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NON-PARAMETRIC METHODS IN DEMAND ANALYSIS

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This note describes techniques developed by Afriat, Diewert, Varian, and others that allow empirical investigation of consumer demand data without imposing any maintained hypotheses concerning the parametric form of the underlying demand or utility functions.

1. Introduction

The received theory of consumer behavior asserts that a consumer chooses a bundle of goods that maximizes utility over all bundles of goods that he can afford. Given some data on actual consumer behavior, the demand analyst can address four sorts of questions concerning this behavior:

(1) Consistency? Is the observed data consistent with the utility maximizing model?
(2) Structure? Is the observed data consistent with a utility function with some special structure?
(3) Recoverability? How can the underlying utility function be recovered?
(4) Extrapolation? How can we forecast behavior in other circumstances?

In the conventional approach to these questions the demand analyst chooses a parametric form for the underlying utility function and derives the associated set of parametric demand functions, which are then fitted to the data using some econometric technique. The resulting parameter estimates can then be tested to see if they conform to the restrictions imposed by the utility maximization model and the special restrictions imposed by various hypotheses concerning functional structure. If the estimated parameters satisfy these tests, they can be used to construct an estimate of the underlying utility function and to forecast demand behavior in new situations.

By now such techniques are quite standard; an excellent account can be found in Deaton and Muellbauer (1980). However, these methods have the inherent defect that they are indeed parametric. The non-parametric approach, which I will describe in this report, avoids the need for these imposed restrictions. Non-parametric techniques in demand analysis have been investigated by Afriat (1967, 1972, 1973, 1976), Diewert (1973), Diewert and Parkan (1978), and Varian (1980a, b, 1981a, b). In this report I will briefly describe how non-parametric techniques can be used to address the four sorts of questions described above. In other works [Varian (1981c, d)] I analyze similar techniques in a production context, and introduce stochastic considerations such as measurement error into the analysis.

2. Testing for consistency

The non-parametric test for consistency with utility maximizing behavior is based on the revealed preference theory of Samuelson (1948), Houthakker (1950), Richter (1966), Afriat (1976) and many others. This theoretical work has indicated that certain inherently finite conditions are necessary *and sufficient* conditions for observed demand behavior to be consistent with the model of utility maximization. It is natural to attempt to see if observed demand behavior is consistent with such conditions. Koo (1963), Landsburg (1981) and others have investigated several sets of demand data with various methods designed to do just that. However, in my opinion, the methods described in Varian (1980a) seem to provide the most efficient and general techniques for addressing these issues.

Definition. Let (p^i, x^i) $i = 1,\ldots,n$ be some observed data. Let x be an arbitrary bundle of goods. Then we say:

(1) x^i is *directly revealed preferred* to x, written $x^i R^0 x$, if $p^i x^i \geqslant p^i x$.
(2) x^i is *strictly directly revealed preferred* to x, written $x^i P^0 x$, if $p^i x^i > p^i x$.
(3) x^i is *revealed preferred* to x, written $x^i R x$, if there is some sequence of observations such that $x^i R^0 x^j$, $x^j R^0 x^k,\ldots, x^m R^0 x$. In this case we say the relation R is the *transitive closure* of the relation R^0.

(4) x^i is *strictly revealed preferred* to x, written x^iPx, if there are some observations j and k such that x^iRx^j, $x^jP^0x^k$, and x^kRx.

It is possible to modify some results originally due to Afriat (1967) and analyzed in Diewert (1973) to prove the following theorem:

Theorem 1. The following conditions are equivalent:

(*1*) *There exists a non-satiated utility function that rationalizes the data.*
(*2*) *The data satisfies the Generalized Axiom of Revealed Preference (GARP) which states: if* x^iRx^j *then* $p^jx^j \leqslant p^jx^i$.
(*3*) *There exist positive numbers* U^i *and* t^i *for* $i = 1, \ldots, n$ *such that:* $U^i \leqslant U^j + t^jp^j(x^i - x^j)$ *for* $i, j = 1, \ldots, n$.
(*4*) *There exists a non-satiated, continuous, concave, monotonic utility function that rationalizes the data.*

The exact statement of the Generalized Axiom of Revealed Preference seems to be new, although it is obviously closely related to earlier revealed preference conditions such as Houthakker's Strong Axiom of Revealed Preference and Afriat's condition of cyclical consistency. In order to test data for consistency with GARP it is necessary to have an efficient way to compute the transitive closure of the revealed preference relation. This problem has been addressed in the computer science literature; for example, Warshall's (1962) algorithm provides an ingenious and efficient technique for computing the transitive closure of an arbitrary relation on n elements in only n^3 computer additions.

I have used this algorithm to check several sets of aggregate demand data for consistency with GARP and therefore, consistency with the model of *individual* utility maximization. Somewhat surprisingly, all aggregate data tested turned out to be consistent with this model. This is a definite contrast with the parametric methods, which often conclude that such data is *not* consistent with the utility maximizing model. The reasons for this discrepancy are discussed in Varian (1980a).

3. Testing for special structure of utility

It is often of interest to determine whether some given demand data is consistent with a utility function with some special structure such as homotheticity, separability, etc. without making any assumptions about the parametric form of utility. The essential logic behind the constructions described below was first investigated by Afriat (1967), and was further elucidated by Diewert (1973), Diewert and Parkan (1978), and Varian (1980b, 1981a, b).

Let us consider the hypothesis of homotheticity. It is possible to prove the following theorem:

Theorem 2. The following conditions are equivalent:

(1) There exists a non-satiated homothetic utility function that rationalizes some data.
(2) The data (p^i, x^i), $i = 1,\dots,n$ satisifies the Homothetic Axiom of Revealed Preference (HARP): $(p^ix^j)(p^jx^k)\dots(p^mx^i) \geq (p^ix^i)(p^jx^j)\dots(p^mx^m)$ for all lists of distinct indices $(i, j,\dots, m)$.
(3) There exists positive numbers U^i $i = 1,\dots,n$ such that: $U^i \leq U^jp^jx^i/p^jx^j$ for all $i, j = 1,\dots,n$.
(4) There exists a non-satiated, homothetic, continuous, concave, monotonic utility function that rationalizes the data.

Condition (3) of Theorem 2 was originally discovered by Afriat (1973) and described by Diewert (1973). Unfortunately it is computationally rather difficult to verify as it involves n^2 inequalities. Condition (2) was first discovered by Afriat but not published until Afriat (1981). Varian (1980b) independently discovered condition (2) and noted that it was very simple to test. Well known graph theory algorithms related to Warshall's algorithm can be used to verify whether or not HARP is satisfied. Afriat (1981) makes a similar point but uses a different algorithm.

Using similar methods I have constructed finite tests for: (1) homotheticity, (2) weak separability, (3) additive separability, (4) quasi-homotheticity, (5) rationing, (6) expected utility, and (7) mean variance utility. These tests are described in detail in Varian (1980b). Afriat (1970) independently discovered the general method and constructed several of the above tests in 1970 in an unpublished paper. Diewert and Parkan (1978) have utilized related techniques on actual demand data.

4. Recovering the underlying preferences

Suppose that we have checked some data (p^i, x^i) $i = 1,\dots,n$ for consistency with the utility maximizing model and that we now have some new, previously unobserved points x^0 and x'. Let us consider the class U of all non-satiated, monotonic, concave utility functions that rationalize the data (p^i, x^i). There are three logical possibilities: (1) all utility functions in U rank x^0 ahead of x', (2) all utility functions in U rank x' ahead of x^0, (3) some utility functions in U rank x^0 ahead of x' and some rank x' ahead of x^0. We would like a computationally feasible method to discover which of these three possibilities is the case.

To formalize this question, let us define $RP(x^0)$ to be the set of all x's 'revealed preferred' to x^0 by all utility functions in U, and let $RW(x^0)$ be the set of x's 'revealed worse' than x^0 by all utility functions in U. The two sets $RP(x^0)$ and the complement of $RW(x^0)$ form an 'inner' and an 'outer' bound to the set of bundles preferred to x^0. What is needed is a practical way to verify whether any given bundle x is in $RW(x^0)$ (for example) or not.

Let us consider all of the possible prices p^0 at which x^0 could be demanded such that the pair (p^0, x^0) would still be consistent with the previous observations (p^i, x^i) for $i = 1,\dots, n$. That is the set of pairs (p^i, x^i) $i = 0,\dots, n$ satisfies the Generalized Axiom of Revealed Preference. If it is the case that for all such prices p^0, $p^0x^0 \geqslant p^0x^i$ for some x^iPx' or $p^0x^0 > p^0x^i$ for some x^iRx', then it is clear that x^0 will have a strictly greater utility than x' for all concave, monotonic, non-satiated utility functions that rationalize the data; that is, x' must be in $RW(x^0)$. The formalization of this argument is the content of Theorem 3.

Theorem 3. A bundle of goods x' is in $RW(x^0)$ if and only if there does not exist a non-negative solution p^0 to the following system of linear inequalities:

$$
\begin{aligned}
&p^0x^i \geqslant p^0x^0 && \textit{for all} \quad x^i \quad \textit{such that} \quad x^iRx^0,\\
&p^0x^i > p^0x^0 && \textit{for all} \quad x^i \quad \textit{such that} \quad x^iPx^0,\\
&p^0x^j \geqslant p^0x^0 && \textit{for all} \quad x^j \quad \textit{such that} \quad x^jRx',\\
&p^0x^j > p^0x^0 && \textit{for all} \quad x^j \quad \textit{such that} \quad x^jPx'.
\end{aligned}
$$

This theorem provides a necessary and sufficient condition for an arbitrary x' to be revealed worse than x^0 that involves solving a small system of linear inequalities. A similar result can be used to decide whether x' is in $RP(x^0)$.

Another way that one might like to recover preferences is in a dual format. Rather than comparing two arbitrary bundles of goods, one might like to compare two arbitrary budgets, (p^0, y^0) and (p', y'). There is a dual version of Theorem 4 that allows for such comparisons.

A third way that one might want to recover preferences is by getting numerical bounds on some cardinal measure of 'willingness to pay' such as the compensating or equivalent variation. It turns out that we can use the 'inner' and 'outer' bounds on the preferred set described above to construct tight upper and lower bounds on such measures.

5. Forecasting demand behavior

This is perhaps the simplest of the four tasks described earlier. Suppose we have observed a consumer's choices (p^i, x^i) $i = 1, \ldots, n$ and we are now given a new previously unobserved budget (p^0, y^0). We can ask for a description of the set of bundles of goods that could be demanded at (p^0, y^0) and still be consistent with all of the previously observed behavior. We denote this set – an 'overestimate' of the demand correspondence – by $S(p^0, y^0)$. A practical way to compute this set is given in the next theorem. In this theorem R stands for the 'indirect revealed preference' relation, which is defined in Varian (1980a) among other places.

Theorem 4. A bundle of goods x^0 is in $S(p^0, y^0)$ if and only if it satisfies the following system of linear inequalities:

$$p^i x^0 \geqslant p^i x^i \quad \textit{for all} \quad (p^i, y^i) \quad \textit{such that} \quad (p^0, y^0)\, R(p^i, y^i),$$
$$p^i x^0 > p^i x^i \quad \textit{for all} \quad (p^i, y^i) \quad \textit{such that} \quad (p^0, y^0)\, P(p^i, y^i).$$

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