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A CONSTRAINT FROM B DECAY ON MODELS WITH NO t QUARK

Gordon L. KANE[1]

Randall Laboratory of Physics, University of Michigan, Ann Arbor, MI 48109, USA

Michael E. PESKIN[2]

Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853, USA

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We show that if the b quark is assigned to a singlet representation of the weak interaction SU(2) symmetry and if it decays through the usual weak bosons $W^{\pm}$ and Z^0, then neutral current decays occur at such a rate that

$$\Gamma(B \to X\ell^+\ell^-)/\Gamma(B \to X\ell^+\nu) \geqslant 0.12 ,$$

even allowing arbitrarily many quarks with which the b can mix. We discuss the manner in which the experimental exclusion of this inequality would constrain the possibilities for t-free models.

1. Introduction

There is no purely theoretical reason why the t quark must exist. A great number of weak interaction models lacking a t quark have been proposed [1–6], and the motivation for considering such models has been strengthened by the failure of the t quark to appear at PETRA [7]. Some of these models require adding to the standard model new gauge bosons with exotic properties [1], but others are relatively conservative in character, insisting that the b quark, though not a member of a weak interaction SU(2) doublet, still decays by exchange of the conventional weak gauge bosons. Theories of this type range from elegant grand unified theories [2, 3] to proposals which are more frankly phenomenological [4–6]. These theories have, however, a common distinguishing characteristic: Just as a model with an s quark but no c quark would have strangeness-changing neutral currents, such models with a b but no t have flavor-changing neutral currents which should be visible in B meson decays. The analysis of B decays is thus capable of providing strong constraints on weak interaction models lacking the t.

The general consequences of flavor-changing neutral current decays of the B have been discussed by several authors [4–6, 8]. We have felt a need, however, for a more precise criterion for the presence – or, more importantly, the absence – of

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such decays. In this paper, we will offer a simple and readily tested criterion. We will show that, in any model in which the b quark is a weak SU(2) singlet which decays by the exchange of the conventional $W^{\pm}$ and Z^{0}, the following inequality is satisfied:

$$\frac{\Gamma(\mathrm{B}\to \mathrm{X}\ell^{+}\ell^{-})}{\Gamma(\mathrm{B}\to \mathrm{X}\ell^{+}\nu)} \geqslant 0.12\,, \tag{1.1}$$

where ℓ is e or μ and only leptons from the direct decay of the B are to be included. Eq. (1.1) holds independently of the number of additional weak-singlet quarks heavier than the b and, within the present exprimental constraints on Cabibbo universality, independently of the values of weak mixing angles.

A test of the inequality (1.1) would confirm or rule out a major class of plausible alternatives to the hypothesis that there is a t quark. It is worth recalling how strongly this can constrain t-free models by reviewing the possibilities which do not require (1.1). If one does not wish to enlarge the standard SU(2)×U(1) guage model, there are only two possibilities: One option would allow the b to decay by Higgs boson exchange. This situation, however, should be readily distinguishable. Since the Higgs couples more strongly to more massive fermions, the semileptonic decay to τ should dominate the decay to a lighter lepton by the ratio $(m_\tau/m_\ell)^2$; effectively, then, μ and e should be produced only indirectly, as τ and c decay products, with correspondingly lowered energies. The other possibility would be to identify the *right-handed* b and c quarks as the members of a weak SU(2) doublet*. The phenomenology of this model has recently been studied by Barger, Keung and Phillips [8]; it is possible, though not easy, to distinguish it from the Kobayashi-Maskawa model with a t [9]. This model has, however, the defect that one can make the cancellation of charm-changing neutral currents (which would induce complete D^0-$\bar{D}^0$ mixing) occur naturally [10] only by forbidding all Cabibbo mixing between u and c.

One can enlarge one's options by adding to the standard model new weak gauge bosons and insisting that only these mediate b decay. Some models of this type have quite characteristic properties, implying, for example, that all B decays are semileptonic [1]. Others produce patterns of B decay quite close to those of the Kobayashi–Maskawa theory; one example would introduce new W bosons with right-handed coupling, with masses several times those of the conventional W's. But all of these possibilities require a major extension of the standard model.

We feel, then, that the class of models to which (1.1) applies contains the only straightforward alternatives to the existence of the t. Even before PETRA searches for higher-energy thresholds, a test of (1.1) will be an important step in deciding whether or not there is a t quark.

* We thank Henry Tye for a discussion of this possibility.

2. General analysis

In this section, we will outline the proof of the inequality (1.1) in weak interaction models in which the b is an SU(2) singlet which decays only through $W^{\pm}$ and Z^0. Our line of argument will be similar to that recently given by Branco and Nilles [5]; essentially, we are seeking to extract a more precise conclusion from their methodology. We will proceed by setting up a description of b decay in this class of models, then reducing the ratio in (1.1) to a ratio of mixing angles. We will then discuss the consequences of a lower bound on this later ratio. The detailed proof of this bound will be given in sect. 3.

To compute the ratio in (1.1), it is appropriate to approximate the B decay as that of a free b quark, treating the antiquark in the B as only a spectator. For semileptonic decays, the leading (multiplicative) QCD corrections are not present; most other strong interaction corrections affect the numerator and denominator of the ratio in (1.1) in the same way. The one exception to this rule is the correction for the finite mass of a c quark which may appear in the final state of a charged-current reaction; we will need to include this correction explicitly into our formulae.

In the standard $SU(2)\times U(1)$ model, the couplings of b to $W^{\pm}$ and Z^0 are given by the lagrangian

$$L=\sqrt{\tfrac{1}{2}}g(W^-_\mu J^{\mu +}+W^+_\mu J^{\mu -})+\frac{g}{\cos\theta_W}A_\mu(J^{\mu 3}-\sin^2\theta_W J^\mu_{EM})\,, \tag{2.1}$$

where $g=e/\sin\theta_W$, J^μ_{EM} is the electromagnetic current, and $(J^+_\mu, J^-_\mu, J^3_\mu)$ is an isospin triplet of the left-handed $(V-A)$ currents. The b-quark appears in J^μ_{EM}; this term is, however, diagonal in flavor and so cannot mediate a decay. If the b quark is not a member of a weak doublet, it can worm its way into the other terms only by Cabibbo mixing with d and s. In this case

$$\begin{aligned} J^+_\mu &= \bar u\gamma^\mu\frac{(1-\gamma^5)}{2}d'+\bar c\gamma^\mu\frac{(1-\gamma^5)}{2}s'\,,\\ J^3_\mu &= \tfrac{1}{2}\left(\bar u\gamma^\mu\frac{(1-\gamma^5)}{2}u+\bar c\gamma^\mu\frac{(1-\gamma^5)}{2}c-\bar d'\gamma^\mu\frac{(1-\gamma^5)}{2}d'-\bar s'\gamma^\mu\frac{(1-\gamma^5)}{2}s'\right), \end{aligned} \tag{2.2}$$

where d', s' are two orthogonal linear combinations of d, s, b, and any other charge $-\frac{1}{3}$ quarks*. Expanding in terms of d, s, b, we may write

$$\begin{aligned} J^+_\mu &= \alpha_u\bar u\gamma^\mu\frac{(1-\gamma^5)}{2}b+\alpha_{\bar c}\gamma^\mu\frac{(1-\gamma^5)}{2}b+\cdots\,,\\ J^3_\mu &= -\tfrac{1}{2}\alpha_d\bar d\gamma^\mu\frac{(1-\gamma^5)}{2}b-\tfrac{1}{2}\alpha_s\bar s\gamma^\mu\frac{(1-\gamma^5)}{2}b+\cdots\,, \end{aligned} \tag{2.3}$$

where the α_q are functions of mixing angles.

* We have assumed that there are no additional charge $+\frac{2}{3}$ quarks. If such a quark appears as a member of a weak doublet, it is the standard t. If such a quark is a weak singlet, it is easy to see that it can affect our analysis only by decreasing the rates of charged-current processes through its mixing with u, c; this would strengthen the inequality (1.1).

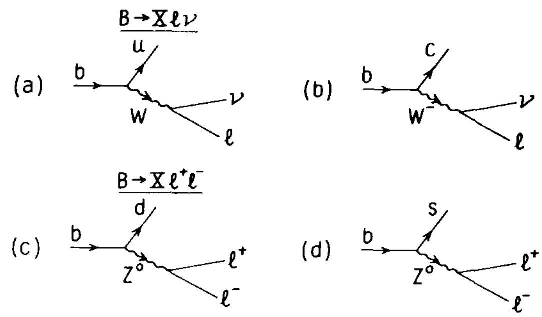

Fig. 1. Diagrams contributing to semileptonic b decay.

The diagrams for $\mathrm{B} \to \mathrm{X}\ell + \nu$ and $\mathrm{B} \to \mathrm{X}\ell^+\ell^-$, in the spectator approximation, are given in fig. 1. If $\ell = \mu$ or e and b decays to a light quark, the kinematics of these two processes are the same. For process (1b), in which b decays to c, the rate should be multiplied by a phase-space suppression factor P. This factor aside, however, these processes have rates in the ratio of their respective couplings. This ratio may be read from the lagrangian (2.1) and the current pieces (2.3), together with the lagrangian coupling a lepton to W and Z:

$$\delta L = \sqrt{\tfrac{1}{2}} g W^+_\mu \bar{l} \gamma^\mu \frac{(1-\gamma^5)}{2} \nu$$

$$+ \frac{g}{\cos\theta_\mathrm{W}} Z_\mu \left[(-\tfrac{1}{2} + \sin^2\theta_\mathrm{W}) \bar{l} \gamma^\mu \frac{(1-\gamma^5)}{2} l + \sin^2\theta_\mathrm{W} \bar{l} \gamma^\mu \frac{(1+\gamma^5)}{2} l \right], \quad (2.4)$$

and the relation $M^2_\mathrm{W} = M^2_\mathrm{Z} \cos^2\theta_\mathrm{W}$. The result is

$$\frac{\Gamma(\mathrm{B} \to \mathrm{X}\ell^+\ell^-)}{\Gamma(\mathrm{B} \to \mathrm{X}\ell^+\nu)} = [(\tfrac{1}{2} - \sin^2\theta_\mathrm{W})^2 + (\sin^2\theta_\mathrm{W})^2] A\,, \quad (2.5)$$

where A is the ratio of mixing angles

$$A = \frac{\alpha_\mathrm{d}^2 + \alpha_\mathrm{s}^2}{\alpha_\mathrm{u}^2 + P\alpha_\mathrm{c}^2}\,. \quad (2.6)$$

The factor P in (2.6) is a rather uncertain one, depending strongly on the ratio $M^2_\mathrm{c}/M^2_\mathrm{b}$. Assuming free-quark kinematics, one finds for $M_\mathrm{b} = 4.7$ GeV and $M_\mathrm{c} = 1.5$ GeV the value $P = 0.49$, and for $M_\mathrm{b} = 5.0$ GeV and $M_\mathrm{c} = 1.2$ GeV the value $P = 0.65$. We take this latter value to give a reasonable upper limit to the $\mathrm{b} \to \mathrm{c}$ transition rate and use this value in the analysis of A.

The term in brackets in (2.5) has its minimum at $\sin^2\theta_\mathrm{W} = \frac{1}{4}$, a point close to the current value of $\sin^2\theta_\mathrm{W} = 0.23 \pm 0.01$ [11]. Setting $\sin^2\theta_\mathrm{W} = \frac{1}{4}$,

$$\frac{\Gamma(\mathrm{B} \to \mathrm{X}\ell^+\ell^-)}{\Gamma(\mathrm{B} \to \mathrm{X}\ell^+\nu)} \geqslant \tfrac{1}{8} A\,. \quad (2.7)$$

To bound the ratio of partial widths it is now only necessary to bound the ratio A. We will save this task for the next section. We will show there that, for one weak singlet b quark,

$$A \geqslant 1.00\,, \tag{2.8}$$

and, for an arbitrary number of weak singlets,

$$A \geqslant 0.98\,. \tag{2.9}$$

These bounds on A, taken together with (2.7), imply inequality (1.1) which we sought. It should also be clear from our discussion that A cannot be made larger tan $1/P$; thus, not even the most optimistic advocate of the b as a weak singlet would expect the ratio of partial widths to exceed 0.25.

Our inequality includes direct leptons only; there remains, therefore, the problem of discriminating these direct leptons from leptons produced in a secondary decay of c or τ. In addition, $B\bar{B}$ events with semileptonic decay on both sides provide a background for measuring the numerator of (1.1). The first background may be eliminated by using the fact that a secondary lepton has a much softer energy distribution than one produced directly. Both backgrounds give equal rates for $\mu^+\mu^-$, $e^+\mu^-$, $e^-\mu^+$, e^+e^-, while the dilepton signal produces e^+e^- and $\mu^+\mu^-$ only. Thus, one can remove both backgrounds by subtracting μe events from $\mu\mu + ee$ events.

It is worth noting one more consequence of the bounds (2.8), (2.9). If the B can decay to a final state with $\ell^+\ell^-$, it can also decay to a state containing $\nu\bar{\nu}$. In the decay $b \to q\nu\bar{\nu}$, where q is d or s, the $\nu\bar{\nu}$ pair carries off, on average, an energy $(0.65)\, M_b \sim (0.6)\, M_B$. The ratio of partial widths for a single species of lepton is:

$$\frac{\Gamma(B \to X\nu_\ell\bar{\nu}_\ell)}{\Gamma(B \to X\ell^+\nu)} = \frac{1}{4}\,, \tag{2.10}$$

so the fraction of the total energy carried away by neutrinos is

$$\frac{E(\nu\bar{\nu})}{E\text{ (total)}} = 3 \cdot (\tfrac{1}{4}A) \cdot (0.6) \cdot \mathrm{BR}(B \to X\ell^+\nu) \geqslant (0.4) \cdot \mathrm{BR}(B \to X\ell^+\nu)\,. \tag{2.11}$$

The factor of 3 counts 3 species of neutrinos. This almost doubles the amount of missing energy expected from normal charged-current semileptonic decay.

3. Lower bounds

The conclusions reached in the previous section depended on the lower bounds (2.8), (2.9) for the ratio A of mixing angles. In this section, we will complete our argument by demonstrating those bounds. To do this, we will analyze the most general Cabibbo-mixing of b with s, d, and possibly other charge $-\frac{1}{3}$ flavors. This mixing will be parametrized by various angles θ_i; we will abbreviate $\cos\theta_i = c_i$,

$\sin\theta_i = s_i$. The usual Cabibbo angle will be denoted θ_C and its cosine and sine c_C, s_C, respectively. Other angles will be chosen in a way which is convenient for this analysis, but which differs from conventions used previously in the literature [4, 5].

Let us consider first the simplest case, in which d, s, and b are the only charge $-\frac{1}{3}$ quarks. This case has been considered previously by Barger and Pakvasa [4]. In this case, we can parametrize the (normalized, but otherwise arbitrary) linear combinations d', s':

$$d' = c_1(c_c d + s_c s) + s_1 b\,, \qquad s' = c_2(-s_{C+3}d + c_{C+3}s) + s_2 b\,, \tag{3.1}$$

where $s_{C+3} = \sin(\theta_C + \theta_3)$, and similarly for c_{C+3}. d' and s' must be orthogonal; this implies the constraint

$$-c_1c_2s_3 + s_1s_2 = 0\,,$$

or

$$s_3 = \frac{s_1s_2}{c_1c_2}\,. \tag{3.2}$$

Inserting (3.1) into (2.2), we find for the parameters of (2.3):

$$\alpha_u = s_1\,, \qquad \alpha_c = s_2\,, \qquad \alpha_d = c_1s_1c_c - c_2s_2s_{c+3}\,, \qquad \alpha_s = c_1s_1c_c + c_2s_2c_{c+3}\,. \tag{3.3}$$

Then, using (3.2)

$$\alpha_d^2 + \alpha_s^2 = c_1^2s_1^2 + c_2^2s_2^2 - (c_1s_1)(c_2s_2)s_3 = c_1^2s_1^2 + c_2^2s_2^2 - s_1^2s_2^2\,,$$

(2.6) becomes

$$A = \frac{(c_1s_1)^2 + (c_2s_2)^2 - (s_1s_2)^2}{s_1^2 + Ps_2^2}\,. \tag{3.4}$$

To minimize A, it is necessary to make s_1 or s_2 as large as possible.

What bounds can be put on s_1, s_2? If $c_1 \neq 1$, Cabibbo universality is violated; thus, tests of Cabibbo universality constrain c_1. The recent Cabibbo fit of Shrock and Wang [12] yields bounds on the mixing angles V_{ud}, V_{us} which appear in the $\bar{u}d$ and $\bar{u}s$ terms of the charged currents. Interpreting their results within this model

$$|V_{ud}| = |c_1c_c| = 0.9737 \pm 0.0025\,, \qquad |V_{us}| = |c_1s_c| = 0.219 \pm 0.011\,. \tag{3.5}$$

Then

$$|\tan\theta_C| = 0.225 \pm 0.11\,, \qquad |c_1| = 0.9980 \pm 0.0034\,. \tag{3.6}$$

Allowing a deviation of 2σ from the preferred value of c_1 gives

$$|s_1| = 0.13 . \tag{3.7}$$

A 3σ deviation gives $|s_1| < 0.16$.

To constrain s_2, we rely on the following observation: The general form (3.1), inserted into (2.2), produced d$\leftrightarrow$s flavor-changing neutral currents. The coefficient of the $\bar{d}s + \bar{s}d$ term in J^3_μ is

$$D = c_1^2 c_C s_{C+3} c_{C+3} . \tag{3.8}$$

This must be very small if the $K_L - K_S$ mass difference is to be kept small; the precise bound is

$$D^2 < 4 \times 10^{-7}.$$

It will be clear from fig. 2, to be discussed in a moment, that such a small deviation of D from zero will be completely negligible in our analysis. Hence, we set $D = 0$. Throwing away all information on the signs of the various angles, this condition may be written

$$|c_1^2 - c_2^2 \cos^2 \theta_3| = |c_2^2 (\cos^2 \theta_C)(\sin^2 \theta_3)| . \tag{3.9}$$

θ_3 is given in terms of θ_1, θ_2 by (3.2).

If θ_2 is raised, for fixed θ_1 satisfying (3.7), the right-hand side of (3.9) can never grow larger than

$$(\cot^2 \theta_C) \frac{s_1}{c_1} < 0.29 ,$$

whereas the left-hand side of (3.9) goes smoothly past 1 as $s_2 \to 1$. Thus (3.9) has solutions only for s_2 small, of order s_1. A graphical solution to (3.9) for the case

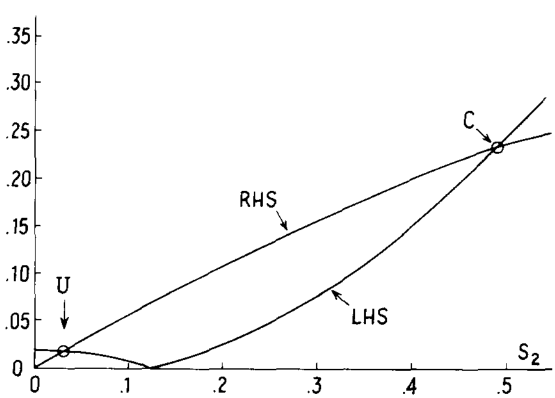

Fig. 2. Graphical solution of eq. (3.9) for $s_1 = 0.13$. RHS and LHS denote the right- and left-hand side of (3.9), respectively. U and C denote the solutions of Barger and Pakvasa.

$s_1 = 0.13$ (with $\tan\theta_C = 0.225$, $P = 0.65$) is given in fig. 2. Notice that there are two solutions, one with $s_2 > s_1$, one with $s_1 > s_2$. Barger and Pakvasa refer to these solutions as solutions C and U, respectively, since, in the former case, final states with c, s dominate. Since, for each solution, the value of A decreases with increasing s_1, s_2, the values of A resulting from $s_1 = 0.13$:

$$A = 1.12, \text{(solution C)}\,,$$
$$A = 1.00, \text{(solution U)}\,. \tag{3.10}$$

are lower bounds for A in this model. Increasing s_1 to 0.16 gives bounds not much worse: $A = 0.99$ for both solutions. The bounds are not far from values one might guess for A; e.g. if one uses $s_1 = 0.13$ and $s_2 = \theta_c$ in 3.4, then $A \simeq 1.3$.

One might make this model more complex by adding additional charge $-\frac{1}{3}$ quarks ℓ_i. The models of refs. [2, 3], in fact, include at least one such extra quark. Before we consider the most general mixing pattern for such quarks, it is useful to examine a particularly simple pattern of mixing proposed by Georgi and Pais [3] as an alternative to the "standard" Kobayashi-Maskawa [10] scheme for insuring the natural absence of d↔s neutral currents. These authors propose the pattern of mixing, involving an additional quark ℓ:

$$d' = c_1(c_C d + s_C s) + c_1(c_4 b + s_4 \ell)\,,$$
$$s' = c_1(-s_C d + c_C s) + s_1(-s_4 b + s_4 \ell)\,. \tag{3.11}$$

In this pattern s_4 is unconstrained, but s_1 still satisfies (3.7). A takes the simple form

$$A = \frac{c_1^2}{c_4^2 + P s_4^2} \geqslant 0.99 \tag{3.12}$$

for $P = 0.65$. (Actually, the specific model proposed by Georgi and Pais has an additional restriction $\theta_4 = \theta_C$, so that $A = 1.0$.)

Now, finally, we will let the number of additional quarks ℓ_i be arbitrary. Let us parameterize d', s' by

$$d' = c_1(c_C d + s_C s) + s_1\Big(c_4 b + \sum_i \beta_i l_i\Big)\,,$$
$$s' = c_2(-s_{C+3} d + c_{C+3} s) + s_2\Big(c_5 b + \sum_i \gamma_i l_i\Big)\,. \tag{3.13}$$

β_i and γ_i are some combinations of additional mixing angles. We may choose phases so c_4, $c_5 > 0$.

d' and s' must still be orthogonal; this implies

$$s_3 = \frac{s_1 s_2}{c_1 c_2}\eta\,, \tag{3.14}$$

where

$$|\eta| = |c_4 c_5 + \sum_i \gamma_i^* \beta_i| < 1 . \tag{3.15}$$

s_1 is still constrained by (3.7). What about s_2? The expression for D in this model is the same as that in case 1; thus s_2 is still related to s_1 through (3.9), though now θ_3 in (3.9) is given by (3.14). Thus, for fixed s_1, the value of s_2 obtained from (3.9) depends on the value of η. Since the left-hand side of (3.9) depends only weakly on η [$c_2 \cos 2\theta_3 = (1-2(s_1 s_2 \eta)^2/c_1^2)$], whereas the right-hand side is almost proportional to η, it is clear that the largest such value of s_2 is obtained by setting $\eta = 1$. This observation informs us that the value of s_2 found as solution C in case 1 is a bound on s_2 in this more general situation. This yields:

$$\begin{aligned} &\text{for } s_1 = 0.13 , \qquad |s_2| < 0.494 , \\ &\text{for } s_1 = 0.16 , \qquad |s_2| < 0.575 . \end{aligned} \tag{3.16}$$

The expression for A, in this more general case, is

$$A = \frac{(s_1 c_4 c_1)^2 + (s_2 c_5 c_2)^2 - s_1 s_2 c_1 c_2 c_4 c_5 s_3}{(s_1 c_4)^2 + P(s_2 c_5)^2} . \tag{3.17}$$

This equality can be converted to a simpler inequality. First note that

$$\begin{aligned} s_1 s_2 c_1 c_2 c_4 c_5 s_3 &\leq |s_1 s_2 c_1 c_2 c_4 c_5| \left|\frac{s_1 s_2}{c_1 c_2}\right| = (s_1 s_2)^2 c_4 c_5 \\ &= \tfrac{1}{2}\xi (s_1^2 c_4)^2 + \frac{1}{2\xi}(s_2^2 c_5)^2 - \tfrac{1}{2}\left(\sqrt{\xi} s_1^2 c_4 - \frac{s_2^2 c_5}{\sqrt{\xi}}\right)^2 \\ &\leq \tfrac{1}{2}\xi (s_1^2 c_4)^2 + \frac{1}{2\xi}(s_2^2 c_5)^2 , \end{aligned} \tag{3.18}$$

so we can replace the first line of (3.18) by the last in (3.17):

$$A \geq \frac{(s_1 c_4)^2 (c_1^2 - \tfrac{1}{2}\xi s_1^2) + (s_2 c_5)^2 (c_2^2 - s_2^2/2\xi)}{(s_1 c_4)^2 + (s_2 c_5)^2 P} . \tag{3.19}$$

This inequality holds for any value of $\xi > 0$. The right-hand side of (3.19) is of the form

$$\frac{x\alpha + y\beta}{x\gamma + y\delta} , \tag{3.20}$$

where x, y, γ, δ are positive. It is easy to see, by differentiating this expression with respect to x, that (3.20) is always larger than the smaller of (α/γ), (β/δ), whatever the ratio x/y. Thus,

$$A \geq \min\left[(c_1^2 - \tfrac{1}{2}\xi s_1^2), 1/P\left(c_2^2 - \frac{s_2^2}{2\xi}\right)\right] . \tag{3.21}$$

Since one of the elements in the brackets increases and one decreases with ξ, the best bound is obtained by choosing ξ so that

$$(c_1^2 - \tfrac{1}{2}\xi s_1^2) = \frac{1}{P}\left(c_2^2 - \frac{s_2^2}{2\xi}\right). \tag{3.22}$$

Solving the quadratic equation, and choosing the root with $\xi > 0$, we find

$$A \geqslant \frac{1}{2}\left[\left(c_1^2 + \frac{1}{P}c_2^2\right) - \left[\left(\frac{1}{P}c_2^2 - c_1^2\right)^2 + (s_1 s_2)^2\right]^{1/2}\right]. \tag{3.23}$$

The right-hand side of (3.23) decreases if either c_1 or c_2 decreases; thus, the worst case is given by using the boundary values (3.16) for θ_1, θ_2. For $s_1 = 0.13$, this yields

$$A \geqslant 0.98\,. \tag{3.24}$$

Extending s_1 to 0.16 decreases the bound to 0.95. The derivation of this bound completes our demonstration of the constraint (1.1) on neutral current B decays.

4. Conclusion

For clarity, we restate our result. If the b quark is assumed to decay via the conventional gauge bosons $W^\pm$, Z^0, then if $\Gamma(B \to X\ell^+\ell^-)/\Gamma(B \to X\ell\nu)$ is less than 0.12, the b quark cannot be a weak SU(2) singlet.

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