, it appears more important now to expose the intricacy of the problem and make some suggestion that might serve as a guide to a prospective solution. In order to make the discussion definitive, we introduce an idealized model of near wake. Consider a sphere of radius R and with a negative surface potential $\phi_s < 0$ moving in a collision-free fully-ionized plasma of singly charged particles. It is assumed that the ambient plasma may be bi-thermal, i.e. having different temperatures (T_e, T_i) for the electrons and ions respectively. The sphere moves at a steady mesothermal speed, namely $(\kappa T_i/m_i)^{1/2} \ll V_\infty \ll (\kappa T_e/m_e)^{1/2}$. It is further postulated that upon collision with the body an electron is absorbed; an ion, neutralized and re-emitted as a neutral. The body size (R) is much larger than the Debye shielding length (λ_D). Magnetic field is absent.

The equations governing the distributions of particles and field in the near wake consist of the Vlasov equation (5.10) for ions, Maxwell–Boltzmann distribution for electrons (7.1) and the Poisson equation (5.27) in dimensionless quantities.

$$\mathbf{v}\cdot\frac{\partial f_i}{\partial \mathbf{x}}-\frac{1}{2}\frac{\partial \phi}{\partial \mathbf{x}}\cdot\frac{\partial f_i}{\partial \mathbf{v}}=0, \quad (7.8)$$

$$n_e = \exp\phi, \quad (7.9)^*$$

$$\left(\frac{\lambda_D}{R}\right)^2\nabla^2\phi = n_e - n_i = \exp\phi - \int f_i\,d\mathbf{v} \quad (7.10)$$

where the superscripts * have been dropped. The set of eqns. (7.8), (7.9) and (7.10) provides a self-consistent system together with the boundary conditions:

$$\phi(R)=\phi_s,\quad \phi(\infty)=0,$$
$$f_i(R, v_r>0)=0,$$
$$f_i(\infty, v)=\left(\frac{1}{2\pi}\right)^{3/2}\exp\left[-\frac{1}{2}(\mathbf{v}-\mathbf{V}_\infty)^2\right]. \quad (7.11)$$

A common practice in the contemporary studies of near wakes is to treat the ions as if they were neutrals as far as the initial determination of ion density $n_i(\mathbf{x})$ in the wake is concerned. Assorted free "ion" trajectories, starting from the free stream, are calculated. The amount of computations thus involved depends on whether the thermal random velocities of the free stream "ions" are taken into account in the parametric trajectory studies. This is important because it is the transversal thermal motions of the free stream particles that are primarily responsible for the filling in the void. Since they are perpendicular to the direction of the free stream, the transversal thermal velocities though much smaller than the longitudinal free stream velocity ($\mathbf{V}_\infty$) must be retained in the trajectory calculations. This neutral particle assumption is tantamount to the neglect of the second term on the left-hand side of eqn. (7.8). In so doing, the particle and field equations are thus decoupled. The population of ions in the wake can be determined by the use of the trajectory results. Once the ion density $n_i(\mathbf{x})$ is known, it is a simple matter to use the Poisson equation (7.10) to evaluate a first approximation to the field potential $\phi(\mathbf{x})$.

In some studies, efforts apparently have been made to iterate for the higher-order approximations to the ion density and the field potential by using eqns. (7.8)–(7.11) in order to obtain a set of self-consistent solutions of particle and field distributions. No published results on such efforts for the near wakes with $R \gg \lambda_D$ seem available. This should not be confused with the works for the studies of far wakes or of near wakes with $R \ll \lambda_D$ which can be done on the basis of entirely different sets of approximations as will be seen later.

The effectiveness of the above-mentioned neutral particle approximation depends on the conditions of the wake in question. With $R \gg \lambda_D$ the formation of a potential valley in the near wake, as depicted earlier, seems inevitable. It might be difficult to recover this result in a computation that starts with neutral particle approximation. The mathematical question of convergency in the numerical iteration of the system of eqns. (7.8)–(7.11) has not been satisfactorily resolved. Suffice to say that there is no guarantee that such an iteration will always converge. It seems obvious that a self-consistent solution of the system must emerge only after a convergent iteration.

It is noted that the difficulty of treating the system of coupled eqns. (7.8)–(7.11) is computational. For instance, if the computer capacity is of no problem, one could start with a guessed initial $\phi(\mathbf{x})$ to perform the trajectory study and then the iteration. It could serve as a test for the convergency of the iteration scheme. In so doing, however, the number of parameters needed to specify an ion is considerably increased as compared with the neutral particle calculations. Attempts have been made to simplify the trajectory specifications by the use of Jeans theorem (5.14) to prescribe the ion distribution in terms of three integrals† of the ion motion[5,36] in order to reduce the number of parameters for specifying an ion trajectory. On the other hand, it is deprived of the flexibility in the method of parametric trajectory calculations. The primary difficulties involved in the application of Jeans theorem to the near wake study are as follows. (1) Two of the integrals, the total energy and the axial angular momentum of a particle, are well known; the third one is not yet available in the general case if at all. There has been some scheme suggested to be used for the purpose of short-range numerical integrations.[5] A special variational method[36] has been proposed to generalize a third integral which is valid for a restricted axisymmetric field.[37] (2) It is difficult to find the function $F(I_1, I_2, I_3)$ tailored to satisfy the boundary conditions (7.11). It goes without saying that the efforts of applying Jeans theorem to the study of near wake could be very rewarding.

A physical interpretation of the use of Jeans theorem in lieu of individual trajectory specification can be made. While the prospective path of a given ion may be specified in terms of the parameters for its initial position, velocity and the field according to the Newtonian equation of motion, it may also be determined in terms of its integrals of motion (5.14).

8. QUASI-NEUTRAL FLOWS AND FAR WAKE

The condition of quasi-neutral, i.e. $n_i(\mathbf{x}) \simeq n_e(\mathbf{x})$, prevails in the plasma flows beyond the sheath and the near wake. Under this simplified condition, it is of interest to investigate the nonlinear interaction between the gas dynamical and the electrostatic phenomena for the mesothermal collision-free flows.

* The use of Maxwell–Boltzmann distribution for field potential with valleys has been cautioned by Lam.[35]

† Also called adiabatic invariants.

8.1. *Isothermal Compressible Flow Analogy*

We restrict the discussion to a negatively charged body of characteristic size R which is much larger than λ_D. The electron temperature (T_e) is assumed to be much larger than the ion temperature (T_i)—a bi-thermal state commonly prevails for the upper ionosphere particularly during the daytime.

We shall use the continuum equations and show that both the ion and the electron flow fields are irrotational and mathematically uncoupled. The governing equation for the uncoupled ion flow velocity potential has been shown[38] to be identical to that of an isothermal irrotational compressible flow in classical gas dynamics.

The continuity equations (5.22) for the steady flows of ions and electrons are

$$\frac{\partial}{\partial \mathbf{x}} \cdot (n_i \mathbf{V}_i) = 0, \tag{8.1}$$

$$\frac{\partial}{\partial \mathbf{x}} \cdot (n_e \mathbf{V}_e) = 0. \tag{8.2}$$

The electron momentum equation (5.23) can be written:

$$\frac{\partial \mathbf{V}_e}{\partial t} + \mathbf{V}_e \cdot \frac{\partial \mathbf{V}_e}{\partial \mathbf{x}} = \frac{e}{m_e}\frac{\partial \phi}{\partial \mathbf{x}} - \frac{1}{m_e n_e}\frac{\partial p_e}{\partial \mathbf{x}} \tag{8.3}$$

where p_e is the scalar electron pressure.

It is noted that in the absence of collisions, the electron gas is essentially isothermal. This and the equation of state for a perfect gas, $p_e = n_e \kappa T_e$, can be introduced to eqn. (8.3) which becomes integrable along a streamline

$$n_e = n_\infty \exp\left\{\frac{e\phi}{\kappa T_e} - \frac{1}{2}\frac{V_\infty^2}{v_e^2}\left(\frac{V_e^2}{V_\infty^2} - 1\right)\right\} \tag{8.4}$$

where V_∞ is the free stream velocity and $v_e = (\kappa T_e/m_e)^{1/2}$ is the electron thermal speed. In the limit of $V_\infty/v_e \ll 1$, eqn. (8.4) is reduced to the Maxwell–Boltzmann distribution (7.9).

The ion momentum equation (5.23) can be written:

$$\frac{\partial \mathbf{V}_i}{\partial t} + \mathbf{V}_i \cdot \frac{\partial \mathbf{V}_i}{\partial \mathbf{x}} = -\frac{e}{m_i}\frac{\partial \phi}{\partial \mathbf{x}}. \tag{8.5}$$

Equation (8.5) is also integrable along a streamline and gives the ion energy equation

$$\tfrac{1}{2} m_i V_i^2 + e\phi = \text{Const.} \tag{8.6}$$

which is valid as long as we can identify ion trajectories with macroscopic ion streamlines.* This condition prevails under the mesothermal condition because the oncoming ion trajectories look essentially like a parallel beam.

It is important to note[38] that both the ion and the electron flow fields are irrotational since taking curl of both eqns. (8.3) and (8.5) we obtain

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_\alpha \cdot \frac{\partial}{\partial \mathbf{x}}\right)\left(\frac{\partial}{\partial \mathbf{x}} \times \mathbf{V}_\alpha\right) = \left(\frac{\partial}{\partial \mathbf{x}} \times \mathbf{V}_\alpha\right) \cdot \frac{\partial \mathbf{V}_\alpha}{\partial \mathbf{x}} - \frac{\partial}{\partial \mathbf{x}} \times \mathbf{V}_\alpha \left(\frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{V}_\alpha\right) \quad (\alpha = i, e). \tag{8.7}$$

Recall that $\partial/\partial\mathbf{x} \times \mathbf{V}_i$ and $\partial/\partial\mathbf{x} \times \mathbf{V}_e$ are initially zero in the oncoming stream and therefore remain so in the flow field. The fact that the flows are irrotational proves to be important in the analysis later because a velocity potential exists for an irrotational flow.

$$\mathbf{V}_i = -\frac{\partial \Phi_i}{\partial \mathbf{x}}, \quad \mathbf{V}_e = -\frac{\partial \Phi_e}{\partial \mathbf{x}}. \tag{8.8}$$

Consider now the Poisson equation (6.2) which can be written:

$$\left(\frac{\lambda_D}{R}\right)^2 \nabla^{*2}\phi^* = n_e^* - n_i^* \tag{8.9}$$

where the superscript * denotes a dimensionless quantity (Section 6.1), and $\lambda_D = (\kappa T_e/4\pi n_\infty e^2)^{1/2}$. In the limiting case of small value for the parameter λ_D/R, we have

$$n_i^* = n_e^* + 0[(\lambda_D/R)^2] \simeq n_e^* \tag{8.10}$$

which is the precise quasi-neutral condition and is valid where ever $\nabla^{*2}\phi^*$ is of order unity or less.

The equations (8.1), (8.2), (8.4), (8.6), (8.8) and (8.10) can be combined for the elimination of all the flow variables except Φ_i and Φ_e. Recognizing the conditions $m_e/m_i \ll 1$ and $V_e^2/v_e^2 \ll 1$ (mesothermal flows), we obtain finally

$$\nabla^2\Phi_i = \frac{m_i V_\infty^2}{\kappa T_e} \nabla\Phi_i \cdot \nabla\left(\frac{1}{2}\nabla\Phi_i \cdot \nabla\Phi_i\right), \tag{8.11}$$

$$\nabla^2\Phi_e = \frac{m_i V_\infty^2}{\kappa T_e} \nabla\Phi_e \cdot \nabla\left(\frac{1}{2}\nabla\Phi_i \cdot \nabla\Phi_i\right). \tag{8.12}$$

Where $m_i V_\infty^2/(\kappa T_e)$ represents the ratio of the directed kinetic energy of the free stream ions to the thermal energy of the electrons it can be interpreted as the square of a flow mach number on the basis of ion-acoustic speed which is equal to $(\kappa T_e/m_e)^{1/2}$ (Section 3.2). Equation (8.11) appears like the potential equation for an isothermal compressible flow in classical gas dynamics.[27] The classical solutions developed for the compressible flows of neutral particles can be adopted with caution for the ion flows in view of the mathematical analogy shown herein. Once the ion flow field is determined, the electron flow field can be obtained from eqn. (8.12) which is then a linear equation. Applications to mesothermal collision-free flows about circular cones and other configurations are available.[35,38]

8.2. *Far-wake Flows*

Another class of quasi-neutral flows that are of aerodynamic interest is that which is far behind a mesothermally moving body. We call it herein the far-wake flows. In contrast to the near-wake flows

* For a more rigorous justification see the discussion in reference 5.

which are governed by a set of nonlinear equations, the gas dynamical as well as electrostatic disturbances are small enough such that perturbation analysis is applicable, hence a method of linear analysis can be used effectively. The structure of the disturbances undergoes sharp variations in the near-wake region which extends a distance of only a few body diameters. Thereafter the disturbed flow becomes essentially quasi-neutral and persists for a relatively long period of time, during which collision-free Landau damping process acts to nullify the disturbances. The body-plasma interaction can also be viewed in a different light: the rapid motion of the charged body can be considered as a generator of low-frequency collective oscillations of the positive ions—the ion acoustic waves.

The unique feature of a great extension of the collision-free plasma disturbances behind a moving body is technically important. For instance, it constitutes an excellent target for radio-wave tracking* of the moving object when the body itself does not provide large enough scattering cross-section for wave-echos. The plasma tail which has perturbation of the electron density that leads to the variation of the plasma dielectric constant which is responsible for the scattering effect. Although the dielectric constant of the trail is lower than that of a metallic body it compensates in total scattering effect many-fold over by its extremely larger size.

The linear theory of mesothermal far wakes in a collision-free plasma without magnetic field has been reviewed recently[5] and will not be repeated herein. It is, however, pertinent to discuss the magnetic field effect on the electron motion in the wake hence the radio-wave scattering by the wake of moving body in the ionosphere. The magnetic field effect on the plasma interaction in the case of a far wake is no longer negligible because the length of the plasma tail could be much larger than the Larmor radii of the ions.

It is expected that the geomagnetic field could critically affect the length of the wake depending on the orientation between the magnetic line of force and the axis of the wake. If the body is moving along the magnetic field lines ambient electrons which are moving into the void with thermal speed will be turned around by the field before they penetrate a distance of the order of its Larmor radius. The ions, having larger Larmor radii, are much less influenced by the magnetic field. As it was depicted earlier, the ions cannot separate from the electrons by more than a distance of the order of Debye shielding length (λ_D). The net effect on the electrons of dragging by the ions and deterring by the magnetic field is to fill in the void with a drift motion of the electrons around the circumference of the cylindrical void which is inconsequential as far as void-filling is concerned. The net effect is that there will be a considerable elongation of the wake if the electron collisions with ions and neutral are neglected. Otherwise, the electrons may diffuse a distance of the order of Larmor electron radius across the field lines as a result of a collision. When the body moves oblique to the magnetic field lines, electrons and ions tend to spread into the void due to their thermal motions.

* Technically known as the RADAR technique.

9. SLENDER-BODY THEORY

The ionospheric aerodynamic problems treated so far are of the type where the characteristic dimension of the moving body is much larger than the Debye shielding length (λ_D). On account of the distinctive differences in flow and field characteristics in front and behind the moving body, they have been treated as separate entities. It is intended now to develop a unified theory of plasma interaction for a slender body, for which $R \ll \lambda_D$, that moves at mesothermal speed in a collision-free fully ionized plasma where magnetic field effects are unimportant.

The dynamical consequences of the motion of a charged body in a plasma can be considered from two different points of view:

(i) *Waves and energy.* A charged body travelling through a tenuous plasma may lose energy to the collective motions of oscillations by the plasma particles in addition to the energy loss by means of aerodynamic drag. When the body speed is mesothermal, the important mode of waves thus excited must be the ion acoustic type (Section 3.2).

(ii) *Disturbed particle and field distribution.* The motion of the body makes it necessary for the ambient plasma in the neighborhood to adjust to new distribution together with a new local field. The self-consistent particle and field distributions under the influence of a prescribed motion of the body are the electrogasdynamic effects of interest. We shall confine our discussions to the latter viewpoint.

9.1. *Linearized Equations of Particles and Field*

The ion distribution, under the influence of a slender body having minute negative charge and moving at mesothermal speeds, is described by the Vlasov equation (5.10):

$$\frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \frac{\partial f_i}{\partial \mathbf{x}} - \frac{e}{m_i} \frac{\partial \phi}{\partial \mathbf{x}} \cdot \frac{\partial f_i}{\partial \mathbf{v}} = 0 \tag{9.1}$$

the electron distribution, as usual, complies with the Maxwell–Boltzmann law:

$$f_e = n_\infty \left(\frac{m_e}{2\pi\kappa T_e}\right)^{3/2} \exp\left[\left(e\phi - \frac{1}{2} m_e v^2\right) / (\kappa T_e)\right] \tag{9.2}$$

and the field potential $\phi(\mathbf{x})$ obeys the Poisson equation:

$$\nabla^2 \phi = -4\pi e \left(\int f_i \, d\mathbf{v} - \int f_e \, d\mathbf{v}\right). \tag{9.3}$$

If the body is sufficiently thin and the charge on the body sufficiently small, the perturbed ion distribution can be represented as

$$f_i(\mathbf{x}, \mathbf{v}, t) = f_\infty(\mathbf{v}) + f'(\mathbf{x}, \mathbf{v}, t) \tag{9.4}$$

where f_∞ is the Maxwell distribution for the undisturbed plasma

$$f_\infty = n_\infty \left(\frac{m_i}{2\pi\kappa T_i}\right)^{3/2} \exp\left(-\frac{mv^2}{2\kappa T_i}\right) \tag{9.5}$$

f', the perturbation. The electron density $n_e(\mathbf{x})$ evaluated from eqn. (9.2) becomes

$$n_e = n_\infty \exp\left(\frac{e\phi}{\kappa T_e}\right) \simeq n_\infty\left(1 + \frac{e\phi}{\kappa T} + \cdots\right) \tag{9.6}$$

where $|e\phi/\kappa T_e| \ll 1$. The substitution of eqns. (9.5) and (9.6) in eqns. (9.1) and (9.3) yields

$$\frac{\partial f'}{\partial t} + \mathbf{v}\cdot\frac{\partial f'}{\partial \mathbf{x}} = \frac{e}{m_i}\frac{\partial \phi}{\partial \mathbf{x}}\cdot\frac{\partial f_\infty}{\partial \mathbf{v}}, \tag{9.7}$$

$$(\nabla^2 - \lambda_D^{-2})\phi = -4\pi e \int f'\, d\mathbf{v} \tag{9.8}$$

where $\lambda_D = (\kappa T_e/4\pi n_\infty e^2)^{1/2}$.

9.2. *Fourier Transform Analysis*

To solve for f' we follow the standard procedure[39,40] with the introduction of a direct velocity U of a particle caused by the field potential $\phi(\mathbf{x})$ as represented by

$$\frac{\partial U}{\partial t} = -\frac{e}{m_i}\frac{\partial \phi}{\partial \mathbf{x}}. \tag{9.9}$$

Let

$$\xi = \mathbf{v} - \mathbf{U}(\mathbf{x}, t). \tag{9.10}$$

Equation (9.7) becomes

$$\frac{\partial f'}{\partial t} + \xi\cdot\frac{\partial f'}{\partial \mathbf{x}} = \xi\cdot\frac{\partial}{\partial \mathbf{x}}\left(U\cdot\frac{\partial f_\infty}{\partial \xi}\right) \tag{9.11}$$

and eqn. (9.8) becomes

$$(\nabla^2 - \lambda_D^{-2})\phi = -4\pi e \int f'\, d\mathbf{v}. \tag{9.12}$$

The method of Fourier transform can be used to solve the system of linear equations (9.11) and (9.12). Let

$$f'(\mathbf{x}, \xi, t) = \int \mathbf{A}(\mathbf{k}, \omega, \xi)\, e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}\, d\mathbf{k}\, d\omega \tag{9.13}$$

and

$$\mathbf{U}(\mathbf{x}, t) = \int \mathbf{B}(\mathbf{k}, \omega)\, e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}\, d\mathbf{k}\, d\omega. \tag{9.14}$$

The ion density perturbation is

$$n'_i = n_i - n_\infty = \int f'\, d\mathbf{v}$$

$$= \frac{m_i}{\kappa T_i}\int e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}\, \mathcal{I}(\mathbf{k}, \omega)\, d\mathbf{k}\, d\omega \tag{9.15}$$

where

$$\mathcal{I}(\mathbf{k}, \omega) = \frac{\mathbf{B}\cdot\mathbf{k}}{k^2}\int \frac{(\boldsymbol{\xi}\cdot\mathbf{k})^2 f_\infty(\xi^2)\, d\boldsymbol{\xi}}{\omega - \boldsymbol{\xi}\cdot\mathbf{k}}. \tag{9.16}$$

Under the mesothermal condition and negligible Landau damping, the integral (9.16) can be simplified[39]

$$\mathcal{I} = 1 + 0\left(\frac{k^2}{\omega^2}\right) \tag{9.17}$$

and eqn. (9.15) becomes

$$n' = \frac{n_\infty m_i}{\kappa T_i}\int (\mathbf{B}\cdot\mathbf{k})\, e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}\, d\mathbf{k}\, d\omega. \tag{9.18}$$

Let $U(\mathbf{x}, t) = \partial/\partial t(\partial G/\partial \mathbf{x})$, the eqns. (9.18), (9.12) and (9.9) can be simplified to the following form:[40,41]

$$\frac{\partial^2 G}{\partial t^2} = \left(\frac{e}{m_i}\right)\phi \tag{9.19}$$

and

$$(\nabla^2 - \lambda_D^{-2})\phi = \frac{4\pi e n_\infty m_i}{\kappa T_i}\nabla^2 G. \tag{9.20}$$

The boundary and initial conditions complying with given physical situations that are imposed on the functions ϕ and G can be formulated as follows: at the surface of the body we may prescribe $\phi = \phi_s$, a constant value for the surface potential in the case of a conducting body.* Far upstream of the body, ϕ must vanish. In the particle aspect, the boundary condition for a two-dimensional body can be prescribed by first letting the equation of the body surface S as

$$f = y - \eta_0(x - V_\infty t) \quad \text{on } S \tag{9.21}$$

where V_∞ is the body velocity which is along the x-axis; x and y measure along the longitudinal and transversal dimensions of the slender body respectively. Differentiating eqn. (9.21) and neglecting the highest-order term, we have the remaining terms:

$$\frac{\partial^2 G}{\partial y \partial t} = -V_\infty \eta_0' \quad \text{on } S. \tag{9.22}$$

Following the usual practice in thin airfoil theory, the condition (9.22) may be evaluated at $y = 0$ instead of at the actual boundary of the body.

Some physical insight into eqns. (9.19) and (9.20) may be gained by introducing an approximation on ϕ, which is appropriate for the slender bodies ($R \ll \lambda_D$)

$$\frac{1}{\lambda_D^2} \gg \left|\frac{\nabla^2\phi}{\phi}\right|. \tag{9.23}$$

Under the condition (9.23), eqns. (9.19) and (9.20)

* In the case of a dielectric body, either ϕ or its normal derivative may be prescribed.

become

$$\phi = -\frac{\kappa T_e}{e}\frac{1}{v_i^2}\nabla^2 G \qquad (9.24)$$

$$\frac{\partial^2 G}{\partial t^2} - \left(\frac{\kappa T_e}{m_i}\right)^2 \nabla^2 G = 0. \qquad (9.25)$$

For coordinates fixed to the body, eqn. (9.25) becomes the usual wave equation with the speed of propagation $(\kappa T_e/m_i)^{1/2}$ equal to the speed of the ion acoustic wave. Here once again we see the fluid-nature behavior of the collision-free plasma due to their collective action through a self-consistent field.

10. AERODYNAMICS OF METEORS

The discussion on aerodynamic problems, so far, has been limited to the cases where the physical conditions of the moving body remain unchanged. In reality, there are many situations, e.g. a missile re-entering into the atmosphere, where the body surface is caused to transform with considerable aerodynamic consequences. The remaining space of the present review will be used for a glimpse at a class of aerodynamic problems for which the moving body of interest is allowed to change as a feedback to the dynamics of flow. The meteor phenomenon will be taken up first and then the comet for which the kinetic processes of interaction between the cometary atmosphere and the solar wind are not yet completely understood.

10.1. *Statement of the Problem*

A meteor is an extraterrestrial body which moves at an extremely high speed, e.g. 30 km/sec, into the Earth's atmosphere. Solid bodies entering the atmosphere at such high speeds will be heated to incandescence by friction with the air and in some cases they are melted or vaporized. When this happens we see a momentary streak of light in the upper atmosphere which is part of a meteor phenomenon. Atoms evaporating from the surface of a meteoroid possess energies as high as hundreds of electron volts. The vaporizing atoms are ionized principally in the first collisions with atoms of the ambient air. The ions and electrons produced thereby form a cloud of quasi-neutral plasma whose concentration at the time of plasma formation is usually higher than the concentration of the ionospheric plasma at those altitudes. The meteor wake formed in the process dissipates by ambipolar and possibly turbulent diffusion. Radar techniques which are adopted to observe echoes from the ionized meteor trails have been useful tools for the study of the upper atmosphere. Various scattering theories have been developed that give the amplitudes and durations of the radar echoes in terms of the ionization densities in the trails among other factors.

From the aerodynamic point of view, the meteor phenomenon is a conglomeration of problems of aerodynamic heat transfer, ablation, plasma dynamics in addition to radar tracking. It has practically all the essentials of the missile re-entry problems and possibly at a higher intensity because meteors move at higher speeds into the Earth's atmosphere. This has been one of the motivations in the earlier studies of meteor aerodynamics. The specific aspects of meteor aerodynamics that fit in the present study are: (i) the problem of the initial radius of ionized meteor trails, i.e. the kinetic process of interaction between the evaporated atoms and the ambient atoms; (ii) the ambipolar diffusion of the trail in the presence of an external magnetic field. The former is germane to the meteor detectability via electromagnetic wave scattering;[42] it also reveals that the evaporated meteor atoms undergo stages of free molecular, transitional and continuum flows—a problem of fundamental importance to rarefied gas dynamics. The latter constitutes an interesting application of the theories developed in Section 5.2.1.

A meteor aerodynamic problem is often made extremely complicated by the nonlinear coupling between the various after-effects of meteor aerodynamics, e.g. ionization, recombination, radiation, etc. The essentials of meteor phenomenon can be effectively treated by dividing it into two parts: the gas kinetics of flow of the evaporated atoms and the microscopic processes of inelastic atomic collisions. It is the first part that we shall be concerned herewith. An idealized and viable model of the meteor trails has been proposed[43] of which the meteor is represented by a point source of evaporated meteor atoms whose intrusion into the ambient atmosphere undergoes stages of free molecular, transitional and diffusional flows. In the initial period prior to their collisions with the ambient air molecules, the atom convection can be considered as a free molecular flow which is related to the determination of the initial radius of a trail; after several collisions the distribution of the evaporated meteor atoms become essentially thermalized and is close to equilibrium with the ambient air—a diffusional flow which is often called a convection–diffusion process. The motion of the ion species, produced by the meteor atomic collisions, obey the law of ambipolar diffusion which is further complicated by the presence of a magnetic field.

10.2. *Formulation of Aerodynamics of the Meteor Trail*

Consider a stationary point source of evaporated meteor atoms at the origin ($\mathbf{x} = 0$). The distribution of meteor atoms having velocity $\mathbf{v}$ at time t and position $\mathbf{x}$ is denoted by $f(\mathbf{x}, \mathbf{v}, t)$. They interact with the ambient atmospheric molecules (or atoms) whose distribution is denoted by $F(\mathbf{v}_1)$ which is assumed stationary. The transfer of the dynamic characteristics of the meteor atoms $Q(\mathbf{v})$ in the phase space $(\mathbf{x}, \mathbf{v})$ can be represented by a moment equation of Boltzmann as given in Section 5.2.1.

$$\frac{\partial(n_m\langle Q\rangle)}{\partial t}+\frac{\partial}{\partial \mathbf{x}}\cdot(n_m\langle vQ\rangle) = \int\int\int [Q(\mathbf{v}')-Q(\mathbf{v})]F(\mathbf{v}_1)f(\mathbf{v}) \times|\mathbf{v}_1-\mathbf{v}|G\,d\Omega\,d\mathbf{v}\,d\mathbf{v}_1 \quad (10.1)$$

where the collision-term $\langle Q(\delta f/\delta t)_c\rangle$ has been prescribed in terms of Boltzmann's binary collision integral[23,43] which is valid for the collisions between meteor atoms and the atmospheric molecules or atoms.

The incomplete understanding of the initial velocity distribution at the source and the interest to obtain a uniform representation including free molecules to diffusional flow prompt us to use the transport equation (5.21) instead of the Boltzmann equation (5.1). With the choice of $Q = 1$ and $Q = \mathbf{v}$, eqn. (10.1) can be simplified into the following forms:[23,43]

$$\frac{\partial n_m}{\partial t}+\frac{\partial}{\partial \mathbf{x}}\cdot(n\mathbf{V}_m)=n_m(\nu_I-\nu_A) \quad (10.2)$$

and

$$\frac{\partial}{\partial t}(mn_m\mathbf{V}_m)+\frac{\partial}{\partial \mathbf{x}}\cdot\overleftrightarrow{\mathbf{P}}_m=-mn_m\nu\mathbf{V}_m \quad (10.3)$$

respectively where ν_I denotes the frequency of ionization of meteor atoms; ν_A that of attachment, $\overleftrightarrow{\mathbf{P}}_m$, the pressure tensor that has diagonal elements* only.

The elimination of $\mathbf{V}_m$ between eqns. (10.2) and (10.3) yields

$$\nabla^2 n_m=\frac{1}{A^2}\frac{\partial^2 n_m}{\partial t^2}+\frac{1}{D}\left(1-\frac{\nu_I-\nu_A}{\nu}\right)\frac{\partial n_m}{\partial t}-\frac{\nu_I-\nu_A}{D}n_m \quad (10.4)$$

where $A^2=\kappa T/m$, $D=\kappa T/m\nu$ with T, assumed constant, denoting an effective kinetic temperature of the evaporated meteor atoms of mass m. The quantity D approximately equals the diffusion coefficient of the meteor atoms in the ambient air. The quantity A represents the propagation speed of a small pressure impulse of meteor atoms assuming an isothermal process during the propagation.

10.3. *Nature of the Solution of Meteor Trail Equations*

Notice that eqn. (10.4) is a so-called telegrapher equation[44] where A denotes the dissipationless propagation speed of a telegraph signal; D, the coefficient of diffusion of the signal during propagation. The solution to eqn. (10.4) will have a well-defined wave front representing an average behavior of meteor atoms prior to their collisions with the ambient molecules.[43] In addition the solution also has terms denoting residual disturbances which persist at all points traversed by the wave front.[43] The telegrapher equation thus lies between the simple wave equation whose solutions have a wave front but no residual disturbances and the classical diffusion equation whose solutions have residual disturbances but no wave front. The difference between them becomes indistinguishable after a few collisions from the initial instant at $t = 0$. The stationary point source solution to eqn. (10.4) has been obtained[43] which gives the distribution of meteor atoms evaporated from a stationary point meteorite. The corresponding distribution for a uniformly moving meteorite can be determined by a simple application of the Galilean transformation to the stationary source solution as has been done in the heat conducting problem for a uniformly moving source.[45]

The present result of the initial expansion of the cloud of evaporated meteor atoms can be used to make a rational determination of the "initial radius" of the ionized meteor trails which were defined on the basis of a variable mean free path approach in the meteor publications, which is an unsatisfactory way of treating nonequilibrium kinetic processes. Considering the high speed with which the evaporated meteor atoms leave the meteor surface, corresponding to a kinetic energy of several electron volts for some meteors, the present theory thus provides an elucidation of an earlier observation which found the initial spreading of a meteor trail "explosively" fast, not at all diffusionlike.[46]

It is also of interest to use the present theory to explain the double trails observed for some fast meteors.[47] It was observed that a meteor at an altitude of 100–110 km has a broad trail that shows an outer trail with a diameter about 20 cm that surrounds a bright inner trail of much smaller width with sharp borders between these concentric trails at the initial instant of the trail's formation. Now consider the atomic streams emitted from the surface of a meteorite; two distinctive groups must exist: (i) the evaporated meteor atoms with a certain kinetic energy of escape from the meteor surface, (ii) the atmospheric molecules which are reflected specularly from the meteor surface with velocities of the order of meteor speed. Besides the difference in their velocities, these streams differ, of course, in composition and ionization collisions as well. Therefore there exist two separate wave fronts with different speeds of propagation into the ambient air that account for the observed double trails according to the present theory of meteor trails.

10.4. *Ambipolar Diffusion of Ionized Meteor Trails*

The ionization collisions between the evaporated meteor atoms and the ambient air are the primary source of ionization production in the meteor trails.

*This assumption implies that the flow of the meteor atoms is frictionless (see Section 5.2.2(b)).

Since the electron density of a meteor trail determines its detectability during a radar observation, it is of interest to study the diffusion process of the positively charged ions and the free electrons in a meteor trail particularly in the presence of an external magnetic field.

In the absence of a magnetic field, it is noted that the large difference in the diffusion coefficients for ions and electrons will lead to their separation. The separation of the charged particles will produce electrostatic forces tending to resist the separation. As a result, the electron diffusion rate is decreased while that for the ions is increased and both particles diffuse at the same rate given by the coefficient of ambipolar diffusion D_A which is a function of the diffusion coefficients and mobilities of ions (D_i, K_i) and electrons (D_e, K_e).

It has been shown[23] that in the absence of a magnetic field, the ambipolar diffusion of a weakly ionized gas obeys the classical diffusion equation:

$$\frac{\partial n}{\partial t} = D_A \nabla^2 n \tag{10.5}$$

where quasi-neutrality is assumed, i.e. $n = n_i = n_e$ with singly-charged particles and

$$D_A = \frac{K_e D_i + K_i D_e}{K_e + K_i}. \tag{10.6}$$

The use of Einstein's relation between the mobility and diffusion coefficient of the particles, namely $K_e/D_e = e/(\kappa T_e)$ and $K_i/D_i = e/(\kappa T_i)$ where T_i and T_e are the ion and the electron temperatures respectively, helps to simplify eqn. (10.6) for D_A. This simplification is found particularly effective considering the large mass ratio of ions and electrons thus $(T_e/D_e)(D_i/T_i) \ll 1$ we have $D_A \simeq D_i(1 + T_e/T_i)$ and with an isothermal plasma $T_i = T_e$, $D_A \simeq 2D_i$.

If we can represent a meteor trail by a line source of strength Q which diffuses into a neutral atmosphere, the solution to eqn. (10.5) becomes

$$n(r, t) = \frac{Q}{4\pi D_A t} \exp\left[-\frac{r^2}{4D_A t}\right] \tag{10.7}$$

where t denotes the time lapse after the passage of the meteor; r, the radius from the trail axis referring to a coordinate system fixed to the meteor in question. It is noted that in the above diffusion analysis, the initial radius effect which accounts for the free molecular and transitional regions of the trail has been ignored.

In the presence of a magnetic field, charged particles tend to gyrate around the magnetic field with their respective Larmor frequencies until these are interrupted by particle collisions. As a result, a magnetic field reduces the ion and electron diffusion coefficients transverse to the field to $D_{i\perp}$ and $D_{e\perp}$, respectively, leaving the longitudinal coefficients unchanged. These transverse diffusion coefficients can be approximated as follows:[23]

$$D_{\alpha\perp} = \frac{\nu_\alpha^2}{\nu_\alpha^2 + \Omega_\alpha^2} D_\alpha \quad (\alpha = i, e) \tag{10.8}$$

where ν_i, ν_e and the ion and electron collision frequencies and Ω_i, Ω_e are the Larmor gyrofrequencies respectively. At meteor altitudes, $D_{i\perp}$ will be close to D_i but $D_{e\perp}$ is greatly reduced at altitude, say, above 95 km. It is reasonable to treat the ambipolar diffusion of a meteor trail above 95 km as an isotropic diffusion with the longitudinal coefficient equal to D_A and the transverse coefficient $D_\perp$.

$$\frac{\partial n}{\partial t} = D_\perp\left(\frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2}\right) + D_A \frac{\partial^2 n}{\partial y^2} \tag{10.9}$$

with the magnetic field in the y-direction and

$$D_\perp = \frac{K_{e\perp} D_{i\perp} + K_{i\perp} D_{e\perp}}{K_{e\perp} + K_{i\perp}}. \tag{10.10}$$

The solution to eqn. (10.9) is

$$n = \frac{Q}{4\pi D_\perp (D_A)^{1/2} t} \exp\left[-\left(\frac{x^2 + z^2}{4D_\perp t} + \frac{y^2}{4D_A t}\right)\right]. \tag{10.11}$$

11. COMETARY GAS DYNAMICS

From the gas dynamics of meteors to that of comets, another degree of complexity is added. While the cause and nature of the meteor atom evaporation and dispersion as a result of aerodynamic heating are well known, the composition of the cometary atmosphere and its microscopic interaction with the oncoming solar wind are yet to be settled. None the less, macroscopic theories of cometary gas dynamics have already served important causes in the contemporary astrophysics.

11.1. *Astrophysical Significance of the Cometary Phenomena*

The primordial origin of comets in the solar system is still being debated among astrophysicists. Suffice to say that comets are astronomical objects that may contain frozen gases like ammonia and probably water. They appear to contain large amounts of hydrogen, trapped in these molecules, or larger molecules that can break up into NH_3, OH, CO_2 and CH on exposure to solar radiation. There may be large amounts of frozen hydrogen gas present as well. Some of the comets at large distances from the sun may represent deep-frozen samples of matter preserved from the early solar system and therefore are interesting objects to study if the history of the solar system is to be reconstructed.

The continual heating by solar irradiation can evaporate most of the short-period comet gases. The comet nucleus itself is too small to hold on to these gases through its gravitation and soon the entire comet disintegrates. If it has a solid core, only that core would remain and become an asteroid or a meteorite in the solar system. Some comets have elliptic orbits about the sun; their periods may range from a few years to many hundreds of years. Other comets have nearly parabolic orbits and must be reaching the sun from the far reaches of the solar system.

A typical comet has a fuzzy head, called the coma, surrounding a bright nucleus and a long tail when its orbit is under closer influence by the sun. The intense interest in the studies of comets stems also from the fact that comets can be considered as natural probes of the interplanetary plasma or solar wind; hence potentially may supply astrophysical information which can be extracted from ground or *in situ* observations. The gasdynamic interaction of solar wind with a comet which has its own atmosphere without its own magnetic field is somewhat like that with a non-magnet planet, e.g. Mars or Venus. In fact, it was Biermann's observation[4] of the comet tails that led him to predict a continuum streaming of plasma from the sun, now known as the solar wind.[4]

11.2. *Nature of Cometary Gas Dynamics*

On the basis of observational data, a gross description of comets can be made. The coma has essentially a spherical volume centered on the cometary nucleus from which neutral molecules appear to be moving away with velocities of about 0.5 km/sec. Charged particles are produced from the neutrals either by absorption of solar radiation or by their interaction with the solar wind. The question pertaining to the mechanism of this interaction is still unsettled. The coma can be detected out to distances of 10^5 to 10^6 km from the nucleus. Another basic structural element of a comet is the tail which can be as long as 10^8 km. There are two distinctive types of tail structures. The Type I tails which point straight away from the sun with filamentary structure in the form of tail rays are composed of ionized gases. The Type II tails, which are composed mainly of dust, are strongly curved, broad and apparently without extensive fine features. The Type II tails are generally shorter than the Type I tails.

It should be noted that the construction of a theory of cometary gas dynamics is still plagued with uncertainties about some basic physical concepts on which it is built. For instance, the hydromagnetic interaction between the solar (magnetoplasma) wind and the cometary atmosphere rests on the assumption that the latter becomes ionized sufficiently fast somehow in order to be coupled to the former by means of a convected or fluctuating field; yet an ionization process suitable for such results has not been established. Besides, the study of cometary gas dynamic interaction depends critically on the rate of ionization of the cometary gas in question. In view of this dilemma, it serves no important purpose by performing detailed exercise of cometary gas dynamic calculations. Instead, we shall dwell on the criterion for the establishment of hydromagnetic interaction between the solar wind and the ionized cometary atmosphere which is relevant to the study of the contemporary cometary gas dynamics.

To simplify matters for the sake of discussion, we consider a cometary atmosphere with sufficient ionized components such that its conductivity is high enough to induce an electric current that interacts with the solar magneto-plasma. Under such conditions, a bow hydromagnetic shock wave can be expected for the interaction between the oncoming solar wind and the comet. With an assumed shock discontinuity in the flow field, the gas dynamic problems becomes a simple hypersonic flow over a blunt body[27] with, of course, proper consideration of the hydromagnetic aspect of the flow. The kernel of the difficulties with such an approach is the irreconcilable time scales: the observed time scales of the variation in cometary structure and of the likely ionization processes either by charge transfer or by absorption of solar ultraviolet light are orders of magnitude apart.

It is illuminating to follow Biermann *et al.*[48] who investigate the macroscopic feature of the plasma flow on the sunward side of a comet by means of a stationary quasi-hydrodynamic model. The hydrodynamic equations are modified by adding source terms to take into account the influence of the added plasma of cometary origin.

The inviscid hydrodynamic equations modified by source terms are as follows:

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial \mathbf{x}}\cdot(n\mathbf{V})=A,$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial \mathbf{x}}\cdot(\rho\mathbf{V})=B,$$

$$\frac{\partial(\rho\mathbf{V})}{\partial t}+\mathbf{V}\cdot\frac{\partial}{\partial \mathbf{x}}(\rho\mathbf{V})+\rho\mathbf{V}\left(\frac{\partial}{\partial \mathbf{x}}\cdot\mathbf{V}\right)+\frac{\partial p}{\partial \mathbf{x}}=C,$$

$$\frac{\partial}{\partial t}\left(\rho\frac{V^2}{2}+\frac{p}{\gamma-1}\right)+\frac{\partial}{\partial \mathbf{x}}\cdot\mathbf{V}\left(\rho\frac{V^2}{2}+\frac{\gamma}{\gamma-1}p\right)=D,$$

where n, ρ, p represent the number density (ions plus electrons) the mass density and the pressure tensor of the plasma (including the magnetic stress tensor) which has only isotropic diagonal components hence have been replaced by a scalar, $\mathbf{V}$ its mass velocity vector and γ denotes the ratio of specific heat ($\gamma = 2$ to include the magnetic pressure $H^2/8\pi$). The source terms A–D describe the local gains which the plasma undergoes in the corresponding quantities due to the (charge) exchange of particles with the neutral cometary gas. Simple order of magnitude analysis shows that in a steady supersonic flow the source term B for the mass conservation equation has the predominant source effect on the system. Using a simplified one-dimensional model, it can be shown[48] that under their assumed ionization rate for the cometary molecules hence the mass accretion rate for the plasma flow, a shock transition from supersonic to subsonic flow is necessary. Biermann *et al.*[48] proceeded to give an elaborate flow-field analysis for the interaction between the solar wind and a comet.

The tentative solution of Biermann *et al.*[48] for comets and their extension to Mars and Venus must be subjected to experimental verification on the basis of *in situ* observations by space probes. Some features of the comet, Mars or Venus that have bearings on the consequence of the theory are experimentally verifiable, e.g. the extent of the upstream disturbance must be limited by the shock, existence of high energy spikes of electrons or ions as a result of a strong shock, etc.

The causes of ionization of the cometary gas are still not clear since all the mechanisms considered are too weak to provide the observed degree of ionization. It was proposed[49] that the motion of the solar magneto-plasma relative to the neutral cometary atmosphere obeys the critical velocity hypothesis of Alfven[50] which implies that when the relative velocity between a neutral gas and a magnetized plasma increases to a value of $(2e\phi_I/m_n)^{1/2}$, then ionization of the gas will increase abruptly. Here ϕ_I and m_n are the ionization potential and mass of the neutral atom respectively. Considerable discussions pertaining to Alfven's hypothesis are available in the literature.[51]

So far our discussion has been limited to the sunward side of the cometary gas dynamics. To study the dynamics of the tails, let us start with the acceleration observed in the comet tails. The Type I tails normally have high accelerations of the order of 10^2 or 10^3 times solar gravity, while the dust tails (Type II) have accelerations of the order of solar gravity (which is equal to 0.6 cm/sec^2 at 1 astronomical unit from the sun). The accelerations in dust tails can be explained by consideration of the solar radiation pressure, the solar gravity and the aerodynamic drag of the free expanding dust particles in a free expanding cometary gas.[52] However, the accelerations in ionized tails are far too high to be explained by this mechanism. It was this observed anomalous high outward acceleration of the comet tails that prompted Biermann to propose the existence of some general streaming of plasma particles outward from the Sun. Biermann has pointed out that the ionization and excitation of the molecular ions can be explained only by some kind of particle flux, presumably the same solar streaming responsible for the acceleration.

.The basic filamentary structure of the Type II comet tails appears to be compelling evidence for magnetic tails in the ionized gas comet tails. Pursuing along this line of argument, one may suggest various modes of plasma instabilities to explain the fine structure of the tails.

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