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SIMPLE PARAMETERIZATION OF NUCLEAR DEFORMATION PARAMETERS ☆

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Analytical expressions are given for the deformation parameters β_2, β_4 and β_6 and compared to the data for $140 < A < 208$.

Semi-empirical expressions are often used to discuss global aspects of certain nuclear properties. Examples are masses [1], Coulomb energies [2] and nuclear radii [3]. The latter have been parameterized on the assumption of constant density in the nuclear interior ("volume conservation"). However, it appears that no simple analytical expressions exist to describe nuclear shapes.

Bertsch [4] has shown that a very simple model can be used to explain qualitatively the dependence on nucleon number of the nuclear quadrupole and hexadecapole deformations. Pairs of protons and neutrons are assumed to fill simultaneously the same major shell, and the residual interaction is introduced to construct aligned wave functions with strong spatial correlations. Orbitals are filled according to their closeness to the body-fixed symmetry axis, with equatorial orbits filled last. The result [4] is

$$\beta_L \propto \int_{1-z}^{1} P_L(\zeta)\, d\zeta ,\qquad (1)$$

where z varies from 0 to 1 when the shell is filled with α-particles. Integrating eq. (1) yields

$$\beta_2 \propto \tfrac{1}{2}z(1-z)(2-z) ,\qquad (2)$$

$$\beta_4 \propto \tfrac{1}{8}z(1-z)(2-z)(7z^2 - 14z + 4) .\qquad (3)$$

$$\beta_6 \propto \tfrac{1}{16}z(1-z)(2-z)[33(z-1)^4 - 30(z-1)^2 + 5].\qquad (4)$$

These equations cannot be expected to describe quantitatively the more realistic situation where protons and neutrons independently fill shell regions composed of different subshells. However, it will be shown below that only minor extensions and modifications of eqs. (2)–(4) will lead to expressions which give a very satisfactory description of the various deformation parameters for the deformed nuclei in the rare-earth region.

Experimental quadrupole deformation parameters β_2 were taken from several published tables [5–7]. One table [5] contains deformation parameters derived from several nuclear and atomic experimental techniques. The parameters from the other tables [6,7] are deduced entirely from experimental $B(E2)$ transition probabilities. (Ref. [6] is a recent update of ref. [7].) The deformation parameters β_2 used in the present work are quadrupoloid deformations obtained under the assumption of "volume conservation". The values of refs. [6,7] therefore have been corrected according to

$$\beta_2 = \beta_2(\text{table})[1 - \tfrac{2}{7}(5/\pi)^{1/2}\beta_2(\text{table})$$

$$+ (73/98\pi)\beta_2(\text{table})^2] .\qquad (5)$$

(See the first relation for the intrinsic electric quadrupole moment Q_0 in table 1 of ref. [5].) Excellent agreement between the tables [5–7] is thus obtained. The data are displayed in fig. 1. A few data points for Z = odd are left out for clarity. Experiments generally determine the quantity $\beta_2 R^2$. A value $R = 1.2\, A^{1/3}$ fm was assumed in the derivation [5–7] of the parameters β_2. The parameterization of r_0 on the basis of

the "constant volume" condition [3] could in principle be used instead.

Much fewer data exist for the deformation parameters β_4 and β_6. These were taken from refs. [8–11] and refs. [9,11], respectively. The accuracy is generally much less than for the β_2 data, and additional uncertainties arise for those data which require the transformation from nuclear to charge deformations. A scaling law has been established [9] to relate β^N to β^C, and they follow roughly the same trend [10] with the latter about 10% larger. Most of the β_4 data represent charge deformation parameters obtained directly from Coulomb excitation, electron scattering and Coulomb/nuclear interference. Many of the β_6 data are corrected nuclear deformation parameters [9]. The averaged data are displayed in figs. 2 and 3. An uncertainty of ±0.003, approximately 30%, has been arbitrarily assigned to the β_6 data.

A generalization of eqs. (2) to (4) requires the introduction of two independent variables to describe the dependence on proton number Z and neutron number N. These were taken as

$$x = (Z - 50)/(82 - 50) , \qquad (6a)$$

$$y = (N - 82)/(126 - 82) , \qquad (6b)$$

with

$$z = \tfrac{1}{2}(x + y) . \qquad (6c)$$

The following semi-empirical expressions were introduced to describe the nuclear deformation parameters β_2, β_4 and β_6,

$$\beta_2 = \beta_{20}x(1-x)(2-x)y(1-y)(2-y) + \beta_{21} , \qquad (7)$$

$$\beta_4 = \beta_{40}x(1-x)(2-x)y(1-y)(2-y)$$
$$\times (7z^2 - 14z + 4.9) , \qquad (8)$$

$$\beta_6 = \beta_{60}x(1-x)(2-x)y(1-y)(2-y)$$
$$\times [33(z-1)^4 - 30(z-1)^2 + 1] . \qquad (9)$$

Here, $\beta_{20} = 1.721$, $\beta_{40} = 0.504$, $\beta_{60} = 0.0215$ and $\beta_{21} = 0.082$. These equations require further justification and discussion. Eq. (7) has been used earlier [2] to describe Coulomb displacement energies of deformed nuclei.

Collectivity results from the cooperative effect of protons *and* neutrons in their respective shells. Indeed,
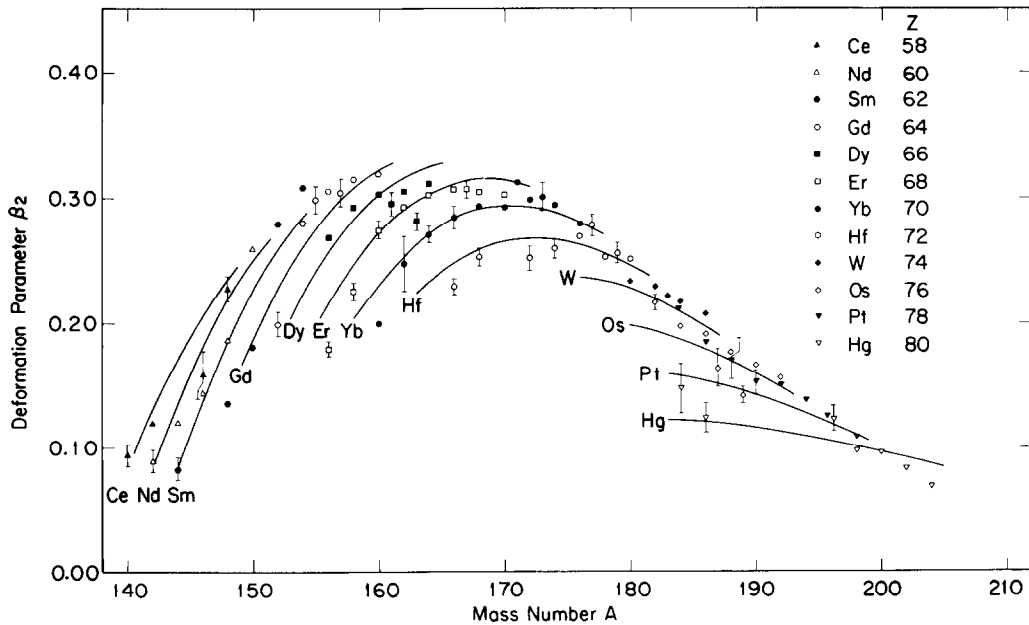

Fig. 1. Experimental deformation parameters β_2 from refs. [5–7]. The curves are calculated from eq. (7).
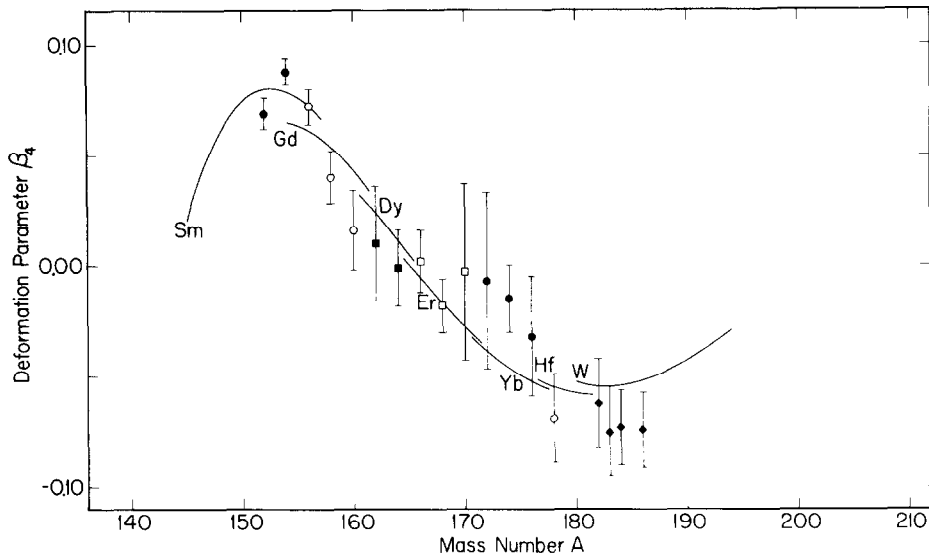
Fig. 2. Experimental deformation parameters β_4 from refs. [8–11]. The curves are calculated from eq. (8). See fig. 1 for an explanation of the symbols used for the various isotopes.

the deformation parameters β_2 are quite small for semi-magic nuclei. Furthermore, since protons and neutrons occupy different major shells, it is not unreasonable to apply eq. (2) to both protons and neutrons. These arguments justify the factor $x(1 - x)$ $\times (2 - x)y(1 - y)(2 - y)$. The observed approximate

linear increase of β_2 with N for constant Z is thus described. The additive constant β_{21} accounts for the fact that the β_2 values for semi-magic nuclei are not zero but about 3 times the respective single-particle value [6]. It should be noted that β_2 represents not only the deformation of statically deformed axially
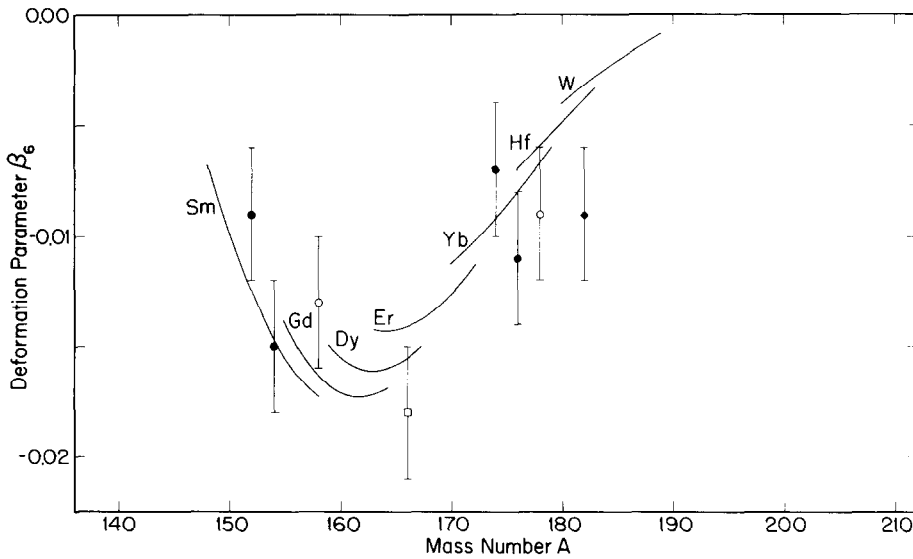

Fig. 3. Experimental deformation parameters β_6 from refs. [9,11]. The curves are calculated from eq. (9). See fig. 1 for an explanation of the symbols used for the various isotopes.

symmetric nuclei but also the "dynamic" deformation of vibrational nuclei expressed in terms of destruction and creation operators as $\beta_2 = (\Sigma_\mu \langle |\alpha_{2\mu}| \rangle^2)^{1/2}$. The parameters β_{20} and β_{21} of eq. (7) were obtained from a least-squares adjustment to the data displayed in fig. 1. Slightly different analytical forms of eq. (7) were also considered but found to lead to increased systematic deviations.

Small additional modifications were introduced in the expressions for β_4 and β_6 by adjusting the additive constants in the quadratic terms. This has the effect of slightly shifting the region where β_4 and β_6 change sign. It was done to achieve better agreement with the data. Eq. (1) based on the original simple assumption of filling in a particular sequence one major L-shell is of course partly invalidated by the presence of subshells which explains the observed presence of shifts.

The simple eqs. (7) to (9) describe the experimental deformation parameters β_2, β_4 and β_6 remarkably well. The standard deviation for the quadrupole deformation is $\sigma_{\beta_2} = 0.019$. Certain systematic differences are also discernible. For example, the initial increase of β_2 at the lower neutron numbers seems to contain a quadratic component, and the maximum values appear to be reached earlier than predicted. Additional, though purely empirical, parameters could easily be introduced to obtain even better agreement with the data.

The parameterizations of the nuclear deformation parameters β_2, β_4 and β_6 should be useful in the discussion of global nuclear properties. It does not replace theoretical approaches [12] based on models such as the Nilsson or modified harmonic oscillator models or the Strutinsky renormalization method.

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