Nuclear Physics B78 (1974) 435–444. North-Holland Publishing Company

LONGITUDINAL MOMENTUM TRANFERS IN THE MULTIPERIPHERAL MODEL*

F.S. HENYEY

Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan 48104

Received 22 April 1974

Abstract: The inclusion of longitudinal momentum transfers in the multiperipheral model is investigated. As recently pointed out by Jadach and Turnau, these longitudinal momentum transfers, if present, have a very large effect on properties of the model. In this paper we find that data rules out such effects. Furthermore, if the longitudinal momentum effects were present, they would in no way solve the serious problem with the multiperipheral model, that average multiplicity and elastic slope ought to be proportional, but experimentally are not. A multiperipheral model using transverse momentum transfers has fewer phenomenological difficulties than one using total momentum transfer.

A powerful constraint on multiparticle models is the requirement that the model simultaneously fit relevant aspects of both multiparticle and elastic data; the elastic scattering is calculated by unitarity. The multiperipheral model has a serious difficulty with this constraint. In this paper the effect of longitudinal momentum transfers is investigated, and two main conclusions are reached:

(i) Longitudinal momentum transfers do not help solve the difficulty.

(ii) Multiparticle data rules out using the entire momentum transfer, but is consistent with using only the transverse part, as the main variable on which the amplitude depends.

Recently, Jadach and Turnau [1] pointed out a feature of some versions of the multiperipheral model (MPM) which had been overlooked in several recent discussions of the MPM [2–4]. This feature is that the longitudinal momentum transfers provide the transverse momentum cutoff in these versions. In the present paper, this feature is explored from both theoretical and experimental viewpoints.

Several points are to be stressed: the phenomenon described by Jadach and Turnau involves a rapid dependence of average transverse momentum on rapidity gap, which can easily be tested experimentally; it is found that there is almost no effect. Even if there were an effect, the random walk description of the MPM would retain its validity. In particular, some peculiar features of the CLA [5] model are

* Research supported in part by the US Atomic Energy Commission.

simply explained by combining the phenomenon of Jadach and Turnau with either the random walk calculation [2] or the more accurate calculation involving the longitudinal momentum contributions [6,7]. Especially, I would like to stress that the proportionality in the MPM between average multiplicity and elastic slope is not at all modified by the phenomenon of Jadach and Turnau; this proportionality is in strong disagreement with the data. The phenomenon of clustering, if it exists, weakens the effect under discussion, and makes the calculations of refs. [2–4] more nearly correct, although still not dominant. A multiperipheral model depending only (or at least mostly) on the transverse momentum transfers (as considered in refs. [2–4]) is favored by the experimental absence of the Jadach–Turnau effect. We consider these points in some detail in what follows.

We first review the effect [1]. The longitudinal momentum transfer of the ith rung of the MP ladder can be expressed as

$$t_i^{\mathrm{L}} = -\left(M\mathrm{e}^{-Y} - \sum_{k=1}^{i} m_k \mathrm{e}^{-y_k}\right)\left(M\mathrm{e}^{-Y} - \sum_{k=i+1}^{n} m_k \mathrm{e}^{y_k}\right) , \tag{1}$$

where M, Y are the mass and rapidity of the incoming particles (equal masses assumed, Y evaluated in the center of mass), and m_k, y_k are the transverse mass, $\sqrt{E_k^2 - P_{lk}^2}$, and rapidity of the kth produced particle.

In the approximation that all rapidity gaps are equal, that all m_k's are equal, and that the link is far from the ends, eq. (1) reduces to

$$t_i^{\mathrm{L}} \approx -\langle m_k\rangle^2 \frac{\mathrm{e}^{-d}}{(1-\mathrm{e}^{-d})^2} , \tag{2}$$

where d is the average rapidity gap. In comparison, the "effective" transverse momentum transfer (when d is large) is given by [2]

$$(t_i^{\mathrm{T}})_{\mathrm{eff}} \approx -\tfrac{1}{2}\langle m_k\rangle^2 . \tag{3}$$

("Effective" here means giving the correct relationship between $\langle t_i^{\mathrm{T}}\rangle$ and $\langle m_k\rangle^2$; the small value of the pion mass squared is ignored.) The relationship between eqs. (2) and (3) is shown as a function of the rapidity spacing d in fig. 1. Two features are apparent: $t^{\mathrm{L}} > t^{\mathrm{T}}$ for all $d < 1.3$ and t^{L} varies extremely rapidly as a function of d.

If we now isolate one particle, $k = K$. letting its transverse mass M_K and the rapidity gaps on either side of it, d_+ and d_-, take on non-average values, eq. (1) becomes

$$\sum_i t_i^{\mathrm{L}} \approx -m_k M_K \frac{\mathrm{e}^{-d_+} + \mathrm{e}^{-d_-}}{(1-\mathrm{e}^{-d})^2} + \text{terms independent of } M_K . \tag{4}$$

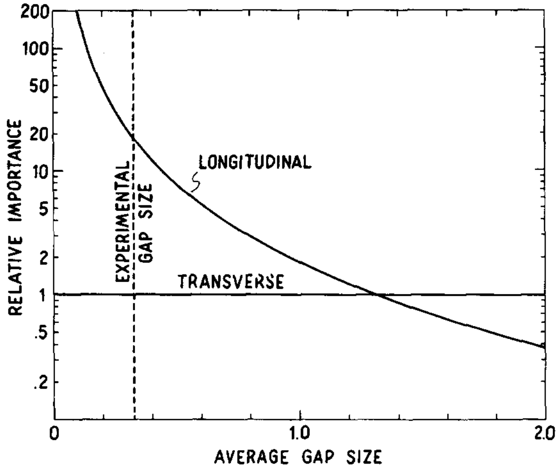

Fig. 1. The relative importance of longitudinal and transverse momentum transfers, as the function of average rapidity gap d. The ordinate is $\langle t^{\mathrm{L}}\rangle / \langle t^{\mathrm{T}}_{\mathrm{eff}}\rangle = 2\mathrm{e}^{-d}/(1-\mathrm{e}^{-d})^2$. For $d < 1.3$, longitudinal momentum transfer is more important.

Because of the rapid dependence on d of eqs. (2) and (4), the presence of a dependence of the multiparticle amplitude on t^{L} can easily be compared to experimental data. In a multiperipheral model one might expect dependence such as

$$|M|^2 \sim \prod_i \mathrm{e}^{2a t_i} = \mathrm{e}^{2a \sum_i t_i} , \tag{5}$$

where a might be a constant, or in a multi-Regge version, such as CLA [5],

$$a = \alpha' \ln \left(\frac{S_i}{S_0} + \text{const.}\right) . \tag{6}$$

If the rapidity gap is small, and experimentally it is, t_i^{L} dominates over t_i^{T}, and the amplitude depends sensitively on the rapidity gaps.

Experimental control over rapidity gaps can be obtained in (at least) two ways. The rapidity gap is approximately inversely proportional to the multiplicity, so the dependence on multiplicity can be examined. Alternatively, the actual charged particle gaps can be measured. Figs. 2 and 3 illustrate these comparisons. In fig. 2, $\langle P_\perp^2\rangle$ is plotted against multiplicity. The data shown is from the Michigan–Rochester 100 GeV bubble chamber experiment at NAL [8]. Theoretically, from eq. (2),

$$\langle P_\perp^2\rangle \approx \frac{(1-\mathrm{e}^{-d})^2}{a\,\mathrm{e}^{-d}} , \tag{7}$$

where d is approximately inversely proportional to n. The actual curve drawn is

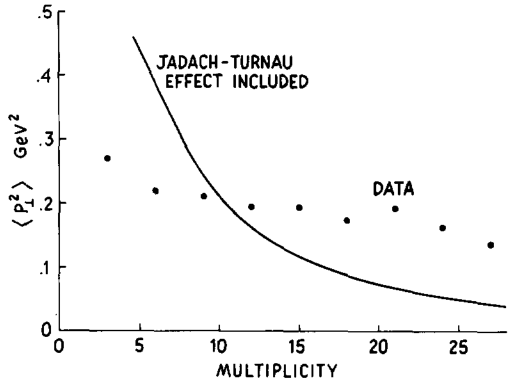

Fig. 2. Average transverse momentum as a function of multiplicity at P_{lab} = 100 GeV/c. The Jadach–Turnau effect predicts a large $\langle P_\perp^2 \rangle$ when the gap is large (low multiplicity) and a small $\langle P_\perp^2 \rangle$ when the gap is small (high multiplicity). The data is from the Michigan–Rochester bubble chamber experiment at NAL [8]. The "multiplicity" plotted for the data are $\frac{3}{2}$ of the charged multiplicity. The small effect in the data might be diffraction (2 prong) and kinematics (16, 18 prong).

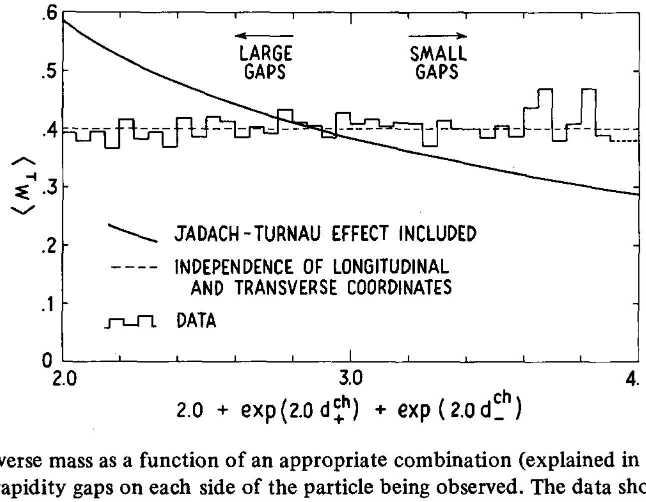

Fig. 3. Transverse mass as a function of an appropriate combination (explained in the text) of the charged rapidity gaps on each side of the particle being observed. The data shown is from the Michigan–Rochester group [8]. The last bin had few events, and was averaged with the next-to-last. The data are consistent with no dependence on gap size, and inconsistent with anywhere near as large a role as expected for the longitudinal momentum transfer.

calculated from a simplified form of the CLA model [5], described below, but the general effect is more or less independent of details.

In fig. 3, the average transverse mass in plotted against an appropriate combination of the charged rapidity gaps on either side. This variable is $e^{-d_+} + e^{-d_-}$, (see eq. (4)) corrected for missing neutrals:

$$\langle e^{-d_\pm}\rangle = \int_0^{d_\pm^{ch}} e^{-\delta} \frac{e^{-\delta/d_0}}{d_0}\, d\delta + \int_{d_\pm^{ch}}^{\infty} e^{-d_\pm^{ch}} \frac{e^{-\delta/d_0}}{d_0}\, d\delta$$

$$= \frac{1 + d_0\, e^{-(1+1/d_0)d_\pm^{ch}}}{1+d_0}\,, \tag{8}$$

where d_0 is the average spacing between neutrals. Experimentally, $d_0 \approx 1$, which gives the variable used in fig. 3. The theoretical curve is calculated as follows: eqs. (2) and (5) imply

$$\frac{1}{2a} = \langle t_i\rangle = \langle m_k\rangle^2 \frac{e^{-d}}{(1-e^{-d})^2}\,. \tag{9}$$

Dividing this into eq. (4) we get

$$\sum t_i^{L} = -\frac{M_K Z}{2a\langle M_k\rangle e^{-d}} + \text{other terms}\,, \tag{10}$$

where

$$Z = e^{-d_+} + e^{-d_-} \approx 1 + \tfrac{1}{2} e^{-2d_+^{ch}} + \tfrac{1}{2} e^{-2d_-^{ch}}\,. \tag{11}$$

Putting this into eq. (5) we get

$$|M|^2 \sim \exp\left\{-\left(\frac{M_K Z}{\langle m_k\rangle e^{-d}}\right)\right\}\,, \tag{12}$$

from which

$$\langle M_K\rangle = \langle m_k\rangle \frac{2e^{-d}}{Z}\,. \tag{13}$$

(Recall that K refers to the observed particle while k refers to all of the other particles.) We take $d = 0.33$, and from the data of fig. 3, read off $\langle m_k\rangle = 0.4$. For comparison we also show $\langle M_K\rangle =$ constant, according to the hypothesis that the longitudinal and traverse coordinates are independent.

From figs. 2 and 3, it is clear that the data is completely inconsistent with the Jadach–Turnau effect. The slow variation in fig. 2 of the data can presumably be explained as the presence of diffractive events in the two-prong data, and kinematic effects at high multiplicity. The direct comparison of fig. 3 is entirely consistent with there being no effect of gap size on transverse momentum, and therefore little dependence of the amplitude on t^{L}.

Although, as we have just seen, models including the Jadach–Turnau effect have serious difficulties when confronted by the experimental data, it is interesting to find out if their effect can help solve another difficulty, that of the elastic slope [2], and whether the random walk description of the MPM [2] is invalidated.

The random walk description give a slope [2],

$$B_{\mathrm{RW}} = \langle (n-1)\chi^2 R^2 \rangle \,, \tag{14}$$

where R is the step size, n is the multiplicity, and

$$\chi_j = \sum_{i=1}^{j} \frac{P_i}{P} \,. \tag{15}$$

The average is taken over j as well as over different events. The exact result, for an amplitude given by eq. (5), is [6]

$$B = \langle (n-1)\chi R^2 \rangle \,. \tag{16}$$

In the multiperipheral limit $\chi \approx 1$, and eqs. (14) and (16) agree. In ref. [7] it is shown that the longitudinal momentum transfers give rise to a contribution (which adds to the slope) of

$$B_{\mathrm{L}} = \langle (n-1)(\chi - \chi^2) R^2 \rangle \,. \tag{17}$$

Eqs. (14) and (17), added together give eq. (16), which provides a check of the approximations used in ref. [2]. From these equations, one finds

$$\frac{B_{\mathrm{RW}}}{B} = \frac{\langle \chi^2 \rangle}{\langle \chi \rangle} \approx \langle \chi \rangle \,, \tag{18}$$

which is almost independent of energy. A reasonable value [7], consistent with experimental data, is $\langle \chi \rangle \approx 0.8$. Thus the random walk picture is not modified very much. The effect of Jadach and Turnau, if it were present, would however, seriously modify the value of R, as they point out [1].

They [1] found that the CLA model [5] has a curious property, namely, that the contribution to the slope decreases as a function of multiplicity. This is known as the "intuitive" dependence of multiplicity on impact parameter. They based an objection to the random walk picture on this property. On the other hand, they found that the Chew–Pignotti (CP) model [9] had the ordinary "multiperipheral" dependence.

One distinction between these models, which is relevant to this point, is that the CLA model is a multi-Regge model, in which R depends on subenergies, while the CP model has R = constant. As the multiplicity increases, the rapidity gaps become smaller, and so do the subenergies. In the CLA model,

$$R^2 \propto \ln(1 + S_i) , \tag{19}$$

(with S_i in GeV units), so that the step size R decreases rapidly as a function of multiplicity. Thus, although the number of steps increases, the total distance walked decreases.

This effect was verified in the following oversimplified approximation to the CLA model: for a given multiplicity, all rapidity gaps were set equal, and all subenergies were set equal. Energy and longitudinal momentum were conserved. The transverse mass of the pions was determined by adding eqs. (2) and (3) (which is not an entirely correct prescription), and using $\langle t \rangle = 1/2a$. This transverse mass is shown in fig. 2. The equations for energy momentum conservation, for transverse mass, for subenergy as a function of rapidity gap, and for R^2 were solved simultaneously. The "partial" slope was calculated from eq. (16). (A "partial" slope is the slope of the contribution of the given multiplicity to the imaginary part of elastic scattering.)

This simplification is not quantitatively accurate, as it does not properly treat end effects caused by the difference in pion and nucleon transverse masses. However, as shown in fig. 4, it does exhibit the decrease of the slope with multiplicity, caused primarily by the extremely rapid decrease of R^2. (χ also decreases with multiplicity.) The magnitude of the decrease depends on the precise form of R^2 as a function of S_i. In particular, if the Regge scale S_0 were taken smaller than the (rather large) value of 1 GeV^2, the decrease would be less, or there would be an increase. In the limit $S_0 \to 0$ (and α' adjusted appropriately) the CLA model becomes the CP model.

The principle conclusion of the second paper of ref. [2] is that, in the MPM, the elastic slope is proportional to multiplicity as a function of energy. The constant of proportionality is mainly R^2, which can be varied by clustering, by including the Jadach–Turneau effect, or by including phases (including phases due to helicity)

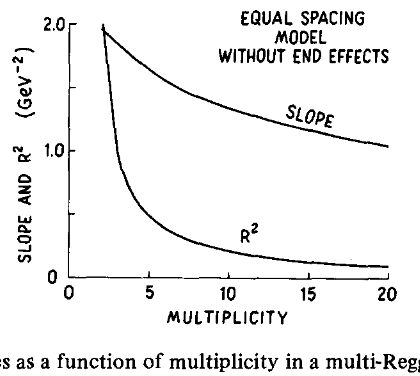

Fig. 4. The slope decreases as a function of multiplicity in a multi-Regge model with a large value of S_0, because the random walk step size R decreases extremely rapidly. The curves are from a (very) simplified CLA model.

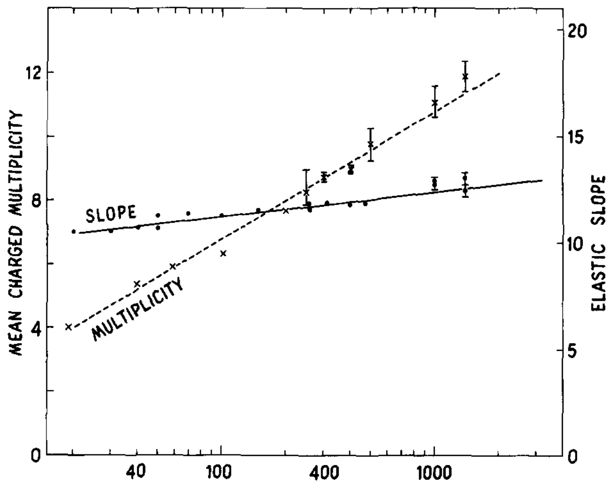

Fig. 5. Selected elastic slope [10] and multiplicity [11] data, showing that they are not proportional, contrary to the prediction of the multiperipheral model. The MPM predicts proportional straight lines for each of these. The lines shown are to guide the eye, and are not a fit, and are not predicted by any known model.

which make the uncertainty principle between impact parameter and transverse momentum an inequality. Thus the value of the slope, at a single energy, only tests details of the model. On the other hand, the energy dependence of the ratio between elastic slope and average multiplicity tests the fundamental assumptions of the MPM. This ratio should be almost constant. Either in the models with [1] or without [2–4] longitudinal momentum, the proportionality between the slope and the multiplicity will hold, provided R^2 and $\langle\chi\rangle$ are independent of energy. Experimentally $\langle\chi\rangle \approx$ constant. Also experimentally the rapidity gap, evaluated at the average multiplicity (not fixed multiplicity), is very nearly energy independent. Therefore, the multi-Regge dependence of R^2 with rapidity gap does not modify the proportionality.

The results of the Monte-Carlo calculations of ref. [1] illustrate the proportionality.

Nevertheless, experimental data are in violent disagreement with the proportionality, as shown in fig. 5. The average multiplicity varies much more rapidly than the slope. This difficulty remains the most serious phenomenological challenge to the multiperipheral model, and does not depend on whether or not longitudinal momentum transfers are present.

The disagreements with the data shown in figs. 2 and 3 are not really in contradiction with the fundamentals of the MPM, although they do not rule out those versions on which most calculations have been performed. Only the large gap limit of the MPM is given by the fundamental assumptions.

In order to continue to small gaps, where the bulk of experimental data is, a choice of variables must be made. The momentum transfer variable can be t_i, or t_i^{T}, or any other variable which becomes equal to t_i in the large gap limit. Some physics prefers t_i, for example the existence of t-channel particle poles and thresholds. Other physics prefers t_i^{T}, for example absorption and double spectral function effects. Still other physics is not yet well enough understood to make a choice, for example, whether $S_i^{\alpha(t_i)}$ or $S_i^{\alpha(t_i^{\mathrm{T}})}$ should be used in the Regge pole formula.

Therefore, no compelling theoretical choice can be made. However, as shown in figs. 2 and 3, a compelling phenomenological choice is made. The correct variable is much closer to t_i^{T} than to t_i. The authors of refs. [2–4], accepting the common lore that longitudinal and transverse coordinates are independent, chose t_i^{T} as the coordinate. The data shown in figs. 2 and 3 justify their choice.

In summary:

(i) The Jadach–Turnau effect, if it were present, would be very strong.

(ii) It is ruled out by the data, which are consistent with no dependence of transverse momenta on rapidity gap.

(iii) If it were present, it would not seriously modify the random walk picture of the multiperipheral model.

(iv) It would not help solve the problem that the multiperipheral model predicts a proportionality between $\langle n \rangle$ and elastic slope, in contradiction to experiment. The energy dependence of the ratio of elastic slope and average multiplicity is a test of the fundamental assumptions of the MPM. The MPM requires a constant ratio, while the data show a very rapid variation.

(v) The transverse part of the momentum transfer is a better variable to use in the multiperipheral model than the entire momentum transfer.

I am grateful to A. Seidl for providing me with the data shown in figs. 2 and 3, and to G. Kane for a careful reading of the manuscript.

References

[1] S. Jadach and J. Turnau, Jagellonian University preprint TPJU 3/74.
[2] F.S. Henyey, Phys. Letters 45B (1973) 363, 469.
[3] R.C. Hwa, Phys. Rev. D8 (1973) 1331.
[4] C.J. Hamer and R.F. Peierls, Phys. Rev. D8 (1973) 1358.
[5] H.M. Chan, J. Loskiewicz and W.W.M. Allison, Nuovo Cimento A57 (1968) 285.
[6] J. Turnau, Acta Phys. Pol. 36 (1969) 7.
[7] F.S. Henyey, Michigan report UM HE 74-2, unpublished.
[8] A. Seidl, Private communication.
[9] G. Chew and A. Pignotti, Phys. Rev. 176 (1968) 2112.
[10] G.G. Beznogikh et al., Phys. Letters 43B (1973) 85;
V. Bartenev et al., Phys. Rev. Letters 31 (1973) 1088;
G. Barbiellini et al., Phys. Letters 39B (1972) 663;
U. Amaldi et al., Phys. Letters 44B (1973) 116.

[11] V. Ammosov et al., Phys. Letters 42B (1972) 519;
C. Bromberg et al., Phys. Rev. Letters 31 (1973) 1563;
G. Charlton et al., Phys. Rev. Letters 29 (1972) 515;
F. Dao et al., Phys. Rev. Letters 29 (1972) 1627;
M. Antinucci et al., Nuovo Cimento Letters 6 (1971) 121.