

RISK ANALYSIS FOR CODE CALIBRATION

Andrzej S. Nowak

Department of Civil Engineering, University of Michigan, Ann Arbor, MI 48109 (U.S.A.)

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ABSTRACT

A procedure has been developed to calibrate safety indices for various types of structures.

The load model is based on normal approximations of non-normal distributions at the design point. For time varying loads, the basic time intervals are considered, with probabilities of occurrence in each interval and conditional distributions of intensity.

Load and resistance factor format and allowable stress design are considered. Computations were carried out for the National Building Code of Canada.

Safety indices are presented in charts and tables. The results can be used to determine optimum values of resistance factors and/or safety factors.

INTRODUCTION

The development of a building code involves the optimization of load and resistance factors. There is a need for an efficient numerical procedure to compare safety of structures corresponding to different code requirements (load and resistance factors). Nowak and Lind [1] developed a procedure, based on Rackwitz and Fiessler's model [2], to optimize resistance factors. Recently, also based on that model, the computer program was developed and safety indices were calculated for various cases of beams and columns. The present paper describes the procedures

and results of these computations.

Five load components are considered: dead load, live load, snow, wind and earthquake. Live load is considered as a sum of sustained live load and transient live load. The load models are based on the presently available data.

Safety indices were calculated for the beams and columns designed according to the National Building Code of Canada [3], for various load cases. The results are presented in the form of charts and tables and can be used in selection of resistance factors or safety factors, as demonstrated.

SAFETY INDEX

The procedure to calculate safety indices is described by Nowak and Lind [1]. A brief summary is given here.

The limit state equation for the considered design cases is

$$Z = R - (D + LS + LT + S + W + E) \quad (1)$$

where R = resistance, or load carrying capacity of the structure, D = dead load effect, LS = sustained live load effect, LT = transient live load effect, S = snow effect, W = wind effect, and E = earthquake effect. $Z \geq 0$ corresponds to safe realizations, while $Z < 0$ corresponds to failure. Probability of failure, P_F , is

$$P_F = \text{Prob}(Z < 0). \quad (2)$$

Safety index, β , is defined as

$$\beta = -\Phi^{-1}(P_F) \quad (3)$$

where $\Phi^{-1}(\cdot)$ = inverse of the standard normal distribution function.

If all the variables in eqn. 1 are independent and normally distributed, then Z is also a normal random variable with the mean \bar{Z} and standard deviation σ_Z given by

$$\bar{Z} = \bar{R} - (\bar{D} + \bar{LS} + \bar{LT} + \bar{S} + \bar{W} + \bar{E}), \quad (4)$$

$$\sigma_Z = (\sigma_R^2 + \sigma_D^2 + \sigma_{LS}^2 + \sigma_{LT}^2 + \sigma_S^2 + \sigma_W^2 + \sigma_E^2)^{1/2}, \quad (5)$$

where

$$\bar{R}, \dots, \bar{E}, \sigma_R^2, \dots, \sigma_E^2 = \text{means and variances of } R, \dots, E.$$

The probability of failure is

$$P_F = \Phi(-\bar{Z}/\sigma_Z) \quad (6)$$

and hence in this case, from eqn. 3,

$$\beta = \bar{Z}/\sigma_Z \quad (7)$$

If the variables in eqn. 1 are not normally distributed, then β can be calculated using Rackwitz and Fiessler's iterative procedure [2]. The method is based on normal ap-

proximations of non-normal distributions. The approximating normal distribution is selected so, that its cumulative distribution function (CDF) and its probability density function (PDF) are equal to the CDF and PDF of the original distribution at the so called "design point". Design point, denoted by R^*, \dots, E^* , are values for R, \dots, E such that

$$R^* - (D^* + LS^* + LT^* + S^* + W^* + E^*) = 0. \quad (8)$$

Initial values of R^*, \dots, E^* , satisfying eqn. 8, are guessed first. The mean, \bar{X}' , and the standard deviation, $\sigma_{X'}$, of the approximating normal distribution are calculated from the following formulas;

$$\sigma_{X'} = \varphi\{\Phi^{-1}[F_X(X^*)]\}/f_X(X^*) \quad (9)$$

$$\bar{X}' = X^* - \sigma_{X'} \Phi^{-1}[F_X(X^*)] \quad (10)$$

where $X = R, \dots, E$, φ = PDF of the standard normal random variable, and

$f_X(\cdot)$ = PDF of X .

For those approximated distributions the mean and standard deviation of Z' can be calculated using eqns. 4 and 5.

$$\bar{Z}' = \bar{R}' - (\bar{D}' + \bar{LS}' + \bar{LT}' + \bar{S}' + \bar{W}' + \bar{E}') \quad (11)$$

$$\sigma_{Z'} = (\sigma_{R'}^2 + \sigma_{D'}^2 + \sigma_{LS'}^2 + \sigma_{LT'}^2 + \sigma_{S'}^2 + \sigma_{W'}^2 + \sigma_{E'}^2)^{1/2} \quad (12)$$

and the safety index is

$$\beta = \bar{Z}'/\sigma_{Z'}. \quad (13)$$

The iteration loop is completed after the new design points are found from the following formula,

$$X^* = \bar{X}' \pm \beta \sigma_{X'}/\sigma_{Z'} \quad (14)$$

where $X = R, \dots, E$. A minus sign is used for R (or negative loads). Then new approximating normal distributions are calculated using eqns. 9 and 10, and a new β is calculated using eqn. 13. The calculations are continued

until β does not differ in the consecutive iterations (within the required accuracy).

The presented procedure allows for the calculation of β for time-invariant loads and resistance. However, earthquake, wind, snow and live load are time-variant. Therefore the model is modified to account for this variation.

For each load component a basic time interval, τ , is determined. It is assumed that load magnitude can be considered constant during this time period.

The occurrence or non-occurrence of the load in each time interval corresponds to repeated independent trials with probability p of occurrence. Given that the load occurs the distribution of its amplitude is $F_{\text{int}}(x)$ and the corresponding PDF is $f_{\text{int}}(x)$.

For n basic time intervals the cumulative distribution function, $F(x)$, and the probability density function, $f(x)$, for each load component can be written as

$$F(x) = \{1 - p[1 - F_{\text{int}}(x)]\}^n \quad (15)$$

$$f(x) = npf_{\text{int}}(x)[F(x)]^{(n-1)/n} \quad (16)$$

The combined effect of the considered load components is calculated as follows. The loads are arranged by increasing τ 's: E , W , LT , S , LS , D . Dead load and resistance are assumed to be time invariant. E is combined with W first. The basic time interval for E is τ_E . The CDF and PDF of E in the time interval τ_W (basic time interval for W) is calculated using eqns. 15 and 16 with $n = \tau_W/\tau_E$. The distributions of E and W , both in time interval λ_W , are approximated by normal distributions at E^* and W^* , respectively. The parameters of the approximating normal distributions are calculated using eqns. 9 and 10. The joint distribution of $E + W$ in time interval λ_W is then normal, with the mean and variance equal to the sum of means and variances of both components.

The joint distribution of E and W , $E + W$, is combined with LT . The value of the CDF

and PDF for $(E + W)$ is calculated at $(E^* + W^*)$, in time interval τ_{LT} (basic time interval for LT). It is approximated by a normal distribution at that point, and a normal distribution approximating LT at LT^* is added.

Then the joint distribution of $(E + W + LT + S)$ is calculated, LS is added, then D , finally the normal approximation is found for Z , as given by eqn. 1 (at the design point). The safety index is calculated from eqn. 13.

New values for R^* and D^* are determined by using eqn. 14, and LS^* becomes

$$LS^* = \overline{LS}' + \sigma_{LS'}^2 \left[(LS + \dots + E)^* - \overline{(LS + \dots + E)}' \right] / \sigma_{(LS + \dots + E)'}^2 \quad (17)$$

where $\overline{(LS + \dots + E)}'$ and $\sigma_{(LS + \dots + E)'}^2$ are the mean and variance of the approximating normal distribution for the joint distribution of $(LS + \dots + E)$, at the previous design point, and $(LS + \dots + E)^*$ is the new design point, calculated from eqn. 14 (by replacing X with $(LS + \dots + E)$). The new values for S^* , LT^* , W^* and E^* are calculated similarly.

RESISTANCE AND LOAD COMPONENTS

Each load component is fully determined by the parameters p and n and the CDF of intensity, $F_{\text{int}}(\cdot)$. The models of load components and resistance are presented below. The nominal values of loads are those specified by the National Building Code of Canada [3].

Resistance It is assumed that the resistance is time invariant, with $n = 1$, and $p = 1$.

The CDF of resistance, $F_R(x)$, is taken as lognormal. The lognormal $F_R(x)$ can be calculated using a standard normal distribution function as follows, see [4],

$$F_R(x) = \Phi(\ln(x/\check{R})/\sigma_{\ln R}) \quad (18)$$

where \check{R} = median of R .

The median, \check{R} , and $\sigma_{\ln R}$ can be expressed in terms of the mean resistance, \bar{R} , and coefficient of variation of resistance, V_R :

$$\check{R} = \bar{R} \exp(-\frac{1}{2}\sigma_{\ln R}^2), \text{ and}$$

$$\sigma_{\ln R}^2 = \ln(V_R^2 + 1) \tag{19}$$

Also the density function of resistance can be expressed in terms of the standard normal density function, φ :

$$f_R(x) = (1/x\sigma_{\ln R})\varphi(\ln(x/\bar{R})/\sigma_{\ln R}) \tag{20}$$

Dead load is considered as time invariant, with $n = 1$ and $p = 1$. The CDF is assumed to be a normal distribution. The ratio of mean-to-nominal is taken as $\lambda_D = 1.0$ and the coefficient of variation as $V_D = 10\%$, including analysis factor.

Live load L is considered as a sum of two components, sustained live load, LS , and transient live load, LT . LS includes the weight of furniture and removable equipment as well as weight of persons usually in the building. LT includes the weight of items and persons in the building only occasionally, for example, people gathering for a special meeting, or weight of furniture stored in an office during remodelling, etc.

The live load model is derived from the results of statistical studies described by Allen [5] and by Ellingwood et al. [6].

Sustained live load It is assumed that changes of LS occur every 5 years, so $n = 10$ for 50 year lifetime, with $p = 1.0$ (LS is always present). The CDF of intensity, F_{LS} , is a gamma distribution,

$$F_{LS}(x) = \Gamma(k, \lambda x) / \Gamma(k) \tag{21}$$

where $\Gamma(k, \lambda x) = \int_0^{\lambda x} e^{-u} u^{k-1} du$, and $\Gamma(k) =$

$\int_0^\infty e^{-u} u^{k-1} du$, k and λ are constants. Consequently the density function of LS is

$$f_{LS}(x) = \frac{\lambda}{L} \left(\frac{\lambda x}{L} \right)^{k-1} e^{-\lambda x/L} / \Gamma(k) \tag{22}$$

where $L =$ nominal live load as specified in the National Building Code of Canada [3]. The parameters k and λ depend on the parameters of the sustained live load distribution.

The statistical studies of live load provide arbitrary-point-in-time values of LS . These values correspond to realizations of intensity of LS . The ratios of mean-to-nominal and the coefficients of variation of LS are given in Table 1.

For the gamma distribution the mean, μ , and variance, σ^2 , are

$$\mu = k/\lambda; \sigma^2 = k/\lambda^2 \tag{23}$$

Assuming nominal live load is 1, μ and σ^2 can be expressed in terms of λ_{LS} and V_{LS} as follows:

$$\sigma^2 = (\lambda_{LS} V_{LS})^2. \tag{24}$$

Then from eqns. 23 and 24, after transformations,

$$k = 1/V_{LS}^2; \lambda = 1/(\lambda_{LS} V_{LS}^2) \tag{25}$$

The resulting values of k and λ are also given in Table 1.

Transient live load It is assumed that the basic time interval for LT is 7 hours, and that LT occurs once per month. The corre-

TABLE 1

Parameters of LS-intensity

Structural Type	Tributary area * (m ²)	Mean-to-nominal ratio (λ_{LS})	Coefficient of variation (V_{LS})	Distribution parameters	
				k	λ
Beam	20	0.24	0.70	2.04	8.50
	100	0.39	0.50	4.0	10.26
Column	100	0.39	0.45	4.94	12.66
	1000	0.55	0.35	8.16	14.84

* Tributary area is the area over which live load is placed to calculate the load effect.

TABLE 2
Parameters of LT for 50 years

Structural Type	Tributary area (m ²)	Mean-to-nominal ratio (λ_{LT})	Coefficient of Variation (V_{LT})	Distribution parameters	
				α	u
Beam	20	0.60	0.19	11.5	0.0
	100	0.40	0.19	17.0	0.0
Column	100	0.40	0.19	17.0	0.0
		0.20	0.19	33.0	0.0

sponding $n = 62571$ (for 50 year lifetime), and $p = 0.008$. The CDF of intensity is an extreme type I distribution, with

$$F_{LT}(x) = \exp\{-\exp[-\alpha(x/L - u)]\} \quad (26)$$

where $L =$ nominal live load, α, u are constants. The corresponding density function $f_{LT}(x)$ is

$$f_{LT}(x) = \frac{\alpha}{L} \exp\left[-\alpha\left(\frac{x}{L} - u\right)\right] \times \exp\left\{-\exp\left[-\alpha\left(\frac{x}{L} - u\right)\right]\right\} \quad (27)$$

The assumed mean-to-nominal ratios and the coefficients of variation for maximum values of LT in 50 years are given in Table 2. The parameters α and u in eqns. 26 and 27 were selected accordingly (see Table 2). The relationship between α, u, λ_{LT} and V_{LT} is shown in Fig. 1.

Snow The basic period is taken as 8 weeks, and it is assumed that snow occurs once in a year. These assumptions correspond to $n = 325$ and $p = 0.15$. The CDF of intensity is taken as an extreme type I distribution with

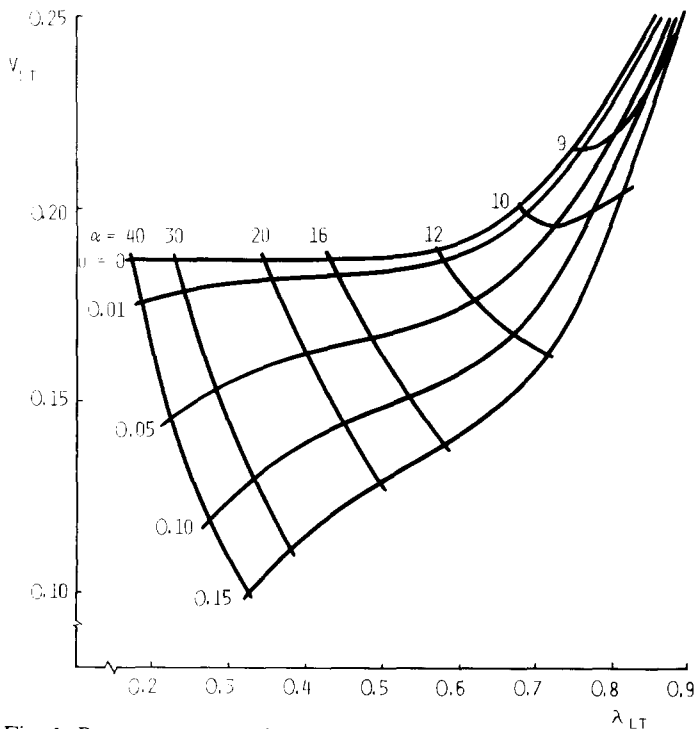


Fig. 1. Parameters u and λ for LT .

$F_S(x)$,

$$F_S(x) = \exp\{-\exp[\alpha(x/S - u)]\} \quad (28)$$

where S = nominal snow load, α and u are constants. The probability density function, $f_S(x)$, is

$$f_S(x) = \frac{\alpha}{S} \exp\left[-\alpha\left(\frac{x}{S} - u\right)\right] \times \exp\left\{-\exp\left[-\alpha\left(\frac{x}{S} - u\right)\right]\right\} \quad (29)$$

The parameters α and u were derived so that the 30 year mean-to-nominal ratio is 0.80 and the coefficient of variation is 0.25. The corresponding 50 year values are 0.88 and 0.227 respectively. The parameters are $\alpha = 6.45$, $u = 0.191$.

Wind It is assumed that wind occurs once in a month and the basic time interval is 4 hours. This corresponds to $n = 109,500$ in 50 years and $p = 0.005$.

The CDF of intensity is taken as an extreme type I distribution with $F_W(x)$ and $f_W(x)$ similar to eqns. 28 and 29, respectively. The parameters α and u were calculated so that 30 year mean-to-nominal and the coefficient of variation were equal to 0.80 and 0.25 respectively (the corresponding 50 year values are 0.875 and 0.177). The parameters are $\alpha = 7.9$, $u = 0.0012$.

Earthquake The basic time interval is $\frac{1}{2}$ minute with a once in a year occurrence. This corresponds to $n = 52.56 \times 10^6$ and $p = 0.9513 \times 10^{-6}$.

The CDF of intensity is taken as an extreme type II distribution with

$$F_E(x) = \exp\left\{-\left(\frac{uE}{x}\right)^k\right\} \quad (30)$$

where E = nominal earthquake load, u and k are constants. The constants were calculated from the assumption of 30 year mean-to-nominal equal to 0.30 and the coefficient of variation equal to 0.70. The parameters are $u = 0.10$ and $k = 4$.

For comparison the 50 year CDF's of all load components are plotted on normal probability paper in Fig. 2.

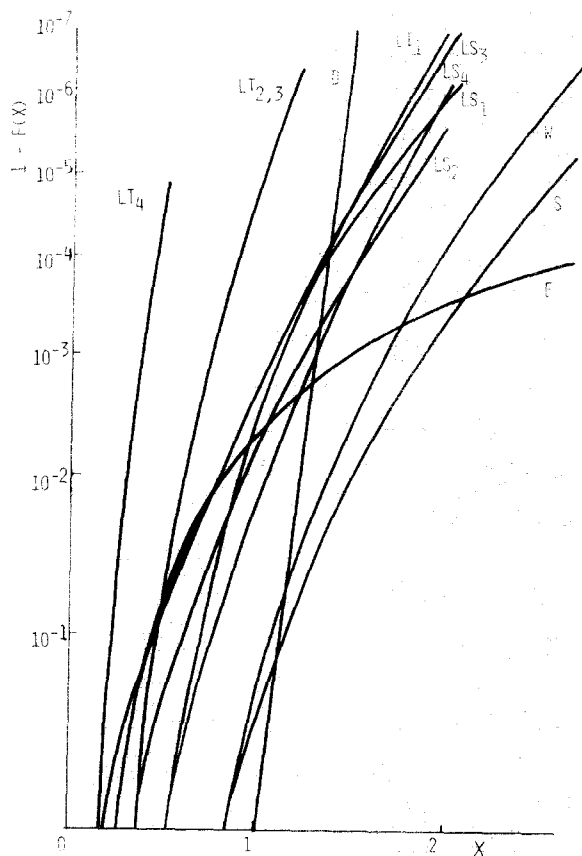


Fig. 2. CDF's for load components (50 years), subscripts at LS and LT : 1-beams with $A_1 = 20 \text{ m}^2$, 2-beams with $A_1 = 100 \text{ m}^2$, 3-columns with $A_1 = 100 \text{ m}^2$, 4-columns with $A_1 = 1000 \text{ m}^2$.

DESIGN FORMULAS

Two design options are considered, as provided by the National Building Code of Canada [3]. The first one is based on allowable stresses and is referred to as Working Stress Design, WSD. The other one is a load and resistance factor format, referred to as Limit State Design, LSD.

WSD is based on comparison of service loads with allowable loads. The design criterion is,

$$R/SF \geq \max \begin{cases} D + L \\ D + Q \\ 0.75(D + L + Q) \end{cases} \quad (31)$$

where D = nominal dead load, L = nominal live load, including snow, Q = nominal wind or earthquake, R = nominal resistance, and SF = safety factor.

In the code no distinction is made between R and SF , and R/SF is specified as the allowable load. Therefore in this study R in eqn. 31 is replaced with \bar{R}/λ_R , and SF is combined with λ_R into $(\lambda_R SF)$, so that eqn. 31 becomes

$$\bar{R}/(\lambda_R SF) = (\text{max. load from eqn. 31}) \quad (32)$$

In LSD the design criterion is

$$\phi R \geq \max \begin{cases} 1.25D + 1.5L \\ 1.25D + 1.5Q \\ 1.25D + 1.05(L + Q) \\ 0.85D + 1.5Q \end{cases} \quad (33)$$

where D , L , Q and R are as in Eq. 31, and ϕ = resistance factor.

In the calculations the CDF of resistance is assumed to be lognormal. The parameters of R , the ratio of mean to nominal, λ_R , and the coefficient of variation, V_R , depend on type of material and limit state (flexure, shear, deflection, etc.). Examples of λ_R and V_R for typical

TABLE 3
Examples of λ_R and V_R , from [6]

Type of Structure and Limit State	Mean to nominal ratio (λ_R)	Coefficient of Variation (V_R)
Steel:		
girders, flexure	1.08	0.12
shear	1.14	0.16
columns	1.08	0.14
Reinforced concrete:		
beams, flexure	1.01–1.22	0.12–0.16
shear	0.93–1.09	0.17–0.21
columns	0.95–1.10	0.14–0.17
Prestressed concrete:		
flexure	1.03–1.06	0.057–0.14
Timber*:		
beams, flexure	1.70–3.85	0.25–0.45
shear		
columns		

* Ellingwood [7].

structures are given in Table 3.

The ratios λ_R depend on the current nominal values of R , and may be subject to change. Therefore in this study the resistance factor, ϕ , and λ_R are combined into ϕ/λ_R , so that eqn. 33 becomes

$$(\phi/\lambda_R)\bar{R} = (\text{max. load from eqn. 33}) \quad (34)$$

Single resistance factors, $(\lambda_R SF)$ for WSD and (ϕ/λ_R) for LSD, allow for more flexibility in the interpretation of results.

CALCULATION OF SAFETY INDICES

Safety indices were computed for beams and columns designed according to the National Building Code of Canada [3]. The considered combinations include four cases with two loads: $D + L$, $D + S$, $D + W$ and $W - D$, and four cases with three loads: $D + L + S$, $D + L + W$, $D + L + E$, and $D + S + W$, each case with a wide range of load ratios. Four

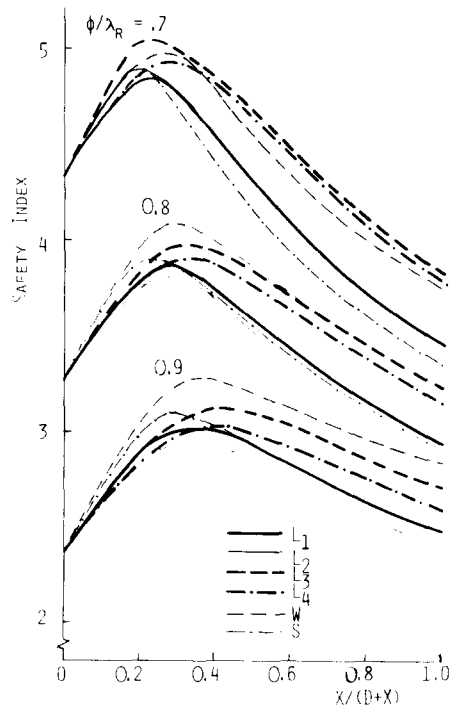


Fig. 3. Safety indices for LSD, $V_R = 0.10$.

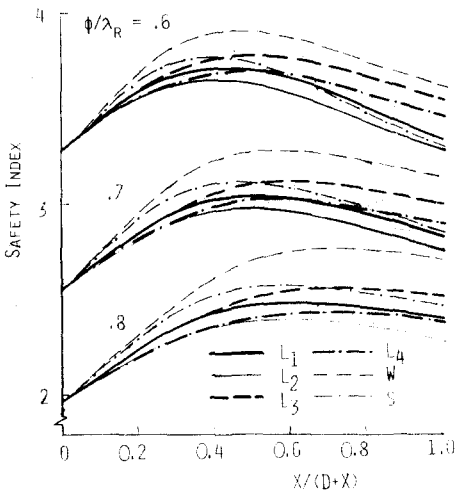


Fig. 4. Safety indices for LSD, $V_R = 0.20$.

models for live load are considered, according to data in Table 1 and 2.

For each set of loads the mean resistance is calculated using eqn. 32 for WSD and eqn. 34 for LSD. The computations were carried out for WSD for $(\lambda_R SF)$ values ranging from 1.60 to 5.0, for LSD for (ϕ/λ_R) from 0.3 to 0.9; and for V_R equal to 10, 20 and 40%.

The results are shown in Figs. 3-8 and in Tables 4-9. In the figures safety indices are plotted versus the ratio of X to $D + X$, where X is L , S or W ; in Figs. 3-5 for LSD and

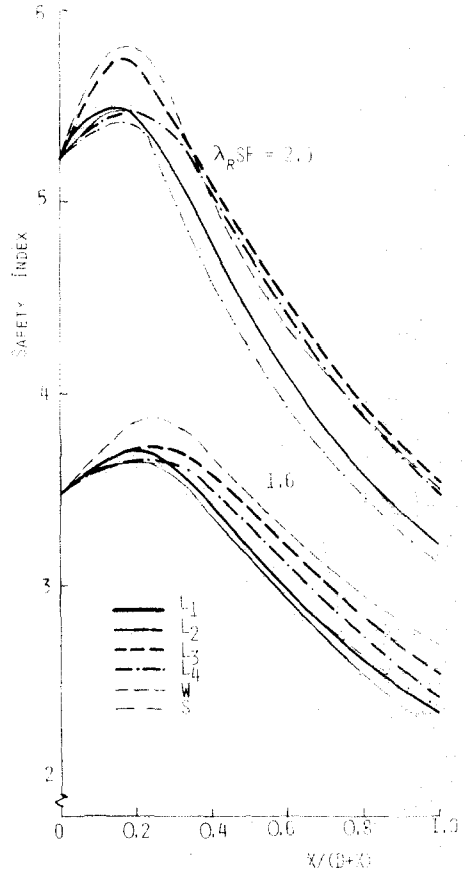


Fig. 6. Safety indices for WSD, $V_R = 0.10$.

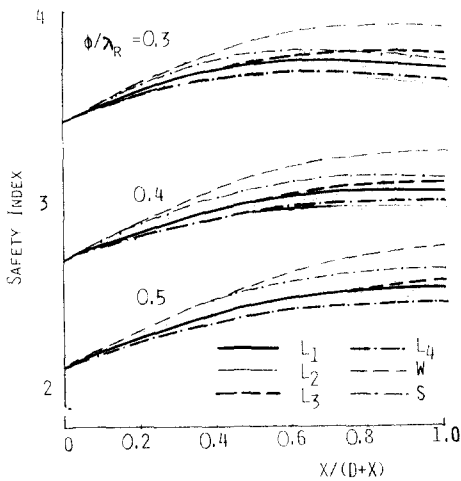


Fig. 5. Safety indices for LSD, $V_R = 0.40$.

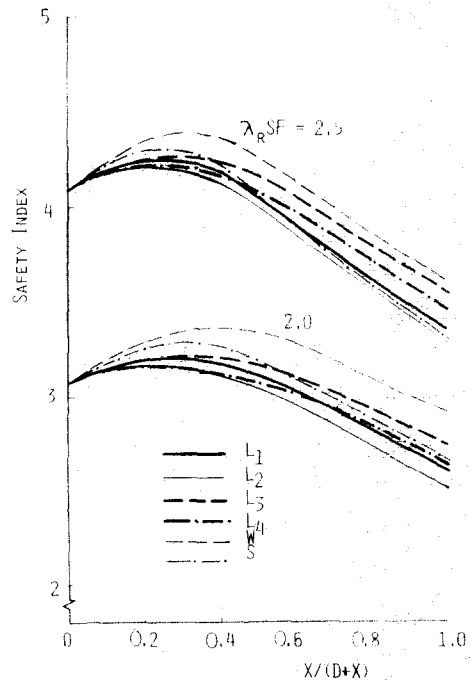


Fig. 7. Safety indices for WSD, $V_R = 0.20$.

TABLE 4
Safety indices for LSD, $V_R = 10\%$

Nominal value of Load Component			Safety indices													
D	L	S	W	E	$\sigma/\lambda_R = 0.90$				$\phi/\lambda_R = 0.80$				$\phi/\lambda_R = 0.70$			
					Beams		Columns		Beams		Columns		Beams		Columns	
					$A_1 = 20$	100	1000	20	100	1000	20	100	1000	20	100	1000
9	4.5	1			3.58	3.52	3.70	3.70	3.57	4.36	4.34	4.57	4.46	5.22	5.24	5.45
3	1.5	2			4.22	4.06	4.08	4.08	3.94	4.86	4.72	4.74	4.62	5.57	5.45	5.36
1	0.5	1			3.86	3.78	3.79	3.79	3.65	4.44	4.37	4.34	4.25	5.08	5.01	4.91
2	1	3			3.57	3.47	3.47	3.47	3.38	4.09	4.00	4.00	3.92	4.67	4.59	4.52
1	0.5	9			2.74	2.72	2.72	2.72	2.70	3.16	3.14	3.14	3.13	3.63	3.62	3.60
9	4.5		1		3.02	2.94	3.10	3.10	3.01	3.83	3.78	3.98	3.90	4.71	4.70	4.90
3	1.5	2			3.41	3.12	3.17	2.88	2.88	4.16	3.91	3.95	3.70	4.96	4.75	4.57
1	0.5	1			3.09	2.88	2.90	2.69	2.69	3.75	3.58	3.59	3.41	4.47	4.32	4.18
2	1	3			2.96	2.82	2.83	2.68	2.68	3.55	3.44	3.44	3.32	4.20	4.10	4.00
1	0.5	9			2.88	2.86	2.86	2.83	2.83	3.34	3.32	3.32	3.30	3.86	3.84	3.82
9	4.5			1	3.02	2.94	3.10	3.10	3.01	3.83	3.78	3.98	3.90	4.71	4.70	4.90
3	1.5	2		2	4.46	3.50	3.50	3.43	3.43	5.22	3.72	3.72	3.67	6.05	3.94	3.90
1	0.5	1		1	3.37	3.32	3.32	3.27	3.27	3.57	3.53	3.53	3.49	3.77	3.74	3.71
2	1	3		3	3.29	3.25	3.25	3.21	3.21	3.47	3.44	3.44	3.41	3.66	3.63	3.61
1	0.5	9		9	3.21	3.20	3.20	3.20	3.20	3.35	3.35	3.35	3.34	3.50	3.50	3.50
9			4.5	1	3.08	3.08	3.08	3.08	3.08	3.81	3.81	3.81	3.81	4.60	4.60	4.60
3			1.5	2	3.70	3.70	3.70	3.70	3.70	4.41	4.41	4.41	4.41	5.18	5.18	5.18
1			0.5	1	3.37	3.37	3.37	3.37	3.37	4.00	4.00	4.00	4.00	4.68	4.68	4.68
2			1	3	3.06	3.06	3.06	3.06	3.06	3.58	3.58	3.58	3.58	4.16	4.16	4.16
1			0.5	9	2.91	2.91	2.91	2.91	2.91	3.36	3.36	3.36	3.36	3.88	3.88	3.88
-1			1	1	3.07	3.07	3.07	3.07	3.07	3.26	3.26	3.26	3.26	3.50	3.50	3.50
-1			2	2	3.00	3.00	3.00	3.00	3.00	3.31	3.31	3.31	3.31	3.67	3.67	3.67
-1			9	9	2.88	2.88	2.88	2.88	2.88	3.28	3.28	3.28	3.28	3.73	3.73	3.73

Note: All tributary areas, A_1 , are in m^2 .

TABLE 5
Safety indices for LSD, $V_R = 20\%$

Nominal value of Load Component		Safety indices												
		$\phi/\lambda_R = 0.80$			$\phi/\lambda_R = 0.70$			$\phi/\lambda_R = 0.60$						
		Beams		Columns	Beams		Columns	Beams		Columns				
D	L	S	W	E	$A_1 = 20$	100	1000	20	100	1000	20	100	1000	
9	4.5	1			2.74	2.70	2.74	2.64	3.35	3.31	3.36	4.05	4.01	4.08
3	1.5	2			3.37	3.27	3.24	3.10	3.95	3.86	3.87	4.61	4.52	4.54
1	0.5	1			3.28	3.20	3.21	3.07	3.82	3.74	3.75	4.42	4.35	4.36
2	1	3			3.17	3.08	3.08	2.98	3.67	3.58	3.58	4.22	4.14	4.14
1	0.5	9			2.64	2.62	2.52	2.60	3.06	3.04	3.04	3.53	3.51	3.50
9	4.5		1		2.37	2.30	2.37	2.31	2.98	2.91	2.99	3.68	3.62	3.70
3	1.5		2		2.66	2.44	2.46	2.25	3.27	3.05	3.08	3.96	3.75	3.59
1	0.5		1		2.55	2.37	2.38	2.21	3.13	2.97	2.98	3.79	3.64	3.50
2	1		3		2.56	2.44	2.44	2.31	3.11	3.00	3.00	3.73	3.62	3.52
1	0.5		9		2.71	2.69	2.68	2.66	3.16	3.14	3.14	3.66	3.65	3.63
9	4.5			1	2.37	2.30	2.37	2.30	2.90	2.91	2.99	3.60	3.62	3.70
3	1.5			2	3.34	3.28	3.36	3.15	3.95	3.89	3.98	4.65	4.60	4.69
1	0.5			1	3.40	3.34	3.34	3.27	3.63	3.58	3.58	3.86	3.83	3.79
2	1			3	3.33	3.27	3.29	3.24	3.53	3.50	3.50	3.75	3.72	3.70
1	5			9	3.26	3.26	3.26	3.25	2.52	3.42	3.42	3.60	3.59	3.59
9				4.5	2.47	2.47	2.47	2.47	3.07	3.07	3.07	3.75	3.75	3.75
3				1.5	2.90	2.90	2.90	2.90	3.50	3.50	3.50	4.18	4.18	4.18
1				0.5	2.80	2.80	2.80	2.80	3.37	3.37	3.37	4.01	4.01	4.01
2				1	2.73	2.73	2.73	2.73	3.27	3.27	3.27	3.87	3.87	3.87
1				0.5	2.73	2.73	2.73	2.73	3.18	3.18	3.18	3.69	3.69	3.69
-1					3.12	3.12	3.12	3.12	3.12	3.30	3.30	3.30	3.54	3.54
-1					2.87	2.87	2.87	2.87	2.87	3.14	3.14	3.14	3.43	3.43
-1					2.56	2.56	2.56	2.56	2.57	2.87	2.87	2.87	3.18	3.18

Note: All tributary areas, A_1 , are in m^2 .

TABLE 6
Safety indices for LSD, $V_R = 40\%$

Nominal value of Load Component				Safety indices												
D	L	S	W	E	$\phi/\lambda_R = 0.50$				$\phi/\lambda_R = 0.40$				$\phi/\lambda_R = 0.30$			
					Beams		Columns		Beams		Columns		Beams		Columns	
					$A_1 = 20$	100	1000	100	1000	20	100	1000	20	100	1000	
9	4.5	1			2.56	2.52	2.40	2.55	2.40	3.12	3.08	3.11	3.06	3.85	3.81	3.79
3	1.5	2			2.94	2.87	2.77	2.88	2.77	3.50	3.43	3.44	2.22	4.22	4.16	4.06
1	0.5	1			2.98	2.88	2.79	2.89	2.79	3.53	3.44	3.44	3.35	4.24	4.15	4.06
2	1	3			2.94	2.87	2.79	2.87	2.79	3.48	3.41	3.41	3.34	4.17	4.10	4.04
1	0.5	9			2.73	2.71	2.70	2.71	2.70	3.22	3.21	3.24	3.19	3.85	3.84	3.86
9	4.5		1		2.35	2.31	2.31	2.35	2.31	2.92	2.88	2.91	2.88	3.64	3.61	3.61
3	1.5	2			2.52	2.39	2.28	2.40	2.28	3.08	2.95	2.97	2.85	3.81	3.68	3.58
1	0.5	1			2.47	2.37	2.27	2.37	2.27	3.03	2.93	2.93	2.83	3.75	3.65	3.56
2	1	3			2.51	2.43	2.35	2.43	2.35	3.06	2.98	2.98	2.91	3.77	3.70	3.62
1	0.5	9			2.72	2.70	2.68	2.70	2.68	3.24	3.22	3.22	3.20	3.89	3.88	3.86
9	4.5		1		2.35	2.31	2.30	2.35	2.30	2.92	2.88	2.91	2.87	3.64	3.61	3.60
3	1.5	2			2.89	2.85	2.77	2.89	2.77	3.46	3.42	3.45	3.33	4.18	4.14	4.07
1	0.5	1			3.23	3.11	2.99	3.13	2.99	3.79	3.67	3.69	3.56	4.52	4.40	4.28
2	1	3			3.52	3.42	3.31	3.42	3.31	3.85	3.93	3.82	3.85	4.20	4.19	4.17
1	0.5	9			3.50	3.49	3.48	3.49	3.48	3.74	3.73	3.74	3.73	4.03	4.06	4.06
9		4.5	1		2.41	2.41	2.41	2.41	2.41	2.98	2.98	2.98	2.98	3.70	3.70	3.70
3		1.5	2		2.66	2.66	2.66	2.66	2.66	3.22	3.22	3.22	3.22	3.94	3.94	3.94
1	0.5	1	1		2.62	2.62	2.62	2.62	2.62	3.18	3.18	3.18	3.18	3.90	3.90	3.90
2	1	3			2.62	2.62	2.62	2.62	2.62	3.17	3.17	3.17	3.17	3.88	3.88	3.88
1		0.5	9		2.74	2.74	2.74	2.74	2.74	3.26	3.26	3.26	3.26	3.91	3.91	3.91
-1			1		3.35	3.35	3.35	3.35	3.35	2.22	2.62	3.64	3.62	2.62	2.62	2.62
-1			2		2.06	2.06	2.06	2.06	2.06	2.15	2.15	2.15	2.15	2.24	2.24	2.24
-1			9		1.83	1.83	1.83	1.83	1.83	1.97	1.97	1.97	1.97	2.10	2.10	2.10

Note: All tributary areas, A_1 , are in m^2 .

TABLE 7
Safety indices for WSD, $V_R = 10\%$

Nominal value of Load component				Safety indices													
D	L	S	W	E	$\lambda_R \times SF = 1.60$				$\lambda_R \times SF = 2.00$				$\lambda_R \times SF = 2.50$				
					Beams		Columns		Beams		Columns		Beams		Columns		
					$A_t = 20$	100	1000	20	100	1000	20	100	1000	20	100	1000	
9	4.5	1			4.04	4.00	4.09	4.21	4.09	4.09	5.47	5.51	5.01	5.74	6.05	6.77	7.36
3	1.5	2			4.45	4.30	4.17	4.30	4.17	4.17	5.63	5.51	5.53	5.42	6.00	6.70	6.62
1	0.5	1			4.00	3.93	3.80	3.93	3.80	3.80	5.00	5.01	5.02	4.71	6.13	6.08	5.77
2	1	3			3.64	3.55	3.45	3.55	3.45	3.45	4.61	4.54	4.54	4.46	5.58	6.52	5.46
1	0.5	9			2.65	2.63	2.61	2.63	2.61	2.61	3.45	3.43	3.43	3.41	4.24	4.23	4.21
9	4.5		1		3.56	3.50	3.68	3.68	3.61	3.61	5.02	5.03	5.31	5.27	6.41	6.50	6.88
3	1.5		2		3.97	3.71	2.96	2.96	2.66	2.66	5.30	5.10	4.42	4.19	6.58	6.41	5.58
1	0.5		1		3.54	3.36	3.01	3.01	2.81	2.81	4.74	4.60	4.28	4.17	5.89	5.77	5.34
2	1		3		3.10	2.79	2.98	2.98	2.84	2.84	4.20	4.10	4.10	4.00	5.26	5.17	5.09
1	0.5		9		2.78	2.76	2.76	2.76	2.74	2.74	3.65	3.64	3.64	3.62	4.51	4.50	4.48
9	4.5			1	3.57	3.53	3.68	3.68	3.57	3.57	5.02	5.03	5.31	5.26	6.41	6.50	6.88
3	1.5			2	4.26	4.23	3.44	3.44	3.36	3.36	5.60	5.35	3.85	3.80	4.20	4.17	4.17
1	0.5			1	3.41	3.33	3.36	3.36	3.30	3.30	3.76	3.73	3.73	3.69	4.05	4.03	4.01
2	1			2	3.33	3.30	3.30	3.30	3.26	3.26	3.66	3.63	3.63	3.61	3.74	3.92	3.90
9		4.5	1		3.56	3.56	3.56	3.56	3.56	3.56	4.00	4.00	4.00	4.00	6.11	6.11	6.11
3		1.5	2		3.51	3.51	3.51	3.51	3.51	3.51	4.04	4.04	4.04	4.04	6.10	6.10	6.10
1		0.5	1		3.48	3.48	3.48	3.48	3.48	3.48	4.64	4.64	4.64	4.64	5.76	5.76	5.76
2		1	3		3.28	3.28	3.28	3.28	3.28	3.28	4.34	4.34	4.34	4.34	5.37	5.37	5.37
1		0.5	9		2.81	2.81	2.81	2.81	2.81	2.81	3.60	3.60	3.60	3.60	4.53	4.53	4.53

Note: All tributary areas, A_t , are in m^2 .

TABLE 8
Safety indices for WSD, $V_R = 20\%$

Nominal value of Load Component			Safety indices															
			$\lambda_R \times SF = 2.00$				$\lambda_R \times SF = 2.50$				$\lambda_R \times SF = 3.00$							
D	L	S	W	E	Beams		Columns		Beams		Columns		Beams		Columns			
					$A_t = 20$	100	1000	100	1000	20	100	1000	20	100	1000	20	100	1000
9	4.5	1			3.53	3.47	3.55	3.45	3.55	4.54	4.49	4.47	5.36	5.32	5.34	5.43	5.32	5.34
3	1.5	2			4.00	3.71	3.92	3.75	3.92	4.94	4.86	4.72	5.69	5.61	5.49	5.63	5.61	5.49
1	0.5	1			3.82	3.74	3.75	3.62	3.75	4.68	4.62	4.52	5.42	5.32	5.22	5.32	5.32	5.22
2	1	3			3.62	3.53	3.53	3.44	3.53	4.42	4.35	4.27	5.07	5.00	4.93	5.00	5.00	4.93
1	0.5	9			3.87	2.88	2.88	2.86	2.88	3.58	3.56	3.55	4.13	4.12	4.11	4.12	4.12	4.11
9	4.5		1		3.20	3.14	3.21	3.16	3.21	4.22	4.16	4.20	5.04	5.00	5.05	5.10	5.10	5.05
3	1.5	2			3.53	2.78	3.35	3.15	3.35	4.53	3.80	4.17	5.33	4.62	5.00	5.18	4.62	5.00
1	0.5	1			3.36	2.93	3.20	3.04	3.20	4.30	3.90	4.16	5.04	4.67	4.80	4.92	4.67	4.80
2	1		3		3.11	3.00	3.00	2.88	3.00	4.00	3.90	3.90	4.70	4.61	4.52	4.61	4.61	4.52
1	0.5		9		2.98	2.96	2.96	2.94	2.96	3.72	3.70	3.68	4.31	4.30	4.28	4.30	4.30	4.28
9	4.5			1	3.20	3.14	3.21	3.15	3.21	4.22	4.16	4.19	5.04	5.00	5.04	5.10	5.00	5.04
3	1.5			2	3.68	3.62	3.70	3.50	3.70	4.69	4.64	4.74	5.51	5.47	5.39	5.58	5.47	5.39
1	0.5			1	3.61	3.57	3.57	3.52	3.57	3.74	3.71	3.88	4.17	4.15	4.13	4.15	4.15	4.13
2	1			3	3.53	3.50	3.50	3.47	3.50	3.83	3.81	3.79	4.05	4.04	4.02	4.04	4.04	4.02
1	0.5			9	3.36	3.36	3.36	3.35	3.36	3.61	3.61	3.61	3.81	3.81	3.80	3.81	3.81	3.80
9		4.5	1		3.28	3.28	3.28	3.28	3.28	4.27	4.27	4.27	5.06	5.06	5.06	5.06	5.06	5.06
3		1.5	2		3.24	3.24	3.24	3.24	3.24	3.24	4.23	4.23	4.23	5.02	5.02	5.02	5.02	5.02
1		0.5	1		3.34	3.34	3.34	3.34	3.34	4.26	4.26	4.26	4.99	4.99	4.99	4.99	4.99	4.99
2		1	3		3.27	3.27	3.27	4.13	3.27	4.13	4.13	4.82	4.82	4.82	4.82	4.82	4.82	4.82
1		0.5	9		3.01	3.01	3.01	3.01	3.01	3.74	3.74	3.74	4.33	4.33	4.33	4.33	4.33	4.33

Note: All tributary areas, A_t in m^2 .

TABLE 9
Safety indices for WSD, $V_R = 40\%$

Nominal value of Load Component				Safety indices												
D	L	S	W	E	$\lambda_R \times SF = 3.00$				$\lambda_R \times SF = 3.50$				$\lambda_R \times SF = 5.00$			
					Beams		Columns		Beams		Columns		Beams		Columns	
					$A_1 = 20$	100	1000	20	100	1000	20	100	1000	20	100	1000
9	4.5	1			2.83	2.79	2.83	2.77	3.22	3.18	3.21	3.16	4.12	4.08	4.12	4.06
3	1.5	2			3.14	3.08	3.09	2.97	3.52	3.46	3.47	3.36	4.42	4.66	4.37	4.26
1	0.5	1			3.15	3.06	3.06	2.97	3.53	3.44	3.44	3.35	4.41	4.32	4.32	4.23
2	1	3			3.08	3.01	3.01	2.94	3.45	3.38	3.38	3.31	4.31	4.24	4.24	4.17
1	0.5	9			2.77	2.75	2.75	2.74	3.10	3.09	3.09	3.07	3.87	3.87	3.87	3.86
9	4.5		1		2.65	2.61	2.65	2.61	3.04	3.00	3.03	3.00	3.94	3.90	3.94	3.91
3	1.5		2		2.84	2.71	2.73	2.61	3.23	3.10	3.11	2.99	4.13	4.01	4.02	3.90
1	0.5		1		2.78	2.67	2.68	2.58	3.16	3.06	3.06	2.96	4.05	3.96	3.96	3.86
2	1		3		2.68	2.60	2.60	2.52	3.06	2.97	2.98	2.90	3.94	3.87	3.87	3.79
1	0.5		9		2.76	2.74	2.74	2.73	3.11	3.10	3.10	3.08	3.93	3.92	3.92	3.90
9	4.5			1	2.65	2.61	2.65	2.61	3.04	3.00	3.03	2.99	3.94	3.98	3.94	3.90
3	1.5			2	2.92	2.88	2.91	2.80	3.30	3.27	3.30	3.18	4.21	4.17	4.21	4.09
1	0.5			1	3.38	3.26	3.28	3.15	3.77	3.65	3.65	3.53	4.67	4.55	4.57	4.44
2	1			3	3.68	3.57	3.57	3.48	3.85	3.93	3.82	3.85	4.28	4.26	4.25	
	0.5			9	3.52	3.51	3.51	3.51	3.68	3.68	3.68	3.67	4.05	4.05	4.05	4.04
9		4.5	1		2.71	2.71	2.71	2.71	3.10	3.10	3.10	3.10	4.00	4.00	4.00	4.00
3		1.5	2		2.68	2.68	2.68	2.68	3.07	3.07	3.07	3.07	4.77	3.97	3.97	3.97
1		0.5	1		2.78	2.78	2.78	2.78	3.16	3.16	3.16	3.16	4.05	4.05	4.05	4.05
2		1	3		2.79	2.79	2.79	2.79	3.17	3.17	3.17	3.17	4.05	4.05	4.05	4.05
1		0.5	9		2.78	2.78	2.78	2.78	3.14	3.14	3.14	3.14	3.95	3.95	3.95	3.95

Note: All tributary areas, A_1 , are in m².

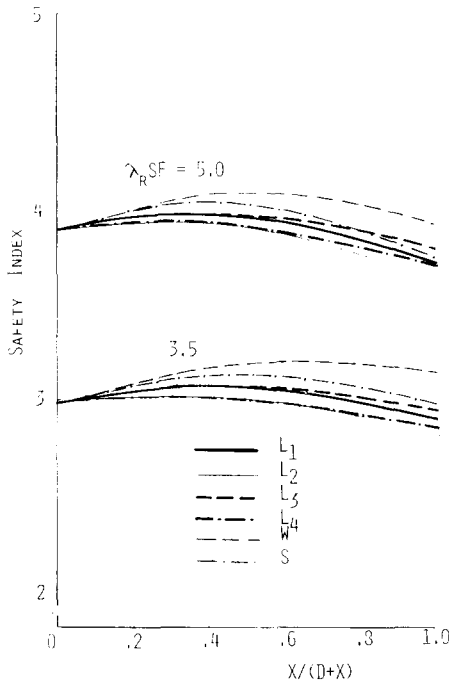


Fig. 8. Safety indices for WSD, $V_R = 0.40$.

$V_R = 10-40\%$, respectively, and in Figs. 6-8 for WSD and $V_R = 10-40\%$. The curves in the figures are denoted as follows: L_1 —live load for beams with tributary area $A_t = 20 \text{ m}^2$, L_2 —live load for beams with $A_t = 100 \text{ m}^2$, L_3 —live load for columns with $A_t = 100 \text{ m}^2$, L_4 —live load for columns with $A_t = 1,000 \text{ m}^2$, S—snow and W—wind. The safety indices computed for the combinations of three loads, and for $W-D$, are presented in the tables; in Tables 4-6 for LSD and $V_R = 10-40\%$ and in Tables 7-9 for WSD and $V_R = 10-40\%$.

SELECTION OF OPTIMUM β 'S AND SF 'S

The basic features of a building code are "goodness-of-fit" and simplicity. Since modern codes are developed based on the safety criteria, the goodness-of-fit is also measured in terms of safety. The perfect fit is achieved if safety of all structures designed using the

code is at an optimum level. Lind [8] considers the optimum safety as a parameter in the economical analysis (trade-off between cost of design and construction and the expected cost of failure).

The user of the code (designer) is interested in a simple format, and the simplicity depends on the number of different sets of load and resistance factors. However, as in the case of the optimum safety, the optimum simplicity is also a parameter of an economical trade-off. The selection of the optimum safety index and the optimum number of different ϕ 's or SF 's is beyond the scope of this paper. However, the calculated safety indices can be used to select the optimum resistance factors and allowable loads for a given target safety index, β_T .

The basic parameter affecting the selection is the coefficient of variation of resistance, V_R . For typical structures and materials V_R 's are well established, see Table 3.

For a given V_R , Tables 4-9 and Figs. 3-8 provide a spectrum of β 's. The optimum $(\phi/\lambda_R)_{opt}$ for LSD, or $(\lambda_R SF)_{opt}$ for WSD, provides the best fit to the preselected β_T . The selection may also be carried out by assigning weighting factors to various β 's in the tables and figures. The weight may represent the importance of the load combination to which it is assigned. Then for each value of (ϕ/λ_R) in LSD, or $(\lambda_R SF)$ in WSD, the weighted sum of the β 's can be calculated. The optimum factor provides the closest fit to β_T .

Finally, the optimum resistance factor, ϕ_{opt} , is

$$\phi_{opt} = (\phi/\lambda_R)_{opt} \lambda_R \quad (35)$$

where λ_R is given in Table 3 for typical structures and materials, and the optimum allowable load is equal to $(\lambda_R SF)_{opt}$.

CONCLUSIONS

A procedure has been developed to calculate safety indices for structures designed on

the basis of given load and resistance factors. The method is based on normal approximations of non-normal distributions at the design point.

Safety indices were calculated for typical cases of load components, including dead load, live load, snow, wind and earthquake. A wide range of resistance parameters (mean-to-nominal ratio and coefficient of variation) was considered. The results are presented in tables and in figures.

The calculated values of β can be used to select the optimum resistance factors and allowable loads according to the National Building Code of Canada.

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