J. D. Murray, *Lectures on Nonlinear Differential-Equation Models in Biology*, Oxford U. P., 1977, ix+370 pp., index; $24.50.

This is a collection of deterministic models of problems which come from the biological sciences, for the most part. The five chapters of the book take up the first three-quarters of the text; the remainder is five mathematical appendices. There is no unifying biological theme; this is a collection which reflects the authors' experience and taste. The mathematical theme is nonlinear differential equations, except for Chapter 3, which deals with a linear problem. Each chapter starts with a presentation of the phenomenon to be modeled to set the stage for the mathematical analysis. The attempt is laudatory, but it is a difficult task; the description of background varies from "barely enough" to "sketchy."

The five chapters are: 1. Enzyme Kinetics; 2. Facilitated Diffusion; 3. Reduction of Dimensionality in Diffusion Processes: Antenna Receptors of Moths; 4. Biological Oscillators I. Homogeneous Temporal Oscillations; 5. Biological Oscillators II. Spatial Structure and Nonlinear Wave Phenomena. The mathematical appendices are: A1. Singular Perturbation Theory: Matched Expansion Procedures; A2. Boundary Conditions and Facilitated Diffusion: Mathematical Analysis; A3. Linear Diffusion Equation: Special Solutions; A4. Hopf Bifurcation Theorem and Limit Cycles; A5. Some Mathematical Results for Reactive-Diffusion Systems.

Chapter 1, on enzyme kinetics, is on the whole fairly well done. The background material is really too brief for the nonbiologist to appreciate the varieties and complexities of enzyme reactions. But the author shows the relation between the classical biochemical approach, the pseudo-stationary-state approximation, and singular perturbation theory and shows that the classical solutions are the outer solutions given by singular perturbation theory. He rightly emphasizes the importance of transformations to dimensionless variables. The definition of cooperative phenomena (p. 33) is so bare and incomplete as to be misleading.

Chapter 2 is on a phenomenon which occurs in some special examples of simultaneous diffusion and reaction of two chemical species. A biologically important example is the facilitation of oxygen diffusion by myoglobin. The description of the background is better than for the chapter on enzyme kinetics, and in fact the whole chapter hangs together better. Again, it illustrates the use of singular perturbation theory, this time for the solution of a two-point boundary-value problem in simultaneous second-order, nonlinear ordinary differential equations. The author contributed significantly to the mathematical

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analysis of this problem, and his involvement and experience comes through. For the most part he presents his work plus references to others'. The formulation of the boundary conditions is not discussed as deeply as I should like to see it, particularly the problem of translating the experimental procedures into the mathematical statement of boundary conditions.

Chapter 3 is concerned with an analysis of how the antennae of the male silk moth can collect and detect the pheromone released by the female. It illustrates the importance of "reduction of dimensionality" in a diffusion-limited phenomenon. The mathematical problem is that of the coupling of two diffusion processes, one of lower space dimensionality than the other. In this example, it is diffusion of pheromone in three-space to the surface of the antennal hairs and adsorption to and diffusion in a surface phase to a pore where the pheromone has access to the sensory cells. Mathematically the problem is one of linear systems that are very difficult to handle. The chapter doesn't seem to quite fit in with the others with their emphases on nonlinear systems arising in reaction-diffusion problems.

Chapter 4 deals with periodic phenomena when there is no spatial dependence. The chapter starts with a description of the action of regulator genes which is so abbreviated I doubt it will be of use to anyone who isn't already acquainted with the subject. The examples presented are the Lotka-Volterra system of equations, the Belousov-Zhabotinskii reaction and some models of the control of enzyme synthesis. The Lotka-Volterra system is so unrealistic, and so many papers have been devoted to the analysis of these unreal equations, that concerned biologists are out of patience with this sort of work. To the author's credit he points this out; I wish he had discussed the modifications (extensive) required to introduce some realism. The Belousov-Zhabotinskii reaction is an example of oscillations in a chemical system, an uncommon phenomenon which has excited much interest over the last few years. The discussion of this and of the characteristics of biological oscillators is good.

In Chapter 5 the discussion of oscillatory phenomena is extended to examples in which there is spatial dependence, so that one enters the domain of propagated waves in nonlinear problems. The examples include Fisher's model of gene propagation in a population, traveling waves in Belousov-Zhabotinskii reaction systems and a little material on traveling waves in reaction-diffusion problems in general.

Of the appendices I found the treatment of singular perturbation theory and of the boundary conditions for the facilitated diffusion problem readable. Appendices A4 and A5 are concerned with proofs and, in conformity with the established canons of mathematics, give up ready understandability for rigorous development.

Who will find this book useful? The applied mathematician who wants an introduction to nonlinear problems, particularly problems of reaction-diffusion type. Another potential customer is the engineer with a good background in applied mathematics and interest in problems that arise from biology. For the biologist it will be useful only to those who have a mathematical background equivalent to that of the engineer. The biologist with a more limited mathematical background will have trouble because the book does not develop the mathematical techniques in sufficient detail or in progressive steps to teach as it goes along; you have to come to this fairly well prepared.

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