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THE SEARCH FOR THE A_1 MESON *

H.E. HABER and G.L. KANE
Physics Department, University of Michigan, Ann Arbor, Michigan 48109

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We review the problem of the apparent experimental absence of the A_1 meson. Although it is required to exist, with rather definite properties, by both the quark model and chiral symmetry, it has not been found. We give a unified set of cross-section predictions for its production in forward and backward non-diffractive πN and KN reactions, $e^+e^- \to \tau^+\tau^- \to \mu^\pm A_1^\mp$ + neutrals, and $\nu N \to \mu + A_1 + X$. Most predictions are comparable to or exceed current sensitivity limits, so a signal could have been seen. Recent indirect evidence about the A_1, such as the relevance of other axial vector particles, is discussed. In particular recent measurements with polarized beam and target imply that A_1 reggeon exchange in pp → pp does occur with the strength expected from meson dominance of the axial vector weak current. All our results are summarized in table 1.

1. Introduction

Both the quark model and chiral symmetry provide compelling reasons why an A_1 meson should exist **. Nevertheless, it has not yet been discovered. If the most straightforward predictions for production cross sections had correctly described the A_1 properties, it would already have been seen in existing data [2,3].

In any quark model, $\bar{q}q$ states with orbital angular momentum $L = 0$ give π (spin singlet) and ρ (spin triplet). With $L = 1$ one has B (spin singlet, $J^{PC} = 1^{+-}$), and triplet states $A_2(2^{++})$, $A_1(1^{++})$, and $\epsilon(0^{++})$. Since B and A_2 are well established, and s-wave resonances and strong scattering occur, it is hard to imagine how the A_1 could not appear. With the spin-spin and spin-orbit mass splittings typical of quark model meson and baryon spectra, one expects m_{A1} not large and relations between the A_2, B, and A_1 widths. For example, if the scalar meson states below the A_2 in mass are connected with the 0^+ quark model states, or if the spin-spin splitting is related to that for ρ-π, then $m_{A1} \lesssim m_{A2}$.

Chiral symmetry suggests that vector and axial vector currents will behave in similar ways. There is evidence (see sect. 3.1) that for coupling strengths to baryons this occurs. The isovector vector current is seen to couple to the ρ meson. Similarly,

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** See ref. [1] for more detail.

the axial current should require a discrete A_1 state. Current algebra arguments give predictions for A_1 mass and width, and $\rho\pi$, $N\bar{N}$, and $N\bar{\Delta}$ couplings. Regardless of the details, an A_1 state should be seen.

Historically, a signal was seen in $\pi^{\pm}N \to (3\pi)^{\pm}N$ diffractive reactions. Then the three-body phase-shift analysis of Ascoli and collaborators [4] was carried out, and it was found that the phase shift did not have a resonance variation in the 1^+ partial wave. Of course, when the large Deck effect background is present [5] in 1^+, it is quite difficult to show no resonance is present [6]; but diffractive reactions cannot be used to establish a resonance unless a clear phase variation happens to be present. The moral is that $\pi^{\pm}N \to A_1^{\pm}N$ data simply is not able to help decide whether or not there is an A_1 state.

Non-diffractive reactions, on the other hand, can provide a clean test [7]. Basically, although there is still a (charge exchange) Deck effect, it is small enough so the resonant signal should dominate [1]. Then no matter what the form of the amplitude or the phase variation, the signal cannot be hidden. If the $A_1\rho\pi$ vertex or the $A_1N\bar{N}$ vertex exists, the A_1 *must* be produced in $\pi^+p \to A_1^0\Delta^{++}$ by ρ exchange, or in $\pi^+p \to pA_1^+$ by N exchange. Present-day two-body reaction technology is good enough to estimate fairly accurately (say better than a factor of two) the expected cross sections. We give estimates for a number of forward and backward two-body reactions, and a detailed discussion of our assumptions and techniques, in sects. 2.1–2.3.

The recent discovery and confirmation of what appears likely to be a heavy lepton [8,9] (denoted by τ) allows another approach to finding an A_1, as discussed in detail by a number of authors [10] some years ago. We give numerical values in sect. 2.4 for these results which suggest that this is a good place to find an A_1. Basically, one should find

$$e^+e^- \to \tau^+\tau^- \to A_1^-\nu$$
$$\quad\;\; \hookrightarrow \mu^+\nu\bar{\nu} \;\; \hookrightarrow \pi^-\pi^+\pi^-\,,$$

which is not too hard to identify experimentally. Approximately one A_1 event should be found for every three μe events.

Another way to produce an A_1 is in ν reactions, diffractively [11,12]. In essence, the axial vector current materializes into an A_1. Since this is known to occur for the electromagnetic current in $ep \to e\rho^0 p$, it is very hard to imagine that $\nu p \to \mu^- A_1^+ p$ would not occur, with a calculable cross section. We summarize existing predictions in sect. 2.5. When the analysis of data already in hand is completed the statistics should suffice to show a signal if one is present.

There are two kinds of experimental results which are indirectly relevant to the A_1 problem. The first comes from hadron reactions where an exchange with the quantum numbers of the A_1 can be isolated, a kind of hadronic axial-vector current. There are two places where such evidence appears to exist, in $\pi N \to \rho N$ with polarized target [13], and in $pp \to pp$ with polarized beam and target [14]. The latter allows a particularly clean test. We find that the recently measured total cross-section difference, $\Delta\sigma_L = \sigma_T(\frac{1}{2}, -\frac{1}{2}) - \sigma_T(\frac{1}{2}, \frac{1}{2})$ [15] for longitudinally polarized protons [14],

Table 1a
A_1 results

Reaction	σ_{tot} (theory)	σ_{tot} (exp.)	Comment
Forward			
$\pi^+ p \to A_1^0 \Delta^{++}$ P_{lab} = 7 GeV/c	(i) 1.7 (ii) 16 (iii) 72	<2 [22]	The theory numbers are the ρ-exchange contributions only; we expect the complete results to be larger. Note that hypothesis (i) is a lower limit for σ_{tot} no matter what the $A_1\rho\pi$ couplings are for fixed $\Gamma(A_1 \to \rho\pi)$. At 15 GeV/c about 0.5 μb of Deck effect background is expected [1].
$\pi^+ p \to A_1^0 \Delta^{++}$ P_{lab} = 15 GeV/c	(i) 0.7 (ii) 6 (iii) 28	<0.5 [3] see fig. 1	
$\pi^- p \to A_1^0 n$ P_{lab} = 8.4 GeV/c	(i) 2 (ii) 16 (iii) 69	see fig. 2 [2]	
$K^- p \to A_1^0 \Lambda^0$ P_{lab} = 4.2 GeV/c	$\dfrac{\sigma(K^- p \to A_1^0 \Lambda^0)}{\sigma(\pi^- p \to A_1^0 n)} \approx 0.04$		
Backward			
$K^- p \to \Sigma^- A_1^+$ P_{lab} = 4.2 GeV/c	2.7	see ref. [31]	The theory numbers are normalized to $\sigma(\pi^- p \to p\pi^-)$. Note that if an A_1 signal is seen in any one of these three reactions, consistency requires that it be seen in the other two (see sect. 2.3).
$K^- p \to \Sigma^+ A_1^-$ P_{lab} = 4.2 GeV/c	2.7		
$\pi^- p \to p A_1^-$ P_{lab} = 4.0 GeV/c	3.9	<5 [29]	
Lepton beams			
$e^+e^- \to \tau^+\tau^- \to A_1^\pm \mu^\mp + \nu$'s	$\dfrac{\sigma(e^+e^- \to A_1^\pm \mu^\mp + \nu\text{'s})}{\sigma(e^+e^- \to e^\pm \mu^\mp + \nu\text{'s})} = 0.4$		First Weinberg sum rule used
νp	$\dfrac{\sigma(\nu p \to \mu^- A_1^+ p)}{\sigma_T(\nu p)} \gtrsim 10^{-3}$		See sect. 2.5 for $\bar{\nu}p$ and inclusive
Other			
$\Delta\sigma_L \equiv \sigma(\frac{1}{2}, -\frac{1}{2}) - \sigma(\frac{1}{2}, \frac{1}{2})$ $pp \to pp$ at P_{lab} = 6 GeV/c	0.9 mb	1.0 ± 0.1 mb [14]	See sect. 3.1

We present a summary of our calculations for $m_{A_1} = 1.1$ GeV/c^2 and $\Gamma(A_1 \to \rho\pi) = 200$ MeV. For other choices of widths, multiply the total cross section by Γ(MeV)/200. In the forward reactions we consider three hypotheses for the $A_1\rho\pi$ couplings:

(i) No D wave in $A_1 \to \rho\pi$;
(ii) broken $SU(6)_W$ result;
(iii) current algebra result.

All total cross sections are in units of μb.

has a magnitude given surprisingly accurately by an $A_1N\bar{N}$ coupling normalized to the strength of the axial vector weak current from neutron β-decay. This is what would be expected if an axial vector meson.dominated the isovector weak axial vector current just as the ρ dominates the electromagnetic and weak vector currents. This result (derived in sect. 3.1) both reinforces expectations for an A_1 to exist, and gives us confidence in our estimates of production in backward hadron reactions, which are normalized by the same coupling used in the estimates which work for $\Delta\sigma_{TL}$.

The second kind of indirect evidence comes from the existence of other axial vector states, both D(1285) and Q mesons which might be in an A_1 SU(3) multiplet [16], and the charmonium $L = 1$ state analogous to the A_1 [17]. The existence of the 1^+ charmonium state has helped to reaffirm the faith of any faint-hearted among

Table 1b
Summary of our calculations for $m_{A_1} = 1.3$ GeV/c^2 and $\Gamma(A_1 \to \rho\pi) = 200$ MeV (see comments made in table 1a)

Reaction	σ_{tot} (theory)
Forward	
$\pi^+p \to A_1^0\Delta^{++}$ $P_{lab} = 7$ GeV/c	(i) 0.8 (ii) 2.4 (iii) 23
$\pi^+p \to A_1^0\Delta^{++}$ $P_{lab} = 15$ GeV/c	(i) 0.3 (ii) 1.1 (iii) 10
$\pi^-p \to A_1^0 n$ $P_{lab} = 8.4$ GeV/c	(i) 0.9 (ii) 2.8 (iii) 26
Backward	
$K^-p \to \Sigma^-A_1^+$ $P_{lab} = 4.2$ GeV/c	0.5
$K^-p \to \Sigma^+A_1^-$ $P_{lab} = 4.2$ GeV/c	1.7
$\pi^-p \to pA_1^-$ $P_{lab} = 4.0$ GeV/c	3.4
Lepton beams	
$e^+e^- \to \tau^+\tau^- \to$ $A_1^\pm\mu^\mp + \nu$'s	$\dfrac{\sigma(e^+e^- \to A_1^\pm\mu^\mp + \nu\text{'s})}{\sigma(e^+e^- \to e^\pm\mu^\mp + \nu\text{'s})} = 0.3$

Table 1c
Summary of our calculations for $m_{A_1} = 1.5$ GeV/c^2 and $\Gamma(A_1 \to \rho\pi) = 200$ MeV (see comments made in table 1a)

Reaction	σ_{tot} (theory)
$\pi^+p \to A_1^0\Delta^{++}$ $P_{lab} = 7$ GeV/c	(i) 0.4 (ii) 0.9 (iii) 9
$\pi^+p \to A_1^0\Delta^{++}$ $P_{lab} = 15$ GeV/c	(i) 0.2 (ii) 0.5 (iii) 4.7
$\pi^-p \to A_1^0 n$ $P_{lab} = 8.4$ GeV/c	(i) 0.6 (ii) 1.2 (iii) 12
$K^-p \to \Sigma^-A_1^+$ $P_{lab} = 4.2$ GeV/c	0.4
$K^-p \to \Sigma^+A_1^-$ $P_{lab} = 4.2$ GeV/c	1.3
$\pi^-p \to pA_1^-$ $P_{lab} = 4.0$ GeV/c	3.3
Lepton beams	
$e^+e^- \to \tau^+\tau^- \to A_1^\pm\mu^\mp + \nu$'s	$\dfrac{\sigma(e^+e^- \to A_1^\pm\mu^\mp + \nu\text{'s})}{\sigma(e^+e^- \to e^\pm e^\mp + \nu\text{'s})} = 0.2$

us who might have doubted the existence of the A_1; witness the admonition [17]: "Don't let anyone tell you there is no A_1 when its $\bar{c}c$ analogue $\chi(3508)$ exists." We agree that this is the proper attitude with which to view the A_1 problem. However, the argument is not compelling. For example, the charmonium states have isospin zero, while the A_1 is an isovector. Further, arguments have been given [18] that SU(3) breaking could take the form of missing members of multiplets. An approach based on the Q and D (mixed with E) [16] puts the A_1 mass at about 1470 MeV and its width over 300 MeV; if such an A_1 were found it would be almost as uncomfortable for serious quantitative predictions of the quark model and chiral symmetry as if no A_1 were found. We discuss these points in more detail in sect. 3.2. Basically, we feel that the existence of other axial vector states is probably not relevant to the existence of the A_1. However, if a situation with apparently magic mixing and an A_1 predicted by mass formulas to be at the chiral symmetry position and width should occur, then it would strengthen the argument for the A_1 to exist.

If the A_1 is not found it is important to know when we can be confident that it is missing. Our predicted numbers depend mainly on coupling constants used to set scales. These coupling constants are determined by symmetry arguments and by comparison with other cross sections. We give, in appendices, detailed descriptions of how we obtain our couplings and of various consistency requirements on them.

Our only conclusion is that it should be possible, through the combined effect of experiments now in progress or recently completed, either to find the A_1 or to be confident that the A_1 needed by the theories is absent. At present it appears that a good A_1 signal should have already been seen if it existed, but soon the experimental situation will become much more compelling. If there is no A_1 state, we do not know whether something complicated and hadronic is happening, such as unitarization effects of insufficient strength [1,18] or whether there will be a basic modification required in our understanding of quark models and chiral symmetry.

Our results are summarized in table 1; the rest of the paper explains how table 1 is obtained.

2. Production cross sections for an A_1 meson

We have every reason to expect the A_1 to be produced in various reactions. The major question before us is whether the lack of experimental confirmation of the A_1 is cause for alarm. To answer this question, we must be able to present reliable estimates of cross sections for the production of the A_1. The procedure required to obtain these estimates is rather involved and is discussed thoroughly in this section. Before getting into the details, we would like to present a brief overview. The key to estimating cross sections is to obtain reliable coupling constants. Then, if one takes ratios with known cross sections, one minimizes the error resulting in an incomplete knowledge of the form of the strong interaction amplitudes. As an illustration, consider the relative production of A_1 and A_2 in $\pi^+ p \to \Delta^{++}(A_1 \text{ or } A_2)$ at $p_{\text{lab}} = 15$ GeV/c. Experimentally [3], $\sigma(A_1)/\sigma(A_2) < 0.03$. If we believe that the A_1 will be produced in this reaction, say, by ρ exchange, then we obtain severe constraints on the $A_1\rho\pi$ coupling. However, these constraints are incompatible with the couplings predicted by the standard symmetry arguments. In this section, we intend to show how compatible the "theoretical A_1" is with present data and what experimental sensitivities are required if a definite conclusion is to be made on the existence of the A_1.

2.1. *Forward non-diffractive reactions*

For reasons stated in the introduction, it is crucial to study non-diffractive reactions to ascertain whether an A_1 is being produced in meson-baryon collisions. We now examine the case of small angle scattering of mesons off baryons to see at what levels the A_1 should be produced.

It is essential to have a reliable model which can predict a large number of different meson-baryon scattering cross sections. However, we wish to keep our amplitudes as simple as possible so that we can understand clearly what factors are causing the predicted cross sections to be of a certain size. We restrict ourselves to a straightforward procedure which can predict the density matrix elements and differential cross section of many types of reactions with few free parameters. We ignore subtleties such as amplitude phases so that we do not expect to calculate correctly quantities which crucially depend on these phases. Our goal is to end up with a procedure that can predict correctly to (say) within a factor of two: the total cross section of a given channel; approximately predict the main features of the differential cross section for $|t'| < 1\ (\mathrm{GeV}/c)^2$; and give a rough estimate of the size of those density matrix elements not sensitive to the phases of the amplitude.

We base our amplitudes on a reggeized absorption model [19]. The phenomeno-

Table 2
Strong interaction vertices

$\rho^0\pi^+\pi^+$	$g_{\rho\pi\pi}(p+p')_\mu$
$\rho^0\pi^+A_1^+$	$m_A[g_s^A\delta_{\mu\nu} + \frac{1}{4}(g_d^A/m_{A_1}^2)(p+q)_\mu(p+q)_\nu]\epsilon_\nu^*(p')$
$\omega^0\pi^+B^+$	$m_B[g_s^B\delta_{\mu\nu} + \frac{1}{4}(g_d^B/m_B^2)(p+q)_\mu(p+q)_\nu]\epsilon_\nu^*(p')$
$\rho^0\pi^+A_2^+$	$(g_{A_2\rho\pi}/m_{A_2}^2)\,\epsilon_{\nu\mu\alpha\beta}p_\alpha q_\beta \varphi_{\nu\rho}^*(p')p_\rho$
$\left\{\begin{matrix}\rho^0 pp\\ \omega^0 pp\end{matrix}\right\}$	$i\bar{u}(p')[G_V\gamma_\mu + (G_T/2m_N)\sigma_{\mu\nu}q_\nu]u(p)$
$A_1^0 pp$	$ig_{A_1NN}\bar{u}(p')\gamma_\mu\gamma_5 u(p)$
$\pi^0 pp$	$g_{\pi NN}\bar{u}(p')\gamma_5 u(p)$
$\rho^- p\Delta^{++}$	$G_{\rho N\Delta}\bar{U}_\mu(p')[\delta_{\mu\nu} - (iq_\mu\gamma_\nu/(m_N + m_\Delta))]\gamma_5 u(p)$
$A_1^- p\Delta^{++}$	$G_{A_1N\Delta}\bar{U}_\mu(p')\delta_{\mu\nu}u(p)$
$\pi^- p\Delta^{++}$	$[G_{\pi N\Delta}/(m_N + m_\Delta)]\,\bar{U}_\mu(p')q_\mu u(p)$

We obey the following conventions: Vertex ABC consists of particles of momenta q, p, and p' respectively, where $q = p - p'$ (p is incoming, p' is outgoing). The coupling constants are dimensionless and are defined for the charge states specified. (For other charge states, one needs to use the appropriate Clebsch-Gordan coefficient). The Lorentz property of the vertex depends on the spin of A. To obtain a Lorentz scalar, multiply the vertex by the appropriate spin polarization tensor for particle A. All coupling constants are real as determined by time reversal invariance [32].

logical amplitudes and our basic procedure are summarized in appendix B. Note that the amplitudes are simple enough so that observables can be calculated with an electronic hand calculator and a table of Bessel functions. We remark that since we are only concerned with non-diffractive reactions, the complications of the pomeron do not enter. The key to our procedure, then, is as follows. Once we are satisfied with the way our amplitudes work in the production of known resonances, we need only to know the value of A_1 coupling constants to predict the expected cross sections of A_1 production.

We now briefly describe the reactions we have studied to obtain the parameters listed in tables 3 and 4. The first step is to study the reactions $\pi^+ p \to \pi^0 \Delta^{++}$ and $\pi^- p \to \pi^0 n$ (for $4 \leqslant P_{LAB} \leqslant 15$ GeV/c) which proceed by ρ exchange only. This allows us to fix the parameters s_0, R_e, and B in our amplitudes [see eq. (B.4)]. Con-

Table 3
Coupling constants

Constant	Value	Where obtained
f_π	0.133 GeV	$\pi \to \mu\nu$ decay
f_ρ	0.147 GeV^2	eq. (A.2) and $\rho \to e^+e^-$ decay
f_A	see table 5	
$g_{\rho\pi\pi}$	6	$\Gamma(\rho \to \pi\pi)$
$g_{A_2\rho\pi}$	34	$\Gamma(A_2 \to \rho\pi)$
g_s^B	3	{ $\Gamma(B \to \omega\pi)$ and the angular distribution of $\pi^+p \to B^+p$
g_d^B	−18	
g_s^A	}see table 5	
g_d^A		
G_V^ρ	1.6	{ either eq. (C.2) or $\pi^-p \to \pi^0 n$
G_T^ρ	11	
G_V^ω	13	{ either vector dominance, $\pi^+p \to B^+p$ or backwards ω production
G_T^ω	−2	
$g_{\pi NN}$	13.5	πN dispersion relations
g_{A_1NN}	see table 5	
$G_{\pi N\Delta}$	33	$\Gamma(\Delta \to N\pi)$
$G_{\rho N\Delta}$	27	either eq. (C.4) or from $\pi^+p \to \pi^0\Delta^{++}$
$G_{A_1N\Delta}$	see table 5	

All coupling constants are dimensionless with the exception of f_x (x = π, ρ, A_1)

Table 4
Forward and backward scattering parameters

Exchange	R_e	B	α_0	α'_0	s_0	r_e
ρ	5.3	1.5–2.0	0.5–0.56	0.9	1.0	
ω	5.3	1.5–2.0	0.4	1.1	1.0	
A_1			0.0		1.0	
K*	5.3	1.5–2.0	0.35	0.9	1.5	
Δ	4.4	0.7	0.05	0.9	0.95	0.11
N	5.5	0.3	−0.39	1.0	0.28	1.37

These parameters are assumed to be approximately valid in reactions where $4 \leqslant P_{lab} \leqslant 15$ GeV/c. In some cases a range of values are given; which either is attributed to different external particles or an energy dependence. As an example, the effective ρ intercept α_0 decreases slowly as the energy increases. All quantities are in units of GeV.

sider the coupling constants involved. In $\pi^- p \to \pi^0 n$; $g_{\rho\pi\pi}$, G_V, and G_T enter (see table 2). We get the value of $g_{\rho\pi\pi}$ from the decay width of the ρ: $\Gamma(\rho \to \pi\pi) = g^2_{\rho\pi\pi} q^3/6\pi m^2_\rho$ (q is the magnitude of the three-momentum of the pion in the ρ-rest frame). Estimates of G_V and G_T are obtained by vector dominance arguments (see appendix C). To settle on the exact numbers to use, we make use of the shape of $d\sigma(\pi^- p \to \pi^0 n)/dt$ which leads to the results in table 3. At this point we note that the coupling constants obtained in eq. (C.2) are fairly consistent with the numbers allowed by the data. Next, in considering $\pi^+ p \to \pi^0 \Delta^{++}$, we use $g_{\rho\pi\pi}$ obtained above and we only need to know $G_{\rho N\Delta}$. We may obtain this by either data fitting or by a vector dominance argument in appendix C. These two methods give consistent values.

Next, we consider $\pi^+ p \to B^+ p$. Both ω and A_2 exchange are allowed. For our purposes here, we neglect the A_2 exchange. Relevant data at present suggests that this is not a bad approximation (for a contrary opinion, see ref. [20]). The ω coupling constants to nucleons are obtained the same way as the ρ couplings. However, this time there are two independent couplings at the $\pi\omega B$ vertex, g^B_s and g^B_d. The width of $B \to \omega\pi$ adds one constraint. The shape of $d\sigma/dt$ $d\sigma(\pi^+ p \to B^+ p)/dt$ adds a second constraint which gives us g_s and g_d listed in table 3. Note that by studying B decay carefully to discover the ratio of S to D wave emission of the π, one gets numbers in fair agreement with the numbers we have obtained [21].

Finally, let us look at $\pi^- p \to A^0_2 n$ and $\pi^+ p \to A^0_2 \Delta^{++}$. There is better data on the latter [3,22] so we concentrate on that reaction. Both ρ and B exchange contribute; it is known that the B exchange is not negligible [23]. Let us calculate the ρ-exchange contribution to A_2 production. Note that the only coupling constant needed is $g_{A_2\rho\pi}$; this is obtained from the partial width of the A_2: $\Gamma(A_2 \to \rho\pi) = g^2_{A_2\rho\pi} q^5/20\pi m^4_{A_2}$.
Since we expect to find both an A_1 and A_2 in a $\rho\pi$ mass plot, it is useful to discuss our A_1 results as the ratio of A_1 and A_2 production in a given reaction. So, we were

Table 5
A_1 couplings

m_{A_1}	f_A	g_{A_1NN}	$G_{A_1N\Delta}$		g_s^A	g_d^A
1.1	0.15	6.9	16.3	(i)	3.2	−4.4
				(ii)	3.0	7.0
				(iii)	2.8	22.3
1.3	0.18	8.1	19.2	(i)	2.5	−3.9
				(ii)	2.4	0.3
				(iii)	1.9	15.1
1.5	0.21	9.3	22.1	(i)	2.2	−3.8
				(ii)	2.1	−1.3
				(iii)	1.4	11.3

f_A is calculated from the first Weinberg sum rule [eq. (A.3)]. g_{A_1NN} and $G_{A_1N\Delta}$ are obtained from eqs. (C.7) and (C.15) respectively.

The $A_1\rho\pi$ couplings g_s and g_d assume $\Gamma(A_1 \to \rho\pi) = 200$ MeV. For fixed g_s/g_d, they are proportional to $\sqrt{\Gamma}$. The following three hypotheses are considered:

(i) No D wave in $A_1 \to \rho\pi$;
(ii) broken $SU(6)_W$ result;
(iii) current algebra result.

m_{A_1} and f_A are in units of GeV; all other coupling constants are dimensionless.

careful in checking our predictions for A_2 production in order that we would have confidence in our A_1 results.

There are experimental results for $\pi^- p \to \Delta^{++} A_2^0$ at 7 [22] and 15 GeV/c [3]. As discussed in ref. [19], the effective intercept decreases with increasing energy, so we use $\alpha_0 = 0.5$ at 15 GeV/c. It is worth noting that the A_1 predictions depend mainly on amplitudes with net helicity flip 0 or 1, whose behavior under absorption is well checked, while the A_2 is sensitive to $(n, x) = (0, 2)$ and $(2, 0)$. The A_2 amplitudes are not so common in other reactions and could be less reliable. So the A_1 predictions should be less model-dependent than those for the A_2.

At this point, we have obtained a set of parameters which can describe ρ-exchange of a large class of non-diffractive reactions. Sample results are displayed in table 6. We now turn to the remaining ingredient needed to predict A_1 production: The $A_1\rho\pi$ coupling. There are two independent couplings: g_s^A and g_d^A (see table 2). Consider the A_1 decay and choose a coordinate system such that the A_1 at rest has helicity λ' and decays into $\rho\pi$. The ρ is emitted at angle θ to the z-direction with helicity λ. Then the helicity amplitude for the decay is

$$M_{\lambda'\lambda} = m_{A_1} F_\lambda d^1_{\lambda\lambda'}(\theta), \tag{1}$$

Table 6
Results for other meson resonances

Reaction	σ_{tot} (theory)	σ_{tot} (exp.)	Comment
Forward			
$\pi^+ p \to A_2^0 \Delta^{++}$ $P_{lab} = 7$ GeV/c	25 (ρ exchange)	76 [22]	We remind the reader that we give only the ρ-exchange contribution in the theory column. It is known that B exchange is important in A_2 production [23]. All A_2 cross sections are corrected for unseen decay modes [28]. See sect. B.3
$\pi^+ p \to A_2^0 \Delta^{++}$ $P_{lab} = 15$ GeV/c	11 (ρ exchange)	16 [3]	
$\pi^- p \to A_2^0 n$ $P_{lab} = 8.4$ GeV/c	27 (ρ exchange)	see fig. 2	
$K^- p \to B^- \Sigma^+$ $P_{lab} = 4.2$ GeV/c	5 (K^* exchange)	14 [33]	
Backward			
$\pi^- p \to p \rho^-$ $P_{lab} = 4.0$ GeV/c	7	8 [29]	Normalized to $\pi^- p \to p\pi^-$ cross section
Lepton beams			
$e^+e^- \to \tau^+\tau^- \to \pi^\pm \mu^\mp + \nu$'s	$\dfrac{\sigma(e^+e^- \to \pi^\pm\mu^\mp + \nu\text{'s})}{\sigma(e^+e^- \to e^\pm\mu^\mp + \nu\text{'s})} = 0.4$		
$e^+e^- \to \tau^+\tau^- \to \rho^\pm \mu^\mp + \nu$'s	$\dfrac{\sigma(e^+e^- \to \rho^\pm\mu^\mp + \nu\text{'s})}{\sigma(e^+e^- \to e^\pm\mu^\mp + \nu\text{'s})} = 1.1$		

We have calculated these numbers to serve as comparison with the results of table 1. Cross sections are in units of μb.

where

$$F_1 = g_s^A ,$$

$$F_0 = \frac{g_s^A m_{A_1} (q^2 + m_\rho^2)^{1/2} + \frac{1}{2} g_d^A q^2}{m_{A_1} m_\rho} .$$

Parity invariance requires $F_\lambda = F_{-\lambda}$. The S and D wave amplitudes are related to F_λ by:

$$D = \tfrac{1}{3}(F_1 - F_0) ,$$

$$S = \tfrac{1}{3}(F_0 + 2F_1) . \tag{2}$$

Hence, no D wave corresponds to $F_0 = F_1$. The width of the A_1 is easily calculated from eq. (1) to be $\Gamma(A_1 \to \rho\pi) = q(2F_1^2 + F_0^2)/12\pi$, where q is the magnitude of the three-momentum of the ρ in the rest frame of the A_1. If we fix Γ, then we have to specify one more relation between g_s and g_d.

First, we consider the current algebra prediction (see appendix C). Suppose we use eqs. (C.16) and (C.17) to get g_s^A and g_d^A. If we pick $\delta = 0$, (so that $g_{\rho\pi\pi}$ given by eq. (C.18) is reasonable) we find $\Gamma(A_1 \to \rho\pi) = 50$ MeV. This corresponds to $g_s^A/g_d^A = \frac{1}{8}$. The quark model can also lead to a constraint on g_s^A and g_d^A. Colglazier and Rosner [24] have used broken $SU(6)_W$ to obtain:

$$2(F_1/F_0)_{A_1\to\rho\pi} = (F_0/F_1)_{B\to\omega\pi} + 1. \tag{3}$$

F_0/F_1 for B decay can be obtained by using our numbers for g_s^B and g_d^B. We get $(F_0/F_1)_B = 0.75$; for $m_{A_1} = 1.1$ GeV$/c^2$, this gives $g_s^A/g_d^A = 0.41$. The measurements of $(F_0/F_1)_B$ obtained from B decay have varied from 0.5 to 0.75 with large errors [25]. We only note that $(F_0/F_1)_B = 0.5$ corresponds to $g_s^A/g_d^A = 0.13$ (which is the current algebra result with $\delta = 0$).

We therefore proceed as follows: we calculate A_1 production assuming $\Gamma(A_1 \to \rho\pi) = 200$ MeV for three hypotheses of g_s^A/g_d^A:

(i) No D wave emission of the π in A_1 decay;
(ii) Broken $SU(6)_W$ result [eq. (3)];
(iii) $g_s^A/g_d^A = \frac{1}{8}$ (current algebra result).

We list the A_1 couplings for these three hypotheses (and various possible A_1 masses) in table 5.

We have included hypothesis (i) because if one fixes Γ and varies g_s/g_d, one essentially minimizes the cross section when g_s/g_d corresponds to no-D-wave. One can therefore take the results of hypothesis (i) as an absolute lower bound for A_1 production.

The results for A_1 production are displayed in table 1a (for $m_{A_1} = 1.1$ GeV) along with the quoted experimental upper limits, if any. Let us consider as an example $\pi^+p \to A_1^0\Delta^{++}$ at 15 GeV$/c$. The upper limit quoted for σ_T is 0.5 μb at $m_{A_1} = 1.1$ GeV$/c^2$. We note in passing that the charge-exchange Deck effect would give a cross section on the order of 0.5 μb [1]. On the other hand, our calculations (displayed in table 1a) show that for "reasonable" values of $\Gamma(A_1 \to \rho\pi)$, e.g. current algebra (with $\Gamma = 50$ MeV) or broken $SU(6)_W$ (with $\Gamma \equiv 200$ MeV), predict an A_1 signal over ten times larger than is seen. If one throws away these expectations and assumes that the A_1 decays with no D wave into $\rho\pi$, then one may just barely avoid significant disagreement with experiment. Notice that one should take the no-D-wave predicted cross section as an absolute lower bound. So, if the upper limit for A_1 production were reduced even more, then we would have a contradiction independent of the g_s^A/g_d^A ratio (for a fixed width). Of course, one might argue that $\Gamma(A_1 \to \rho\pi) \lesssim 20$ MeV but why this should be so would be another puzzle.

Suppose that we agree that the A_1 exists with no D wave decay into $\rho\pi$. Let us see what this implies for the current algebra results. Putting in $g_s^A/g_d^A = -0.72$, and assuming that $m_{A_1} \approx \sqrt{2}m_\rho$ and $\Gamma(A_1 \to \rho\pi) = 200$ MeV, we obtain [using eqs. (C.16) and (C.17)] $\delta = -0.35$. [Note that although δ is fixed in ref. [26], Weinberg [27] obtains eqs. (C.16) and (C.17) without any restrictions on δ. We take this attitude here.] Inserting into eq. (C.18), one obtains $g_{\rho\pi\pi} = 3.8$ which leads to $\Gamma(\rho \to \pi\pi) = 60$ MeV. This is to be compared with 150 MeV in the particle data tables [28].

Before concluding this section, we should make some remarks about the A_1 at other masses. A standard hypothesis, once people began to imagine $m_{A_1} \neq \sqrt{2}m_\rho$ was that maybe $m_{A_1} \approx m_{A_2}$ [6]. This would then hide the A_1 under the more copiously produced A_2. What is needed here, of course, is sufficiently high statistics to separate out the two effects. If one works at separating out the partial wave contributions in the 3π system, then the 1^+ contamination of the A_2 mass can be separated. We present [3] fig. 1 as an illustration. If a partial wave analysis is not possible, there are other ways to determine whether there is an A_1 hiding under an A_2. For example, we have no reason to assume that the ratio of unnatural parity exchange (UPE) to natural parity exchange (NPE) is the same in A_1 and A_2 production. Since UPE will die out faster than NPE as the energy increases, one should compare the ratio of UPE/NPE as a function of energy at the A_2 mass in $\eta\pi$ and $\rho\pi$. No A_1 is possible in $\eta\pi$ so that a difference would suggest an unnatural parity object in $\rho\pi$ at the A_2 mass. Such a technique could also be used by studying the $\rho\pi$ final state alone. Since A_1 and A_2 production will have different t'-distributions, one can investigate UPE/NPE ratios as a function of energy, cutting on intervals of t'.

Masses for the A_1 as high as 1.5 GeV/c^2 [6,15] have been proposed. We repeated our calculations for $m_{A_1} = 1.3$ and 1.5 GeV/c^2 and display the results in table 1b and 1c.

Fig. 1 [3] has shown the absence of A_1 in $\pi^+p \to A_1^0\Delta^{++}$, with a partial wave analysis employed to separate out the 1^+ partial wave. One expects about $\frac{1}{2}\mu$b of charge-exchange Deck effect which should be subtracted. Over a wide mass range, no signal is seen.

Fig. 2 shows one recent preliminary result [2], which will soon be available in

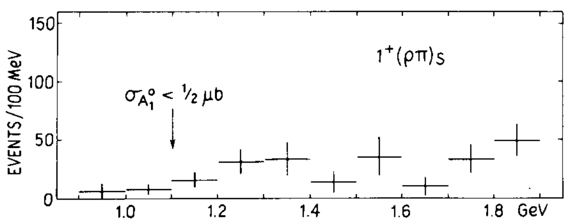

Fig. 1. The 1^+ partial wave of the $\rho\pi$ system in $\pi^+p \to \rho^\pm\pi^\mp\Delta^{++}$ at $p_{lab} = 15$ GeV/c, from ref. [3]. No resonant signal is seen. See table 1a and sect. 2.1.

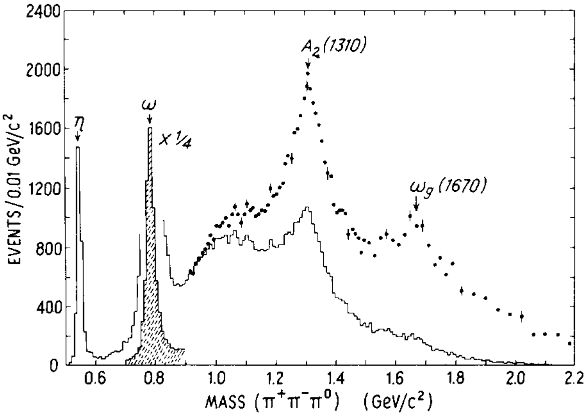

Fig. 2. $\pi^+\pi^-\pi^0$ mass spectrum in $\pi^-p \to \pi^+\pi^-\pi^0 n$ at 8.4 GeV/c, with acceptance corrected events indicated by large dots (arbitrarily normalized at 0.9 GeV/c^2), from ref. [2]. The ω-signal corresponds to 40 μb. We estimate the A_2 signal to be 30 μb. If one takes $m_{A_1} = 1.1$ GeV/c^2, then we estimate an upper limit allowed by the data is $\sigma(A_1)/\sigma(A_2) < \frac{1}{4}$. See table 1a and sect. 2.1.

final form, for $\pi^-p \to A_1^0 n$. Again, no signal is seen. We predict that about $\frac{1}{2}$ of the A_2 peak is due to ρ exchange, and that $\sigma(A_1)/\sigma(A_2) \simeq 0.6$ (for hypothesis (ii) so an A_1 peak of about 0.3 of the A_2 peak was expected.

2.2. Backward reactions

All the logic used in sect. 2.1 can be repeated in the case of backward A_1 production. The amplitude is more complicated as discussed in appendix B, but the procedure is the same. The key observation is that coupling constants obtained in the analysis of forward reactions can be used in the backward direction analysis. We begin by studying πN backward scattering. Since $g_{\pi NN}$ and $G_{\pi N\Delta}$ are known, we can obtain all other parameters by fitting to the three reactions $\pi^{\pm}p \to p\pi^{\pm}$, $\pi^-p \to n\pi^0$. By making some simplifying assumptions (see appendix B), we can immediately calculate cross sections for the production of resonances. Unfortunately, the data of resonance production in the backward direction is less precise than in the forward production. At this point, since we know the ρ and ω coupling constants best, we consider $\pi^-p \to p\rho^-$, $\pi^-p \to n\rho^0$ and $\pi^-p \to n\omega^0$. Having reproduced cross sections to within a factor of two, we calculate A_1 production in the backwards direction. Note that we have used A_1NN and A_1NΔ couplings obtained from current algebra calculations (appendix C) which are displayed in table 5.

Experimental results to date [29] are not quite sensitive enough for any claims of non-existence to be made. But, we note that experiments are not far from the required sensitivity. Backwards scattering experiments are notoriously difficult, but we would like to reinforce the notion that they are a useful way to study the existence of resonant states. Some effort to push the current limits by a factor of five or so would be well worthwhile.

2.3. Production of A_1 in K^- beams

We briefly look at the possibility of A_1 production in K beams. If we consider forward reactions, we see that hypercharge exchange is required to produce non-strange meson resonances. As an example, we compare the K* exchange contribution to $K^- p \to A_1^0 \Lambda^0$ with the ρ exchange contribution to $\pi^- p \to A_1^0 n$. We use SU(3) symmetry to relate the couplings * and a K* Regge intercept of 0.35. Both the couplings and the lower Regge intercept lead to decreasing $\sigma(K^- p \to A_1^0 \Lambda^0)$ with respect to $\sigma(\pi^- p \to A_1^0 n)$; putting in the kinematics, we find, e.g. a ratio of 0.04 at $P_{lab} = 4.2$ GeV/c.

Let us consider backwards production. Here, one still has N and Δ exchange, and it turns out that one has as good a chance to see A_1^0 production in backward $K^- p$ scattering as in backward πN scattering. To see this, first consider the comparison of $\pi^\pm p \to p\pi^\pm$ and $K^- p \to \Sigma^\pm \pi^\mp$. Using eq. (B.10), we see that exact SU(3) symmetry implies that $\sigma(\pi^- p \to p\pi^-) = \sigma(K^- p \to \Sigma^+ \pi^-)$ which is approximately satisfied by the data [30]. In the reactions where N exchange is allowed, SU(3) symmetry gives relations which depend on the D/F ratio. We can use data from these reactions to fix this ratio as discussed at the end of appendix B. Although the angular distributions are different, integrating over the backwards peak, we find $\sigma(K^- p \to \Sigma^- \pi^+) \approx \sigma(K^- p \to \Sigma^+ \pi^-)$ at 4 GeV/c [30]. We now apply factorization in regard to A_1 production. This immediately leads to $\sigma(K^- p \to \Sigma^- A_1^+) \approx \sigma(K^- p \to \Sigma^+ A_1^-) \approx \sigma(\pi^- p \to pA_1^-)$ at 4 GeV/c. When we put in the kinematics, we obtain the results listed in table 1. Preliminary results by the Amsterdam-CERN-Nijmegen-Oxford Collaboration [31] report an A_1 signal in $K^- p \to \Sigma^- A_1^+$. An important consistency check on this result is the detection of an A_1 signal at the same level in $K^- p \to \Sigma^+ A_1^-$ and $\pi^- p \to pA_1^-$.

2.4. $e^+e^- \to \tau^+\tau^-$

There is now fairly good evidence for the existence of a lepton τ with mass $m_\tau \approx 1.95$ GeV/c^2 [9]. This gives us a unique opportunity to test current algebra predictions by studying the decays $\tau \to$ meson + ν_τ. In particular, one can compare the two decays $\tau^\pm \to \pi^\pm \nu_\tau$ and $\tau^\pm \to A_1^\pm \nu_\tau$; if at the same time $\tau^\mp$ decays leptonically, then the signal for $\tau^\pm$ decay should be fairly clean. We regard this as a viable test for the existence of the A_1 since the signal is easily seen by studying $e^+e^- \to \pi^+\pi^-\pi^-\mu^+$ + neutrals. The predictions of these decays have been worked out by a number of authors [10].

* The two K*NΛ couplings, G_V and G_T, have different D/F ratios. SU(6) suggests that G_V is pure F, while $G_V + G_T$ has $D/F = \frac{3}{2}$. We use these for our calculations.

Let us review some of the results obtained. First, the cross section for $e^+e^- \to \tau^+\tau^-$ at c.m. energy $\sqrt{s}$ is (in the one γ approximation):

$$\sigma(s) = \frac{4\pi\alpha^2}{3s}\left(1 + \frac{2m_\tau^2}{s}\right)\left(1 - \frac{4m_\tau^2}{s}\right)^{1/2}, \tag{4}$$

where we have neglected the mass of the electron. Let us then consider the branching ratios of the decays of τ. Neglecting the mass of the muon,

$$\Gamma(\tau^+ \to \mu^+\bar{\nu}_\tau\nu_\mu) = \frac{G^2 m_\tau^5}{192\pi^3} \approx 10^{12}\ \mathrm{sec}^{-1}. \tag{5}$$

We also need to know the total decay rate of τ. This can be estimated quickly by arguing that the total decay rate for τ is n times $\Gamma(\tau^+ \to \mu^+\bar{\nu}_\tau\nu_\mu)$ where n is the number of fermion weak isospin doublets into which τ decays. Taking three colors into account, and noting that τ decay into a charmed hadron is negligible (due to phase space), we find that $n = 5$. Thus the total rate of τ is predicted to be $5 \times 10^{12}\ \mathrm{sec}^{-1}$; this estimate is confirmed by a more detailed calculation. Next, consider the result:

$$\Gamma(\tau^- \to \nu_\tau + A_1^-) = \frac{G^2 f_A^2 \cos^2\theta_c}{16\pi m_{A_1}^2}\ m_\tau^3\left(1 - \frac{m_{A_1}^2}{m_\tau^2}\right)^2\left(1 + \frac{2m_{A_1}^2}{m_\tau^2}\right). \tag{6}$$

If we take $m_{A_1} = 1.1\ \mathrm{GeV}/c^2$, then we find (using the numbers given in tables 3 and 5):

$$\Gamma(\tau^- \to \nu_\tau + A_1^-) \approx 0.4\,\Gamma(\tau^- \to \mu^-\nu_\tau\bar{\nu}_\mu). \tag{7}$$

Experimentally, $e^+e^- \to \tau^+\tau^-$ events are seen by looking for $e^+e^- \to e^\pm\mu^\mp$ + neutrals. Using the result that each leptonic decay mode of $\tau^\pm$ is one fifth its total decay rate, we obtain:

$$\frac{\sigma(e^+e^- \to e^\pm\mu^\mp + \mathrm{neutrals})}{\sigma(e^+e^- \to \tau^+\tau^-)} = 0.08. \tag{8}$$

Eqs. (7) and (8) allow us to conclude that approximately one out of every 30 $\tau^+\tau^-$ events will consist of an $A_1^\pm$ produced opposite a $\mu^\mp$.

It is crucial to see other mesons produced from a decaying τ in order to draw any conclusions if no A_1 is seen. For example,

$$\Gamma(\tau^- \to \nu_\tau + \pi^-) \approx 0.4\,\Gamma(\tau^- \to \mu^-\nu_\tau\bar{\nu}_\mu), \tag{9a}$$

$$\Gamma(\tau^- \to \nu_\tau + \rho^-) \approx 1.1\,\Gamma(\tau^- \to \mu^-\nu_\tau\bar{\nu}_\mu). \tag{9b}$$

Thus, if ρ and π are seen produced opposite a μ in e^+e^- collisions, then one must expect to see A_1 produced at an observable level.

We should remark that $\Gamma(\tau^- \to \nu_\tau + A_1^-)$ decreases as m_{A_1} is increased. But, e.g. $m_{A_1} = 1.5\ \mathrm{GeV}/c^2$ leads to $\Gamma(\tau^- \to \nu_\tau + A_1^-) \approx 0.2\,\Gamma(\tau^- \to \mu^-\nu_\tau\nu_\mu)$. We conclude that for "reasonable" A_1 masses we expect to see as many A_1's as π's (to within a factor of 2) from τ decays.

2.5. $\nu p \to \mu^- A_1^+ p$, $\mu^- A_1^+$ + anything

This process and other vector meson production has been discussed in detail in refs. [11,12]. Here we just include the results for completeness.

The basic method is to take the data for $ep \to e\rho^0 p$, where the electromagnetic current diffracts into a ρ. Then use CVC to go to ν reactions, and the V–A theory to get the axial vector current prediction. In so doing one assumes A_1 dominance of the axial vector current. Previously this did not have directly relevant tests, but the analysis of sect. 2.1 explicitly implies that the numbers used in ref. [11] to estimate the A_1 contribution should be valid.

To within about a factor of two, in the energy range $E_\nu \gtrsim 10$ GeV, one expects from ref. [11]:

$$\sigma(\nu p \to \mu^- A_1^+ p)/\sigma_T(\nu p) \approx 10^{-3} . \tag{10}$$

For $\bar{\nu}p$ the numerator is unchanged in a vector dominance model, while the denominator is 2–3 times smaller so the ratio is correspondingly increased. If it is possible experimentally to look inclusively one should essentially multiply by $\sigma_T(A_1 p)/\sigma_{EL}(A_1 p) \simeq 5$ if a quark model estimate could be trusted. Thus several thousand events are needed to look at the exclusive process, somewhat less if it can be inclusive.

The predictions for A_1 production from ref. [12] are somewhat larger, about 8×10^{-3} for the ratio of eq. (10). Such a large rate should surely be observable in data which will be available this summer.

The A_1^0 could also be produced in neutral current ν reactions, with a cross section ratio of about 10^{-3} relative to the total neutral current cross section. Amusingly, it is the only neutral vector meson not suppressed by some other mechanism.

The kinematic region where production occurs is clear from the physics. In the exclusive case the momentum transfer at the hadronic vertex (t) must be small in order to have had a diffractive scattering. Similarly, q^2 will not be too large. Consequently the production peaks at low x. A significant energy loss to hadrons is required, so at lower energies the production peaks at high y. But for A_1 the extra mass is not too large, so at higher energies the y peak moves in and is near $y = 0.2$ at 50 GeV/c. Further details are available in refs. [11,12].

3. Indirect evidence relevant to the A_1

3.1. Evidence for A_1 exchange

NN elastic scattering gives a unique opportunity to extract evidence for A_1 exchange. Consider pp elastic scattering; there are five independent helicity amplitudes. Using the notation $M_{\lambda_c\lambda_d;\lambda_a\lambda_b}$ for a + b → c + d, we look at $\varphi_1 \equiv M_{1/2,1/2;1/2,1/2}$ and $\varphi_3 \equiv M_{1/2,-1/2;1/2,-1/2}$. Both amplitudes are $n = 0, x = 0$. In a Regge-pole model including pomeron, ρ, A_2, f, ω, π, and B exchange, $\varphi_1 = \varphi_3$. This result is not changed by absorption. However, for A_1 exchange, $\varphi_1 = -\varphi_3$ (as can be seen from the rules in table 7). Thus, any observable that is sensitive to $\varphi_1 - \varphi_3$ will measure the presence of an unnatural parity exchange which does not flip nucleon helicity.

One can show that [15]

$$\sigma_T(\tfrac{1}{2}, \tfrac{1}{2}) = -\frac{1}{4q\sqrt{s}} \operatorname{Im} \varphi_1|_{t=0} \,, \tag{11a}$$

$$\sigma_T(\tfrac{1}{2}, -\tfrac{1}{2}) = -\frac{1}{4q\sqrt{s}} \operatorname{Im} \varphi_3|_{t=0} \,, \tag{11b}$$

where $\sigma_T(\lambda_a, \lambda_b)$ is the total cross section for the scattering of protons in a definite initial helicity state and q is the c.m. momentum. Therefore,

$$\begin{aligned} \Delta\sigma_L &\equiv \sigma_T(\tfrac{1}{2}, -\tfrac{1}{2}) - \sigma_T(\tfrac{1}{2}, \tfrac{1}{2}) \\ &= \frac{1}{4q\sqrt{s}} \operatorname{Im} (\varphi_1 - \varphi_3)|_{t=0} \,. \end{aligned} \tag{12}$$

Suppose that we take A_1 exchange to be the major contribution to $\Delta\sigma_L$. If we take $m_{A_1} \approx \sqrt{2} m_\rho$, $\alpha_0 \approx 0$ and $s_0 = 1$ for the trajectory, we can use eq. (B.4) to obtain:

$$\operatorname{Im}(\varphi_1 - \varphi_3)|_{t=0} = \frac{4C_0 g^2_{A_1NN}}{m^2_{A_1}} \,.$$

We have $C_0 \approx 0.32$ [see eq. (B.5); this accounts for the effects of absorption according to our general procedure] and g_{A_1NN} is listed in table 5. The result is

$$\operatorname{Im}(\varphi_1 - \varphi_3)|_{t=0} \approx 50,$$

which should be approximately valid for $4 \lesssim P_{lab} \lesssim 15$ GeV/c. This gives at $P_{lab} = 6$ GeV/c:

$$\Delta\sigma_L = 0.9 \text{ mb},$$

which is to be compared with the experimental result [14] $\Delta\sigma_L = 1.0 \pm 0.1$ mb. Our calculation is admittedly crude, but we expect that a result obtained more carefully won't be much different. Note the following two ingredients in obtaining this result. First is the value of g_{A_1NN} obtained from current algebra arguments (see appendix C). Second is the value of α_0 which was estimated by supposing $m_{A_1} \approx \sqrt{2} m_\rho$ and a typical Regge slope for the A_1 trajectory. The validity of the above calculation suggests that using g_{A_1NN} to estimate backward A_1 production should be reliable.

3.2. The existence of other axial vector states

Two other kinds of axial vector states have been found whose existence might be taken to be relevant to the A_1. One is the equivalent charmonium state [17], $\chi(3508)$, identified as a 1^{++} meson. Since the $\bar{c}c$ system binds in this state, presumably the $\bar{u}d$ system does also.

The second kind are the isoscalar (D(1285)) and strange 1^+ states. The latter may have been found recently [34]. Two strange axial vector mesons have been reported, presumably the Q_A and Q_B associated with A_1 and B multiplets. Because they are produced diffractively other interpretations are possible, and Basdevant and Berger [35] have argued that only one resonance is needed to explain the data, the Q_B.

In our view, as we remarked in the introduction, the existence of the above states is not to be interpreted as directly relevant to the existence of the A_1. The quantum numbers of the A_1 channel are different. If it should happen that binding forces or unitarization effects are relevant to the existence of the state, then it would be easy to have quite different effects in the A_1 channel from the others.

Indeed, while it is less often emphasized than the A_1 case, there is another missing axial vector state, the isoscalar partner of the B (often called H). It is also supposed to have mainly a $\rho\pi$ decay, and every experiment that did not observe an A_1^0 (such as $\pi^+ p \to A_1^0 \Delta^{++}$ or $\pi^- p \to A_1^0 n$) also has not observed an H. From the quark model point of view, its absence would be as mysterious as that of the A_1.

The basic reasons for expecting to find an A_1 (and an H) remain the quark model and chiral symmetry arguments. Given A_2 and B, no stronger arguments are needed. Information on other axial states should of course be explored [16] for any hints as to where A_1 might be found and with what properties, but all such results should be interpreted as suggestive rather than compelling.

4. Discussion

Let us review the evidence. The existence or non-existence of the A_1 has implications both for the quark model and current algebra. If we assume the standard results, then we obtain a set of couplings of the A_1 in various situations. This allows us to predict the expected A_1 signal in $e^+e^- \to \tau^+\tau^-$, $\tau \to A_1\nu$, forward non-diffractive and backward resonance production, neutrino production of A_1, and the level of A_1 exchange in various reactions. The predicted magnitude of A_1 exchange is indeed observed, but so far the A_1 is not produced. The limits set by recent experiments continue to provide evidence that the A_1 is not being produced for reasons that seem not to apply to the well established meson resonances.

It is natural then to question the "standard results" of the quark model and current algebra. An interesting hypothesis has been advanced by Caldi and Pagels [36] in regard to the standard current algebra approaches. They set out to put vector meson dominance on the same footing as PCAC (i.e. a result of the spontaneously broken symmetry). They put the ρ in a $(3, \bar{3}) \oplus (\bar{3}, 3)$ representation of SU(3) $\times$ SU(3) which makes the chiral partner of the ρ a 1^{+-} meson (i.e. the B). With this the case, the A_1 is not needed (though not precluded, of course) in the current algebra calculations; and the results for A_1 couplings obtained in appendix C no longer follow.

As far as the quark model is concerned, there are two possibilities. One is that the A_1 couples very weakly to $\rho\pi$, although one is at a loss to explain how this can be made consistent in light of, for example, broken $SU(6)_W$ calculations. The other possibility is that for some reason, the A_1 doesn't exist as a particle state; this could have extremely important consequences in our future dealings with the quark model. Contrary to the situation with quarks, it would not make theories more attractive if the A_1 were confined.

Appendix A

Notation and convention

(a) Metric. We use the Pauli metric [37]. Note that we take $e^2/4\pi \approx \frac{1}{137}$.

(b) Scattering. We consider meson-baryon scattering where a, c are mesons and b, d are baryons. One defines s-channel helicity amplitudes $M_{\lambda_c\lambda_d;\,\lambda_a\lambda_b}$ [38].

(i) Forward scattering $a + b \to c + d$

$$m_a = \lambda_c - \lambda_a,$$

$$m_b = \lambda_d - \lambda_b.$$

Then $n = |m_b - m_a|$ is the net helicity flip. We also define:

$$x = |m_b| + |m_a| - n.$$

In general x is an even non-negative integer. If $x \neq 0$ then the pole term is evasive (i.e. vanishes faster than $(-t')^{n/2}$ at $t' = 0$).

(ii) Backward scattering $a + b \to d + c$

$$m_a = \lambda_d - \lambda_a\ ,$$

$$m_b = \lambda_c - \lambda_b.$$

Then $n = |m_b - m_a|$ is the net helicity flip in the backward direction. We also define

$$x = |m_b| + |m_a| - n - 1.$$

In general, $x = -1, 0$ for the non-evasive case and $x > 0$ for the evasive case (vanishes faster than $(-u')^{n/2}$ at $u' = 0$).

One useful relation is the parity relation:

$$M_{-\lambda_c-\lambda_d;-\lambda_a-\lambda_b} = \eta_a\eta_b\eta_c\eta_d(-1)^{s_a+s_b-s_c-s_d}$$

$$\times (-1)^{(\lambda_c-\lambda_a)-(\lambda_d-\lambda_b)} M_{\lambda_c\lambda_d;\lambda_a\lambda_b}\ ,$$

where s, η are the spin and parity of an external particle.

(c) Normalization. Our spinors are normalized:

$$\bar{u}(p)\, u(p) = 2m .$$

Thus, for any process a + b → c + d,

$$\frac{d\sigma}{dt} = \frac{1}{(2s_a + 1)(2s_b + 1)} \frac{1}{64\pi q^2 s} \sum_{\lambda_c \lambda_d \lambda_a \lambda_b} |M_{\lambda_c \lambda_d ; \lambda_a \lambda_b}|^2 ,$$

where q is the magnitude of the incoming c.m. three-momentum and $s = -(p_a + p_b)^2$.

(d) Current algebra. We define the following matrix elements of the vector and axial vector currents.

$$\langle 0 | A_\mu^{1+i2}(0) | \pi^- \rangle = i f_\pi q_\mu , \tag{A.1a}$$

$$\langle 0 | V_\mu^{1+i2}(0) | \rho^- \rangle = f_\rho \epsilon_\mu(q) , \tag{A.1b}$$

$$\langle 0 | A_\mu^{1+i2}(0) | A_1^- \rangle = f_A \epsilon_\mu(q) , \tag{A.1c}$$

where q is the four-momentum of the meson and ϵ_μ is its spin-1 wave function.

When one states the vector meson dominance hypothesis, one writes the current field identity [39]

$$\langle 0 | V_\mu^3(0) | \rho^0 \rangle = \frac{m_\rho^2}{\gamma_\rho} \rho_\mu^3 , \tag{A.1d}$$

where ρ_μ^3 is the neutral ρ field. We then identify:

$$f_\rho = \frac{\sqrt{2}\, m_\rho^2}{\gamma_\rho} . \tag{A.2}$$

Finally, let us recall the first Weinberg sum rule [40]. If we saturate the sum rules with ρ, π, and A_1 we obtain:

$$\frac{f_\rho^2}{m_\rho^2} - \frac{f_A^2}{m_{A_1}^2} = f_\pi^2 . \tag{A.3}$$

Appendix B

Phenomenological amplitudes for forward and backward scattering

Ref. [19] gives a rather complete description of the absorption model approach to the two-body hadron reactions. We use here an approximation to that model which we expect will give cross sections correctly to within a factor of two. Since we ignore subtleties of amplitude phases [by simply using the phases predicted by a strict Regge pole approach] we don't expect to reproduce characteristics of the data which crucially depend on these phases (e.g. polarizations).

B.1. Forward non-diffractive scattering

Consider the pole term of reggeon exchange of spin J and mass m_e. Then the amplitude is:

$$R^e_{n,x}(s,t) = |R^e_{n,x}|\, e^{-i\pi[\alpha(t)-J]/2} . \tag{B.1}$$

We take:

$$|R^e_{n,x}| = (-t')^{(n+x)/2}\, g_{\mathrm{eac}} g_{\mathrm{ebd}} \left(\frac{s}{s_0}\right)^{\alpha(t)} \frac{s_0^J}{m_e^2 - t} , \tag{B.2}$$

which has the correct form at $\alpha(t = m_e^2) = J$. s_0 is taken to be a free parameter.

At $t' = 0$, we define:

$$R^e(s) \equiv \lim_{t'\to 0} \left| \frac{R^e_{n,x}(s,t')}{(-t')^{(n+x)/2}} \right| = \frac{g_{\mathrm{eac}} g_{\mathrm{ebd}} s^{\alpha_0} s_0^{J-\alpha_0}}{m_e^2} . \tag{B.3}$$

To simulate the absorption correction, the complete amplitude used in sect. 2.1 was taken to be:

$$M^e_{n,x}(s,t) = \pm \frac{C_{n,x} 2^{n+x} (n+x)!}{R_e^{n+x}}\, e^{Bt} J_n(R_e\sqrt{-t'})\, R^e(s)\, e^{-i\pi(\alpha(t)-J)/2} . \tag{B.4}$$

$C_{n,x}$ sets the scale for the effect of the absorption with $C = 1$ indicating no decrease in $|M_{n,x}|$ at $t' = 0$ from the pure Regge-pole result.

By studying the full absorption model, we have noted a phenomenological formula for $C_{n,x}$:

$$C_{n,x} = \frac{\xi[1 - A^{n+1}]}{x+1} , \tag{B.5}$$

where

$$A = \frac{m_e}{m_e + 0.515\ \mathrm{GeV}} ,$$

$\xi = -1$ for $x > 0$ and $n = 0$; $\xi = +1$ otherwise. This seems to work well in the energy range we have been considering.

In table 4, we present the values we have used for meson-baryon scattering in the $P_{\mathrm{lab}} = 4$–15 GeV/c range. We now make a brief remark about the parameters. R_e is determined by the zero structure of the various amplitudes since $M_n \sim J_n(R_e\sqrt{-t})$. s_0 is determined in order that we obtain correct absolute cross sections. We anticipate that it is constant for a given exchange, but it might vary from exchange to

Table 7
SCHA factors in the forward direction

	$\lambda''\lambda'\lambda$	SCHA factor
$\rho\pi\pi$	0	$\sqrt{2}\, g_{\rho\pi\pi}$
$\rho\pi A_1$	1	$g_d^A/4m_{A_1}$
	0	$-\frac{1}{\sqrt{2}}\,[g_s^A + g_d^A\,(m_\rho^2 + m_{A_1}^2 - m_\pi^2)/4m_{A_1}^2]$
	-1	$-g_d^A/4m_{A_1}$
$\omega\pi B$	1	$g_d^B/4m_B$
	0	$-\frac{1}{\sqrt{2}}\,[g_s^B + g_d^B(m_\omega^2 + m_B^2 - m_\pi^2)/4m_B^2]$
	-1	$-g_d^B/4m_B$
$\rho\pi A_2$	$+2$	$g_{A_2\rho\pi}/2\sqrt{2}\, m_{A_2}^2$
	$+1$	$-g_{A_2\rho\pi}(m_{A_2}^2 + m_\rho^2 - m_\pi^2)/4\sqrt{2}\, m_{A_2}^3$
	0	0
	-1	$-g_{A_2\rho\pi}\;[(m_{A_2}^2 + m_\rho^2 - m_\pi^2)/4\sqrt{2}\, m_{A_2}^3]$
	-2	$-g_{A_2\rho\pi}/2\sqrt{2}\, m_{A_2}^2$
$\left\{\begin{matrix}\rho pp\\ \omega pp\end{matrix}\right\}$	$\frac{1}{2}\,\frac{1}{2}$	$-\sqrt{2}\, G_V$
	$-\frac{1}{2}\,\frac{1}{2}$	$-\sqrt{2}\, G_T/2m_N$
$\rho p\Delta$	$\frac{3}{2}\,\frac{1}{2}$	$G_{\rho N\Delta}/(m_N + m_\Delta)$
	$\frac{3}{2}\,-\frac{1}{2}$	0
	$\frac{1}{2}\,\frac{1}{2}$	0
	$\frac{1}{2}\,-\frac{1}{2}$	$m_N G_{\rho N\Delta}/\sqrt{3}\, m_\Delta(m_N + m_\Delta)$
A_1pp	$\frac{1}{2}\,\frac{1}{2}$	$\sqrt{2}\, g_{A_1NN}$
	$-\frac{1}{2}\,\frac{1}{2}$	0

We list the rules for forward scattering which express $g_{eac}g_{ebd}$ in terms of the basic coupling constants defined in table 2. All expressions are calculated to leading order in s. We give the two vertices separately with the following conventions: vertex ABC indicates incoming B and outgoing C with the exchange of A. We define λ'' to be the helicity of the outgoing meson and λ', λ to be the outgoing and incoming baryons. We assume a c.m. coordinate system for the scattering where the baryon is moving forward. The charge states above are the same as those in table 2.

exchange (cf. backward results). B represents the fact that hadrons have rounded edges in impact parameter space. We find that by allowing B to vary within the range given in table 4, we get reasonable results for our calculations. We should also point out that α_0 is the effective Regge trajectory intercept which is expected to be higher than the one obtained from the Chew-Freutschi plot due to effects of absorption. Since absorption effects will decrease with increasing energy, α_0 should also be considered a mildly energy-dependent parameter which will decrease with increasing energy.

We now say some words about $g_{\text{eac}}g_{\text{ebd}}$. Note that this product has dimensions of $[m]^{2-2J-n-x}$. Starting from phenomenological Lagrangians for the various vertices of interest [see table 2], we derive expressions for $g_{\text{eac}}g_{\text{ebd}}$ at the pole $t = m_{\text{e}}^2$ in terms of basic coupling constants to leading order in s. Note that we use s-channel helicity amplitudes so that $g_{\text{eac}}g_{\text{ebd}}$ will depend on the helicities of the vertex. We will call these expressions SCHA factors; they are listed in table 7. To get values for these coupling constants, there are three methods:

(i) From decays; for example $g_{\rho\pi\pi}$ is obtained from $\rho \to \pi\pi$.

(ii) From vector meson dominance and current algebra; see appendix C.

(iii) From using eq. (B.4) and requiring consistency with experimental cross sections.

Note that since we fix our coupling constants and s_0 by using many reactions the numbers we get are often very well constrained and in some cases overdetermined.

B.2. Backward scattering

The pole term of fermion reggeon exchange is more complicated. We parametrize the pole couplings away from the pole as linear functions of $\sqrt{u}$ and take [41]

$$|R_{n,x}^{\text{e}}| = \frac{(-u')^{n/2}}{m_{\text{e}}^{n+x-1}} \left(\frac{s}{s_0}\right)^{\alpha(t)} \left(\frac{s_0}{m_{\text{e}}^2}\right)^J$$

$$\times \frac{1}{2}\left[\frac{(\sqrt{u})^x(A_1 + B_1\sqrt{u})(A_2 + B_2\sqrt{u})}{m_{\text{e}} - \sqrt{u}} + [\sqrt{u} \to -\sqrt{u}]\right]. \tag{B.6}$$

We can write this as (with y defined below):

$$|R_{n,x}^{\text{e}}| = \frac{(-u')^{(n+y)/2} g_{\text{ead}}g_{\text{ebc}}}{m_{\text{e}}^{n+y-2}} \left(\frac{s}{s_0}\right)^{\alpha(t)} \left(\frac{s_0}{m_{\text{e}}^2}\right)^J \frac{f_x(u)}{m_{\text{e}}^2 - u}\,, \tag{B.7}$$

where $f_x(m_{\text{e}}^2) = 1$. Note that $g_{\text{ead}}g_{\text{ebc}}$ is dimensionless.

If we take $r_i = (A_i - B_i m_{\text{e}})/(A_i + B_i m_{\text{e}})$, $i = 1,2$, then for example:

$$f_{-1}(u) = \tfrac{1}{4}[3 + r_1 + r_2 - r_1 r_2 + u(1 - r_1)(1 - r_2)/m_{\text{e}}^2]\,,$$

$$f_0(u) = \tfrac{1}{4}[(1 + r_1)(1 + r_2) + u(3 - r_1 - r_2 - r_1 r_2)/m_{\text{e}}^2]\,. \tag{B.8}$$

Now since non-evasive amplitudes correspond to $x = -1$ or 0, we define

$$y = \begin{cases} x & \text{if } x \text{ is even} \\ x+1 & \text{if } x \text{ is odd,} \end{cases}$$

then we take

$$M^{\mathrm{e}}_{n,x}(s,u) = \pm \frac{C_{n,y} 2^{n+y}(n+y)!}{R_{\mathrm{e}}^{n+y}} \, \mathrm{e}^{Bu} J_n(R_{\mathrm{e}}\sqrt{-u'})$$

$$\times \frac{g_{\mathrm{ead}} g_{\mathrm{ebc}} s^{\alpha_0} s_0^{J-\alpha_0}}{m_{\mathrm{e}}^{n+y+2J}} f_{\mathrm{x}}(0)\, \mathrm{e}^{-i\pi(\alpha(u)-J)/2} \,. \tag{B.9}$$

$C_{n,y}$ is the same as the one that appears in eq. (B.5). Actually, r_1 and r_2 can be regarded as free parameters for each different vertex and exchange. We make a simplifying assumption that

$$r_1 = r_2 \equiv r_{\mathrm{e}} \,,$$

which could be different for each possible exchange. In practice we only consider N and Δ exchange for the reactions we have studied. See table 4 for the backwards parameters. As in the forward direction, we derive expressions for the SCHA factors $g_{\mathrm{eac}} g_{\mathrm{ebd}}$ at the pole $u = m_{\mathrm{e}}^2$ in terms of basic coupling constants to leading order in s (see table 8).

B.3. Strange particles

All the rules in tables 2 and 3 are for the scattering of non-strange mesons and baryons. Note that the derivation of the rules depends only on the spin and parity of the particles involved, so that the same rules can be used in the case of strange particles. We will need to obtain coupling constants of strange particles. These may be obtained by the techniques already mentioned. Alternatively, we may use SU(3) symmetry to express strange particle couplings in terms of non-strange particle couplings. There is an ambiguity in how one applies SU(3) to the rules in tables 2 and 3 when one takes the mass differences within an SU(3) multiplet into account. We take the following approach: We use SU(3) symmetry to obtain coupling constants and use the actual particle masses when using the rules.

We briefly discuss the relations between couplings in the SU(3) limit. Vertices involving three octet mesons are simple since charge conjugation invariance always requires that either the F or D coupling vanish. We concentrate on baryon-baryon-meson vertices where all hadrons are members of an octet. Then, the D/F ratio is needed. We use the following notation $\alpha = F/(D+F)$.

We obtain the following results:

$$g_{\mathrm{KN}\Sigma} \equiv g_{\mathrm{K^-p}\Sigma^0} = (2\alpha - 1) g_{\pi\mathrm{NN}} \,, \tag{B.10a}$$

$$g_{KN\Lambda} \equiv g_{K^- p\Lambda^0} = \sqrt{\tfrac{1}{3}}(2\alpha + 1)\, g_{\pi NN} \,, \tag{B.10b}$$

$$g_{K\Sigma\Delta} \equiv g_{K^-\Delta^{++}\Sigma^+} = G_{\pi N\Delta} \,, \tag{B.10c}$$

where the couplings on the right are defined in table 2. We obtained a value for α by fitting the backward cross sections $K^-p \to \Sigma^\pm\pi^\mp$ and $K^-p \to \Lambda^0\pi^0$ [30]. The cross section most sensitive to α is $K^-p \to \Sigma^-\pi^+$ which crucially depends on $g_{KN\Sigma}$. We find that $\alpha = 0.3$ gives a reasonable fit to the data and is in fair agreement with other determinations of α [42].

Table 8
SCHA factors in the backward direction

N exchange

(a) Vertex ead

	λ'	SCHA factor
$\pi^- n$	$\frac{1}{2}$	$\sqrt{2}\, g_{\pi NN}$
$\pi^-\Delta^0$	$\frac{3}{2}$	$G_{\pi N\Delta} m_N/\sqrt{6}\,(m_N + m_\Delta)$
	$\frac{1}{2}$	$\sqrt{2}\, G_{\pi N\Delta} m_N \left[1 + \dfrac{m_N^2 + m_\Delta^2 - m_\pi^2}{m_N m_\Delta}\right] / 6(m_N + m_\Delta)$

(b) Vertex ebc

	$\lambda\ \lambda''$	
$p\pi^0$	$\frac{1}{2}\ 0$	$g_{\pi NN}$
	$-\frac{1}{2}\ 0$	$-g_{\pi NN}$
$p\rho^0$	$\frac{1}{2}\ {+1}$	$(2G_V + G_T)/\sqrt{2}$
	$\frac{1}{2}\ 0$	$m_\rho G_T/2m_N$
	$\frac{1}{2}\ {-1}$	$-G_T/\sqrt{2}$
	$-\frac{1}{2}\ {+1}$	$G_T/\sqrt{2}$
	$-\frac{1}{2}\ 0$	$m_\rho G_T/2m_N$
	$-\frac{1}{2}\ {-1}$	$(2G_V + G_T)/\sqrt{2}$
pA_1^0	$\frac{1}{2}\ 1$	$-\sqrt{2}\, g_{A_1NN}$
	$\frac{1}{2}\ 0$	$2m_N g_{A_1NN}/m_{A_1}$
	$\frac{1}{2}\ {-1}$	0
	$-\frac{1}{2}\ 1$	0
	$-\frac{1}{2}\ 0$	$-2m_N g_{A_1NN}/m_{A_1}$
	$-\frac{1}{2}\ {-1}$	$+\sqrt{2}\, g_{A_1NN}$

Table 8 (continued)

Δ exchange
(a) Vertex ead

	λ'	
$\pi^- n$	$\frac{1}{2}$	$\frac{1}{\sqrt{6}} \frac{m_\Delta}{m_N + m_\Delta} G_{\pi N\Delta}$

(b) Vertex ebc

	$\lambda\ \lambda''$	
$p\pi^0$	$\frac{1}{2}\ 0$	$\frac{1}{\sqrt{3}} \frac{m_\Delta}{m_N + m_\Delta} G_{\pi N\Delta}$
	$-\frac{1}{2}\ 0$	$\frac{1}{\sqrt{3}} \frac{m_\Delta}{m_N + m_\Delta} G_{\pi N\Delta}$
$p\rho^0$	$\frac{1}{2}\ 1$	$\sqrt{\frac{2}{3}} \frac{m_\Delta}{m_N + m_\Delta} G_{\rho N\Delta}$
	$\frac{1}{2}\ 0$	0
	$\frac{1}{2}\ -1$	0
	$-\frac{1}{2}\ 1$	0
	$-\frac{1}{2}\ 0$	0
	$-\frac{1}{2}\ -1$	$-\sqrt{\frac{2}{3}} \frac{m_\Delta}{m_N + m_\Delta} G_{\rho N\Delta}$
pA_1^0	$\frac{1}{2}\ 1$	0
	$\frac{1}{2}\ 0$	$m_\Delta G_{A_1 N\Delta}/\sqrt{3}\, m_{A_1}$
	$\frac{1}{2}\ -1$	0
	$-\frac{1}{2}\ 1$	0
	$-\frac{1}{2}\ 0$	$m_\Delta G_{A_1 N\Delta}/\sqrt{3}\, m_{A_1}$
	$-\frac{1}{2}\ -1$	0

We list the rules for backward scattering which express $g_{ead}\, g_{ebc}$ in terms of the basic coupling constants defined in table 2 to leading order in s. Conventions are as follows: we choose the c.m. coordinate system where the initial baryon, b, is in the forward direction. Let λ'' be the meson helicity and let λ', λ be the final and initial baryon helicities.

Appendix C

Coupling constants from vector meson dominance and current algebra

We begin by investigating the couplings of the ρ-meson. The statement of ρ-vector meson dominance of the neutral isovector current is [39]:

$$\langle B(p')\,|\,j_\mu^{\mathrm{V}}\,|\,A(p)\rangle = \frac{m_\rho^2}{m_\rho^2+q^2}\,\frac{e}{\gamma_\rho}\,\langle B(p')\,|\,J_\mu^{(\rho)}\,|\,A(p)\rangle\,, \tag{C.1}$$

where $q = p - p'$. $\langle B|J_\mu^{(\rho)}|A\rangle$ is the ρAB strong interaction vertex (see table 2). γ_ρ is defined in eq. (A.1d).

Applications:

(i) $\langle \pi(p')\,|\,j_\mu^{\mathrm{V}}\,|\,\pi(p)\rangle = e(p+p')_\mu\,F_\pi(q^2)$,

where $F_\pi(0) = 1$. One finds simply using eq. (C.1) that $g_{\rho\pi\pi} = \gamma_\rho$.

(ii) $\langle p(p')\,|\,j_\mu^{\mathrm{V}}\,|\,p(p)\rangle = \frac{1}{2}ie\;\bar{u}(p')[F_1^{\mathrm{V}}(q^2)\gamma_\mu + \sigma_{\mu\nu}q_\nu F_2^{\mathrm{V}}(q^2)]\,u(p)$,

where F_i^{V} are the isovector form factors. Note that if we denote F_i^{S} to be the isoscalar form factors, then

$$F_i^{\mathrm{p}} = \tfrac{1}{2}(F_i^{\mathrm{S}} + F_i^{\mathrm{V}}).$$

Rotating in isospin space,

$$F_i^{\mathrm{n}} = \tfrac{1}{2}(F_i^{\mathrm{S}} - F_i^{\mathrm{V}})\,.$$

From the normalization conditions:

$$F_1^{\mathrm{p}}(0) = 1, \qquad F_2^{\mathrm{p}}(0) = \mu_{\mathrm{p}} - 1,$$

$$F_1^{\mathrm{n}}(0) = 0, \qquad F_2^{\mathrm{n}}(0) = \mu_{\mathrm{n}}\,,$$

where μ is the magnetic moment in units of $e/2m_{\mathrm{N}}$, we find:

$$F_1^{\mathrm{V}}(0) = 1,$$

$$F_2^{\mathrm{V}}(0) = \mu_{\mathrm{p}} - \mu_{\mathrm{n}} - 1.$$

Therefore, eq. (C.1) implies that

$$G_{\mathrm{V}}^{\rho} = \tfrac{1}{2}\gamma_\rho\,, \tag{C.2a}$$

$$G_{\mathrm{T}}^{\rho} = \tfrac{1}{2}\gamma_\rho(\mu_{\mathrm{p}} - \mu_{\mathrm{n}} - 1)\,. \tag{C.2b}$$

Putting in the numbers [$\mu_{\mathrm{p}} = 2.79$, $\mu_{\mathrm{n}} = -1.91$], we get:

$$G_{\mathrm{V}}^{\rho} = 2.86,$$

$$G_{\mathrm{T}}^{\rho} = 10.57.$$

(iii) $\langle p(p')\,|\,j_\nu^{\mathrm{V}}\,|\,\Delta^+(p)\rangle = G_{\gamma\mathrm{N}\Delta}\bar{u}(p')\left[\delta_{\mu\nu} - \dfrac{iq_\mu\gamma_\nu}{m_{\mathrm{N}}+m_\Delta}\right]\gamma_5 U_\mu(p)\,,$

which defines $G_{\gamma N\Delta}$. We have assumed a Stodolsky-Sakurai [43] form for the decay $\Delta^+ \to p\gamma$. Computing the width, we obtain:

$$\Gamma(\Delta^+ \to p\gamma) = \frac{G_{\gamma N\Delta}^2}{16\pi m_\Delta^2}\left(\frac{m_\Delta^2 - m_N^2}{2m_\Delta}\right)(m_\Delta - m_N)^2\,[1 + m_N^2/3m_\Delta^2]\ . \tag{C.3}$$

Using $\Gamma = 0.71$ MeV [28], we get $G_{\gamma N\Delta} = 1.43$.

Now, we apply eq. (C.1) which leads to:

$$G_{\gamma N\Delta} = \frac{e}{\gamma_\rho}\, G_{\rho N\Delta}\ . \tag{C.4}$$

We find then that $G_{\rho N\Delta} \approx 27$.

We next study the couplings of the A_1. Consider the matrix element of the weak axial vector current between nucleon states:

$$\begin{aligned}&\langle p(p')|A_\lambda^{1+i2}(0)|n(p)\rangle \\ &\quad = \bar{u}(p')[i\gamma_\lambda\gamma_5 g_A(q^2) - q_\lambda\gamma_5 f_A(q^2)]\,u(p)\,,\end{aligned}$$

where the momenta are given in parenthesis and $q = p - p'$. We have dropped the second class current.

Consider the A_1 contribution to the matrix element. Using the A_1NN vertex given in table 2 and eq. (A.1c), we obtain

$$\langle p'|A_\lambda^{1+i2}(0)|p\rangle_{A_1} = i\sqrt{2}\, f_A g_{A_1NN}\left[\frac{\delta_{\mu\lambda} + q_\mu q_\lambda/m_{A_1}^2}{m_{A_1}^2 + q^2}\right]\bar{u}(p')\gamma_\mu\gamma_5 u(p). \tag{C.5}$$

Thus, we can immediately identify:

$$g_A(0) = \sqrt{2}\, f_A g_{A_1NN}/m_{A_1}^2\,. \tag{C.6}$$

Using the first Weinberg sum rule [eq. (A.3)] leads to

$$g_{A_1NN} = \frac{m_{A_1} m_\rho g_A(0)}{\sqrt{2}\,(f_\rho^2 - m_\rho^2 f_\pi^2)^{1/2}}\ . \tag{C.7}$$

Experimentally, [28], $g_A(0) = 1.2$. The values of g_{A_1NN} for various choices of m_{A_1} are given in table 5.

We next study the matrix element of the axial current between the nucleon and Δ states [44]:

$$\begin{aligned}&\langle \Delta^{++}(p')|A_\lambda^{1+i2}(0)|p(p)\rangle \\ &\quad = \bar{U}_\mu(p')[b_1(q^2)\delta_{\mu\lambda} + ib_2(q^2)q_\mu\gamma_\lambda + b_3(q^2)q_\mu(p+p')_\lambda + b_4(q^2)q_\mu q_\lambda]\,u(p).\end{aligned} \tag{C.8}$$

By considering the π contribution we find:

$$\langle \Delta^{++}(p')|A_\lambda^{1+i2}|p(p)\rangle_\pi$$

$$= \frac{G_{\pi N\Delta}}{m_N + m_\Delta} \bar{U}_\mu(p') q_\mu\, u(p) \frac{1}{m_\pi^2 + q^2} f_\pi q_\lambda\,, \tag{C.9}$$

which implies:

$$b_4(q^2) = \frac{G_{\pi N\Delta}}{m_N + m_\Delta} \frac{f_\pi}{m_\pi^2 + q^2}\,. \tag{C.10}$$

Applying the standard PCAC argument of a conserved axial vector current when $m_\pi^2 = 0$,

$$q_\mu \bar{U}_\mu(p')[b_1 + b_2(m_\Delta - m_N) + b_3(m_\Delta^2 - m_N^2) + b_4 q^2]u(p) = 0\,,$$

where $q^2 b_4 = f_\pi G_{\pi N\Delta}/(m_N + m_\Delta)$ from eq. (C.10). If we neglect the b_2 and b_3 contributions, we find that:

$$b_1(0) = \frac{-f_\pi G_{\pi N\Delta}}{m_N + m_\Delta}\,. \tag{C.11}$$

We next compute the A_1 contribution. We note that the dominant contribution to the $A_1 N\Delta$ vertex will be:

$$G_{A_1 N\Delta} \bar{U}_\mu(p')\delta_{\mu\lambda} u(p)\,. \tag{C.12}$$

Therefore,

$$\langle \Delta^{++}(p')|A_\lambda^{1+i2}|p(p)\rangle_{A_1}$$

$$= G_{A_1 N\Delta} f_A \bar{U}_\mu(p')\delta_{\mu\nu} \frac{\delta_{\nu\lambda} + q_\nu q_\lambda/m_{A_1}^2}{m_{A_1}^2 + q^2} u(p)\,. \tag{C.13}$$

This allows us to read off

$$b_1(0) = \frac{G_{A_1 N\Delta} f_A}{m_{A_1}^2}\,. \tag{C.14}$$

Using eqs. (A.3), (C.11) and (C.14), we get:

$$|G_{A_1 N\Delta}| = \frac{m_{A_1} m_\rho f_\pi G_{\pi N\Delta}}{(m_N + m_\Delta)(f_\rho^2 - f_\pi^2 m_\rho^2)^{1/2}}\,. \tag{C.15}$$

Using the numbers from table 3, we obtain the values of $G_{A_1 N\Delta}$ for various choices of m_{A_1} which are given in table 5.

Finally, we turn our attention to the $A_1 \to \rho\pi$ decay. The $A_1\rho\pi$ vertex depends on two coupling constants g_s and g_d. That the standard current algebra result lead leads to untenable results was well known in 1967. The solution then obtained was to remove the constraint that all form factors be unsubtracted dispersion relations. For example, the results of refs. [26,27] then become:

$$g_s^A = \frac{m_{A_1}^2 - m_\rho^2}{\sqrt{2}\, m_{A_1} f_\pi} \left(\frac{m_\rho^2}{m_{A_1}^2} - \delta \frac{m_{A_1}^2}{m_\rho^2} \right) , \tag{C.16}$$

$$g_d^A = \frac{2 m_{A_1} \sqrt{2}}{f_\pi} \left(\frac{m_\rho^2}{m_{A_1}^2} + \delta \frac{m_{A_1}^2}{m_\rho^2} \right) , \tag{C.17}$$

$$g_{\rho\pi\pi} = \frac{f_\rho}{\sqrt{2} f_\pi^2} (1 + \delta) . \tag{C.18}$$

In some models, δ is a free parameter [27]. In other models δ is fixed; e.g., in ref. [26] one uses the Weinberg sum rules to obtain $\delta = 1 - 2m_\rho^2/m_{A_1}^2$. In any case, from eq. (C.18), one has fairly strict limits on what δ can be from $g_{\rho\pi\pi}$ in table 3.

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