

Three Contraceptive Acceptance Strategies*

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ABSTRACT

Three main classes of contraceptive acceptance strategy may be distinguished: "fixed duration T " (women counseled to accept T months after childbirth); "postmenorrhagic" (accept directly after the first postpartum menses), and "mixed T " (accept T months after childbirth or after first menses, whichever occurs sooner). Any two strategies may be compared by means of a probability model simulating the first passage times from childbirth to next pregnancy of two cohorts of mothers identical in their fecundity and in the effectiveness and continuation with which contraception is practiced, but contrasting in their acceptance regimens. Of particular interest is the class of mixed- T strategies, which have not previously been analyzed. The efficiency of the mixed- T rule at least equals, and for most T -values exceeds that of the corresponding fixed-duration rule both in the short run (lower cumulative pregnancy rate during the first few months) and in the long run (greater mean interval to next conception). Conditions for the superiority of the mixed- T over the postmenorrhagic are also given. Several results are illustrated with reference to a Bangladesh subpopulation.

1. INTRODUCTION

The exact duration after childbirth when a lactating mother will resume menstruation and ovulation cannot be predicted exactly, and hence contraception is often initiated during the postpartum period of amenorrhea and anovulation. However, to the extent that contraceptive practice and anovulation overlap, the former is redundant inasmuch as the woman is already protected. Moreover, when discontinuation rates are high, a significant fraction of practice time may be of this character [3]. An extreme case

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has been encountered in an area served by the Cholera Research Laboratory (CRL) in Bangladesh [5]. Here amenorrhea is extremely long, averaging about 19 months [2]. At the same time, usage of the pill is brief enough so that only 35 percent are continuing after one year. Nor in a context of incipient birth control is reacceptance of the pill or some other method at all assured after a discontinuation. In this situation, it is far from clear which among many possible acceptance strategies is the best one.

Three main types of acceptance strategies following childbirth may be distinguished. Subject to a *fixed duration* T strategy, women are encouraged to accept T months after childbirth. As a special case, when T is set at 0 or 1, it may be called a *postpartum* strategy. Under a *postmenorrhoeic* strategy, mothers are urged to accept right after their first postpartum menses. What will be called a *mixed- T* strategy is one in which women are counseled to accept either T months after childbirth or right after their first postpartum menses, whichever occurs sooner. This third strategy is really a mixture of the first two. Moreover, as T is made close to zero, the mixed regimen converges to a postpartum strategy; whereas if T is made large, the mixed strategy becomes essentially a postmenorrhoeic policy. CRL staff are currently conducting a field experiment to compare the mixed 6-month and 18-month rules [4].

Plainly one cannot rely on field experiments to evaluate all possible acceptance strategies. The relative efficiencies of pairs of acceptance strategies are much more conveniently assessed by means of a probability model which compares the expected intervals from childbirth to next conception of two cohorts of women sharing a common fecundity and practicing contraception of identical effectiveness and continuation, but differing in their acceptance rules and consequently contrasting in the proportion who accept at all before another conception and among those accepting, contrasting also in the proportion whose practice either extends into, or starts during, the fecundable period.

Two previous studies, [9] and [10], have utilized this approach, the first exploring optimal T -values among fixed-duration- T strategies and the second comparing the postmenorrhoeic and postpartum strategies. To the writers' knowledge, no previous analysis of the mixed- T strategies has been published, despite its being possible, as is done below, to demonstrate that they yield longer average intervals to next conception than corresponding fixed-duration- T policies.

One might think that the postmenorrhoeic acceptance strategy would always yield the longest interval to next conception. By waiting until her first postpartum menses, the woman is avoiding overlap with amenorrhea. At the same time she is using her first menstrual onset as an indicator of when she is no longer protected. However, more often than not, an ovulation precedes the first potential menses by about two weeks [8], which

under the postmenorrhic rule means an unprotected month. Indeed, it will be demonstrated below that for any given set of fecundity parameters (barring sterility) there is an efficiency of contraception above which the mixed-T strategies for some subset of T values yields longer mean intervals to next conception than the postmenorrhic rule.

The present paper grew out of a request by CRL staff to the senior author to extend previous algebra in order (1) to encompass the family of mixed-T acceptance strategies and (2) to derive the probability distribution function (pdf) as well as the mean of the interval to next conception. One acceptance rule may generate a longer average delay to next conception than another but in the short run allow more pregnancies. Thus it might find favor with the family-planning leadership charged with reducing the country's growth rate by a targeted amount, but seem disadvantageous to the field personnel responsible for distributing the contraceptive. A thorough mathematical appraisal of two acceptance rules should cover short-range as well as long-range efficiency.

The main objective of this paper is to present a systematic statement of this extended algebra together with principal analytical results. To add interest, some of these results are illustrated in terms of the Bangladesh context cited above. For a more detailed application to the Bangladesh case, see the policy-oriented paper of Langsten et al. [5].

Assumptions are enumerated in the next section. Sections 3–5 present derivations for the postmenorrhic, the fixed-duration, and the mixed strategies, in that order. Section 6 is concerned with criteria to facilitate comparing pairs of strategies. The numerical illustrations of Sec. 7 deal with relative efficiencies of different acceptance rules in the Bangladesh context. The generalizability of these results is discussed in the last section. An appendix touches on computer aspects.

2. ASSUMPTIONS

Each cohort of women is represented by a discrete-time, expected-value probability model designed to follow the woman from childbirth through anovulatory and fecundable periods to next conception. The women are homogeneous with respect to their fecundity and contraceptive practice. The time unit is one month, which all menstrual cycles are assumed to equal.

Respecting fecundity, care must be taken to distinguish between amenorrhea and anovulation. Both lengths are measured in whole months. Amenorrhea is the interval from childbirth to first menses postpartum; anovulation stretches from childbirth to start of the first ovulatory cycle. If the first menstrual cycle is anovulatory and the second ovulatory, then the two intervals, amenorrhea and anovulation, coincide. When the first

menstrual cycle is ovulatory, with an ovulation preceding by roughly two weeks the first menses, amenorrhea exceeds anovulation by one month.

It is necessary to fix the range in number of anovulatory cycles that may precede the first ovulatory one, thereby limiting the possible differences of length between amenorrhea and anovulation. Specifically, it is assumed that anovulatory cycles number 0 or 1 with probabilities of λ and $1-\lambda$.

Women share a common pdf of anovulation, defined later. They also share a common natural fecundability f , that is, monthly chance of conception when fecundable and not protected by contraception. During anovulation, the monthly chance of conception is zero. During the last month of amenorrhea, it is f of 0 according as the first cycle is ovulatory or not.

Additional assumptions simplifying the algebra but scarcely affecting results are:

- (i) women always initiate contraception near the start of a month;
- (ii) they discontinue near the end of the month;
- (iii) they conceive about the middle of the month;
- (iv) they do not accept during a recognizable pregnancy.

Respecting (iv), the position is taken that if a woman has not accepted prior to conceiving, then the continuing absence of menses for more than two weeks after conception alerts her not to accept during the pregnancy.

A contraceptive is defined by its monthly discontinuation rate d' and its effectiveness e , both parameters having the range of 0 to 1. During an anovulatory month the risk of discontinuation is d' . During a fecundable month, there is a probability $p=(1-e)f$ of accidental pregnancy, a risk $d=(1-p)d'$ of discontinuing usage for a reason other than pregnancy, and a chance $1-d-p$ of continuing usage into the next month. One may think of p as a "residual fecundability," namely, that remnant $(1-e)$ of natural fecundability f left by contraception of effectiveness e .

It is assumed that after discontinuing contraception once, a woman does not reaccept before the next conception. If it is desired to allow for reacceptances, this can be done crudely by appropriately lowering the discontinuation rate d' .

Another basic simplifying assumption is that whatever the acceptance rule, it is followed scrupulously, with all women accepting at prescribed times without either procrastination or advance anticipation.

A last simplification is that the analysis deals solely with women whose infant survives the period of lactation amenorrhea; and in Sec. 7 a pdf of anovulation is chosen accordingly. When an infant dies in the early months of its life and breastfeeding is suspended, the still amenorrheic women may expect the return of menses usually within a couple of months. Consequently, any acceptance strategy carries the implicit proviso that the still-amenorrheic mother whose breastfeeding is abruptly terminated should seek

contraceptive protection right away if she wants to be confident about delaying the next pregnancy.

With respect to the fixed-duration and mixed strategies, the symbol T demands careful definition. T signifies the total number of completed months elapsing from childbirth to the prescribed time to initiate contraception, which time falls near the start of the $(T+1)$ th ordinal month. For example, if anovulation is two months and T is set at 4, the first two months are months of anovulation, the next two are unprotected months, and the start and end of month 5 [spanning the exact interval $(T, T+1)$] are the earliest possible times to initiate and discontinue the method respectively.

Defined as a measure of long-run efficiency of an acceptance strategy is the mean interval M from childbirth to next conception, including into the average those women who conceive before the prescribed time for acceptance. An equivalent measure is the added months Δ to next conception over the duration to be expected in the absence of contraception. The cumulative pregnancy proportions $U(i)$, $i=1, \dots, 18$, will serve as the measure of short-range efficiency.

For two reasons the relative efficiencies of the various acceptance strategies work out differently for the subset of acceptors than for the entire set of postnatal women. First of all, the acceptors include no women whose acceptance is precluded by conception. Secondly, under all rules except the postamenorrhic, acceptance before conceiving again is selective of lengthy anovulation. Subject to a common probability distribution of anovulation, some women experience by chance anovulation shorter than T months and therefore, either under the mixed- T or the fixed-duration- T strategy, run a risk of conceiving before having an opportunity to accept contraception; whereas the remaining women who by chance experience anovulation of at least T months are assured of accepting. Thus, the selection in question is a matter of conditional probabilities. Being an acceptor is certain if anovulation is T months or more, but less than certain if anovulation is shorter. Accordingly, it will be worthwhile to obtain a parallel set of results for acceptors alone.

In the formulas below, the minimum length of anovulation is set at 2.0, but otherwise the pdf is left general. A minimum length of one month is appropriate for nonlactators; but for women prolonging anovulation by breastfeeding, it becomes about eight weeks [8].

3. POSTAMENORRHEIC STRATEGY

The rule having the simplest algebra is the postamenorrhic. Women are asked to accept right after their first postpartum menses. For any given anovulation length $j=2, 3, \dots$, the natural fecundability retains a value of f , while the chances of the first cycle being ovulatory are λ . Therefore, for any

anovulation length, a proportion λf conceive before acceptance; $\lambda(1-f)$ survive one unprotected fecundable month before accepting; and $1-\lambda$ accept at the end of anovulation thanks to the first menstrual cycle being anovulatory. Overlap between contraception and anovulation is zero, because either contraception is not accepted at all or else it is accepted at end of anovulation or one month later. Taken across all possible anovulatory lengths, the proportion of postnatal women conceiving before the prescribed time to accept is

$$P_c = \sum_{j=2}^{\infty} a_j \lambda f = \lambda f,$$

while the complementary proportion P_a accepting is $1-\lambda f$, which is also the proportion P_f of women accepting during a fecundable month.

Among the P_f women, the expected length of contraceptive practice is $1/(d+p)$ months regardless of outcome, accidental pregnancy, or discontinuation for another reason. That fraction $d/(d+p)$ of the P_f women who stop contraception for a reason other than pregnancy can expect an additional $1/f$ months to conception.

From these considerations, it follows that the mean interval from birth to next conception of all women is

$$M = \lambda f \left(\sum_{j=2}^{\infty} a_j j + 1 \right) + \lambda(1-f) \left[\sum_{j=2}^{\infty} a_j j + 1 + \frac{1}{d+p} + \frac{d}{d+p} \left(\frac{1}{f} \right) \right] \\ + (1-\lambda) \left[\sum_{j=2}^{\infty} a_j j + \frac{1}{d+p} + \frac{d}{d+p} \left(\frac{1}{f} \right) \right],$$

which simplifies to

$$M = \sum_{j=2}^{\infty} a_j j + \frac{1}{f} + (1-\lambda f) \left[\frac{1}{d+p} - \frac{P}{d+p} \left(\frac{1}{f} \right) \right].$$

Now $\sum_{j=2}^{\infty} a_j j + 1/f = M - \Delta$ constitutes the mean interval to conception to be expected in the absence of contraception. Moreover, the two components of the remaining term in the expression for M have interpretations worth noting. $1-\lambda f = P_f$ signifies the proportion of women "benefitting" from contraception in the sense of practicing it during one or more fecundable months and therefore having the possibility of lengthening their interval to next conception through contraception. The last factor, which equals

$$\frac{1}{d+p} + \left(\frac{d}{d+p} \right) \frac{1}{f} - \frac{1}{f},$$

may be construed as the mean duration added by contraception per woman "benefitting." In view of its importance, this last term is assigned the symbol C to give $\Delta = P_f C$.

Useful as a short-term measure of efficiency is the cumulative rate of pregnancy by end of the i th ordinal month after childbirth, $U(i) = \sum_{j=2}^i C(j)$, $i = 2, \dots, 18$. During the i th month, a proportion $A(i) = \sum_{j=2}^i a_j$ of the post-natal women remain anovulatory; at its start $F(i)$ are fecundable and unprotected, and $P(i)$ are fecundable and protected by contraception. With anovulation at least two months long,

$$C(i) = F(i)f + P(i)p, \quad i = 3, 4, \dots, \\ = 0, \quad i = 1, 2.$$

Because the expressions for $A(i)$ and $P(i)$ are so much simpler than for $F(i)$, it is convenient to derive the latter as a residual, namely,

$$F(i) = 1 - A(i) - P(i) - U(i-1).$$

To be fecundable and protected by contraception at the start of ordinal month i —which spans exact interval $(i-1, i)$ —requires having an anovulatory length $j \leq i-2$, accepting at times j or $j+1$ with probabilities $1-\lambda$ and $\lambda(1-f)$ respectively, (or else having an anovulatory length $j=i-1$ and accepting at time $i-1$) and continuing to practice contraception up to duration $i-1$. In symbols,

$$P(i) = \sum_{j=2}^{i-1} a_j (1-\lambda)(1-d-p)^{i-j-1} + \sum_{j=2}^{i-2} a_j \lambda(1-f)(1-d-p)^{i-j-2}, \\ = a_2(1-\lambda), \quad \begin{matrix} i \geq 4, \\ i = 3, \\ i = 1, 2. \end{matrix} \\ = 0,$$

An asterisk will be used to distinguish coefficients pertaining just to acceptors rather than all postnatal women. Among acceptors there can be no selection toward longer than average anovulation, since the risk λf of pregnancy before starting contraception is constant over all anovulatory lengths. Thus $a_j^* = a_j$ for all j .

By definition $P_c^* = 0$, while $P_f^* = 1 - P_c^* = 1$.

Independent of length of anovulation, a proportion $\lambda(1-f)/(1-\lambda f)$ of acceptors each contribute one fecundable month before acceptance, and the remainder, $(1-\lambda)/(1-\lambda f)$ contribute zero. Collecting results,

$$M^* = \sum_{j=2}^{\infty} a_j j + \frac{\lambda(1-f)}{1-\lambda f} \left(1 + C + \frac{1}{f} \right) + \frac{1-\lambda}{1-\lambda f} \left(C + \frac{1}{f} \right).$$

Let us define $\Delta^* = M^* - \sum a_j - 1/f$, to assure that $M^* - M = \Delta^* - \Delta$. Then

$$\Delta^* = \frac{\lambda(1-f)}{1-\lambda f} + C,$$

which, incidentally, simplifies to $\lambda(1-f)/(1-\lambda f) + 1/d'$ if $e=1$. Note that in this special case, the gain in duration to next conception is slightly greater than $1/d'$ because the fact of acceptance insures that if the first cycle had been ovulatory, conception did not occur during that unprotected month.

4. FIXED-DURATION STRATEGIES

Most of the treatment of this section is predicated on T , the prescribed time for acceptance, being three months or longer, thereby exceeding the minimum length of anovulation. The consequences of setting T at 0, 1, or 2 are noted at the end of this section.

Barring a prior pregnancy, contraception under a fixed-duration rule is initiated T months after childbirth. For anovulatory lengths less than T , there is a risk of conceiving before acceptance. Given that $j = T - k$, this risk equals

$$\begin{aligned} P_c(T-k) &= f + (1-f)f + (1-f)^2f + \cdots + (1-f)^{k-1}f \\ &= 1 - (1-f)^k. \end{aligned}$$

The range of k is 1 to $T-2$, yielding

$$P_c = \sum_{j=2}^{T-1} a_j [1 - (1-f)^{T-j}].$$

Two groups benefit from contraception. First are those who, though starting practice during anovulation, continue it into the fecundable period. Their proportion is

$$P_x = \sum_{j=T+1}^{\infty} a_j (1-d')^{j-T}.$$

Second are those whose anovulation lasts less than T months, but who thanks to escaping pregnancy during $T-j$ unprotected months accept contraception at duration T in a fecundable state. Their proportion is

$$P_f = \sum_{j=2}^T a_j (1-f)^{T-j}.$$

Of the total proportion $P_\alpha = \sum_{j=T+1}^{\infty} a_j$ who accept while still anovulatory, a fraction $P_\alpha - P_x$ also discontinue during anovulation and therefore draw no benefit from contraception.

The mean interval from childbirth to next conception may be derived as the mean length of anovulation plus the expected fecundable months contributed by the four subpopulations having the proportions P_c , $P_\alpha - P_x$, P_x , and P_f (which sum to 1.0), giving

$$\begin{aligned} M &= \sum_{j=2}^{\infty} a_j j + \sum_{j=2}^{T-1} a_j \sum_{i=0}^{T-j-1} (1-f)^i f(i+1) + (P_\alpha - P_x) \left(\frac{1}{f} \right) \\ &\quad + P_x \left(C + \frac{1}{f} \right) + \sum_{j=2}^T a_j (1-f)^{T-j} \left\{ (T-j) + C + \frac{1}{f} \right\} \\ &= \sum_{j=2}^{\infty} a_j j + \frac{1}{f} + (P_f + P_x) C, \end{aligned}$$

or

$$\Delta = (P_f + P_x) C.$$

Once again the mean duration Δ added by contraception may be construed as the proportion benefitting from contraception times the per capita gain per benefitter.

When anovulation exceeds T , then overlap occurs from T onward, lasting until discontinuation or end of anovulation whichever transpires first. Accordingly, the overlap for given anovulation length $j > T$ is

$$\begin{aligned} \phi(j) &= \phi(T+k) \\ &= d' + 2(1-d')d' + \dots + k(1-d')^{k-1}d' + k(1-d')^k \\ &= \frac{1 - (1-d')^{j-T}}{d'} \end{aligned}$$

and taken across all anovulatory lengths, the mean overlap is

$$\phi = \sum_{j=T+1}^{\infty} a_j \phi(j).$$

Adopting the approach taken in Sec. 3 to $U(i)$, one need derive a new expression only for $P(i)$. To be fecundable and practicing contraception near the start of the i th ordinal month—spanning exact interval $(i-1, i)$ —implies acceptance at T and continued practice $i-1-T$ months, the

probability of which is affected by how many of those months are anovulatory. Thus,

$$\begin{aligned} P(i) &= \sum_{j=2}^{T-1} a_j (1-f)^{T-j} (1-d-p)^{i-1-T} \\ &\quad + \sum_{j=T}^{i-1} a_j (1-d')^{j-T} (1-d-p)^{i-1-j}, \quad i > T, \\ &= 0, \quad 0 \leq i \leq T. \end{aligned}$$

Acceptance selects against anovulatory lengths shorter than T , since if $j < T$, there is only a $(1-f)^{T-j}$ chance of escaping pregnancy during $T-j$ unprotected months. It follows that

$$\begin{aligned} a_j^* &= (1-f)^{T-j} a_j / P_a, \quad j = 2, 3, \dots, T-1 \\ &= a_j / P_a, \quad j = T, T+1, \dots, \end{aligned}$$

with $P_a = 1 - P_c = 1 - \sum_{j=2}^{T-1} a_j [1 - (1-f)^{T-j}]$. Clearly, the selection intensifies as T is made greater or the larger is f . Also, plainly, if $T \geq 3$,

$$\phi^* = \sum_{j=2}^{\infty} a_j^* \phi(j) > \sum_{j=2}^{\infty} a_j \phi(j) = \phi.$$

P_f^* differs from P_f , and P_x^* differs from P_x , only with respect to the pdf of anovulation. Given that anovulation lasts j months ($2 \leq j \leq T-1$), the number of fecundable months before acceptance per acceptor is $T-j$. From these considerations,

$$\begin{aligned} M^* &= \sum_{j=2}^{\infty} a_j^* j + (P_a^* - P_x^*) \frac{1}{f} + \sum_{j=2}^{T-1} a_j^* (1-f)^{T-j} (T-j) \\ &\quad + (P_f^* + P_x^*) \left(C + \frac{1}{f} \right). \end{aligned}$$

Recalling our definition $\Delta^* = M^* - \sum a_j j - 1/f$,

$$\begin{aligned} \Delta^* &= \left(\sum_{j=2}^{\infty} a_j^* j - \sum_{j=2}^{\infty} a_j j \right) + \sum_{j=2}^{T-1} a_j^* (1-f)^{T-j} (T-j) \\ &\quad + (P_f^* + P_x^*) C. \end{aligned}$$

Thus the difference $\Delta^* - \Delta$ has three sources: longer average anovulation, a selection for not conceiving before acceptance when anovulation terminates earlier than T , and a higher proportion benefitting from contraception.

Finally, if T is set at 0, 1, or 2, there can be no conceptions before acceptance. All women accept at T , thereby eliminating any selection on anovulation. Accordingly, several of the above formulas simplify. For example, the second and the last term in the expansion of M vanish.

5. MIXED STRATEGIES

If T is set at 0, 1, or 2 months, the mixed strategy becomes indistinguishable from the corresponding fixed-duration rule, since it is being assumed that menses cannot occur earlier than end of the second month. Accordingly, the following discussion is predicted on $T \geq 3$.

Under any mixed strategy, $P_a = \sum_{j=T+1}^{\infty} a_j$ women will be accepting at time T in an anovulatory state. If anovulation lasts exactly T months, women will be starting contraception just after end of anovulation, at the start of the first fecundable month. If anovulation lasts $T-k$ months ($k = 1, 2, \dots, T-2$), there is a $1-\lambda$ chance of accepting contraception at time $T-k$ at the start of the first fecundable month, a probability λf of conceiving during ordinal month $T-k+1$, and a probability $\lambda(1-f)$ of starting contraception (at the start of the second ovulatory cycle) at time $T-k+1$. From these considerations follow the formulas

$$P_c = \sum_{j=2}^{T-1} a_j \lambda f,$$

$$P_f = \sum_{j=2}^{T-1} a_j (1-\lambda f) + a_T,$$

and

$$P_x = \sum_{j=T+1}^{\infty} a_j (1-d')^{j-T}.$$

Note that the expression for P_x is the same as in Sec. 4.

The same approach to M as employed in Sec. 4 yields

$$M = \sum_{j=2}^{\infty} a_j j + \sum_{j=2}^{T-1} a_j \lambda f + (P_a - P_x) \frac{1}{f} + P_x \left(C + \frac{1}{f} \right) \\ + P_f \left(\frac{\lambda(1-f)}{1-\lambda f} + C + \frac{1}{f} \right),$$

a formula which can be shown to translate into $\Delta = (P_f + P_x)C$.

There being overlap with contraceptive practice only if anovulation lasts $T+1$ months or longer, the value of ϕ is identical for mixed- T and fixed-duration- T rules.

Necessary to the derivation of $U(i)$ is an expression for $P(i)$. If $i \leq T$, the expression for $P(i)$ under a mixed strategy is that obtained for the post-amenorrhoeic strategy, any acceptance prior to T being prompted by the occurrence of menses. The T th month spans $(T-1, T)$. To be practicing contraception during this month, or an earlier one, implies an acceptance triggered by menses before time T .

To be fecundable and practicing contraception during the i th month when $i > T$ implies acceptance at T or earlier and continued practice into the i th month, the probability of which depends both on the month of acceptance and the number of previous months of anovulation. Thus, if $i > T+1$,

$$P(i) = \sum_{j=2}^{T-1} a_j [(1-\lambda)(1-d-p)^{i-j-1} + \lambda(1-f)(1-d-p)^{i-j-2}] \\ + \sum_{j=T}^{i-1} a_j (1-d')^{j-T} (1-d-p)^{i-j-1}.$$

A selection, typically weak because of the small value of λf , operates in favor of anovulatory lengths T months or longer. Only if $j < T$ is there a chance of conceiving before accepting contraception. Thus,

$$a_j^* = (1-\lambda f)a_j/P_a, \quad j=2,3,\dots,T-1, \\ = a_j/P_a, \quad j \geq T,$$

with $P_a = 1 - P_c$.

Arguments which by now are familiar lead to

$$M^* = \sum_{j=2}^{\infty} a_j^* j + (P_a^* - P_x^*) \frac{1}{f} + \sum_{j=2}^{T-1} a_j^* \frac{\lambda(1-f)}{1-\lambda f} \\ + (P_f^* + P_x^*) \left(C + \frac{1}{f} \right),$$

and by definition, $\Delta^* = M^* - \sum_{j=2}^{\infty} a_j j - 1/f$.

6. COMPARING STRATEGIES

Other things equal, the expected interval M to next conception generated by any acceptance strategy increases as either e or $\sum_{j=2}^{\infty} a_j j$ is augmented or as either d' or f is reduced. Lowering λ also increases M for the post-amenorrhoeic and mixed- T strategies. However, of more central concern to the present paper are the relative efficiencies of acceptance rules, which have been shown to be directly proportional to $P_f + P_x$ terms, since $\Delta = (P_f$

$+ P_x)C$ with C a strict function of the parameters d' , e , and f . Hence the ranking of acceptance strategies with respect to M or Δ depends on the fraction $P_f + P_x$ of postnatal women whose practice of contraception extends into, or commences during, the fecundable period. That the parameter e does not appear in any expression for $P_f + P_x$ means that the common level of contraceptive effectiveness does not affect the ranking of acceptance strategies.

It is also worth remarking that the absolute reduction $L/(P_f + P_x)$ per benefitter due to contraception resulting from accidental pregnancies is also independent of acceptance strategy. The quantity L signifies the change in M (or equivalently Δ) when 1 is substituted for e , all other factors remaining the same. That is,

$$\begin{aligned} L &= (P_f + P_x) \left[\frac{1}{d'} + \frac{1}{f} - \frac{1}{d+p} - \frac{d}{d+p} \left(\frac{1}{f} \right) \right] \\ &= (P_f + P_x) \left(\frac{p}{d+p} \right) \left(\frac{1-d'}{d'} + \frac{1}{f} \right). \end{aligned}$$

This formula makes intuitive sense. An accidental pregnancy can only occur if use of contraception extends into, or starts during, fecundable exposure, the chances of which are $P_f + P_x$. Subject to this condition, the rate of accidental pregnancy is $p/(d+p)$; and the cost per accidental pregnancy is $(1/d' + 1/f) - 1$ or $(1-d')/d' + 1/f$ fecundable months.

Several relationships and formulas useful for comparing acceptance rules are covered in the remainder of this section.

We start by considering the value of T , denoted as T_{\max} , which maximizes Δ among possible fixed-duration strategies for any given set $\{d', e, f$ and pdf $\langle a_j \rangle\}$. We exploit the identity $P_f + P_x = 1 - P_c - (P_\alpha - P_x)$, valid for any T . To maximize Δ , one must maximize $P_f + P_x$, or what is the same, minimize $P_c + (P_\alpha - P_x)$. Now as T is progressively incremented by one from an initial value of 2, P_c increases monotonically from a starting value of zero and approaches 1.0 at values of T high enough to exceed almost all anovulatory lengths. Contrariwise, the proportion $P_\alpha - P_x$ decreases from its initial value $1 - P_x$ monotonically toward zero. Whatever makes P_c rise less rapidly as a function of T or starts $P_\alpha - P_x$ at a higher level, allowing it a larger possible decline, will favor a later T_{\max} . Contributing to the first condition is a lower f or longer anovulation; contributing to the second is a higher d' or, again, longer anovulation. In sum, when natural fecundability is low, the discontinuation rate high, and anovulation long, prescribing a relatively late time of acceptance will optimize the fixed-duration approach.

An analogous argument applies to the behavior of T_{\max} for the class of mixed strategies, though with an additional parameter relevant, namely λ . A

lower λ favors a slower rise of P_c , which incidentally has a ceiling of λf for T large. Now the same values of $P_\alpha - P_x$ are shared by mixed- T and fixed-duration- T strategies, but for $T \geq 3$, P_c rises more slowly for the former than the latter; thus T_{\max} must be at least as high for the mixed- T as the fixed-duration- T strategy.

Another analytical result of some importance is that for $T \geq 3$, the mixed- T rule is always more efficient than the corresponding fixed-duration- T rule. It has been seen that the two rules share the same P_x -value and both P_f terms include a_T . The difference in P_f terms (mixed less fixed) is

$$(1-\lambda f) \sum_{j=2}^{T-1} a_j - \sum_{j=2}^{T-1} a_j (1-f)^{T-j} = \sum_{j=2}^{T-1} a_j [1 - (1-f)^{T-j} - \lambda f],$$

which is positive inasmuch as

$$\begin{aligned} 1 - (1-f)^{T-j} - \lambda f &\geq 1 - (1-f) - \lambda f \\ &= (1-\lambda)f. \end{aligned}$$

Moreover, each increment of T adds to the size of the difference.

A second key comparison is between the mixed- T and the post-amenorrhagic strategy for modest levels of T . From the expressions given for P_f and P_x in Secs. 3 and 5, the difference $\Delta_p - \Delta_M$ is positive only if

$$1 - \lambda f > \sum_{j=2}^{T-1} a_j (1-\lambda f) + a_T + \sum_{j=T+1}^{\infty} a_j (1-d')^{j-T},$$

which reduces to the criterion

$$\lambda f < \frac{\sum_{j=T}^{\infty} a_j [1 - (1-d')^{j-T}]}{\sum_{j=T}^{\infty} a_j}.$$

Plainly the relative efficiency of the postamenorrhagic versus the mixed- T rule is reduced by a higher λ , higher f , or lower d' . It is also apparent that for any given set $\{\langle a_j \rangle, f > 0, T, \lambda > 0\}$, there is a d' -value below which the criterion fails.

Attention now shifts from a consideration of long-term efficiency as measured by M and Δ to an appraisal of short-term utility measured by means of the cumulative pregnancy function $U(i)$, $i = 1, 2, \dots, 18$. What can be said of the relative efficiencies of the various acceptance rules in the short run?

Let $U(i|F, T)$, $U(i|M, T)$, and $U(i|P)$ denote the proportions already having conceived by the end of the i th month conditional on the designated acceptance strategy. Because of the unimodal pdf of anovulation, it may be conjectured that the comparative behavior of $U(i)$ for any pair of acceptance rules reduces to one of three simple forms. First, respecting mixed T and fixed duration T when $T=0, 1, \text{ or } 2$, the two U -functions are identical. Second, for mixed T and fixed duration T ($T \geq 3$), $U(i|M, T) - U(i|F, T)$ is zero for $i=1, 2$; rises during months $i=3, 4, \dots, T_0$ for $T_0 \leq T$; and then declines asymptotically to zero as i becomes large. During months $3, 4, \dots, T$, the risks of a conception before acceptance are higher under the fixed-duration- T strategy than under the mixed- T , but are operating on a progressively smaller residual population in the fixed-duration case than the mixed one. From month $T+1$ onwards, the duration-specific average risks of pregnancy are slightly higher under the mixed- T strategy owing to a lesser selection toward anovulation longer than T months, and furthermore are operating upon a larger residual proportion of postnatal women not yet pregnant.

This second pattern of behavior may be expected of the two U -functions belonging to fixed-duration- T and $-(T+k)$ strategies, provided that the former yields the larger M -value. Then $U(i|F, T) - U(i|F, T+k)$ remains zero for $i=1, 2, \dots, T$; increases during $i=T+1, T+2, \dots, T+k_0$ ($k_0 \leq k$); and later approaches zero from above as i becomes large. The same expectation applies to mixed- T and $-(T+k)$ strategies, for which the former vouchsafes the higher M -value.

A third pattern, in which the two U -functions cross once, obtains when the fixed-duration- T (or mixed- T) strategy yields a smaller M than does its $T+k$ counterpart. Here either $U(i|M, T+k) - U(i|M, T)$ —or $U(i|F, T+k) - U(i|F, T)$ —remains zero up to $i=T$; becomes negative over some range $i=T+1, \dots, T_c - 1$; and then becomes positive for $i \geq T_c$, rising to a maximum followed by a monotonic decline toward zero.

One may expect $U(i|P)$ to behave as $U(i|M, \infty)$, and on this account may anticipate that depending on whether the postamenorrhic or the mixed- T rule yields the higher M -value, $U(i|P) - U(i|M, T)$ will behave according to the third or the second pattern respectively.

To return to the question posed earlier, the mixed- T rule ($T \geq 3$) proves more efficient than the corresponding fixed-duration rule in the short as well as the long run. The mixed- T strategy always possesses an advantage over the postamenorrhic during at least a short interval starting with ordinal month $T+1$, but often forfeits its advantage in the longer run.

A final concern is the expected interval to next conception of acceptors and its relation to that of all postnatal women. It is easily shown that (i) $M^* - M = \Delta^* - \Delta$; (ii) $\Delta^* - \Delta \geq 0$ for all acceptance strategies ($\Delta^* - \Delta = 0$

only for mixed and fixed duration T when $T=0, 1, \text{ or } 2$); (iii) M^* (or Δ^*) monotonically increases as T is incremented for either the fixed-duration or mixed strategies.

Result (iii) is of enough interest to explore further. The progressive increase of M^* with increasing T has three sources. First, an intensifying selection toward lengthy anovulation is increasing $\sum a_j^* j$. Second, depending on choice of strategy (mixed or fixed-duration),

$$\sum_{j=2}^{T-1} a_j^* \frac{\lambda(1-f)}{1-\lambda f} \quad \text{or} \quad \sum_{j=2}^{T-1} a_j^* (1-f)^{T-j} (T-j)$$

is growing with T . Third, $P_f^* + P_x^*$ rises with T because $P_c^* = 0$ means that $P_f^* + P_x^* \geq 1 - P_a^*$; but P_a^* must be declining, since the later mandatory acceptance at T is, the less chance there is of accepting while still anovulatory.

7. A NUMERICAL ILLUSTRATION

In the area of Bangladesh cited earlier as being served by CRL, two acceptance rules are being experimented with, the mixed 6-month and the mixed 18-month. To be included in the comparisons below are these two mixed rules, the postpartum ($T=1$), the postamenorrhic, and the fixed-duration 6- and 18-month rules.

The only published details respecting the postpartum anovulation of the population of interest, taken from [2], consist of two parts. First is a life-table distribution of amenorrhea lengths among 83 mothers whose infants survive. Because of small numbers, it yields useful percentiles only for the first 9 or 12 months. Second is a partially overlapping sample of 88 partly retrospective and entirely prospective intervals, also pertaining to mothers whose infants survive. Estimated for this second sample is a mean of 18.9 and a standard deviation of 8.0. To fit this combined experience, the percentiles up to the ninth month of the first sample are smoothed, and then joined to them is that uniform distribution which yields for the overall distribution the reported mean of 18.9. The advantage of this empirical uniform distribution is, first, that it provides control over the right tail of the distribution, setting an upper limit of 33 months. Secondly, it avoids the underestimation of menstrual onsets during the first six months that characterizes the use of curves such as the Pascal or Weibull that have been previously proposed. Thirdly, it assures an ascending monthly hazard function, which is appropriate because as the sample of mothers progress from full to partial nursing and thence to its complete suspension, the mean of their conditional monthly probabilities of resuming menstruation can be expected to rise.

The detailed pdf, which is

$$\begin{aligned}
 a_j &= 0.0150, & j &= 2, 3, \\
 &= 0.0167, & j &= 4, 5, 6, \\
 &= 0.0200, & j &= 7, 8, 9, \\
 &= 0.0358, & j &= 10, \dots, 33, \\
 &= 0 & & \text{otherwise,}
 \end{aligned} \tag{1}$$

yields a standard deviation of 8.2 about 2 percent larger than the reported 8.0.

The salient characteristics of the rural population at issue—notably very lengthy anovulation, low natural fecundability, and a high discontinuation rate with the pill—are such as to favor prescribing delayed rather than postpartum acceptance. Natural fecundability f is estimated at 0.10, well under the 0.20–0.30 levels typical of industrialized countries. With only 35 percent continuing the pill as long as one year, the monthly rate d' of discontinuation may be put at 0.0838, satisfying $(1 - d')^{12} = 0.35$.

On the other hand, a relatively high rate of ovulatory first cycles tends to penalize the postmenorrhagic and high- T strategies while benefitting the low- T fixed-duration or mixed rules. From the analysis of Chen et al. [2] comes a λf -estimate of 0.07 based on the proportion of women becoming pregnant without menstruating.

With a monthly discontinuation rate as high as 0.0838, no acceptance rule can produce a long average interval to next conception. To show that acceptance strategies can produce increasingly divergent intervals to next conception as continuation improves, four discontinuation rates are considered altogether, namely, $d' = 0.0838, 0.0561, 0.0289,$ and 0.0119 , equivalent to 12, 25, 50, and 75 percent continuing for as long as two years.

Some preliminary observations about the consequences of the above parameter assignments are afforded by Table 1. Without any contraception at all, the expected interval to next conception is $\sum a_j j + 1/f = 29$ months. If the effectiveness of contraception is 0.95 and the natural fecundability 0.10, then the residual fecundability is $p = (1 - e)f = 0.005$. Given such a low monthly risk of accidental pregnancy, the discontinuation rate $d = (1 - p)d'$ during a fecundable month [column (1) of Table 1] is scarcely different than during an anovulatory month. That fraction $P_f + P_x$ of postnatal women whose practice of contraception starts during, or extends into, the fecundable period has probability $d/(d + p)$, or $p/(d + p)$, of ending that practice still fecundable or accidentally pregnant. These probabilities are given in columns (2) and (3). Among the $P_f + P_x$ benefitting from contraception, the average lengths of contraceptive practice during the fecundable period if $e = 1.00$ and if $e = 0.95$, and the difference between the two, are tabulated in

TABLE 1
 Functions of the Monthly Discontinuation Rate d' , Given Effectiveness $e=0.95$
 and Natural Fecundability $f=0.10$

Discontinuation rate d'	$d=(1-p)d'$ (1)	$\frac{d}{d+p}$ (2)	$\frac{p}{d+p}$ (3)	$\frac{1}{d'}$ (4)	C^a (5)	$\frac{L}{P_f+P_x}^b$ (6)
.0838	.0834	.943	.057	11.9	10.7	1.2
.0561	.0558	.918	.082	17.8	15.6	2.2
.0289	.0288	.852	.148	34.6	28.1	6.5
.0119	.0118	.702	.298	84.0	56.3	27.7

$$^a C = \frac{1}{d+p} - \frac{p}{d+p} \left(\frac{1}{f} \right). \text{ (Note that } C=1/d' \text{ if } e=1.)$$

$$^b L = (P_f + P_x) \left(\frac{p}{d+p} \right) \left(\frac{1-d'}{d'} + \frac{1}{f} \right).$$

columns (4)–(6). When the discontinuation rate is high, accidental pregnancy plays a minor role, but exercises a progressively larger influence as d' is lowered.

In Table 2, the mean months added (Δ) as the result of exercising each of six acceptance rules are compared. When the discontinuation rate d' is high, the postmenorrhagic followed by the mixed 18-month rule produce the longest pregnancy intervals on average; the postpartum ($T=1$) convention, the shortest. Thanks to long anovulation and low natural fecundability, the postmenorrhagic retains its advantage even when d' is set at the relatively low value of 0.0119. The 6-month fixed-duration and mixed rules are made nearly identical by the fact that so few women menstruate earlier than the sixth month—less than 8 percent.

TABLE 2
 Mean Months Δ Added to the Interval to Next Conception, by Acceptance
 Strategy and Monthly Discontinuation Rate d'^a

Acceptance strategy	Monthly discontinuation rate d'			
	.0838	.0561	.0289	.0119
Postpartum ($T=1$)	2.905	6.230	17.161	45.783
Fixed duration ($T=6$)	4.181	7.943	19.341	47.707
Mixed ($T=6$)	4.282	8.088	19.604	48.233
Fixed duration ($T=18$)	5.993	9.647	19.473	42.193
Mixed ($T=18$)	7.895	12.410	24.452	52.172
Postmenorrhagic	9.997	14.527	26.174	52.463

^a Other assumptions: $e=0.95$, $f=0.10$, $\lambda=0.70$; and pdf $\langle a_i \rangle$ as described in (1).

TABLE 3
Correlates of Long-Range Efficiency and Two Measures of Short-Range
Efficiency, by Acceptance Strategy^a

Acceptance strategy	$P_f + P_x$	P_c	ϕ	$U(6)$	$U(18)$
Postpartum ($T=1$)	.270	.0	8.712	.0038	.1426
Fixed duration ($T=6$)	.389	.014	7.127	.0140	.1164
Mixed ($T=6$)	.398	.004	7.127	.0055	.1136
Fixed duration ($T=18$)	.558	.208	2.802	.0140	.2077
Mixed ($T=18$)	.734	.031	2.802	.0055	.0908
Postamenorrhic	.930	.070	0.	.0055	.0908

^a Assumptions: $d' = 0.0838$, $f = 0.10$, $e = 0.95$, $\lambda = 0.70$; and pdf $\langle a_j \rangle$ as described in (1).

Since $\Delta = (P_f + P_x)C$, an immediate explanation for the profile of Δ -values in Table 2 is the fractions $P_f + P_x$ enumerated in Table 3. The distinctively high value of P_c (proportion conceiving before acceptance) associated with the 18-month fixed duration rule helps to explain its relatively weak performance in Table 2. The mean overlap ϕ varies inversely with T ; for given T , it is the same for fixed-duration and mixed strategies.

At the end of six months, the cumulative pregnancy rate $U(6)$ is low regardless of strategy, owing to the protection conferred by anovulation upon the great majority of women. By 18 months, the rates are consequentially high and well differentiated by strategy, with the mixed ($T=18$) and

TABLE 4
Increases, and Their Sources, in the Gain in Mean Interval to Next Conception
of Acceptors versus All Postnatal Women, by Acceptance Strategy^a

Acceptance strategy	Components ^b of $\Delta^* - \Delta$					
	Δ^*	$\Delta^* - \Delta$	C_1	C_2	C_3	$\phi^* - \phi$
Postpartum ($T=1$)	2.901	0.00	0.00	0.00	0.00	.00
Fixed duration ($T=6$)	4.579	0.40	0.22	0.13	0.05	.10
Mixed ($T=6$)	4.409	0.13	0.07	0.04	0.02	.03
Fixed duration ($T=18$)	11.658	5.67	2.55	1.55	1.57	.74
Mixed ($T=18$)	8.680	0.79	0.25	0.28	0.26	.09
Postamenorrhic	11.43	1.43	0.00	0.68	0.75	.00

^a Assumptions are: $f = 0.10$, $\lambda = 0.70$, $e = 0.95$, $d' = 0.0838$; a_j -distribution as in Tables 2 and 3.

^b C_1 refers to the difference in mean anovulation, $\sum a_j^* j - \sum a_j j$; C_2 to the differential benefit from contraception, $\{(P_f^* + P_x^*) - (P_f + P_x)\}C$; and C_3 to the residual $(\Delta^* - \Delta) - C_1 - C_2$. $C = 10.74$.

postmenorrhagic rules already expressing their long-run advantages.

Table 4 directs attention to the subset of acceptors. Strikingly enough, for $d' = 0.0838$ the highest Δ^* is generated by the 18-month fixed-duration regimen, which produced an intermediate Δ in Table 2. The next four columns of Table 4 resolve the differences $\Delta^* - \Delta$ into three components. The distinctively large $\Delta^* - \Delta$ differences for the 18-month fixed-duration rule reflect the elimination of the 20.8 percent who conceive unprotected by month 18 or earlier, thereby adding 2.55 months to mean anovulation and 1.55 to mean months gained from contraception. The difference $\phi^* - \phi$ in overlap (last column of Table 4) can be sizable only when acceptance strongly selects for lengthy anovulation.

8. DISCUSSION

The finding in Sec. 7 of such an advantage for high- T acceptance strategies over low- T ones depends very much on the population in question having unusually long anovulation, a relatively low natural fecundability, and a very high monthly rate of discontinuation. With better continuation, a natural fecundability nearer 0.2, and an average anovulation length of 10–12 months or less, the advantage passes to intermediate- or even low- T acceptance rules; but this is a topic whose proper exploration is beyond the scope of the present paper.

Pure acceptance strategies exist only conceptually. Some mothers in their anxiety to avoid pregnancy will insist upon accepting right after childbirth or at least before the time prescribed. Others, presumably a larger minority, procrastinate; and high- T strategies are more vulnerable than low- T rules to any tardiness of acceptance, inasmuch as the months immediately following the prescribed time to accept are more likely to be fecundable ones under a high- T policy than under a short-duration one.

An important and general result, not dependent on particular parameter assignments, is the superiority of the mixed- T over fixed-duration- T strategies for any T exceeding the minimum length of postpartum anovulation. However, the administrative feasibility of the mixed- T and postmenorrhagic rules depends in part on a capacity for frequent contact with the client population, which certainly prevails in the population treated by Sec. 7. When that ready access is lacking, program personnel must depend wholly on the initiative of the women themselves, who alone know when they have resumed menstrual cycles; or else they must shift to a fixed-duration strategy which, given knowledge of birth dates, permits knowing also when to send reminders to those clients failing of their own volition to seek contraceptive assistance by the prescribed time. Thus, an added latent function of frequent contact with a population for health-care purposes is the possibility of employing an inherently more efficient contraceptive-acceptance approach than a fixed duration one. It also follows that despite

its inferiority to the class of mixed- T acceptance strategies, the set of fixed-duration strategies retains interest for its wider administrative practicability.

Juxtaposed, the results of Tables 2 and 4 serve a timely warning to those who would evaluate acceptance strategies by means of a field experiment. If they attend to only the experience of acceptors, they may be badly misled. The experience of acceptors is such a biased sample of all postnatal women that in an extreme case, such as illustrated by Sec. 7, the least efficient of three high- T acceptance rules can appear as the most efficient.

The models in this paper have been kept simple in the hope of grasping more readily the general conditions for one acceptance rule being more efficient than another. Perhaps the most serious simplifications have been to ignore age variation and the finiteness of the reproductive period, as a result of which some intervals to conception remain open. The results presented are most nearly valid for cohorts of young fecund mothers practicing contraception at low or intermediate levels of continuation. For purposes of simulating contraception accepted late in the reproductive period or practiced at very high levels of continuation, it becomes advantageous, as Mode and his co-workers have shown [6, 7], to treat the interval to conception as an improper random variable, which, with its mean undefined, is necessarily studied in terms of quantiles.

APPENDIX

The calculations of Sec. 7 are done by three computer programs—POST, MIXEDT, and FIXEDT—written in FORTRAN IV, listings of which are available upon request.

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