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INERTIAL PROPERTIES OF COMMERCIAL VEHICLES

Descriptive Parameters Used in Analyzing
the Braking and Handling of Heavy Trucks

Volume 2
2nd Edition

Christopher B. Winkler

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16. Abstract This volume is one of a set of five volumes which provided a compilation of heavy vehicle parameter data. The original editions of these volumes were prepared under support from the Motor Vehicle Manufacturers Association and were published in 1981. The five volumes address (1) brakes, (2) inertial properties, (3) tires, (4) steering and suspension systems, and (5) antilock systems. This volume is one of two "2nd Editions" providing updated information on (1) inertial properties, and (2) steering and suspension systems.					
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1.0 INTRODUCTION

Computerized models for analyzing the braking and handling performance of commercial vehicles require two types of input information. First, a physical description of the vehicle to be studied is needed. Second, control inputs must be specified for the type of maneuver to be simulated. This volume deals with obtaining the parametric information necessary for describing the inertial properties of heavy vehicles.

The inertial properties of commercial vehicles can be difficult to determine because of the sizes and weights of these vehicles. Nevertheless, UMTRI has measured certain of these inertial properties for a rather large number of vehicles. This document presents the methods used and the results obtained by UMTRI in an effort to assist simulation users in describing vehicles of their interest.

Hence, the purposes of this report are to (1) provide example data and (2) describe devices that can be used to measure inertial properties. This information is presented in the sections that follow. The UMTRI Pitch Plane Inertial Properties Facility is described in the next section of this report. In addition, Section 2.0 gives pitch plane data for over thirty heavy vehicles. Section 3.0 discusses additional inertial measurement techniques and provides data in all three planes for a limited number of heavy vehicles. The last section (Section 4.0) describes measurements of the inertial properties of vehicle components such as axles and wheels.

2.0 THE PITCH PLANE INERTIAL TEST FACILITY

The heart of the UMTRI inertial properties measurement capability is the Pitch Plane Inertial Test Facility. This is a permanently installed facility which allows for the measurement of total vehicle inertial properties in the pitch plane, i.e., longitudinal and vertical center of gravity position and the polar moment of inertia in pitch.

The facility, shown in Figure 1, is best described as a "swing" on which the vehicle is mounted for testing. Center-of-gravity positions are measured using static moment balance techniques. Moment of inertia is measured by treating the vehicle as a compound pendulum. Following a physical description of the facility in Section 2.1, a simplified mathematical explanation of these test methods will be given in Section 2.2. Section 2.3 contains a listing of data gathered on the facility.

2.1 The Facility

The Pitch Plane Inertial Test Facility is pictured in Figure 1. The facility can normally accept two- or three-axle vehicles with a gross test weight of up to 25,000 lb. and with no axle located more than 12 feet (longitudinally) from the vehicle center of gravity. (Special modifications can be made to accept some vehicles outside of these specifications.)

The facility is basically a swing-like fixture upon which the test vehicle rests and which, in turn, is supported by knife-edge pivots such that the swing and vehicle may rotate freely about a lateral axis. More specifically, the facility is composed of:

- (1) Static frame
- (2) Two lift and tilt mechanisms
- (3) Two knife-edge pivot assemblies
- (4) Two side members
- (5) Two (or three) cross members for use with two- (or three-) axle trucks
- (6) Instrumentation

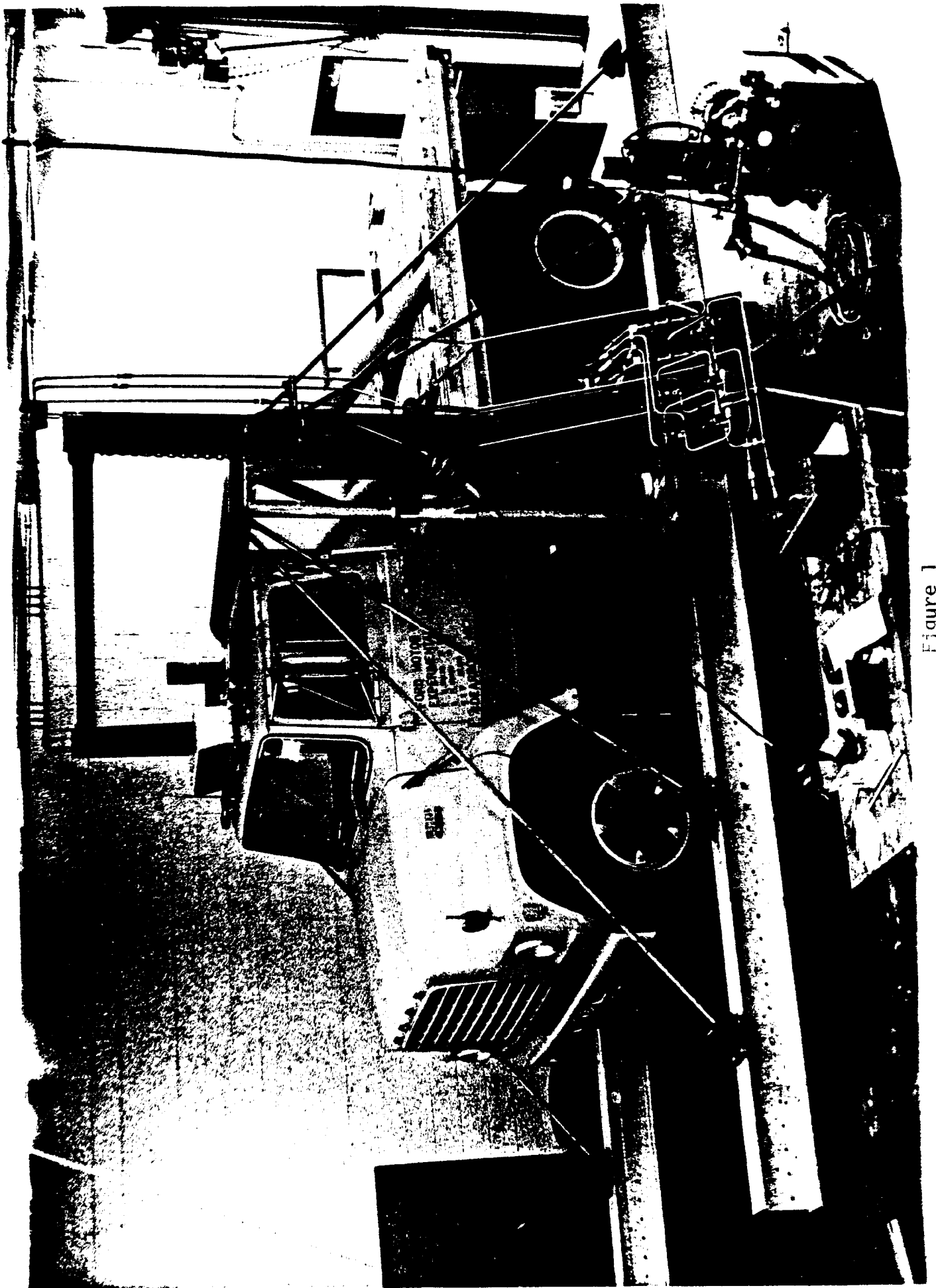


Figure 1

Referring to Figure 1, the static frame is fixed in the ground and provides vertical guides for the hydraulically powered lifting mechanism. The lifting mechanism supports the swing and vehicle on two knife-edge bearings, which are secured in the upright portion of the side members. The elevation of the knife-edge assemblies is adjustable over virtually the full height of the side members. (This feature leads to enhanced measurement accuracy, as explained in Section 2.2.)

The side members are each composed of a 25-foot aluminum I-beam located longitudinally beside the vehicle, the center upright structure, and four bracing rods. The cross members span the distance between the side members, resting on the lower, inside flange of the side member I-beams. The webs of these beams are perforated along their entire length allowing the cross members to be securely bolted in place anywhere along the length of the beams. The test vehicle rests on its tires on top of the cross members.

During c.g. position testing, the hydraulically motivated tilting mechanisms apply an external torque to the swing through a pair of strain-gauged load cells. The static angular position of the swing, with and without applied torque, is measured using an inclinometer placed on the upper surface of both side member I-beams. Tilt angles used do not exceed 8 degrees. It has been found that at this level of tilt, vertical tire and suspension deflections for heavy vehicles are insignificant, and so artificial constraints are rarely used. However, small, albeit significant, longitudinal deflections of the vehicle on its tires and of the swing members can and do occur during tilting. Longitudinal constraints are used to reduce the magnitude of these deflections and dial gauges are employed to measure those that do occur.

For moment of inertia measurements, the tilting mechanisms are decoupled from the swing. The swing is allowed to oscillate freely (from an initial deflection of less than 5 degrees) and the total period of 50 cycles is measured.

Additional geometric measurements which locate the various adjustable members of the swing, as well as the position of the vehicle on the swing, are made.

The data are reduced by computer program and are presented as shown in Figure 2. The "estimated error" values are determined by a linear error analysis explained in the following section and represent the maximum (plus or minus) error expected for the given measurement. Notice that the vertical position of the c.g. is referenced to an easily defined point in the sprung mass, thus avoiding the need to specify tire radius in the test condition. (Vertical c.g. position with respect to ground is given only for convenience.)

2.2 Test Techniques: A Mathematical Explanation

To provide for clarity in the following discussions, the inertial properties of the swing itself have been ignored. In the actual treatment of test data, the combined properties of the vehicle and swing are determined and the known properties of the swing are then "subtracted" to determine vehicle properties.

Figure 3 illustrates a generalized c.g. test arrangement. The upper portion of the figure illustrates the test vehicle resting on the swing with no external torque applied. As is generally the case, the vehicle c.g. is not directly aligned with the centerline of the swing, and so this static condition involves a non-zero angular position of the swing, θ_0 (which has a negative value as shown in the figure). In the lower portion of the figure, the system is shown at static equilibrium with a known, external torque applied to the swing. The swing has assumed the angular position, θ_T (after moving through the differential rotation, $\theta_T - \theta_0$). During a c.g. test series, measurements of θ_0 , T , θ_T , and W are taken (as mentioned previously) and calculations of the following nature are made to determine c.g. position:

The equation for the summation of moments about the pivot point, for the lower figure, is

$$T = W l_4 \quad (1)$$

and from the geometry of the lower figure

$$l_4/l_1 = \sin(\theta_T - \theta_0) \quad (2)$$

UMTRI PITCH PLANE INERTIAL
PROPERTIES TEST

TEST NO. KH-2

DATE 10-26-72 TIME _____ OPERATOR D Brown

I. VEHICLE ID

MANUFACTURER AMC WHEELBASE 109
MODEL NO. _____ SERIAL NO. _____

II. BODY ID

MANUFACTURER _____ DESCRIPTION AWH Dr.
MODEL NO. EAGLE
SERIAL NO. _____

III. VEHICLE CONDITION

FRS PULL LOADING EMT

IV. RESULTS

	TEST			
	1	2	3	AVG
C.G. POSITION (INCHES)				
AFT OF FRONT AXLE	43.52	43.55	43.53	43.54
EXPECTED ERROR	0.167	0.170	0.168	
STANDARD DEVIATION				0.012
HEIGHT ABOVE VEHICLE				
REFERENCE♦	11.50	11.27	11.78	11.52
EXPECTED ERROR	0.489	0.442	0.448	
STANDARD DEVIATION				0.206
HEIGHT ABOVE GROUND				22.64
PITCH MOMENT OF INERTIA				
ABOUT CG (IN-LB-SEC♦♦2)	23308.	23326.	22973.	23202.
EXPECTED ERROR	1873.	1874.	1860.	
STANDARD DEVIATION				162.
WEIGHT (LBS)	3448.			

♦ REFERENCE POINT IS FRAME RAIL AT REAR OF T. BRACK.

Figure 2

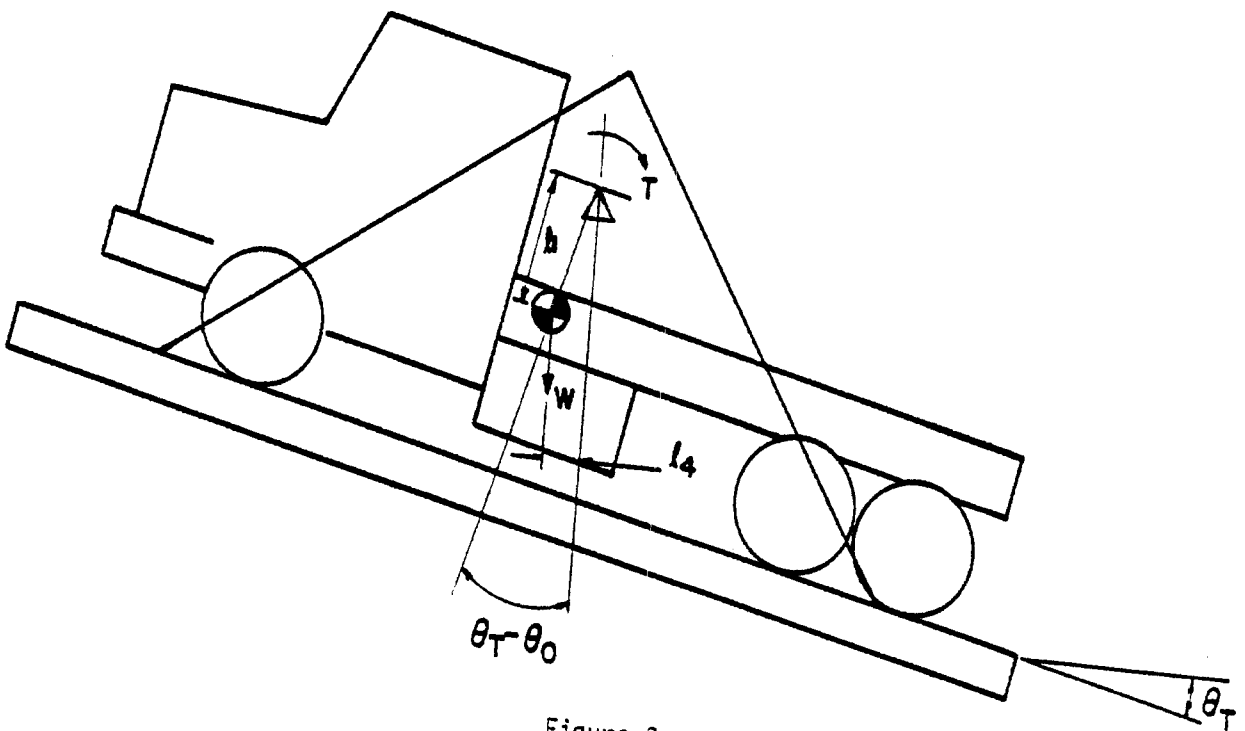
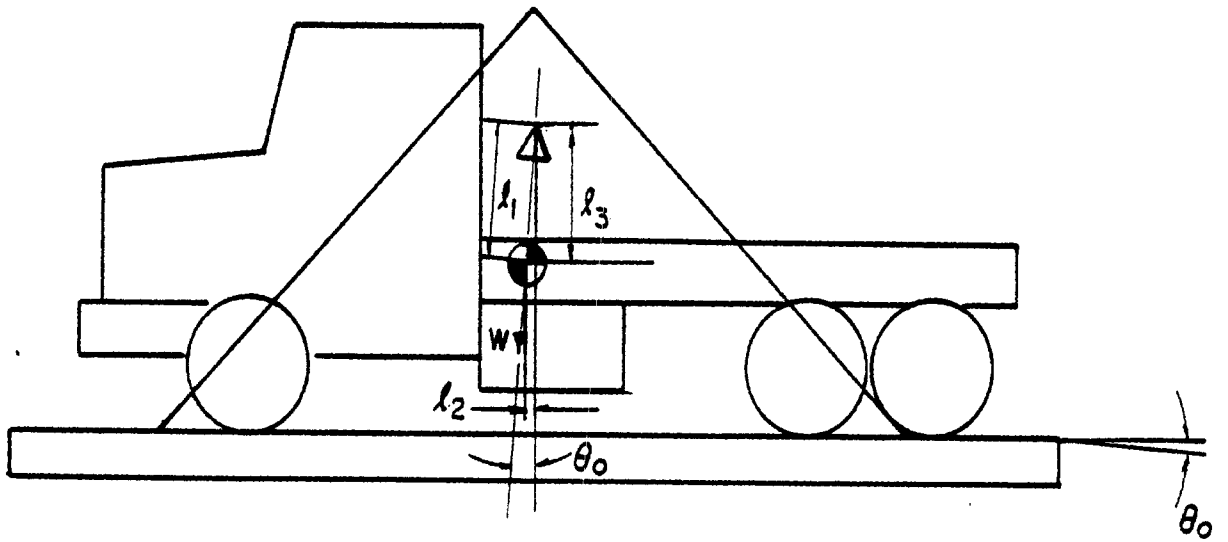


Figure 3

Combining (1) and (2) yields the expression for l_1 , viz.:

$$l_1 = \frac{T}{W \sin (\theta_T - \theta_0)} \quad (3)$$

From the geometry of the upper figure

$$l_3 = l_1 \cos \theta_0 \quad (4)$$

$$l_2 = l_1 \sin \theta_0 \quad (5)$$

These values, l_2 and l_3 , are the fundamental experimental measurements which, when combined with straightforward geometric measurements locating the vehicle on the swing, lead to the determination of c.g. height and longitudinal position.

As mentioned above, the actual data reduction process is more complex. A number of geometric measurements are made to determine the relative position of the various adjustable members of the facility. These are used to determine the inertial properties of the swing assembly in its specific test condition. These properties are ultimately removed from the measured properties to determine the vehicle properties. Also, since the vehicle rests on the swing on its own tires and since the swing structure is, of course, somewhat flexible, a small longitudinal motion of the masses occurs (in an axis system with its origin at the pivot point and which rotates with the swing) between the "tilted" and "non-tilted" test conditions. Experimental results are very sensitive to this shift in position, and so these motions are measured (with dial gauges) and are treated appropriately in the calculations.

To determine the pitch moment of inertia, the vehicle is supported as shown in the upper portion of Figure 3 and is allowed to oscillate freely, through small angles, as a compound pendulum. The period of oscillation (P) is measured. Again, ignoring swing properties, the pitch moment of inertia (I_y) can be calculated from the equation

$$I_y = \frac{W l_1 P^2}{4\pi^2} - \frac{W l_1^2}{g} \quad (6)$$

where "g" is the gravitational constant and l_1 has been determined from the c.g. test procedure. (Equation (6) derives from the linear analysis of a compound pendulum.)

As mentioned previously, the vertical position of the knife-blade pivot is adjustable relative to the vehicle in order that the geometry of the test set-up may be arranged to enhance measurement accuracy. The following presentation, although greatly simplified relative to the actual calculations, serves to illustrate the principles involved.

If we assume that the vehicle is precisely located on the swing so that $l_2 = 0$, then $l_3 = l_1$, then Equation (3) serves as an exact formulation of the primary c.g. height measurement. That is:

$$l_3 = \frac{T}{W \sin \theta_T}$$

where we have also assumed $\theta_0 = 0$ for simplicity. A first-order estimate of the measurement error on l_3 , that is, Δl_3 , is given by

$$\Delta l_3 = \frac{\partial l_3}{\partial T} \Delta T + \frac{\partial l_3}{\partial W} \Delta W + \frac{\partial l_3}{\partial \theta_T} \Delta \theta_T \quad (7)$$

where $\partial l_3 / \partial T$, $\partial l_3 / \partial W$, and $\partial l_3 / \partial \theta$ are the partial derivatives of l_3 with respect to T , W , and θ , respectively, and ΔT , ΔW , and $\Delta \theta$ are the measurement errors on T , W , and θ_T , respectively. Calculating the partial derivatives and substituting them into Equation (7) yields:

$$\Delta l_3 = \left[\frac{1}{W \sin \theta_T} \right] \Delta T + \left[- \frac{T}{W^2 \sin \theta_T} \right] \Delta W + \left[- \frac{2 T^2}{W \sin \theta_T \sin (2\theta_T)} \right] \Delta \theta_T \quad (8)$$

To reduce experimental error, it is generally desirable to reduce the absolute value of each of the partial derivatives (the bracketed terms). Equation (8) shows that

- 1) The absolute value of each of the partial derivatives is reduced by increasing the absolute value of θ_T .

- 2) The absolute value of two of the partial derivatives is reduced by reducing the absolute value of T.

In practice, Point (1) can be used by employing the maximum tilt angle available with the facility. Point (2) can be used by adjusting the knife-edge pivot point to provide the shortest ℓ_3 length which is practical. That is, the value of T required to produce a given tilt angle (θ_T) becomes smaller for smaller values of ℓ_3 .

In theory, if the knife-edge position was adjusted so that it aligned directly with the c.g., the affected partials would become zero, eliminating these sources of error. In practice, we have found that it is best to maintain a minimum value of 6-8 inches on ℓ_3 . Smaller values produce such a low level of system stability that the static condition is difficult to attain due to normally present air currents, etc.

The same analysis technique provides a first-order error estimation for pitch moment of inertia as follows:

$$\Delta I = \left[\frac{I}{W} \right] \Delta W + \left[\frac{I}{\ell_3} - \frac{W \ell_3}{g} \right] \Delta \ell_3 + \left[\frac{W \ell_3^{1/2}}{\pi} \sqrt{\frac{\ell_3^2}{g} + \frac{I}{W}} \right] \Delta P \quad (9)$$

where the bracketed quantities are the appropriate partial derivatives and the " Δ " terms are experimental measurement errors. Equation (9) shows that:

- 1) $\frac{\partial I}{\partial W}$ is beyond the experimenter's control
- 2) $\frac{\partial I}{\partial \ell_3}$ is minimized (at zero) when

$$\ell_3 = \left(\frac{I g}{W} \right)^{1/2} \quad (10)$$
 (i.e., when ℓ_3 equals the radius of gyration)
- 3) $\frac{\partial I}{\partial P}$ is minimized (also at zero) when $\ell_3 = 0$

Points (2) and (3) imply that a compromise is in order when choosing ℓ_3 for moment of inertia testing. Equation (10) produces values on the order of 1/2 of the wheelbase for typical trucks. For the UMTRI facility, analysis has shown that values of ℓ_3 on the order of 25 inches produce minimized error estimates.

Although the major sources of error have been included here, the actual error estimate calculations used in data reduction are far more complex. Terms are included that represent potential errors deriving from errors in a host of swing inertial and geometric properties which may be "built in," as well as the specific measurement errors deriving from each test sequence. The "estimated errors" which appear on the data sheet, as shown previously in Figure 2, are calculated according to the form:

$$\left[\sum \left(\frac{\partial M}{\partial x_i} \Delta x_i \right)^2 \right]^{1/2} \quad i = 1, 2, \dots$$

where

M is the measurement of interest

$\partial M / \partial x_i$ is the partial derivative of M with respect to parameter x_i

Δx_i is an estimate of the accuracy of parameter x_i

i varies sufficiently to include virtually all parameters used in the calculation of M.

In addition to calculating the "estimated errors," each test procedure is repeated three times and a standard deviation of the three results of each measurement is computed. Commonly, standard deviations are an order of magnitude smaller than the "estimated errors" indicating that test repeatability errors are small relative to our estimates of potential parametric errors (i.e., the Δx_i quantities used).

2.3 Pitch Plane Data

The following pages contain a tabular presentation reviewing the vehicles measured and measurements made on the Pitch Plane Facility.

An additional numeric, the normalized pitch radius of gyration, is also presented for trucks and tractors. This numeric is the radius of gyration in pitch divided by the wheelbase, viz.:

Normalized Radius of Gyration \equiv

$$\left[\frac{I_y \times g}{W} \right]^{1/2} / WB$$

where

I_y is the pitch moment of inertia, in-lb-sec²

g is the gravitational constant, 386 in/sec²

W is the weight, lbs

WB is the wheelbase, in

For single rear-axle trucks, except for one (short wheelbase) outlier, these parameters range from .46 to .53 with an average value of .48 and a standard deviation of .027. For dual rear-axle vehicles, the parameter averages .50 with a standard deviation of .013. These results suggest that wheelbase and weight may be used to produce reasonably good estimates of pitch moment of inertia.

TYPE OF VEHICLE: Single Rear-Axle Trucks (Tractors)

Name	Vehicle Load Distribution	Test Weight lbs	Wheelbase in	Tandem Spacing in	Longitudinal Position Aft of Ft. Axle in	C.G. Hgt. Above Ground in	Pitch Moment of Inertia in-lb-sec ² /rad	Normalized Radius of Gyration in
GMC 6500 V-8	Dump Empty Springs Compressed	11,920	125		68.06	35.43	131,634	.52
GMC T3060 6500 V-8	Iron Blocks in Body Axle. Loads - 8700-F; 17,800-R	26,325	125		84.12	64.12	259,832	.49
GM Astro 95	Dump Empty Springs Compressed	15,749	143		57.37	38.83	176,556	.46
GM Astro 95	Dump Empty Springs Free	15,749	143		57.39	39.56	177,109	.46
GMC 6500 V-8	Dump Empty Springs Free	11,525	125		67.82	36.64	129,654	.53
GM 9500 Astro 95	Dump Roll Bar	14,415	139		69.76	36.24	159,824	.47
GMC 6500 V-8		10,770	109		47.68	35.42	123,358	.61
Ford 9000	Empty Flat Short Bed, Roll Bar	17,850	137		67.43	37.61	187,166	.46
Ford		10,828	156		83.18	37.84	145,327	.46
Ford 9000	Empty Bed Roll Bar	16,314	138		72.35	38.99	220,044	.52
Ford 9000	Sleeper Cab	13,861	134.5		49.6	37.21	138,510	.46
Freightliner	Load Rack	17,194	190		82.11	37.61	341,362	.46
GMC	5th Wheel	10,875	150		64.70	32.89	138,559	.47
IH, 1954	Bare Frame	9,656	140		55.04	32.54	88,192	.42

TYPE OF VEHICLE: Dual Rear-Axle Trucks (Tractors)

Name	Vehicle Load Distribution	Test Weight lbs	Wheelbase in	Tandem Spacing in	Longitudinal Position of Ft. Axle in	C.G. Hgt. Above Ground in	Pitch Moment of Inertia in-lb-sec ² /rad	Normalized Radius of Gyration in
GMC 9500	Diesel Springs Compressed	17,277	146	50	82.27	34.15	235,750	.50
GMC Astro 95		19,250	151.5	55.5	80.22	35.68	284,134	.50
GMC 9500 Diesel	Dump Empty	25,945	146.6	50	97.83	50.53	362,949	.50
GMC Astro 95	5th Wheel Empty	17,389	150.75	49	68.58	32.64	241,479	.49
GMC	High Box Partial Filled (Sand)	25,945	147	50	97.80	50.5	372,714	.51
Ford 9000	Flat Bed Roll Bar	20,975	144.5	50.12	77.76	35.95	302,995	.52
Ford A73 LNT 9000	Flat Bed Roll Bar	25,215	170	55	97.7	39.79	491,364	.51
White Freight	Empty	21,255	175	72	88.25	36.37	383,829	.48
International Harvester	Bare Frame	14,761	143	49	65.58	44.52	176,762	.48
Ford 9000	5th Wheel	17,135	185.75	50.5	90.48	30.80	318,715	.46
Ford 800	Bare Frame	11,383	150	50	81.8	30.82	161,347	.49

TYPE OF VEHICLE: Four-Wheel Drive

Ford Bronco	Ranger with Spare Tire & Top	3,793	92.5		39.59	27.19	21,212	
Ford Bronco	Modified Roll Bar - 2 Full Gas Cans - No Spare	3,673	92.5		41.17	28.62	24,963	

TYPE OF VEHICLE: Four-Wheel Drive

Name	Vehicle Load Distribution	Test Weight lbs	Wheelbase in	Tandem Spacing in	Longitudinal Position Aft of Ft. Axle in	C.G. Hgt. Above Ground in	Pitch Moment of Inertia in-lb-sec ² /rad	Normalized Radius of Gyration in
AMC Jeep CJ-5	No Spare	2,852	83		37.92	26.45	10,455	.45
AMC Jeep CJ-7	No Spare	2,756	93.5		40.99	24.8	11,853	.44
Chevrolet Blazer K-5 1974	With Spare	5,005	107		51.16	27.14	39,628	.52
AMC Jeep CJ-5	With Spare 8 Cyl.	3,895	83		39.92	24.12	14,710	.46
AMC Eagle	2-Door Empty	3,448	109		43.54	22.64	23,202	.47

TYPE OF VEHICLE: Passenger Cars

GM Nova	4-Door Empty	3,773	111		48.68	19.03	31,269	.51
AMC Pacer	2-Door Empty	3,275	104		43.23	21.38	20,275	.47
AMC Concord	2-Door Empty	3,244	108		45.89	21.14	23,731	.49
AMC Spirit	4-Door Empty	3,125	96.5		38.9	21.00	18,005	.49

TYPE OF VEHICLE: Pickup/Van

Dodge Van Sportsman	Empty	5,163	127.5		69.46	28.54	66,777	.55
Dodge Van Sportsman	Ambulance Equipped	6,050	127.5		65.08	33.14	78,000	.55

TYPE OF VEHICLE: School Bus

Name	Vehicle Load Distribution	Test Weight lbs	Wheelbase in	Tandem Spacing in	Longitudinal Position Aft of Ft. Axle in	C.G. Hgt. Above Ground in	Pitch Moment of Inertia in-lb-sec ² /rad	Normalized Radius of Gyration in
Ford		25,625	242		74.95 Forward of Rear Axle	47.74	801,654	.45
Ford		14,738	260		94.19	43.80	688,748	.52

TYPE OF VEHICLE: Travel Trailers

Airstream	31' Travel	5,545	At Support Position 169	33.125	42.6 Ahead of Rear Axle	42.38	123,352	
Holiday Rambler	31' Travel #1 Stock	6,950	At Support Position 167.5	39.8	73.8 Ahead of Rear Axle	49.9	253,119	
Holiday Rambler	31' Travel #2 Loaded with Sand Bags	4,540	At Support Position 169	32.5	33.6 Ahead of Rear Axle	35.07	87,584	
Holiday Rambler	31' Travel #2 Stripped	3,193	At Support Position 169	32	40.67 Ahead of Rear Axle	38.13	78,969	
Prowler Fleetwood H	Standard Equip.	3,238	At Support Position 139	30	30.54 Ahead of Rear Axle	38.9	40,758	
Starcraft 6	Empty Stan. Equip.	1,479	At Support Position 174.9	30	15.46	28.11	7,858	

TYPE OF VEHICLE: Equipment Trailers

Name	Vehicle Load Distribution	Test Weight lbs	Wheelbase in	Tandem Spacing in	Longitudinal Position Aft of Ft. Axle in	C.G. Hgt. Above Ground in	Pitch Moment of Inertia in-lb-sec ² /rad	Normalized Radius of Gyration in
Donahue Farm Trailer	3-Axle	5,961		36	34.75	34.86	79,229	
Ford Backhoe 555	With Backhoe & Front Loader		6'8"		65.32	24.8	262,130	
Ford Backhoe 555	Without Backhoe With Front Loader		6'8"		27.56	35.7	70,136	

3.0 ADDITIONAL VEHICLE INERTIAL MEASUREMENTS

UMTRI uses two additional arrangements to conduct moment of inertia measurements of heavy vehicles. These test set-ups are both much more temporary in nature than the facility of Section 2.0.

First is a swing facility designed to perform inertial measurements in either roll or pitch. The facility is shown in these two modes, respectively, in Figures 4 and 5. The test technique and data handling methods for this device are analogous to those describing moment of inertia testing in Section 2.0.

The second test method, used for yaw moment of inertia testing is shown in Figures 6 and 7. With the suspensions constrained to their static positions by cables, the vehicle is primarily supported at a pivot point located slightly aft of the vehicle c.g. This leaves a small portion of the vehicle weight (a few hundred pounds) to be supported by the front wheels. Under each of the front wheels are placed two steel plates separated by a number of ball bearings. Thus, the front wheels are free to move about on a horizontal plane. A grounded coil spring is attached at right angles to the vehicle frame some distance from the pivot point. With this arrangement, a small oscillation in yaw may be introduced and the period of oscillation, τ , determined. Using the notation of Figure 6, yaw moment of inertia of the vehicle, I_{zz} , may be determined using Equation (11).

$$I_{zz} = \frac{K_s \ell^2 \tau^2}{4\pi^2} - \frac{W}{g} \ell \text{ cg}^2 \quad (11)$$

The supporting pivot is illustrated in Figure 8. Unrestrained yaw motion results from the use of a hydrostatic bearing. Vehicle weight is supported by a single 3/4-inch ball atop the bearing in order to limit pitch and roll moments passed to the bearing. The test arrangement is shown in Figure 9.

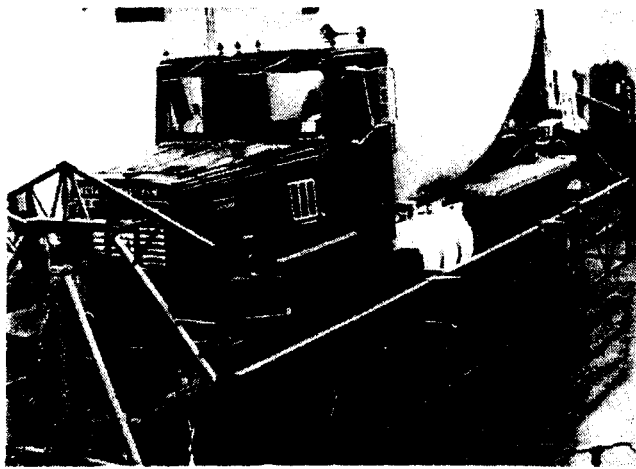


Figure 4

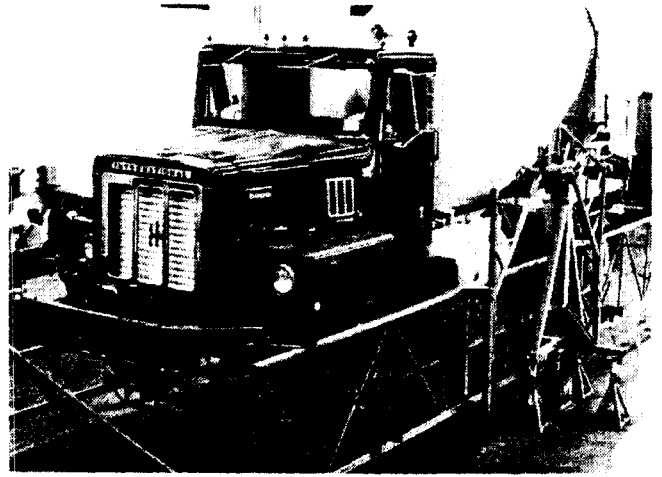


Figure 5

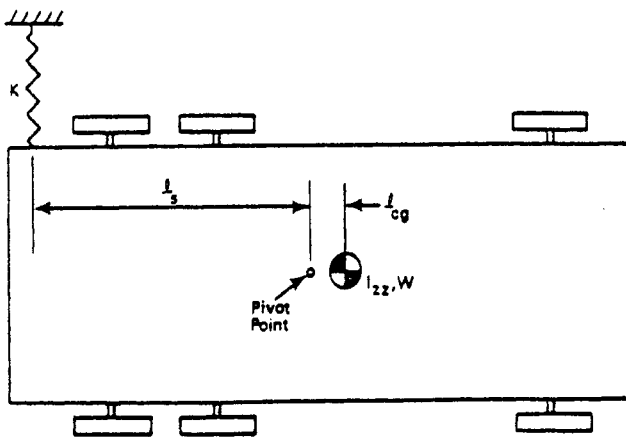


Figure 6

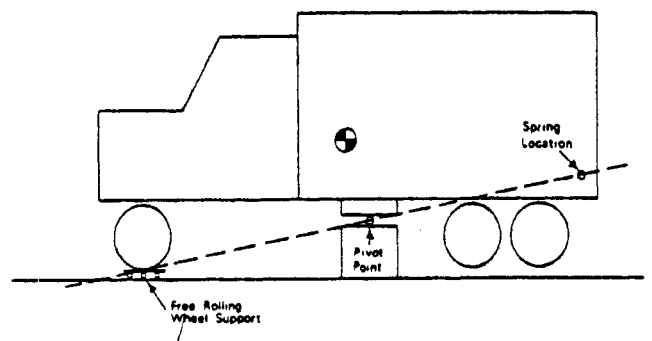


Figure 7

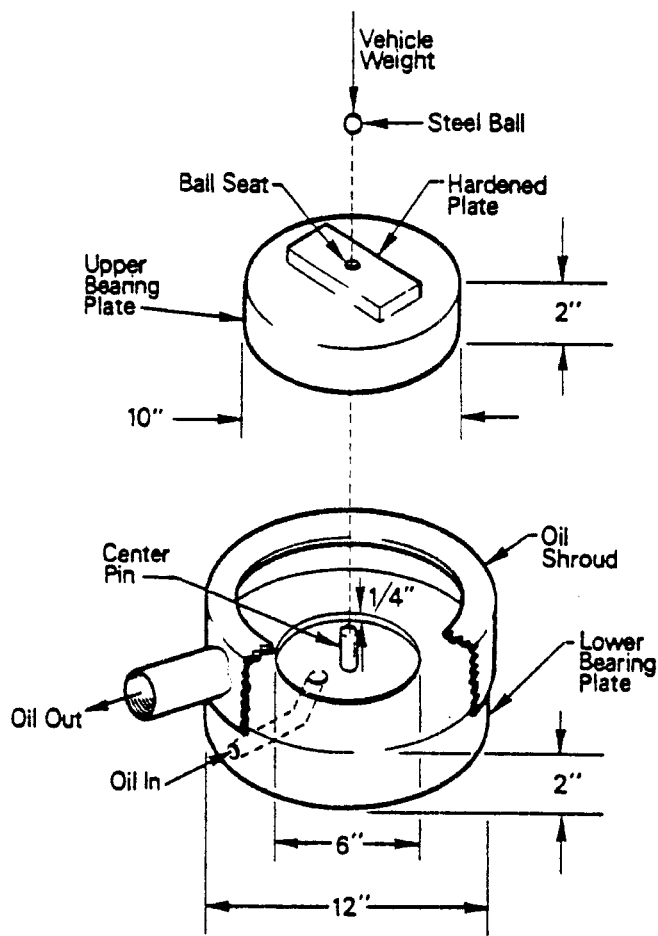


Figure 8

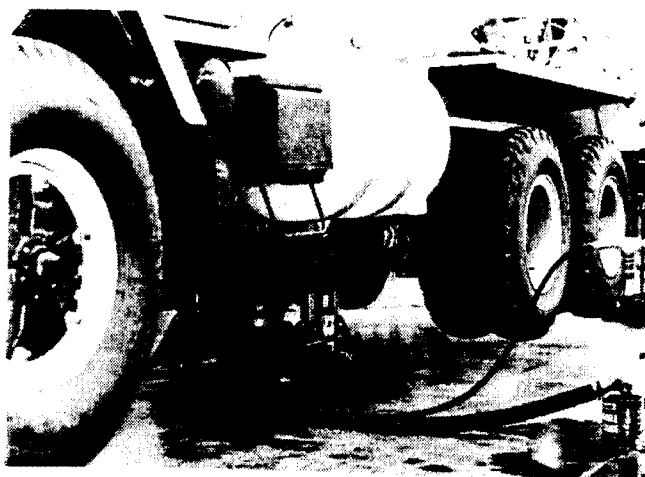


Figure 9

Under certain conditions, unwanted oscillations tend to appear during yaw inertia testing. A tendency for the vehicle to oscillate slightly in roll was noted. As it is supported during testing, the vehicle may roll about an axis passing through the ball bearing at the pivot point and the front tire contact point. (The front suspension is effectively rigid due to the constraining cables.) This axis is shown by the dashed line in Figure 7. To minimize the excitation of roll oscillations, the coil spring was anchored to the vehicle as close to this roll axis as possible. Furthermore, the spring constant, K , and the length, l_s , were chosen such that the natural yaw frequency of the system was considerably different from the roll frequency, thus reducing the tendency for yaw oscillations to excite roll oscillations.

An additional mode of oscillation was observed during yaw inertia tests. The construction of commercial vehicles typically results in considerable torsional compliance of the frame. Consequently, some vehicles showed a tendency to oscillate in a twisting manner along the length of their frames. This problem was effectively reduced by locating the spring near the horizontal centerline of the frame rails, thus reducing the moment resulting from the spring force which was passed into the frame.

Results for vehicles tested in roll and/or yaw are given in Table 2. Note that, for estimating purposes, yaw moment of inertia tends to be nearly equal to the pitch moment. Further, the radius of gyration in roll, for the two "bare-frame" heavy vehicles measured, is near 30 inches.

TABLE 2

Vehicle Description	Weight lbs	Wheelbase in	C.G. Height Above Ground in	C.G. Aft of Front Axle in	Moment of Inertia, in-lb-sec ² I_x I_y I_z	Radius of Gyration in Roll (in)(R_{xx})	Normalize Radius of Gyration in Pitch (in)(R_{yy}/R_{xx})
Packer Truck: Hill Model #A615E850 GMC 8500 V-6	18,000	150	55 1/4	131	71,400 476,800 453,500	39.1	.67
Concrete Mixer REX, 9-yd. drum I.H. F5070-6x4	23,600	102	58 1/3	103 1/2	95,500 505,000 536,500	39.5	.50
Medium Duty Van Truck GM CE 56703	9,380	168	44 1/4	98 1/4	61,800 296,800 308,300	50.4	.66
Motor Home Chevrolet (PA 31442)	9,395	157	44.3	91 1/4	33,639 205,900	37.2	.59
Straight Truck Ho Body Diamond Reo 6x4	13,740	189	31 1/4	85 1/4	28,000 283,200	28.0	.47
Tractor White 6x4	14,270	142	39 3/4	66 1/2	36,000 178,760	31.2	.49

4.0 VEHICLE COMPONENT MEASUREMENTS

Moments of inertia of vehicle components, particularly the roll inertia of unsprung masses and the polar inertia of tire/wheel/drum assemblies, have been measured for two heavy vehicles.

The test technique used is illustrated in Figure 10. As shown in this figure, the component of interest was suspended on a three-cable, multifilar pendulum. A small rotational oscillation about the vertical axis was introduced and the period determined.

The equation for calculating the moment of inertia, I , about the c.g. of the assemblies is

$$I = \frac{Wr^2\tau^2}{4\pi^2\ell} + \frac{W_b r^2}{4\pi^2\ell} (\tau^2 - \tau_0^2) \quad (12)$$

where

W = test weight of the assembly

W_b = weight of the supporting platform

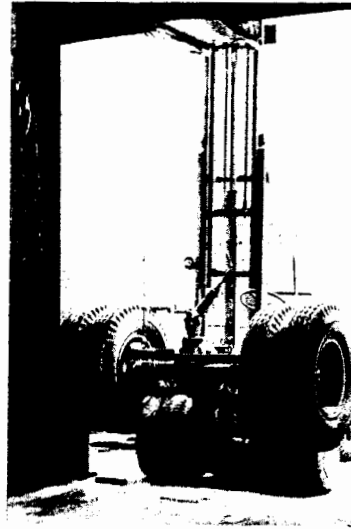
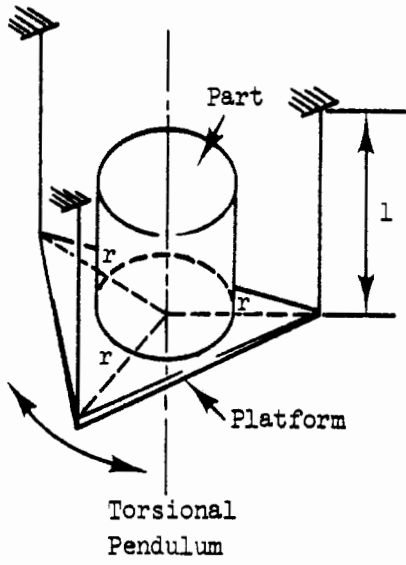
ℓ = length of the supporting cables

r = horizontal distance from center of platform to supporting cables

τ = period of oscillation of platform plus assembly

τ_0 = period of oscillation of platform only.

Data gathered in this manner is presented in Table 3.



Pendulum Suspended by Means of Lift Truck

Figure 10

Table 3

Polar Moment of Inertia of Wheel/Tire/Drum Assemblies:

<u>Description</u>	<u>Inertia (in-lb-sec²)</u>	<u>Weight (lb)</u>	<u>Radius of Gyration (in)</u>
18 x 22.5 tire, spoke wheel, 15" drum	163		
10 x 20 duals, spoke wheels, 15" drum	205		
10 x 20 tire, spoke wheel, 15" drum	103		
10 x 20 duals, spoke wheels, 16.5" drum	231		
10 x 20 tire, disk wheel 15 x 1.43 disk brake (294 lb)	99	294	11.4
10 x 20 duals, disk wheels 15 x 1.43 disk brake (513 lb)	211	513	12.6
10 x 20 tire, spuke wheel 15 x 4 drum (319 lb)	115	319	11.8
10 x 20 duals, spoke wheel 15 x 6.5 drum (569 lb)	241	569	12.8

Roll Moment of Inertia, Unsprung Mass*

Front unsprung mass, 10 x 20 tires, spoke wheels, 15" drum	3719	1321	33.0
Drive axle assembly; 10 x 20 duals, 15" drum, spoke wheels	4458	2330	27.2
Trailer axle assembly; 10 x 20 duals, 15" drum, spoke wheels	4100	1520	32.3
Front unsprung mass, 10 x 20 tires, disk wheel, 15 x 1.43 disk brake		1093	
Drive axle unsprung mass, 10 x 20 duals, disk wheels, 15 x 1.43 disk brake		2133	

*Includes both leaf springs.