Clarification of the Reinvestment Assumption in Capital Analysis

C. Robert Carlson, *The University of Michigan*
Michael L. Lawrence, *University of Missouri**
Donald H. Wort, *University of Illinois*

The treatment of the reinvestment rate assumption in the literature is sufficiently ambiguous that many conclude that reinvestment is included in the mechanics of calculating internal rate of return and net present value while others assume that reinvestment is only an implicit assumption necessary when two or more alternatives are compared [7]. Because of this confusion, many students experience difficulty in grasping the basic concepts of capital analysis. The purpose of this paper is to help clear up this confusion by answering the question: Is a reinvestment rate assumed in the mechanics of calculating the internal rate of return?

The Problem

To highlight the issue, consider the following investment opportunity in which an initial outlay of $2,880 is made at time $t = 0$, and cash inflows of $2,000 occur at the end of each of the next two years.

($2,880)	$2,000	$2,000
$t = 0$	$t = 1$	$t = 2$

The typical form of the net present value equation is:

$$NPV = \sum_{t=0}^{n} A_t/(1+i)^t$$

where A_t = the net cash flow relating to the investment in year t;
i = the appropriate discount rate, usually assumed to be the "cost of capital"; and,
n = the total number of years for the investment project.

If the internal rate of return approach is used, $NPV = 0$ and

$$0 = \sum_{t=0}^{n} A_t/(1+IRR)^t$$

where IRR = the discount rate which will make the total discounted

cash inflows just equal the total discounted cash outflows. Applying the IRR equation to the data at hand:

$$2880 = 2000/(1+IRR) + 2000/(1+IRR)^2$$
$$2880 = 2000/(1+.25) + 2000/(1+.25)^2$$
$$2880 = 1600 + 1280 = 2880.$$

The IRR is 25 percent. In this computation, the \$2000 cash flow at the end of year one was not reinvested at the internal rate of return or at any other rate.

Now, assume a 25 percent reinvestment rate for interim cash flows to compute the terminal value and then discount this value at the IRR back to year zero. At $t = 1$, reinvest \$2000 at 25 percent to get $(2000)(1+.25) = \$2500$ at $t = 2$. Including the cash flow at $t = 2$ of \$2000, the terminal value is \$4500. Discounting back to time zero at 25 percent, $NPV = 4500/(1+.25)^2 = \$2880$. Using this approach, it is apparent that the intermediate cash flows must be reinvested at the internal rate of return in order to get an unbiased measure of internal rate of return.

The Basic Question

The above example illustrates the confusion often confronting students of finance as they try to understand the reinvestment assumption. The basic question involved is whether the intermediate cash flows generated by a capital asset must be reinvested to get a true measure of IRR. One popular explanation is that no explicit reinvestment of the cash flows is made but that an implicit assumption of a reinvestment rate equal to IRR is inherent in the comparison of two alternative projects [7]. For example, consider projects A and B:

	Net Cash Flow						
Project	t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	IRR
A	(1000)					1610	10%
B	(1000)			1404			12%

To decide that B with a three-year life and IRR = 12 percent is preferred to A with a five-year life and IRR = 10 percent includes the implicit assumption that the \$1404 recovered in $t = 3$ by B will be reinvested in some Project C at a rate sufficiently attractive that the combined return on B and C over five years will be greater than 10 percent. (In this case the return on C must be greater than 7.1 percent.) To use 12 percent as the appropriate return for B in comparing it to A, the analyst implicitly assumes that the \$1404 is reinvested at exactly 12 percent. The proponents of this explanation suggest that the reinvestment rate assumption is only a problem when two or more mutually exclusive employments of capital are considered.

Another explanation of the reinvestment assumption, which is mathematically more appealing, is that the typical form of the NPV equation is merely a special case of a more general form which stipulates reinvestment [1]. By stipulating the reinvestment of intermediate cash flows, the basic formulation of the NPV equation can be expressed as follows:

$$NPV = [A_t \prod_{\alpha=t+1}^{n} (1+r_\alpha)] \ [\prod_{\alpha=1}^{n} (1+d_\alpha)]^{-1}$$

where r_α == reinvestment rate from time $\alpha-1$ to α;

d_α == discount rate from time α to $\alpha-1$;

A_t == the net cash flow relating to the investment in year t;

and, π == symbol for multiplication of terms that follow.

For example, let n = 3. Thus,

$$NPV = \sum_{t=0}^{3} [A_t \prod_{\alpha=t+1}^{3} (1+r_\alpha)] \ [\prod_{\alpha=1}^{3} (1+d_\alpha)]^{-1}$$

$NPV = A_0 \ (1+r_1)(1+r_2)(1+r_3)[(1+d_1)(1+d_2)(1+d_3)]^{-1} +$

$\quad A_1 \ (1+r_2)(1+r_3)[(1+d_1)(1+d_2)(1+d_3)^{-1} +$

$\quad A_2 \ (1+r_3)[(1+d_1)(1+d_2)(1+d_3)]^{-1} +$

$\quad A_3 \ [(1+d_1)(1+d_2)(1+d_3)]^{-1}.$

If we assume that r and d are constant over the life of the investment, i.e., $r_\alpha = r$ and $d_\alpha = d$, we have

$$NPV = \sum_{t=0}^{n} A_t(1+r)^{n-t}/(1+d)^n.$$

When we also assume that r = d, we have the familiar NPV equation:

$$NPV = \sum_{t=0}^{n} A_t/(1+d)^t.$$

The standard IRR equation can be similarly derived by merely setting NPV equal to zero. Thus, for IRR, $0 = \sum_{t=0}^{n} A_t/(1+d)^t.$ The only difference is that in the IRR equation, d is the unknown rather than NPV as in the NPV equation. By including the reinvestment assumption in the basic NPV and IRR formulations and then simplifying to the more typical equations, all of the necessary assumptions can be explicitly considered.[3]

Both of the foregoing explanations recognize the inconsistency of the reinvestment rate assumptions when comparing two or more projects using NPV or IRR. However, the second approach assumes that reinvestment is explicitly included in the mechanics of calculating IRR and NPV while the first suggests it is only implicitly included. Which is the correct explanation?

Historical Perspective

The origin of the phrase "internal rate of return" appears to be the Austrian concept "ursprunglicher Zins," loosely translated as internal rate of return, which Bohm-Bawerk, an eighteenth century Austrian economist, described as the average rate of return earned in the process of production by the capital tied up in it. It was a static measure assuming a single time period and somewhat analogous to what we now call the accounting return on investment over a single time period. At a later date Wicksell also referred to the internal rate of return as a simple rate of interest over a single time period, the length of which was determined by the life of the project [5, p. 12].

The origin of the modern concept of internal rate of return appears to be the writings of Irving Fisher, who defined internal rate of return as that rate of interest at which the present values of all future outlays and inflows on a project are made equal to each other [3]. This rate of return is obviously more dynamic than the simple rate which was used by Bohm-Bawerk and by Wicksell. But it is not clear from the writings of Fisher that this was a compound rate of return. He certainly did not intend that the intermediate cash flows would be reinvested. As Hirshleifer [4] has explained, the purpose of investment to Fisher was to spread consumption over time so as to maximize the marginal utility of consumption. That is, consumption is deferred by the investor to a time period in which consuming would be more highly valued.

Indeed, Fisher could have attained the same measurement of IRR if he had assumed that the intermediate cash flows are reinvested out to the end of the project, and then solved for the interest rate which would equate the compound sum to which the investment would grow over n years with the compound sum to which the intermediate cash flows would grow over n years. We conclude that Fisher selected discounting the cash flows instead of compounding them because the former did not require the reinvestment of cash flows. Reinvestment of cash flows would have defeated Fisher's purpose of spreading consumption so as to increase its marginal utility to the individual.

A Suggested Clarification

It is essential here that the reader recognize that discounting is the inverse of compounding. They are not the same process. The internal rate of return, if calculated by compounding out to a terminal time period, is a compound rate of return; if calculated by discounting back to the present, it is a discount rate of return. Since the two processes are the inverse of each other, the same "rate" of return on invested capital can be calculated with either technique. However, if

the intermediate cash flows are reinvested, the rate of return includes more absolute interest recovery because the investment base is increasing over time, while under discounting the investment base is being reduced as the cash flows occur. Under the assumption of reinvestment the accumulated wealth of the individual at the termination of the project will be greater than if reinvestment does not take place.

Since the compound rate of return and the discount rate of return do not measure the same characteristic of an investment, which is the characteristic which should be measured? This is the source of the confusion surrounding the reinvestment assumption. As Conrad Doenges [2] has pointed out, what the fair measure of an investment is depends upon the objectives of the investor. To Fisher, the objective of investment was to spread consumption and maximize utility. Thus to him the appropriate measure was the "discount" internal rate of return, the mechanics of which did not require the reinvestment of intermediate cash flows.

In modern capital analysis, the objective of investment is appropriately viewed as maximization of the wealth of the investor at some future point in time.[4] Thus, modern capital theorists view an individual project as meaning little as it stands by itself, and as requiring analysis in conjunction with the investments to which capital can be applied as it is recovered from the investment under study. The IRR measure consistent with their objectives is the "compound" internal rate of return, which requires reinvestment of the intermediate cash flows.

It should be noted carefully that when IRR is used in decision analysis, at least an implicit assumption concerning reinvestment is always involved. The purpose of this paper was not to disclaim a reinvestment assumption problem but rather to clarify the mechanics of calculating IRR. While the IRR, in and of itself, does not necessarily require reinvestment, it means very little if not compared to some standard. There are two general types of comparisons in which the IRR of some Project A can be involved:

1. the IRR on Project A might be compared to an appropriate hurdle rate (cost of capital) and accepted or rejected based on this comparison; and,
2. mutually exclusive projects A and B might be compared to one another on the basis of IRR.

As demonstrated earlier, when the timing of cash flows from two projects differ, the decision of which to select involves an implicit assumption concerning the rate at which the cash flows in the two projects will be reinvested. The cost of capital reflects the return which the investors could make on the capital in an alternate employ-

ment of similar risk characteristic. Unless we accept the extremely naive assumption that these alternate employments generate cash flows exactly of the same timing and magnitude as Project A, the problem of comparability also exists in testing Project A against a hurdle rate and an implicit assumption is made concerning reinvestment of the differential cash flows. Thus, the problem of the reinvestment rate assumption exists any time the internal rate of return is used to judge the desirability of an investment.

In general these same conclusions can be applied to the NPV method. Whether explicit specification of a reinvestment rate is neces-sary in the mechanics of the NPV method depends upon the purposes of the investor. Of course, in comparing the NPV of a proposed investment to some standard (NPV = 0 or the NPV of a mutually ex-clusive alternative), the problem of inconsistency of reinvestment rates does not exist, as the assumed reinvestment rate is the cost of capital. While the IRR presents both consistency and accuracy prob-lems concerning reinvestment, the NPV method presents only the problem of accuracy. Of the IRR and the cost of capital, either of the two might be more accurate depending upon the specific situation.[5]

Note carefully that the most desirable solution to the reinvest-ment rate problem is not in selecting either the IRR or NPV metho-dology, depending upon the situation. Rather, a much better solution is to explicitly select a consistent and accurate reinvestment rate for the alternatives under consideration [8, p. 77].

Conclusion

To summarize, whether the mechanics of calculating an NPV or IRR involves the explicit reinvestment of intermediate cash flows depends upon the objective of the investor.

If the objective is to defer and spread consumption, the NPV or IRR does not necessarily involve an explicit reinvestment rate. It should be recognized here that if consumption occurs, it is done at the opportunity cost of reinvestment. If the objective is to maxi-mize investor wealth as of some terminal point in time, a rein-vestment rate must be specifically included in the calculation. How-ever, once an IRR is calculated, it can be used at face value in deci-sion analysis only if there is at least an implicit assumption that the intermediate cash flows could have been reinvested at a rate equal to IRR. It is this assumption that causes the IRR measure and the alter-nate measure to which it is compared (the cost of capital or the IRR on another project) to make inconsistent assumptions concerning the use of the capital in question.

Footnotes

*Michael L. Lawrence is currently on one-year leave from University of Missouri and is Manager of Profit Planning and Analysis at Spector Industries.

[1]For example, see Van Horne [9, p. 63].

[2]For example, see Porterfield [7, p. 37].

[3]When computing the IRR by solving for the discount rate, d=IRR, the above equation shows that the reinvestment rate, r, must also equal IRR in order to use the standard formulation. However, when computing NPV, d (and thus r) would not equal IRR, but would ordinarily be set equal to the cost of capital rate or other discount rate. This reinvestment rate discrepancy can clearly lead to conflicting ranking decisions for the NPV and IRR techniques.

This inconsistency can be reconciled by specifying a common reinvestment rate in the general formulation. For a more detailed discussion of this problem, see Carlson, Lawrence and Wort [1].

[4]For example, see Hirshleifer [4, p. 95].

[5]A number of problems in capital analysis have been ignored in our treatment. These include: capital rationing, investments with unconventional cash flow patterns, the presence of a rapidly rising marginal cost of capital curve, and the possibility of multiple rates of return. The reader is referred to Mao [6] and Weston and Brigham [10] for detailed treatments of these problems.

References

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