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FREE CONVECTIVE HEAT TRANSFER FROM SLENDER CYLINDERS SUBJECT TO UNIFORM WALL HEAT FLUX

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ABSTRACT For laminar flows over slender cylinders, the boundary-layer equations do not admit similarity solutions as do the laminar layers over cylinders with large radius. Consequently, the prediction of heat transfer from such surfaces requires the solution of a system of partial differential equations for different boundary conditions and Prandtl number. In the present paper we study the free-convective heat transfer from slender cylinders subject to uniform wall heat flux. This is done by solving the boundary-layer equations by an efficient numerical method described in reference (1).

Nomenclature

Gr_{r_0}	Grashof number based on r_0, $Gr_{r_0} \equiv (\dot{q}_w \bar{g} \beta r_0^4 / k\nu^2)$
$\bar{g}$	acceleration of gravity
k	thermal conductivity
Pr	Prandtl number
Re	Reynolds number, $Re \equiv u_c r_0/\nu$
$\dot{q}_w$	wall heat flux
r	radial distance from axis of revolution, $r = r_0 + y$
r_0	cylinder radius
t	static temperature
u, v	x- and y-components of velocity, respectively
u_c	a characteristic velocity, $u_c = (\dot{q}_w \bar{g} \beta r_0^2/k)^{1/2}/(Re)^{1/4}$
x, y	rectangular coordinates
α	thermal diffusivity
β	coefficient of thermal expansion
ν	kinematic viscosity
ρ	density
ψ	stream function

Governing Equations

We consider the boundary-layer equations for an axisymmetric flow with a body force. For an incompressible, steady, laminar flow they are:

Continuity:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0 \tag{1}$$

Momentum:

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \bar{g}\beta(t - t_\infty) + \frac{\nu}{r}\,\frac{\partial}{\partial y}\left(r\,\frac{\partial u}{\partial y}\right) \tag{2}$$

Energy:

$$u\,\frac{\partial t}{\partial x} + v\,\frac{\partial t}{\partial y} = \frac{\alpha}{r}\,\frac{\partial}{\partial y}\left(r\,\frac{\partial t}{\partial y}\right) \tag{3}$$

We consider the following boundary conditions:

$$y = 0 \qquad u = v = 0, \qquad \left(\frac{\partial t}{\partial y}\right)_w = -\frac{\dot{q}_w}{k} \tag{4a}$$

$$y \to \infty \qquad u = 0 \qquad t = t_\infty \tag{4b}$$

The introduction of the following dimensionless quantities

$$\begin{gathered}\bar{x} = \frac{x}{r_o}, \qquad \bar{y} = \frac{y}{r_o}\sqrt{Re}, \qquad \bar{u} = \frac{u}{u_c}, \qquad \bar{v} = \frac{v}{u_c}\sqrt{Re} \\ \bar{r} = \frac{r}{r_o}, \qquad u_c = \left(\bar{g}\beta\,\frac{r_o^2 \dot{q}_w}{k\,\sqrt{Re}}\right)^{1/2} \\ Re = \frac{u_c r_o}{\nu}, \qquad g = \frac{(t - t_\infty)k\,\sqrt{Re}}{\dot{q}_w r_o}\end{gathered} \tag{5}$$

into (1), (2), (3) allows them to be written as

$$\frac{\partial}{\partial \bar{x}}(\bar{r}\bar{u}) + \frac{\partial}{\partial \bar{y}}(\bar{r}\bar{v}) = 0 \tag{6}$$

$$\bar{u}\,\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\,\frac{\partial \bar{u}}{\partial \bar{y}} = g + \frac{1}{\bar{r}}\,\frac{\partial}{\partial \bar{y}}\left(\bar{r}\,\frac{\partial \bar{u}}{\partial \bar{y}}\right) \tag{7}$$

$$\bar{u}\,\frac{\partial g}{\partial \bar{x}} + \bar{v}\,\frac{\partial g}{\partial \bar{y}} = \frac{1}{Pr}\,\frac{1}{\bar{r}}\,\frac{\partial}{\partial \bar{y}}\left(\bar{r}\,\frac{\partial g}{\partial \bar{y}}\right) \tag{8}$$

We next introduce the Mangler transformation

$$dx^* = d\bar{x}, \qquad dy^* = \bar{r}\,d\bar{y} \tag{9}$$

and by using the definition of stream function

$$\bar{r}\bar{u} = \frac{\partial \psi}{\partial \bar{y}}, \qquad \bar{r}\bar{v} = -\frac{\partial \psi}{\partial \bar{x}} \tag{10}$$

and by noting that

$$\bar{r} = \left(1 + \frac{2y^*}{\sqrt{Re}}\right)^{1/2} \tag{11}$$

we can write (7) and (8) as

$$\frac{\partial\psi}{\partial y^*}\frac{\partial^2\psi}{\partial x^*\partial y^*} - \frac{\partial\psi}{\partial x^*}\frac{\partial^2\psi}{\partial y^{*2}} = g + \frac{\partial}{\partial y^*}\left(\bar{r}^2\frac{\partial^2\psi}{\partial y^{*2}}\right) \tag{12}$$

$$\frac{\partial\psi}{\partial y^*}\frac{\partial g}{\partial x^*} - \frac{\partial\psi}{\partial x^*}\frac{\partial g}{\partial y^*} = \frac{1}{Pr}\frac{\partial}{\partial y^*}\left(\bar{r}^2\frac{\partial g}{\partial y^*}\right). \tag{13}$$

Equations (12) and (13) now can be expressed in similarity variables ξ,η defined by

$$\xi = x^*, \qquad \eta = \frac{y^*}{(x^*)^{1/5}} \tag{14}$$

and by dimensionless parameters $f(\xi,\eta)$ and $G(\xi,\eta)$ defined by

$$\psi = (x^*)^{4/5}f(\xi,\eta), \qquad g = (x^*)^{1/5}G(\xi,\eta)\ . \tag{15}$$

They become

$$(bf'')' + \frac{4}{5}ff'' - \frac{3}{5}(f')^2 + G = \xi\left(f'\frac{\partial f'}{\partial\xi} - f''\frac{\partial f}{\partial\xi}\right) \tag{16}$$

$$(cG')' + \frac{4}{5}fG' - \frac{1}{5}f'G = \xi\left(f'\frac{\partial G}{\partial\xi} - G'\frac{\partial f}{\partial\xi}\right) \tag{17}$$

Here

$$b = 1 + \Lambda\eta \qquad c = b/Pr \tag{18a}$$

The parameter Λ represents the slenderness of the cylinder and will be referred to as the transverse curvature parameter. It is defined by

$$\Lambda = 2(x^*)^{1/5}/(Re)^{1/4} \tag{18b}$$

The boundary conditions (4) become

$$\eta = 0 \qquad f = f' = 0 \qquad G'_w = -1 \tag{19a}$$

$$\eta = \eta_\infty \qquad f' = 0 \qquad G = 0 \tag{19b}$$

Calculated Results

The solution of the system given by (16), (17), (19) is obtained by an efficient numerical method described in ref. (1). According to this method, at first the governing equations (16) and (17) are written as a first-order system. Then the derivatives are approximated by centered difference quotients and averages centered at the midpoints of net rectangles or net segments. A nonuniform grid described in ref. (2) is used in the η-direction. The nonlinear difference equations are solved by Newton's method using an efficient block-tridiagonal factorization technique. For details see refs. (1) and (3).

Table 1 shows the derivation of the cylinder local Nusselt number, $(Nu_x)_{cyl}$, from that of a flat plate, $(Nu_x)_{f.p.}$, for various values of Λ and Pr in tabular form. Here the ratio of two Nusselt numbers is found from

$$\frac{(Nu_x)_{cyl}}{(Nu_x)_{f.p.}} = \frac{G_w(0)}{G_w(x)}$$

TABLE 1

Deviation of Local Nusselt Number of a Slender Cylinder Subject to Constant Wall Heat Flux From That of a Flat Plate for Various Values of Pr.

$(Nu_x)_{cyl}/(Nu_x)_{f.p.}$

Λ	Pr=0.01	Pr=0.1	Pr=0.72	Pr=1.0	Pr=10	Pr=100
0.000	1.000	1.000	1.000	1.000	1.000	1.000
0.200	1.521	1.360	1.294	1.283	1.231	1.220
0.275	1.627	1.367	1.280	1.265	1.203	1.185
0.317	1.692	1.412	1.315	1.299	1.231	1.207
0.437	1.844	1.477	1.347	1.328	1.243	1.211
0.502	1.939	1.526	1.379	1.359	1.263	1.226
0.693	2.161	1.631	1.439	1.412	1.293	1.243
0.751	2.250	1.675	1.466	1.439	1.310	1.254
0.796	2.301	1.700	1.481	1.451	1.316	1.257
0.914	2.444	1.771	1.523	1.491	1.341	1.273
1.099	2.654	1.872	1.583	1.545	1.372	1.291
1.191	2.770	1.929	1.618	1.577	1.392	1.303
1.262	2.848	1.967	1.640	1.598	1.406	1.310
1.450	3.062	2.070	1.703	1.655	1.441	1.333
1.572	3.200	2.135	1.742	1.692	1.461	1.345
1.741	3.391	2.226	1.797	1.743	1.493	1.364
1.888	3.553	2.303	1.843	1.787	1.518	1.380
2.000	3.681	2.363	1.880	1.820	1.539	1.393
2.297	4.318	2.512	1.970	1.903	1.589	1.424
2.491	4.221	2.615	2.031	1.959	1.625	1.445
2.639	4.379	2.687	2.076	2.001	1.650	1.460
2.759	4.515	2.749	2.114	2.034	1.672	1.475
2.993	4.764	2.863	2.183	2.098	1.709	1.497
3.170	4.964	2.952	2.237	2.148	1.740	1.516

References

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