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LETTER TO THE EDITOR

Comment on: **Aspects of the mechanics of driving nails into wood** by S. A. L. SALEM, S. T. S. AL-HASSANI and W. JOHNSON, *Int. J. mech. Sci.* **17**, 211 (1975).

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THE WORK of nailing seems to be bound up principally in "indentation" of the wood and friction work at the tip and along the nail shank. The fracture work of splitting the wood is very small. For example, the fracture toughness (R) of many timbers is perhaps $0{\cdot}2 < R < 1$ lb-in./in^2. Penetration to a distance x, with a split of width $2c$ alongside the nail of diameter D, generates a new crack area of some $2c\,x$. If $x = 3$ in. and $2c = 4D$ (? on high side), we have for a $\frac{1}{4}$ in. dia. nail,

$$\text{fracture work} = (0{\cdot}2 \sim 1)\ 4\ (\tfrac{1}{4})\ 3 = (0{\cdot}6 \sim 3) \text{ lb-in.}$$

Fig. 4(a) in Salem *et al.* gives, for a 3-in. penetration, total work of at least

$$(\sim 190 \times 3) = 570 \text{ lb-in.}$$

with a reduced shank nail. With a full diameter nail, the work is greater. Thus fracture work is trivial. However, the mechanics of cracking seem to play a part in determining the sideways pressure (p) and particularly the extent of circumferential contact between the nail and wood (i.e. $4\theta_0$ in the notation of Salem *et al.*), where the contact stresses may be relieved by splitting alongside the nail.

Let us model the process of *frictionless* nailing in terms of a deep elliptical part-through crack, as shown in Fig. 1, which grows because of the strip of pressure p contained by $2\theta_0$ on both faces. The penetration x is the semi-major axis of the ellipse. Following Newman,[1] the *remote boundary* stresses σ that will cause propagation of a semi-elliptical surface crack (Fig. 2(a)) are given by

$$\sigma = \frac{K_I}{\sqrt{(\pi x)}}\frac{1}{M_e/\sqrt{(Q)}}, \tag{1}$$

where K_I is the stress intensity factor, M_e an "elastic magnification factor on stress intensity" and Q is an elastic shape factor for an elliptical crack. M_e and Q are functions of x/t and x/c. The factor $M_e/\sqrt{(Q)}$ is given graphically in ref. (1) for various (x/t) and (x/c) combinations. For $x/c > 1$, which is the range of interest for nailing, $M_e/\sqrt{(Q)}$ is approximately independent of (x/t), i.e. of the penetration relative to the block thickness. Calculations show that for $x/c > 1$, σ decreases slightly as x increases (cf. $M_e/\sqrt{(Q)}$ decreases as x/c increases), but it is permissible to take an average value for σ which is independent of x/t, and depends only on K_I and c.

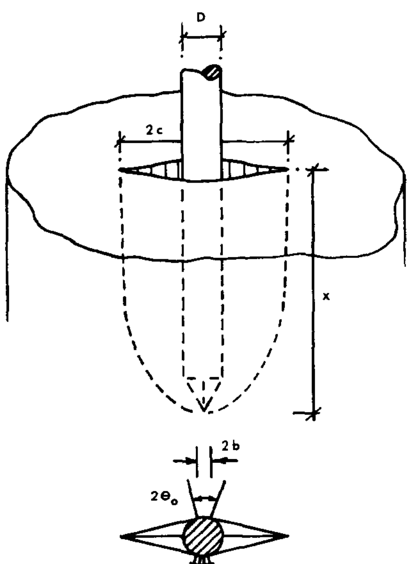

FIG. 1. Frictionless nailing modelled as propagation of a "part-through" semi-elliptical crack of semi-major axis x (the nail penetration) and minor axis c (the length of split accompanying nailing).

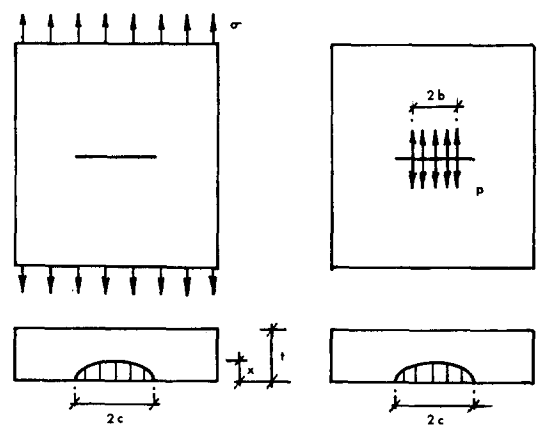

FIG. 2. (a) The part-through crack with remotely applied stresses. (b) The part-through crack with stresses applied on the crack faces.

The nailing problem concerns stresses (p) applied over part of the crack faces (Fig. 2(b)), where $2c > 2b = D \sin\theta_0$. We are not aware of a fracture mechanics solution for this problem with a part-through crack. However, the solution for the corresponding through crack problem is known[2], viz.

$$\sigma = \frac{K_I}{(2/\pi)\sin^{-1}(b/c)\sqrt{(\pi c)}}, \qquad (2)$$

where σ acts over a strip of width $2b$ symmetrically within the crack length $2c$, with the crack going right through the block. Note that when $b = c$,

$$\sigma = K_I/\sqrt{(\pi c)}, \qquad (3)$$

which is the standard solution for *remote* stresses in a through cracked plate. Assuming that equations (1) and (3) may be thought of as a corresponding pair, let us heuristically modify equation (1) to determine the part-through equivalent of equation (2), i.e.

$$p = \frac{K_I}{(2/\pi)\sin^{-1}(b/c)\sqrt{(\pi x)}\,(M_e/\sqrt{(Q)})}.$$

Then, p/K_I may be worked out for various values of $2c/D$ (assuming straight sided crack openings tangential to the nail, Fig. 1, to determine the associated θ_0 and b). Such results are used later in this note.

Returning to "real" nailing, which involves considerable work of indentation and work of friction, penetration by the nip is similar to cone indentation, where the pressure resisting indentation (p') is given by $p''(1+\mu\cot\alpha)$, where p'' is pressure in the absence of friction.[3] This latter pressure must be some $2{\cdot}8\ \sigma_y$,[4] say, where σ_y is some average yield strength of the wood beneath the indentation, which fudges anisotropic effects. Because of compressibility effects in timber, the conversion factor may be too large. The nip angle (2α) is about 60°, and the coefficient of friction between wood and steel is perhaps 0·2.[5] Thus

$$p' \approx 2{\cdot}8\ \sigma_y(1+0{\cdot}2\cot 30°)$$
$$\approx 3{\cdot}8\ \sigma_y.$$

Consequently, the force F_0 resisting nip indentation is $p'(\pi D^2/4)$. Note that this is constant, and independent of depth of penetration (because successive indentations are supposedly geometrically similar). Displacements x_p (Fig. 2 in Salem *et al.*) are elastic or compressibility effects presumably. Fig. 5 in Salem *et al.* gives $p' = F_0/(\pi D^2/4) =$ 3100 lb/in² (along the grain) and 4500 lb/in² (radially normal to the grain). Thus σ_y is some 820 lb/in² (beneath a nail driven in along the grain) and 1200 lb/in² (beneath a nail driven in radially normal to the grain), noting that these values may really be greater if $p'' < 2{\cdot}8\ \sigma_y$.

With a full shank nail, the friction *force* from the sides of the nail upon further indentation is

$$[\mu p(4\theta_0/2\pi)\,\pi D]\,x$$

(neglecting height of nip). Values for p and θ_0 depend upon the assumed geometry of splitting alongside the nail, i.e. $2c/D$. However, $p \ngtr 1{\cdot}8\ \sigma_y$ from considerations of indentation of rigid cylinders against incompressible flat plates,[6] noting again that compressibility in wood may reduce the constraint. Here σ_y is the mean indentation yield or crushing stress *perpendicular* to the direction of nail travel. The estimates determined earlier for σ_y are in the direction of nail travel, so that to determine the limiting sideways pressure ($p \ngtr 1{\cdot}8\ \sigma_y$) the previous σ_y values should be swapped it seems, i.e.

$p \ngtr 1{\cdot}8\ (1200) = 2200$ lb/in² (alongside a nail driven in along the grain)

and

$p \ngtr 1{\cdot}8\ (820) = 1500$ lb/in² (alongside a nail driven in radially normal to the grain).

Again, the real values of σ_y would vary with grain direction in the plane of penetration.

The contact angle associated with this sideways pressure, and also the extent of sideways splitting alongside the nail ($2c$), may be determined from the fracture analysis discussed earlier. The critical stress intensity factor for many woods at incipient splitting is some (200–400) (lb/in²)√in,[7] precise values varying with orientation, temperature and moisture content.

Thus, if $p \ngtr 2200$ lb/in², $p/K_I \ngtr 11$ or 5·5, and fracture calculations show that $2c/D$ then is 2·5 or 1·7 (i.e. the wood splits $(2{\cdot}5-1)\,D/2 \approx 0{\cdot}7D$ on either side of the nail, or for $K_I = 400$ lb/in²√in about $0{\cdot}3D$, and also that $(4\theta_0/2\pi) = 0{\cdot}25$ or 0·4, respectively). Consequently,

$$[\mu p(4\theta_0/2\pi)\,\pi D] = 82 \text{ or } 131 \text{ lb/in}$$

(nail driven along the grain), for a 0·236-in. nail assuming $\mu = 0{\cdot}2$.[5] Similarly, for $p \ngtr 1500$ (nail driven radially normal to the grain) $p/K_I \ngtr 7{\cdot}5$ or 3·8, and $2c/D = 2$ or 1·4 with $(4\theta_0/2\pi) = 0{\cdot}33$ or 0·55. Thus,

$$[\mu p(4\theta_0/2\pi)\,\pi D] = 74 \text{ or } 122 \text{ lb/in.}$$

Now these values represent the slopes in the force–penetration diagrams in Salem *et al.*, which we have measured from their Fig. 4(a) as 89 lb/in. (radially normal penetration) and 84 lb/in. (along the grain); elsewhere in the paper a figure of 140 lb/in. is quoted. The agreement with our calculated values is remarkably reasonable. The calculated values show the "wrong trend", i.e. the radially normal slope is predicted to be *less* than that along the grain slope. Had we not inverted the sideways σ_y values, the trend would have had the correct sense (cf. we have no information for penetration tangentially normal to the grain, the results of which would influence anisotropic calculations).

We also note that the slope of the force–penetration diagram varies directly with D. This trend is seen in Fig. 4(b) of Salem *et al.* where nails varying in diameter from 0·236 to 0·1155 in. were driven along the grain (diameters given in the legend of the figure in terms of length). Now if the slope is 84 lb/in for $D = 0{\cdot}236$ in., we would expect a slope of (0·155/0·236) 84 = 55 lb/in. for the 0·155-in. nail driven along the grain. The experimental value is rather smaller (~40 lb/in), which perhaps is to be expected because the spacing of the wood grain is likely to be important, and smaller diameter nails fit more easily in between the spacing of the grain.

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