EXPERIMENTAL PREDICTIONS FROM TECHNICOLOR THEORIES

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We calculate detailed predictions for masses, decay branching ratios, and production cross sections for new dynamical bosons expected as physical particles in a technicolor theory The colored technieta state ($m \simeq 240$ GeV) has a significant production cross section at Isabelle or the Tevatron collider. Light, neutral, spin-zero axions ($m \lesssim 2.5$ GeV) are expected, and also charged, spin-zero, axions (analogous to charged Higgs bosons) with mass 8 GeV The latter should be seen at PETRA/PEP.

1. Introduction

The standard model of electroweak [1] and strong interactions is by now well established at low energies. A serious flaw of the standard model is the existence of fundamental scalars [2]. It limits the predictive power of the model by introducing numerous arbitrary parameters. In addition, scalars cannot naturally account for the fact that the electroweak breaking scale (300 GeV) is much less than the cutoff scale ($\sim 10^{16}$ GeV).

Recently some attempts have been made to construct theories without fundamental scalars [2–17]. A common feature of these attempts is the existence of a new strong interaction, called technicolor, at the scale of ~ 1 TeV. An immediate consequence of this scenario is the existence of technihadrons (technirho, technisigma, technibaryons, etc.) at the 1 TeV scale.

The purpose of this paper is to discuss some possible consequences of these theories at energies between a few GeV and several hundred GeV. These consequences are very different from what one expects to see in theories with fundamental scalars. They are associated with the pseudo-Nambu-Goldstone bosons that may occur in such theories [4, 8–14]. These pseudos have masses ranging from a few GeV to about 250 GeV. Consequently they may show up in the next generation of accelerators, or even at present machines.

In sect. 2 we discuss the quantum numbers and masses of these pseudos in a simple model. In sect. 3 we consider their dominant decay modes and evaluate their decay rates. In sect. 4 we compute some interesting production cross sections for these pseudos, and in sect. 5 we discuss the consequences for experiment. Details of mass calculations are in the appendices.

Our main results are a set of masses and decay widths for technihadrons that are much more detailed and complete than in previous work. Hopefully this will assist in relating the technicolor ideas to experiment. In particular, two results should be emphasized. First, the colored technieta can be produced with a detectable cross section at Isabelle or the Tevatron collider. Second, and most important for now, the masses of the charged axions have been carefully estimated and should be about 8 GeV, so there is good reason to hope for their discovery at PETRA or PEP. We also expect light neutral axions (2 states) with masses of about 2.5 GeV; they will decay dominantly to $s\bar{s}$ with at least a 10% branching ratio to $\mu^+\mu^-$.

2. Pseudotypes

In this section we wish to exhibit the quantum numbers and the masses of the pseudos that occur in a large class of models. We shall assume that there exists one complete family of technifermions

$$
\begin{pmatrix} U_L \\ D_L \end{pmatrix}_{r,y,b} \qquad \begin{pmatrix} N_L \\ E_L \end{pmatrix} \qquad \begin{matrix} U_{R r,y,b} & N_R \\ D_{R r,y,b} & E_R \end{matrix} \tag{2.1}
$$

where the technifermions have the standard $SU(3) \times SU(2)_L \times U(1)_Y$ quantum numbers.

Let us also assume that the technicolor group is $SU(N)$ and the technifermions belong to the N-dimensional representation[*].

In the absence of the $SU(3) \times SU(2)_L \times U(1)_Y$ forces there exists an $SU(8)_L \times SU(8)_R$ chiral symmetry.

Following the scenario of refs. [2–13] we assume that when the technicolor forces become strong the following condensates form:

$$
\langle \bar{U}_L U_R \rangle_{r,y,b} = \langle \bar{D}_L D_R \rangle_{r,y,b} = \langle \bar{N}_L N_R \rangle = \langle \bar{E}_L E_R \rangle . \tag{2.2}
$$

These condensates break the $SU(8)_L \times SU(8)_R$ chiral symmetry down to $SU(8)_{L+R}$. The $8^2 - 1$ broken generators then produce 63 Nambu-Goldstone bosons. Note that three of these bosons give mass to the $W^\pm$ and Z^0 bosons. In table 1 we list these bosons with their $SU(3) \times SU(2)_L \times U(1)_Y$ quantum numbers as well as their masses. Their mass is generated by the $SU(3) \times SU(2)_L \times U(1)_Y$ forces as well as the extended technicolor and Pati-Salam forces. Detailed calculations of the masses of these bosons have been done in refs. [8, 9, 11–13]. We believe that the contributions of the $SU(3) \times SU(2)_L \times U(1)_Y$ forces to the masses of these bosons are reliable. In contrast, the contribution of the extended technicolor-Pati-Salam forces are model dependent. As a result the masses of the light bosons are uncertain. We shall, however, estimate the contribution to the mass of the light pseudos coming from Pati-Salam forces at the end of this section.

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[*] Our discussion does not sensitively depend on the assumption that the techni-fermions belong to the N-dimensional representation of $SU(N)$. It is, however, strongly dependent on the assumption of the existence of one and only one complete techni-family

TABLE 1

Pseudo Nambu–Goldstone bosons of the model

Pseudo	Color	Charge	Mass (GeV)	Name
$\bar{U}U - \bar{D}D + \bar{N}N - \bar{E}E$	1	0		Technipions
$\bar{U}D + \bar{N}E$	1	-1		(eaten by gauge bosons)
$\bar{U}\frac{1}{2}\lambda^a U + \bar{D}\frac{1}{2}\lambda^a D$	8	0		colored technieta η_T^a
$\bar{U}\frac{1}{2}\lambda^a U - \bar{D}\frac{1}{2}\lambda^a D$	8	0	$240\sqrt{\dfrac{4}{N}}$	colored technipions
$\bar{U}\frac{1}{2}\lambda^a D$	8	-1		$\pi_T^{a3}, \pi_T^{a\pm}$
$\bar{E}U$	3	$\frac{5}{3}$	$165\sqrt{\dfrac{4}{N}}$	
$\sqrt{\frac{1}{2}}(\bar{E}D - \bar{N}U)$	3	$\frac{2}{3}$	$160\sqrt{\dfrac{4}{N}}$	technileptoquark
$\bar{N}D$	3	$-\frac{1}{3}$	$155\sqrt{\dfrac{4}{N}}$	
$\sqrt{\frac{1}{2}}(\bar{N}U + \bar{E}D)$	3	$+\frac{2}{3}$	$160\sqrt{\dfrac{4}{N}}$	
$\bar{U}U + \bar{D}D - (\bar{N}N + \bar{E}E)$	1	0	see table 4	paraxion a_T^0
$\bar{U}U - \bar{D}D + 3(\bar{E}E - \bar{N}N)$	1	0		axion a_T^3
$\bar{U}D - 3\bar{N}E$	1	-1		charged axion $a_T^\pm$

Before we do so we must discuss the relevant scales of the problem. The scale of symmetry breaking as measured by the technipion decay constant F_T is determined by the requirement that the $W^\pm$ and Z^0 bosons have the right mass.

Let ψ be the column of 4-component Dirac fields:

$$\psi = \left\{ \begin{array}{c} U_r \\ U_y \\ U_b \\ N \\ D_r \\ D_y \\ D_b \\ E \end{array} \right\} \tag{2.3}$$

The broken axial generators are defined by

$$J^a_{\mu 5} = \bar{\psi}\gamma_\mu\gamma_5 t^a \mathbb{1}_T \psi , \tag{2.4}$$

where t^a are the SU(8) generators normalized by $\text{Tr}\,(t^a t^b) = \frac{1}{2}\delta^{ab}$ and $\mathbb{1}_T$ is the unit matrix in technicolor space.

The technipion decay constant is defined by

$$\langle 0|J^a_{\mu 5}(0)|\pi_T^b(q)\rangle = iF_T q_\mu \delta_{ab} . \tag{2.5}$$

The conventions are those of Bjorken and Drell except for the normalization of the single-particle states which is taken to be

$$\langle \pi_T^a(q) | \pi_T^b(k) \rangle = (2\pi)^3 2q_0 \delta^3(q-k)\delta_{ab}.$$ (2.6)

The $SU(2)_L$ currents are given by

$$J_{\mu L}^i = \bar{\psi}\gamma_\mu \left(\frac{1-\gamma_5}{2}\right)\frac{\tau^i}{2}\mathbb{1}_T\psi.$$ (2.7)

where

$$\tau^1 = \begin{pmatrix} 0 & \mathbb{1}_p \\ \mathbb{1}_p & 0 \end{pmatrix}, \qquad \tau^2 = \begin{pmatrix} 0 & -i\mathbb{1}_p \\ i\mathbb{1}_p & 0 \end{pmatrix}, \qquad \tau^3 = \begin{pmatrix} \mathbb{1}_p & 0 \\ 0 & -\mathbb{1}_p \end{pmatrix}$$

and $\mathbb{1}_p$ is a 4×4 unit Pati-Salam matrix. Note that $\mathrm{Tr}\,(\frac{1}{2}\tau^i\frac{1}{2}\tau^j) = \frac{1}{2}4\delta_{ij}$.

The hypercharge current is

$$Y_\mu = \bar{\psi}\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)\left\{\begin{matrix} \frac{1}{3} & & & & & & & 0 \\ & \frac{1}{3} & & & & & & \\ & & \frac{1}{3} & & & & & \\ & & & -1 & & & & \\ & & & & \frac{1}{3} & & & \\ & & & & & \frac{1}{3} & & \\ & & & & & & \frac{1}{3} & \\ 0 & & & & & & & -1 \end{matrix}\right\}\mathbb{1}_T\psi$$

$$+ \bar{\psi}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\left\{\begin{matrix} \frac{4}{3} & & & & & & & 0 \\ & \frac{4}{3} & & & & & & \\ & & \frac{4}{3} & & & & & \\ & & & 0 & & & & \\ & & & & -\frac{2}{3} & & & \\ & & & & & -\frac{2}{3} & & \\ & & & & & & -\frac{2}{3} & \\ 0 & & & & & & & -2 \end{matrix}\right\}\mathbb{1}_T\psi,$$

or

$$Y_\mu \equiv \bar{\psi}\gamma_\mu\gamma_5\tfrac{1}{2}\tau^3\mathbb{1}_T\psi + \bar{\psi}\gamma_\mu\left[\tfrac{1}{2}\tau^3 + \tfrac{1}{3}\left\{\begin{matrix} 1 & & & & & & & 0 \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & -3 & & & & \\ \hline & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & 1 & \\ 0 & & & & & & & -3 \end{matrix}\right\}\right]\mathbb{1}_T\psi.$$ (2.8)

With these definitions in hand let us compute the electroweak vector boson mass matrix. As a result we shall obtain the value of F_T in this model.

Consider the axial current

$$J^l_{\mu 5} = \bar{\psi}\gamma_\mu \gamma_5 \frac{\tau^l}{2} \mathbb{1}_T \psi . \tag{2.9}$$

Expand it in the basis of the currents $J^a_{\mu 5}$ of eq. (2.4):

$$J^l_{\mu 5} = \sum_a \text{Tr}\,(\tau^l t^a) J^a_{\mu 5} . \tag{2.10}$$

In order to obtain the mass matrix for the weak vector bosons, consider the $q \to 0$ limit of the Fourier transform of the electroweak and axial current-current correlation function,

$$\text{FT}\langle 0|\text{T}\,(J^l_{\mu 5}(x) J^l_{\mu 5}(0))|0\rangle . \tag{2.11}$$

Using

$$\underset{q\to 0}{\text{FT}}\,\langle 0|\text{T}\,(J^a_{\mu 5}(x) J^b_{\mu 5}(0))|0\rangle = F^2_T \delta_{ab} \tag{2.12}$$

and the expansion (2.10), we obtain

$$\underset{q\to 0}{\text{FT}}\,\langle 0|\text{T}\,(J^l_{\mu 5}(x) J^l_{\mu 5}(0))|0\rangle = 4F^2_T \delta_{lj} . \tag{2.13}$$

The factor of 4 is simply the number of electroweak technidoublets. This axial current-current correlation function together with eqs. (2.7) and (2.8) yields the following weak vector boson mass matrix:

$$M^2 = \begin{array}{cccc} W^+ & W^- & W^3 & B \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & t \\ 0 & 0 & t & t^2 \end{pmatrix} \tfrac{1}{4}g^2_2 F^2_T N_{\text{TD}} , \tag{2.14}$$

where $t = g_1/g_2 = \tan\theta_W$ and $N_{\text{TD}} = 4 =$ number of technidoublets.

From this we obtain $M_W = \tfrac{1}{2}g_2 F_T \sqrt{N_{\text{TD}}} = g_2 F_T$ and $M_W = M_Z \cos\theta_W$. Using the correct value for the $W^\pm$ and Z^0 masses, we obtain

$$F_T \simeq 125 \text{ GeV} . \tag{2.15}$$

Next we want to find the mass of the technirho. This mass sets the scale of the technihadron masses. To find it consider the dimensionless ratio m_ρ/f_π. For QCD this ratio has the value

$$\frac{m_\rho}{f_\pi} \simeq \frac{773}{93} \simeq 8.3 . \tag{2.16}$$

However, this number depends on the size N of the color group $SU(N)$ like

$$\frac{m_\rho}{f_\pi} \propto \frac{1}{\sqrt{N}}, \tag{2.17}$$

where $N = 3$ for QCD. The reason for this is that m_ρ is independent of N, whereas f_π grows as $\sqrt{N}$ [15]. The latter is easily seen to follow from the QCD version of eq. (2.5). Therefore in order to find m_{ρ_T} we have to scale the N-independent ratio

$$\frac{m_\rho}{f_\pi}\sqrt{3} \simeq 14.4 \simeq \frac{m_{\rho_T}}{F_T}\sqrt{N}. \tag{2.18}$$

Thus,

$$m_{\rho_T} \simeq \frac{14.4}{\sqrt{N}}F_T \simeq \sqrt{\frac{4}{N}}\,900\text{ GeV}, \tag{2.19}$$

where again N is the size of the technicolor group.

Note that the N-dependence of the pseudo-Nambu-Goldstone masses in table 1 arises from the N-dependence of m_{ρ_T}. Also, since F_T is fixed [eq. (2.15)] it should not be considered to scale with N in the results.

Armed with the previous definitions and scales we can now estimate the masses of the four lightest pseudos. The neutral pseudos (axion, paraxion) do not receive any mass from $SU(3) \times SU(2)_L \times U(1)_Y$ forces. In order to avoid massless axions they must, however, receive mass from other sources [16]. The simplest possible sources of axionic mass are the standard $SU(4)$ Pati-Salam generators which mediate both quark-lepton and techniquark-technilepton transitions [7]. The charged axions, on the other hand, receive their mass from both Pati-Salam and electroweak forces. The contribution of the electroweak forces to their mass has been calculated in refs. [4, 11, 12] (see table 1). (See appendix A for the details of this calculation.) Now we shall evaluate the contribution of the Pati-Salam generators to the mass of these pseudos.

Using Dashen's formula [17], we have

$$M^2_{ab,p} \equiv \langle \pi_T^a | \delta \mathcal{L}^p | \pi_T^b \rangle$$

$$= \frac{1}{F_T^2}\langle 0 | [Q_5^a, [Q_5^b, \delta\mathcal{L}^p]] | 0 \rangle, \tag{2.20}$$

where

$$\delta\mathcal{L}^p \equiv \tfrac{1}{2}g_{PS}^2 \int \frac{d^4q}{(2\pi)^4} \frac{-g^{\mu\nu}}{(q^2 - m_p^2)} \int d^4x\, e^{iqx}\, T(J_\mu^p(x)J_\nu^p(0)). \tag{2.21}$$

$J_\mu^p(x)$ is a leptoquark Pati-Salam current, g_{PS} is the Pati-Salam coupling constant, m_p is the leptoquark mass (p labels the massive leptoquark generators), Q_5^a is the axial

charge constructed from the currents of eq. (2.4), and $\delta \mathcal{L}^p$ is the second-order Pati-Salam lagrangian. In the limit that m_p is much larger than the technihadronic scale (1 TeV) [which is justified in the discussion preceding eq. (2.36)], we can approximate eq. (2.21) by

$$\delta \mathcal{L}^p \equiv \frac{g_{\text{PS}}^2}{2m_p^2} \, \mathrm{T}(J_\mu^p(0) J_\mu^p(0)) \, . \tag{2.22}$$

After applying the double commutator of eq. (2.20), we obtain

$$M_{ab,p}^2 = \frac{4g_{\text{PS}}^2}{F_{\text{T}}^2 m_p^2} \sum_{cd} \{ \mathrm{Tr}\,(t_c[t_b,[t_a,V_p]])\,\mathrm{Tr}\,(t_d V_p) I_{cd}^1$$

$$+ \mathrm{Tr}\,(t_c[t_a,V_p])\,\mathrm{Tr}\,(t_d[t_b,V_p]) I_{cd}^2 \} \, , \tag{2.23}$$

where V_P is defined by

$$J_\mu^p \equiv \bar{\psi}\gamma_\mu V_p \mathbb{1}_{\text{T}} \psi \tag{2.24}$$

and $I^{1,2}$ are given by

$$I_{cd}^1 \equiv \langle 0|\bar{\psi}\gamma_\mu t_c \mathbb{1}_{\text{T}}\psi\bar{\psi}\gamma^\mu t_d \mathbb{1}_{\text{T}}\psi|0\rangle \, ,$$

$$I_{cd}^2 \equiv \langle 0|\bar{\psi}\gamma_\mu\gamma_5 t_c \mathbb{1}_{\text{T}}\psi\bar{\psi}\gamma^\mu\gamma_5 t_d \mathbb{1}_{\text{T}}\psi|0\rangle \, . \tag{2.25}$$

We now assume that the vacuum state dominates the sum over intermediate states in eq. (2.25). This approximation has been substantiated by Shifman et al. [18] in QCD. We find

$$I_{cd}^1 \simeq \tfrac{1}{2}\delta cd\,(\tfrac{1}{8}\langle\bar{\psi}\mathbb{1}_{\text{T}}\psi\rangle)^2 \, ,$$

$$I_{cd}^2 = -I_{cd}^1 \, . \tag{2.26}$$

Substituting (2.26) into (2.23) and using the algebraic identities

$$\mathrm{Tr}\,(t_c[t_a,V_p]) = -if_{acd}\,\mathrm{Tr}\,(t_d V_p) \, ,$$

$$\mathrm{Tr}\,(t_c[t_b,[t_a,V_p]]) = -f_{bcd}f_{ade}\,\mathrm{Tr}\,(t_e V_p) \, , \tag{2.27}$$

we obtain

$$M_{ab,p}^2 = \frac{g_{\text{PS}}^2 C_{ab,p}}{F_{\text{T}}^2 m_p^2}\,(\tfrac{1}{8}\langle\bar{\psi}\mathbb{1}_{\text{T}}\psi\rangle)^2 \, , \tag{2.28}$$

where the Clebsch $C_{ab,p}$ is given by

$$C_{ab,p} \equiv 4f_{acd}f_{bce}\,\mathrm{Tr}\,(t_d V_p)\,\mathrm{Tr}\,(t_e V_p) \, . \tag{2.29}$$

Let us now use the above formalism to evaluate the contribution of the Pati-Salam forces to the light pseudos. Consider first the neutral paraxion a_{T}^0. In this case

$$t^a \equiv \sqrt{\tfrac{1}{2}}\left(\begin{matrix} \tfrac{1}{2}\lambda_{15} & 0 \\ 0 & -\tfrac{1}{2}\lambda_{15} \end{matrix}\right) , \tag{2.30}$$

where $\frac{1}{2}\lambda a (a = 1, \ldots, 15)$ are the 4×4 Pati-Salam matrices, i.e.

$$\tfrac{1}{2}\lambda_{15} \equiv \frac{1}{2\sqrt{6}} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix}, \tag{2.31}$$

and the leptoquark generators V_p are

$$V_p \equiv \begin{pmatrix} \tfrac{1}{2}\lambda_p & 0 \\ 0 & \tfrac{1}{2}\lambda_p \end{pmatrix}, \qquad (p = 9, \ldots, 14). \tag{2.32}$$

Using (2.30)–(2.32) we find $\sum_p C_{ab,p} = 8$ and thus

$$M^2_{a^0_T} \equiv \sum_{p=9}^{14} m^2_{aa,p} = \frac{8 g^2_{PS}}{F^2_T m^2_p} (\tfrac{1}{8}\langle \bar{\psi} \mathbb{1}_T \psi \rangle)^2. \tag{2.33}$$

In order to evaluate the above expression numerically we need to know both (i) $\tfrac{1}{8}\langle \bar{\psi} \mathbb{1}_T \psi \rangle$ and (ii) g^2_{PS}/m^2_p.

(i) We obtain $\tfrac{1}{8}\langle \bar{\psi} \mathbb{1}_T \bar{\psi} \rangle$ by simple scaling arguments. We have

$$\tfrac{1}{8}\langle \bar{\psi} \mathbb{1}_T \psi \rangle = N \left(\frac{F_T}{f_\pi} \sqrt{\frac{3}{N}} \right)^3 \tfrac{1}{3}\langle \bar{q} \mathbb{1}_c q \rangle. \tag{2.34}$$

Using $\langle \bar{q} \mathbb{1}_c q \rangle \simeq (250 \text{ MeV})^3$ we find

$$\tfrac{1}{8}\langle \bar{\psi} \mathbb{1}_T \psi \rangle \simeq \sqrt{\frac{3}{N}} (330 \text{ GeV})^3 \tag{2.35}$$

(ii) A lower bound on m_p can be estimated as follows. The leptoquark Pati-Salam generators are expected to mediate the highly suppressed decay $K_L \to \mu^{\pm} e^{\mp}$. Thus, the absence of such a decay sets a lower bound on the mass m_p of the leptoquark generator. This bound is estimated in detail in appendix B. Here we only quote the result which is

$$m_p / g_{PS} > 310 \text{ TeV}. \tag{2.36}$$

We thus obtain the following contribution to the a^0_T mass

$$M^2_{a^0_T} \simeq \frac{3}{N} \frac{g^2_{PS}}{m^2_p} (6.6 \times 10^{11} \text{ GeV}^4)$$

$$\leqslant \left(\sqrt{\frac{4}{N}} 2.54 \text{ GeV} \right)^2. \tag{2.37}$$

A similar computation for the contribution of Pati-Salam forces to the mass of the axion $M^2_{a^0_T}$ and to the mass of the charged axion $\delta M^2_{a^\pm_T}$ yields

$$\delta M^2_{a^\pm_T} = M^2_{a^0_T} = \tfrac{3}{4} M^2_{a^0_T}. \tag{2.38}$$

Thus we see that the neutral axions are expected to have a mass which does not exceed 2.5 GeV. Moreover, this contribution to the mass of the charged axions is negligible compared to their electroweak mass.

We should emphasize that the lightness of the axions is not necessarily inescapable. For example, if we had assumed that the Pati-Salam generators which cause transitions between techniquarks and technileptons are different than those which cause transitions between quarks and leptons, then the mass of the axions is *a priori* arbitrary, because we cannot impose the limit 2.36.

3. Decays of pseudos

In this section we discuss the dominant decay modes of the pseudos listed in table 1. We shall consider the pseudos in the following order: color octets, triplets, and singlets.

3.1. COLOR OCTETS

In table 2 we list some possible decay modes of the color octet pseudos. We shall evaluate the decay rates for the two-body decays which are expected to be dominant.

Let us first consider the decays of the octets into two gauge bosons (fig. 1a). To do this we shall use a more general formalism which applies to the decay of any pseudo into two gauge bosons. Let Π_T^a be the pseudo defined by eq. (2.5). Let $J_\mu = V_\mu + A_\mu$ be any $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge current where

$$V_\mu = \bar\psi \gamma_\mu \Gamma_V \mathbb{1}_T \psi$$

and   (3.1)

$$A_\mu = \bar\psi \gamma_\mu \gamma_5 \Gamma_A \mathbb{1}_T \psi$$

and Γ_V, Γ_A are matrices in the SU(8) space defined by eq. (2.3). (Note that the matrices Γ_V and Γ_A also include the gauge coupling constant.) Consider now the decay of Π_T^a into two gauge bosons that couple to J_μ^1 and J_μ^2. The amplitude T for this process is given by

$$T = \lim_{q \to 0} \frac{1}{F_T} \int d^4x \, d^4y \, e^{iqx} \partial^\mu \langle 0|T(J_{\mu 5}^a(x)J_\alpha^1(y)J_\beta^2(0))|0\rangle \varepsilon_1^{*\alpha}(p_1)\varepsilon_2^{*\beta}(p_2) \, e^{-ip_1 y}.$$

(3.2)

The Green function $\langle 0|T(J_{\mu 5}^a(x)J_\alpha^1(y)J_\beta^2(0))|C\rangle$ to lowest order in electroweak interactions is given by the Adler–Bell–Jackiw anomaly [19]. Notice that only the VVA and AAA terms contribute. Following ref. [19] we obtain

$$T = \frac{S}{2\pi^2 F_T} \varepsilon_{\mu\nu\rho\sigma} p^{1\mu} p^{2\sigma} \varepsilon_1^{*\nu} \varepsilon_2^{*\rho},$$

(3.3)

<div align="center">Table 2</div>

Dominant decay modes of color octet pseudos which conserve
C-conjugation, parity and the $SU(2)_V$ techni-isospin symmetry

Pseudo	Decay modes	Rates (MeV)
η_T^a	$\bar{t}+t$	$1100\sqrt{\dfrac{4}{N}}$
	$G+G$	$60\sqrt{\frac{1}{4}N}$
	$G+\gamma$	$0\ 6\sqrt{\frac{1}{4}N}$
	$G+Z_0$	$2\sqrt{\frac{1}{4}N}$
	$G+G+G$	
	$G+G+a_T^0$	
	$G+Z^0+Z^0$	
	$G+W^++W^-$	
	$G+Z^0+\gamma$	
π_T^{a3}	$G+\gamma$	$5.4\sqrt{\frac{1}{4}N}$
	$G+Z_0$	$2.4\sqrt{\frac{1}{4}N}$
	$G+a_T^{3(0)}+\gamma$	
	$G+a_T^{3(0)}+Z^0$	
	$G+a_T^-+W^+$	
	$\bar{t}+t$	$1100\sqrt{\dfrac{4}{N}}$
π_T^{a+}	$G+W^+$	$48\sqrt{\frac{1}{4}N}$
	$t+\bar{b}$	$375\sqrt{\dfrac{4}{N}}$

G stands for gluon, t and b are the top and bottom quarks
respectively.

where S is given by

$$S = \tfrac{1}{2}N \, \mathrm{Tr}\, t^a (\{\Gamma_V^1, \Gamma_V^2\} + \{\Gamma_A^1, \Gamma_A^2\}) \,, \tag{3.4}$$

and $N = \mathrm{Tr}\, \mathbb{1}_T$. The rate is then given by

$$\Gamma = S^2 \frac{1}{256\pi^5} \frac{m^3}{F_T^2} \,, \tag{3.5}$$

where m is the mass of the pseudo[*,**].

(a)   (b)

Fig. 1 Relevant graphs for the two-body decays, and (backwards) the production, of the color octet and singlet pseudos

[*] Eq. (3 5) is correct for decays into two identical gauge bosons. For decays into non-identical particles
we must multiply by a factor of 2.

[**] Throughout this paper we always approximate all phase spaces contributions by those of massless
particles.

We can check this formalism for the classic case of $\pi^0 \to \gamma\gamma$ where in this case

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix}; \qquad t^3 \otimes \mathbb{1}_{\text{color}} = \tfrac{1}{2}\tau^3 \otimes \mathbb{1}_{\text{color}} = \begin{pmatrix} \tfrac{1}{2} & 0 \\ 0 & -\tfrac{1}{2} \end{pmatrix}_{\text{flavor}} \otimes \mathbb{1}_{\text{color}},$$

$$\Gamma_V^1 = \Gamma_V^2 = \begin{pmatrix} \tfrac{2}{3}e & 0 \\ 0 & -\tfrac{1}{3}e \end{pmatrix}_{\text{flavor}} \otimes \mathbb{1}_{\text{color}}, \qquad \Gamma_A^1 = \Gamma_A^2 = 0.$$

$$(3.6)$$

Then S is equal to $\tfrac{1}{2}e^2$ and the decay rate is

$$\Gamma_{\pi^0 \to \gamma\gamma} = \frac{\alpha^2}{64\pi^3} \frac{m_\pi^3}{f_\pi^2} = 7.47 \text{ eV}. \tag{3.7}$$

Let us now calculate the total decay rate for the process $\eta_T^a \to G^b + G^c$. In this case

$$t^a = \sqrt{\tfrac{1}{2}} \begin{pmatrix} \tfrac{1}{2}\lambda_a & & & \\ & 0 & & \\ & & \tfrac{1}{2}\lambda_a & \\ & & & 0 \end{pmatrix},$$

$$\Gamma_V^b = \sqrt{2}g_3 t^b, \qquad \Gamma_A^b = 0, \tag{3.8}$$

where $a, b, c = 1, 2, \ldots, 8$; λ^a are the Gell-Mann SU(3) color matrices and g_3 is the SU(3) coupling constant. Thus,

$$S_{abc} = \frac{N g_3^2}{2\sqrt{2}} d_{abc}, \tag{3.9}$$

$$\Gamma_{\eta_T^a \to GG} = \sum_{b,c} \Gamma_{\eta_T^a \to G^b G^c} \simeq \frac{5N^2}{384\pi^3} \alpha_3^2 \frac{m_{\eta_T^a}^3}{F_T^2} \simeq 60 \text{ MeV} \sqrt{\tfrac{1}{4}N}. \tag{3.10}$$

Note that here the loop is made of technifermions, and the number of loops contributing to the amplitude is (2 techniflavors) × (3 colors) × (N technicolors). We took $\alpha_3 \simeq 10^{-1}$ and $m_{\eta_T} \simeq 240\sqrt{4/N}$ GeV.

In table 2 we have listed the analogous results for the two gauge boson decays of all the color octets. Let us now consider the decay of the color octet pseudos into a pair of fermions (fig. 1b) which is expected to be the dominant decay mode. To estimate this process we have assumed that the effective Yukawa coupling g_y of the octets to a pair of fermions f and f' is given by (using either a Goldberger–Treiman type argument, or from models which give quark masses and couplings)

$$g_y \sim \frac{m_f + m_{f'}}{F_T}. \tag{3.11}$$

In order to evaluate this process more accurately, however, one needs more model-dependent information on the extended technicolor sector of this theory. In the absence of such information we believe, nevertheless, that the assumption (3.11)
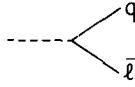
Fig. 2. Decay (production) of the leptoquark bosons.

for the value of g_y is a valid order of magnitude estimate. With this caveat, we find the rate to be

$$\Gamma_{\text{pseudo}\rightarrow\text{ff}'} = \frac{g_y^2 m_{\text{pseudo}}}{16\pi}. \tag{3.12}$$

The results are listed in table 2.

3 2. COLOR TRIPLETS

The dominant decay mode of the color triplet pseudos is into a quark and an antilepton (fig. 2). The computation is identical to that of subsect. 3.1. The results are listed in table 3.

3 3. COLOR SINGLETS

There are 4 color singlet pseudos in this scenario $[a_T^0, a_T^3, a_T^\pm]$. They are especially interesting since they are expected to be light (see table 4). An important difference between this scenario and the standard Higgs scenario is that all of the light axions are pseudoscalars. In fact, in these theories the standard Higgs particle is replaced by

TABLE 3

Decays of color triplet pseudos

Pseudo	Decay modes	Rates (MeV)
$\bar{E}U$	$t + \tau^+$	$190\sqrt{\dfrac{4}{N}}$
$\sqrt{\tfrac{1}{2}}(\bar{E}D - \bar{N}U)$	$t + \bar{\nu}$	$95\sqrt{\dfrac{4}{N}}$
	$b + \tau^+$	$4\sqrt{\dfrac{4}{N}}$
$\bar{N}D$	$b + \bar{\nu}$	$8\sqrt{\dfrac{4}{N}}$
$\sqrt{\tfrac{1}{2}}(\bar{E}D + \bar{N}U)$	$t + \bar{\nu}$	$95\sqrt{\dfrac{4}{N}}$
	$b + \tau^+$	$4\sqrt{\dfrac{4}{N}}$

TABLE 4

Contributions to the masses squared of the color singlet pseudos

Pseudo	M^2(GeV^2)		Total
	Electroweak	Pati-Salam	
a_T^0	0	$(2.54)^2$	$m \leqslant 2.54$ GeV
a_T^3	0	$(2.16)^2$	$m \leqslant 2$ 16 GeV
$a_T^\pm$	$(7.7)^2$	$(2.16)^2$	$m \simeq 8$ GeV
	(ref. [11] and appendix A)		

a composite scalar with a mass of order 1 TeV. Thus the discovery of a light scalar would be strong evidence against theories of dynamical symmetry breaking.

In table 5 we list some possible decay modes of the color singlet pseudos. The calculation of the decay rates into two body modes is identical to those discussed in subsect. 3.1. For example, for the total decay rate for $a_T^0 \rightarrow G^a + G^b$ we find explicitly

$$\sum_{ab} \Gamma_{a_T^0 \rightarrow G^a G^b} = \left\{ \frac{\alpha_3^2}{6\pi^3 F_T^2} \right\} m_a^3 (\tfrac{1}{4}N)^2 . \tag{3.13}$$

The results are given in table 5. The following comments are in order: this model has the same number of color singlet spinless bosons as the non-minimal two-Higgs,

TABLE 5

Dominant decays of light color singlet pseudos

Pseudo	Decay modes	Rates general form	Magnitude
a_T^0	$s + \bar{s}$	$g_\gamma^2 \dfrac{1}{16\pi} m_{a_T^0}$	300 eV
(rates computed for $m_{a_T^0} = 2.54$ GeV)	$G + G$	$\dfrac{\alpha_3^2}{6\pi^3} \dfrac{1}{F_T^2} m_{a_T^0}^3 (\tfrac{1}{4}N)^2$	60 eV
	$\gamma + \gamma$	negligible	
	$a_T^3 + \pi^0$	negligible	
	$\mu^+ \mu^-$	branching ratio m_μ^2/m_s^2, possible color factor	$\sim$125 eV
a_T^3	$s + \bar{s}$	$\dfrac{g_\gamma^2}{16\pi} m_{a_T^3}$	260 eV
(rates computed for $m_{a_T^3} = 2.16$ GeV)	$\gamma + \gamma$	negligible	
	$\mu^+ \mu^-$	see above	
a_T^+	$c + \bar{b}$	$\dfrac{(m_b + m_c)^2}{16\pi F_T^2} m_{a_T^+} \sin^2 \theta$, θ = appropriate mixing angle	0.5 MeV
(rates computed for $m_{a_T^+} = 8$ GeV)	$c + \bar{s}$	$\dfrac{m_c^2}{16\pi F_T^2} m_{a_T^+}$	1 MeV
	$\tau \nu_\tau$	branching ratio $\geqslant \tfrac{1}{3}$	
	$\ell \nu_\ell (\ell = \mu, e)$	negligible	

Glashow–Weinberg–Salam model [1]. The important difference is that the two-Higgs model has two neutral scalars and one neutral pseudoscalar whose tree-level Yukawa couplings are parity conserving. In contrast, in a dynamical scenario some of the Yukawa couplings of the light neutral axions are expected to violate parity. This is in fact a mechanism for splitting up and down quark masses in extended technicolor scenarios [20, 7]. Thus one would expect to see some parity-violating fermionic decays of light, neutral axions. In contrast to the fermionic decays, the two-photon and two- and three-gluon decay modes of the neutral axion and paraxion are to lowest order parity conserving. These decays can thus be used to determine the parity of the neutral axions [10, 14]. For example, since they are pseudoscalars they cannot decay into four gluons.

4. Production cross sections

In this section we consider some processes which give rise to single production of technihadrons.

We discuss the following processes:

(a) $p\bar{p}(pp) \rightarrow \eta_T^a + X$,

(b) $p\bar{p}(pp) \rightarrow a_T^0 + X$,

(c) $p\bar{p} \rightarrow \rho_T^a + X$.

These processes may lead to the production of technihadrons in the colliding Tevatron machines.

4 1. $p\bar{p}(pp) \rightarrow \eta_T^a + X$

The cross section $d\sigma/dy$ for this process (see fig. 3) is given by [21][*]

$$\frac{d\sigma}{dy} = \pi^2 \frac{\Gamma(\eta_T^a \rightarrow GG)}{m_{\eta_T}^3} G(x_1)G(x_2), \tag{4.1}$$

where $G(x)$ is the gluon distribution function in the proton and the kinematic variables (fig. 3) are defined in the standard fashion, i.e. x_1 and x_2 are the fraction of
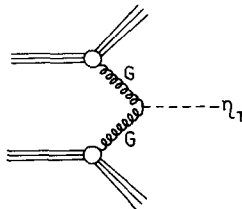

Fig 3   Production mechanism for η_T^a.

[*] In order to obtain the decay rate into any specific channel using our production cross sections, one simply multiplies our result by the branching ratio for the particular channel of interest.

longitudinal momentum carried by the gluons and y is the rapidity of η_T. They are related by (see fig. 3).

$$y \equiv \tfrac{1}{2} \ln \frac{E_{\eta_T} + P^L_{\eta_T}}{E_{\eta_T} - P^L_{\eta_T}} \simeq \tfrac{1}{2} \ln \frac{x_1}{x_2},$$

$$x_1 = \frac{m_{\eta_T}}{\sqrt{s}} e^y, \qquad x_2 = \frac{m_{\eta_T}}{\sqrt{s}} e^{-y}, \tag{4.2}$$

where $\sqrt{s}$ is the total c.m. energy and

$$\int_0^1 dx \, G(x) \simeq \tfrac{1}{2}. \tag{4.3}$$

Although $G(x)$ is not known we shall use the form

$$G(x) = 3(1-x)^5. \tag{4.4}$$

Using eqs. (4.1) and (3.10) we find

$$\left. \frac{d\sigma}{dy} \right|_{y=0} = \frac{15}{128\pi} \frac{\alpha_3^2}{F_T^2} \left(1 - \frac{m_{\eta_T}}{\sqrt{s}} \right)^{10} N^2$$

$$\simeq 160 \times 10^{-36} \, \text{cm}^2 \left(1 - \frac{m_{\eta_T}}{\sqrt{s}} \right)^{10} (\tfrac{1}{4}N)^2 \tag{4.5}$$

For $N = 4$ and $m_{\eta_T} = 240$ GeV we obtain the following values for $d\sigma/dy|_{y=0}$:

$$\sqrt{s} = 500 \, \text{GeV}: \quad \left. \frac{d\sigma}{dy} \right|_{y=0} = 2 \times 10^{-37} \, \text{cm}^2,$$

$$\sqrt{s} = 800 \, \text{GeV}: \quad \left. \frac{d\sigma}{dy} \right|_{y=0} = 4.5 \times 10^{-36} \, \text{cm}^2, \tag{4.6}$$

$$\sqrt{s} = 2000 \, \text{GeV}: \quad \left. \frac{d\sigma}{dy} \right|_{y=0} = 44 \times 10^{-36} \, \text{cm}^2.$$

4.2. $p\bar{p}(pp) \to a_T^0 + X$

This single paraxion production cross section is given by

$$\left. \frac{d\sigma}{dy} \right|_{y=0} = \frac{3\alpha_3^2}{16\pi} \frac{1}{F_T^2} \left(1 - \frac{m_{a_T^0}}{\sqrt{s}} \right)^{10} (\tfrac{1}{4}N)^2 \simeq 15 \times 10^{-36} \, \text{cm}^2 \left(1 - \frac{m_{a_T^0}}{\sqrt{s}} \right)^{10} (\tfrac{1}{4}N)^2. \tag{4.7}$$

4 3. $p\bar{p} \to \rho_T^a + X$

The cross section for this process is given by

$$\left. \frac{d\sigma}{dy} \right|_{y=0} = \frac{16\pi^2}{m_{\rho_T^a}^3} \{ \tfrac{2}{3} \Gamma_{\rho_T^a \to \bar{u}u} u(x)^2 + \tfrac{2}{3} \Gamma_{\rho_T^a \to \bar{d}d} d(x)^2 + \tfrac{3}{16} \Gamma_{\rho_T^a \to GG} G(x)^2 \}, \tag{4.8}$$

where $\Gamma_{\rho_T \to \bar{u}u}$ is the total decay rate for ρ_T^a into a $u\bar{u}$ pair, $u(x)$ is the up quark distribution function in a proton and $x = m_{\rho_T}/\sqrt{s}$. In order to evaluate this cross section we must compute the decay rates $\Gamma_{\rho_T^a \to \bar{u}u}$, $\Gamma_{\rho_T^a \to \bar{d}d}$ and $\Gamma_{\rho_T^a \to GG}$. This calculation is done in appendix C. We find

$$\Gamma_{\rho_T^a \to \bar{u}u} = \Gamma_{\rho_T^a \to \bar{d}d} = \frac{4\pi}{3} \alpha_3^2 \frac{F_{\rho_T}^2}{m_{\rho_T}},$$

$$\Gamma_{\rho_T^a \to GG} = 2\pi\alpha_3^2 \frac{F_{\rho_T}^2}{m_{\rho_T}}.$$ (4.9)

Using the result of Weinberg's sum rules discussed previously in eq. (A.16) we can relate F_ρ to F_T. We find

$$F_\rho^2 \equiv \frac{1}{1-\varepsilon} F_T^2,$$

where

$$\varepsilon \equiv \frac{m_{\rho_T}^2}{m_{A_T}^2} \simeq \frac{1}{2}.$$

Finally combining the results of eqs. (4.8) and (4.9) we find the differential cross section for $\sqrt{s} = 2000$ GeV, using $u^2(x) + d^2(x) \simeq 0.06$:

$$\left.\frac{d\sigma}{dy}\right|_{y=0} \simeq 6 \times 10^{-36} \text{ cm}^2.$$ (4.10)

Note this scales as $u^2(m_{\rho_T}/\sqrt{s})/m_{\rho_T}^4$ so it would get considerably larger if m_{ρ_T} decreased a little.

5. Implications for experiment

Here we use the numbers computed in the previous sections to emphasize the consequences for experiment.

5.1. CHARGED AXIONS

We expect $m_{a_T^\pm} \simeq 8$ GeV.

If the basic ideas of technicolor physics are correct it is likely that charged dynamical scalar bosons (pointlike until a TeV mass scale) exist[*] with about this mass. They should be detectable at PETRA/PEP[!]

This number is made up of 7.7 GeV from electroweak contributions, which should be reliably calculated, and up to 2.2 GeV from Pati-Salam flavor-changing interactions, adding in m_a^2. The absence of such charged dynamical spin-zero bosons

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[*] One interesting consequence of this is that the decay $t \to b a_T^+$ will dominate t decay since it is semiweak. Then a_T^+ mainly gives $\bar{b}c$, $\bar{s}c$ and $\tau^+ \nu_\tau$, so there is a violation of $\tau/\mu/e$ universality (hard to see since τ^+ gives equal μ^+, e^+) and the t decay pattern is not as expected

would be a strong constraint on constructing theories of dynamical symmetry breaking. So far no model has been constructed without them, but since there is no realistic model yet it is not clear how to interpret this. The situation is somewhat similar to that with nucleon decay and grand unification. There, the simplest ideas suggest the nucleon should decay, and in the simpler theories one can calculate the lifetime. Similarly, here, the simplest scenarios of dynamical symmetry breaking require charged axions with mass about 8 GeV. On the other hand, it is possible to construct grand unification schemes with a stable nucleon, and it is possible that much lighter Pati-Salam bosons could exist, increasing the mass of $a_T^\pm$. But it is most appropriate to be hopeful that $a_T^\pm$ will exist as predicted, and to take the steps to find them.

To detect them the following properties (see also refs. [10, 14, 22]) are relevant for $e^+e^- \to a_T^+ a_T^-$.

(a) Scalar bosons have $\frac{1}{4}$ unit of R, a $\sin^2\theta$ production angular distribution, and a β^3 threshold behavior.

(b) In this mass range the largest decay modes of $a_T^\pm$ will be $a_T^+ \to c\bar{b}, c\bar{s}, \tau\nu_\tau$. Whether $c\bar{b}$ or $c\bar{s}$ dominates depends on the quark mixing angles. The ratio will be

$$\frac{\Gamma(c\bar{b})}{\Gamma(c\bar{s})} \simeq \frac{m_b^2}{m_c^2} \sin^2\theta,$$

where θ is an appropriate quark mixing angle ($\theta = \theta_2$ in the most usual parameterization). For $m_b^2/m_c^2 \approx 10$, $c\bar{s}$ is probably the larger mode for typical $\theta (\theta \lesssim 0.05)$, and could dominate for small $\sin^2\theta$. Then a_T will look like a charmed $F^\pm$ at a higher mass, with all the modes expected for $F^\pm$ decay. The $\tau\nu_\tau$ mode (or any violation of $\tau/\mu/e$ universality) could be a helpful signal – any unusual source of τ's should be looked for, and this mode may be the dominant one.

(c) In both hadronic modes one gets 2 strange quarks from each of a_T^+, a_T^-, so four (strange quarks) altogether, which will produce four strange particles and/or ϕ, η.

(d) Specific exclusive modes such as $a_T^\pm \to \psi F^\pm$, $\psi K\bar{K}$, $\psi D\bar{K}$, $\phi\pi^\pm$, DK, DKπ, $F^\pm\eta$, $F^\pm\phi$, D$\bar{B}$, etc. (with $D \to D^*$ and other generalizations as well) could be very clear signals, with little background. They also allow spin determination. The last five listed modes are expected to be the dominant ones.

(e) Situations where a_T^+ (say) decays hadronically, giving a core of hadrons with half the beam energy, and a_T^- decays into a semileptonic or leptonic or low multiplicity mode, could perhaps be exploited as a signature. There is room for clever analysis here.

5.2. PRODUCTION OF COLORED TECHNIETA

As seen from sect. 4, a lower limit on the cross section for producing the colored technieta state can be reliably calculated through the anomaly contribution. The

resulting cross sections are big enough to be observable at the next hadron colliders, with about 440 events at colliding Tevatrons (assuming $\mathcal{L} = 10^{30}/\mathrm{cm}^2$ sec) in 10^7 sec, and about 3000 events at Isabelle (assuming $\mathcal{L} = 10^{32}/\mathrm{cm}^2$sec) in 10^7 sec.

It will dominantly decay to $t\bar{t}$, where t is the heaviest quark, assuming there is one heavier than the b; if the b is the heaviest quark the $b\bar{b}$ and GG rates are about equal. To find η_T^a it will be necessary to do effective mass work with jets. For the GG modes there are two-jet events which must be separated from large background. For $t\bar{t}$ there will be four- and six-jet events, since t will give 2 or 3 jets depending on its mass, with a background of order the signal[*]. In both cases it will be necessary to reconstruct the mass of η_T^a (about 240 GeV) to show convincingly that such a state is found. Since the total widths of η_T^a is $\leqslant 1$ GeV, the achievable experimental resolution is an important aspect of determining what can be detected. If the theory is qualitatively correct, such a state should appear, with essentially the properties described here.

It is useful to compare the situation with η_T^a and a standard fundamental Higgs h^0 (say of the same mass). The production cross section for η_T^a is larger by a factor,

$$\sigma(\eta_T^a)/\sigma(h^0) \simeq 8N^2 .$$

N is the dimension of the technicolor group. In summing over fermion loops in the anomaly contribution one has number of loops = (number of techniflavors) × (number of colors) × (number of technicolors) for η_T^a, versus (number of heavy quark flavors) × (number of colors) for h^0, and appropriate factors of 2 from the relation between $\langle\phi\rangle$ and F_T. The anomaly contribution for η_T^a is (approximately) larger by a factor N in amplitude, N^2 in rate. Further, the production cross section is larger by a factor 8 since η_T^a is in a color octet and it is sufficient to produce any of the 8 states. Thus (if, say, $N = 4$) it is $8 \times 16 = 128$ times easier to produce η_T^a (and test technicolor) than to produce h^0! For decays, the GG mode is N^2 larger for η_T^a, so it is expected to be negligible for h^0 compared to $q\bar{q}$, while for η_T^a the GG mode is at least 5%, and perhaps large if $t\bar{t}$ is absent.

We have ignored scaling violations in these calculations, which is a reasonable approximation. They have little effect since the important x values are in the range 0.1–0.25 where the scaling violations are minimal, even for $Q^2 \simeq m^2(\eta_T^a)$. The scaling violations could even enhance the results slightly at colliding Tevatrons, where one might be on the rising part of the curve, while at 400 GeV hadron colliders there will be perhaps a factor of 2 suppression.

5.3 RECOGNIZING TECHNICOLOR

There are a number of other ways that the heavy technicolor singlets of table 1 could be produced. We have given an estimate for production of the technirho in

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[*] G L K. thanks S. Wolfram for a helpful conversation about this point

sect. 4. That estimate, and others one could make, are very model dependent and not necessarily reliable, in contrast to the lower limit on production of technieta from the anomaly contribution. Note, for example, that the technirho rate goes as $1/m_{T_\rho}^4$, so it increases significantly if m_{T_ρ} is overestimated. With events such as

$$\rho_T^a \to \pi_T^a \pi_T^a,$$
$$\llcorner GG \searrow t\bar{t}$$

one would see dramatic multijet structure with little QCD background. With the branching ratios and masses of tables 1–5, and the extensive discussion of refs. [8, 9] possible scenarios for hadron collider experiments can be discussed and any interesting events can be interpreted.

5 4. HIGGS VERSUS TECHNICOLOR

Distinguishing whether spontaneous symmetry breaking occurs with fundamental scalar bosons (Higgs) or dynamically will not be straight-forward, even after candidate states are found. The light axion states, charged and neutral, will show no structure until probed on a TeV scale. The differences expected are significant, however, in the mass spectrum, parity structure, and decays, so a detailed study of these properties should suffice. We have remarked on some features in sects 3, 4. Some discussion is given in refs. [10, 14]. The presence of a light, neutral scalar would probably confirm the Higgs alternative, while finding the charged states around 8 GeV and no neutral scalar would favor technicolor strongly (but charged scalars do occur in both forms of the theory). Finding the technieta with a cross section of at least that of sect. 4 (over 100 times that for a fundamental Higgs) would strongly favor technicolor.

Appendix A

CALCULATION OF THE ELECTROWEAK CONTRIBUTION TO THE MASS OF THE AXIONS

$$M_{ij}^2 = \langle a_T^i | \delta \mathcal{L}_{EW} | a_T^j \rangle, \tag{A.1}$$

where

$$\delta\mathscr{L}_{EW} = \frac{1}{2}\int \frac{d^4q}{(2\pi)^4} \frac{-g_{\mu\nu}}{(q^2-m_W^2)} \int d^4x\, e^{iqx} T(J_\mu^+(x)J_\nu^-(0))$$

$$+ \frac{1}{2}\int \frac{d^4q}{(2\pi)^4} \frac{-g^{\mu\nu}}{(q^2-m_Z^2)} \int d^4x\, e^{iqx} T(J_\mu^z(x)J_\nu^z(0))$$

$$+ \frac{1}{2}\int \frac{d^4q}{(2\pi)^4} \frac{-g^{\mu\nu}}{q^2} \int d^4x\, e^{iqx} T(J_\mu^\gamma(x)J_\nu^\gamma(0)). \tag{A.2}$$

Using Dashen's formula we have

$$M_{ij}^2 = \frac{1}{F_T^2} \langle 0|[Q_5^i,[Q_5^i,\delta\mathscr{L}_{EW}]]/0\rangle, \tag{A.3}$$

where

$$Q_5^i \equiv \int d^3x\, J_{05}^i(x)$$

and

$$J_{\mu 5}^i \equiv \bar{\psi}\gamma_\mu\gamma_5 \frac{\sigma^i}{4} \mathbb{1}_T\psi, \tag{A.4}$$

where

$$\sigma^1 = \begin{pmatrix} & & & & 1 & & & \\ & 0 & & & & 1 & & \\ & & & & & & 1 & \\ & & & & & & & -1 \\ 1 & & & & & & & \\ & 1 & & & & 0 & & \\ & & 1 & & & & & \\ & & & -1 & & & & \end{pmatrix},$$

$$\sigma^2 = \begin{pmatrix} & & & & -i & & & \\ & 0 & & & & -i & & \\ & & & & & & -i & \\ & & & & & & & i \\ i & & & & & & & \\ & i & & & & 0 & & \\ & & i & & & & & \\ & & & -i & & & & \end{pmatrix},$$

$$
\sigma^3 = \begin{pmatrix}
1 & & & & & & & \\
& 1 & & & & & & \\
& & 1 & & & & 0 & \\
& & & -1 & & & & \\
& & & & 1 & & & \\
& & & & & 1 & & \\
& 0 & & & & & 1 & \\
& & & & & & & -1
\end{pmatrix}.
$$

In order to evaluate the commutator it is convenient to resort to a more general formalism. Consider thus

$$
M_{ab}^2 \equiv \langle \pi_T^a | \delta\mathscr{L}^x | \pi_T^b \rangle , \tag{A.5}
$$

where the states π_T^a have been defined in eqs. (2.4) and (2.5) and

$$
\delta\mathscr{L}^x \equiv \tfrac{1}{2} \int \frac{d^4q}{(2\pi)^4} \frac{-g^{\mu\nu}}{(q^2 - m_x^2)} \int d^4x \, e^{iqx} T(J_\mu^x(x) J_\nu^x(0)), \tag{A.6}
$$

$$
J_\mu^x \equiv \bar\psi \gamma_\mu \Gamma_x \psi ,
$$

where Γ_x is an 8×8 matrix (including the gauge coupling constant) and

$$
\Gamma_x \equiv \Gamma_V^x + \gamma_5 \Gamma_A^x .
$$

We then have

$$
M_{ab,x}^2 = \frac{1}{F_T^2} \langle 0 | [Q_5^b, [Q_5^a, \delta\mathscr{L}^x]] | 0 \rangle , \tag{A.7}
$$

where

$$
Q_5^a \equiv \int d^3x \, J_{05}^a , \qquad J_{\mu 5}^a \equiv \bar\psi \gamma_\mu \gamma_5 t^a \psi
$$

[eq. (2.4)]. Upon evaluating the double commutator we obtain

$$
M_{ab,x}^2 = \frac{1}{F_T^2} \int \frac{d^4q}{(2\pi)^4} \frac{-1}{q^2 - m_x^2} \int d^4x \, e^{iqx}
$$

$$
\times \langle 0 | T \left\{ \begin{array}{l} \bar\psi \gamma_\mu [t^b, [t^a, \Gamma_x]] \psi(x) \bar\psi \gamma^\mu \Gamma_x \psi(0) \\ + \bar\psi \gamma_\mu \gamma_5 [t^a, \Gamma_x] \psi(x) \bar\psi \gamma^\mu \gamma_5 [t^b, \Gamma_x] \psi(0) \end{array} \right\} | 0 \rangle . \tag{A.8}
$$

Now following Das et al. [23] we define the vector (axial vector) spectral function $\rho_V(\rho_A)$ by the relation

$$
\int d^4x \, e^{iqx} \langle 0 | \bar\psi \gamma_\mu t^a \psi(x) \bar\psi \gamma_\nu t^b \psi(0) | 0 \rangle \equiv (q_\mu q_\nu - g_{\mu\nu} q^2) \delta_{ab} \int \frac{dm^2 \rho_V(m^2)}{q^2 - m^2} \tag{A.9}
$$

and similarly for ρ_A. Using the definition (A.9) and the algebraic relations of eq. (2.27) we obtain

$$M^2_{ab,x} = \frac{3}{4F^2_F\pi^2} \int_0^\infty \frac{q^4\,dq^2\,dm^2\,\rho(m^2)}{(q^2+m^2_x)(q^2+m^2)}\,C_{ab,x}\,, \tag{A.10}$$

where we have made the standard Wick rotation into euclidean space; $\rho(m^2) \equiv \rho_A(m^2) - \rho_V(m^2)$ and the Clebsch $C_{ab,x}$ is given by

$$C_{ab,x} \equiv f_{acd}f_{bce}[\text{Tr}\,(t^d\Gamma^x_V)\,\text{Tr}\,(t^e\Gamma^x_V)$$
$$- \text{Tr}\,(t^d\Gamma^x_A)\,\text{Tr}\,(t^e\Gamma^x_A)]. \tag{A.11}$$

We note that the spectral function $\rho(m^2)$ appears explicitly in the polarization tensor $\pi_{\mu\nu}(q^2)$ defined by

$$\delta_{ab}\pi_{\mu\nu}(q^2) \equiv \int d^4x\,e^{iqx}\langle 0|T(\bar{\psi}\gamma_\mu(1-\gamma_5)t^a\psi(x)\bar{\psi}\gamma_\nu(1+\gamma_5)t^b\psi(0)|0\rangle$$
$$= (q_\mu q_\nu - g_{\mu\nu}q^2)\int \frac{dm^2\,\rho(m^2)}{q^2-m^2}\,. \tag{A.12}$$

With the use of the operator product expansion Bernard et al. [24] have shown that

$$\delta_{ab}\pi_{\mu\nu}(q^2) \sim \sum_i \langle 0|\bar{\psi}\Gamma_i\psi\bar{\psi}\Gamma_i\psi|0\rangle/q^4\,, \tag{A.13}$$

and thus we obtain the Weinberg sum rules:

$$\int dm^2\,\rho(m^2) = 0,$$
$$\int dm^2\,m^2\rho(m^2) = 0\,. \tag{A.14}$$

If we now saturate the Green function, eq. (A.12), with the narrow resonance π^a_T, ρ^a_T, A^a_T we have

$$\rho(m^2) = F^2_T\delta(m^2) + F^2_{A_T}\delta(m^2-m^2_{A_T}) - F^2_{\rho_T}\delta(m^2-m^2_{\rho_T}), \tag{A.15}$$

or

$$F^2_T + F^2_{A_T} = F^2_{\rho_T}\,,$$
$$m^2_{A_T}F^2_{A_T} = m^2_{\rho_T}F^2_{\rho_T}. \tag{A.16}$$

We may now evaluate the integral in eq. (A.10). We obtain

$$I \equiv \int_0^\infty dm^2\,\rho(m^2)\int_0^{\Lambda^2} \frac{q^4\,dq^2}{(q^2+m^2_x)(q^2+m^2)}$$
$$\simeq \int_0^\infty dm^2\,\rho(m^2)\frac{m^4_x\ln m^2_x - m^4\ln m^2}{m^2_x - m^2}\,, \tag{A.17}$$

where we have used the sum rules eq. (A.14). Now using eqs. (A.15) and (A.16) we find

$$I = F_T^2 \left\{ m_x^2 \ln m_x^2 + \frac{\varepsilon}{1-\varepsilon} \left(\frac{m_x^4 \ln m_x^2 - (m_{\rho T}^4/\varepsilon^2) \ln (m_{\rho T}^2/\varepsilon)}{m_x^2 - m_{\rho T}^2/\varepsilon} \right) \right.$$
$$\left. - \frac{1}{1-\varepsilon} \left(\frac{m_x^4 \ln m_x^2 - m_{\rho T}^4 \ln m_{\rho T}^2}{m_x^2 - m_{\rho T}^2} \right) \right\}, \tag{A.18}$$

where $\varepsilon \equiv m_{\rho T}^2 / m_{A T}^2$.

In the case of interest $x = \{W^\pm, Z, \gamma\}$ and $m_x \ll m_{\rho T}$. In this limit we have

$$I \sim F_T^2 \left\{ \frac{m_{\rho T}^2}{1-\varepsilon} \ln \frac{1}{\varepsilon} + m_x^2 \left(\ln \frac{m_x^2}{m_{\rho T}^2} + \frac{\varepsilon}{1-\varepsilon} \ln \frac{1}{\varepsilon} \right) \right. . \tag{A.19}$$

Finally, combining eq. (A.19) with (A.10) we obtain

$$M_{ab,x}^2 = \frac{3}{4F_T^2 \pi^2} I C_{ab,x} . \tag{A.20}$$

Let us now evaluate the Clebsch $C_{ab,x}$ for $x = \{W^\pm, Z, \gamma\}$. For the axions $t^a \equiv \frac{1}{4}\sigma^i$, $i = 1, 2, 3$, and the gauge currents eq. (A.6) are given by

$$\Gamma_V^{W^\pm} = g_2 \left(\frac{\tau^1 \pm i\tau^2}{2} \right) = -\Gamma_A^{W^\pm} ,$$

$$\Gamma_V^Z = \frac{g_2}{\cos \theta_W} (\tfrac{1}{4}\tau^3 - \sin^2 \theta_W Q) ,$$

$$\Gamma_A^Z = \frac{-g_2}{\cos \theta_W} (\tfrac{1}{4}\tau^3) , \tag{A.21}$$

$$\Gamma_V^\gamma = g_2 \sin \theta_W Q ,$$

$$\Gamma_A^\gamma \equiv 0 ,$$

where

$$Q \equiv \begin{pmatrix} \tfrac{2}{3} & & & & & & & \\ & \tfrac{2}{3} & & & & & & \\ & & \tfrac{2}{3} & & & & & \\ & & & 0 & & & & \\ & & & & -\tfrac{1}{3} & & & \\ & & & & & -\tfrac{1}{3} & & \\ & & & & & & -\tfrac{1}{3} & \\ & & & & & & & -1 \end{pmatrix} .$$

We see immediately that the contribution of $W^\pm$ to the axion mass vanishes. Thus we need only consider Z^0 and γ. Putting eq. (A.11) into the more tractable form

$$C_{ij,x} \equiv \tfrac{1}{2}\{\mathrm{Tr}\,([\tfrac{1}{4}\sigma^i,[\tfrac{1}{4}\sigma^j,\Gamma_V^x]]\Gamma_V^x) - \mathrm{Tr}\,([\tfrac{1}{4}\sigma^i,[\tfrac{1}{4}\sigma^j,\Gamma_A^x]]\Gamma_A^x)\,, \tag{A.22}$$

and using eq. (A.21) we obtain

$$C_{33,Z} \equiv C_{33,\gamma} \equiv 0\,,$$

$$C_{11,Z} = -\tfrac{1}{4}e^2 = -C_{11,\gamma}\,, \tag{A.23}$$

$$C_{22,Z} = -\tfrac{1}{4}e^2 = -C_{22,\gamma}\,,$$

or finally

$$M_{a_T^3}^2 = 0\,,$$

$$M_{a_T^\pm}^2 = \frac{3\alpha}{4\pi}\,m_{Z_0}^2\left(\ln\frac{m_{\rho_T}^2}{m_{Z_0}^2} - \frac{\varepsilon}{1-\varepsilon}\ln\frac{1}{\varepsilon}\right)\,, \tag{A.24}$$

where

$$\varepsilon \equiv m_{\rho_T}^2/m_{A_T}^2\,,$$

Taking $\varepsilon = \tfrac{1}{2}$ and $m_{\rho_T} = 900$ GeV we find

$$M_{a_T^\pm}^2 = (7.7\ \mathrm{GeV})^2\,. \tag{A.25}$$

Appendix B

In this appendix we estimate a lower bound on the mass of the Pati-Salam generator based on the fact that

$$\frac{\Gamma(\mathrm{K_L} \to \mu^+e^-)}{\Gamma(\mathrm{K}^+ \to \mu^+\nu)} < (2.5 \times 10^{-5})^2\,. \tag{B.1}$$

To do this we note that the Pati-Salam generators mediate the process $\mathrm{K_L} \to \mu^+e^-$ via the graph shown in fig 4. The amplitude for this process is given by

$$\langle \mu^+e^-|H_I|\mathrm{K_L}\rangle = \langle \mu^+e^-|\frac{g_{\mathrm{PS}}^2}{M_p^2}\,\bar{d}\gamma_\mu e\bar{\mu}\gamma_\mu s|\mathrm{K_L}\rangle$$

$$= \frac{g_{\mathrm{PS}}^2}{M_P^2}\{\tfrac{1}{2}\langle 0|\bar{d}\gamma_\mu\gamma_5 s|\mathrm{K_L}\rangle\langle\mu^+e^-|\bar{\mu}\gamma_\mu\gamma_5 e|0\rangle\} \tag{B.2}$$

$$-\langle 0|\bar{d}\gamma_5 s|\mathrm{K_L}\rangle\langle\mu^+e^-|\bar{\mu}\gamma_5 e|0\rangle\,.$$

The second equation is the result of a Fierz rotation. Now the matrix elements involving $\mathrm{K_L}$ are estimated by using PCAC techniques as follows:

$$\langle 0|\bar{d}\gamma_\mu\gamma_5 s|\mathrm{K_L}\rangle = if_\mathrm{K}q_\mu\,. \tag{B.3}$$
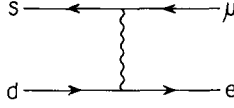
Fig. 4. The decay $K^0 \to \mu e$ mediated by a t-channel Patı-Salam boson

To estimate $\langle 0|\bar{d}\gamma_5 s|K_L\rangle$ we use kaon pole dominance for the matrix element of the operator

$$\mathop{FT}_{q \to 0} \partial_\mu \langle 0|T\{\bar{s}\gamma_\mu\gamma_5 d(x)\bar{d}\gamma_5 s(0)\}|0\rangle = \langle 0|[Q_5^{K^0}, \bar{d}\gamma_5 s]|0\rangle$$

$$= \langle 0|(\bar{s}s + \bar{d}d)|0\rangle$$

$$\simeq \mathop{FT}_{q \to 0} \sum_{i=L,S} \partial_\mu \langle 0|T\{j_{\mu 5}^{K^0}|K_i\rangle\langle K_i|\bar{d}\gamma_5 s\}|0\rangle \tag{B.4}$$

$$= 2\langle 0|\bar{d}\gamma_5 s|K_L\rangle f_K .$$

These equations together with the standard relation for the kaon mass

$$m_K^2 f_K^2 = (m_s + m_d)\langle 0|\bar{q}q|0\rangle , \tag{B.5}$$

result in the following form for $K_L \to \mu^+ e^-$:

$$\langle \mu^+ e^-|H_I|K_L\rangle = \frac{g_{PS}^2}{M_p^2} \left\{ \tfrac{1}{2}f_K(m_\mu + m_e) - \frac{m_{K^0}^2 f_K}{m_s + m_d} \right\} \bar{\mu}\gamma_5 e \tag{B.6}$$

The second (pseudoscalar) term clearly dominates. Thus the rate for $K_L \to \mu^+ e^-$ is given by

$$\Gamma(K_L \to \mu^+ e^-) \simeq \left\{ \frac{g_{PS}^2}{M_p^2} \frac{m_{K^0}^2 f_K}{(m_s + m_d)} \right\}^2 2m_K^2 \times \left\{ \begin{matrix} \mu^+ e^- \\ \text{phase space} \end{matrix} \right\} . \tag{B.7}$$
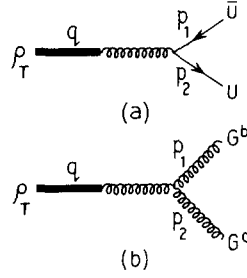
This is to be compared to the standard rate

$$\Gamma(K^+ \to \mu^+ \nu) \simeq \tfrac{1}{2}f_K^2 m_K^2 \left\{ \frac{g_2^2}{M_W^2} \sin\theta_C \right\}^2 \times \left\{ \begin{matrix} \mu^+ \nu \\ \text{phase space} \end{matrix} \right\} . \tag{B.8}$$

Dividing (B.7) by (B.8) and using (B.1), we obtain

$$\frac{\Gamma(K_L \to \mu^+ e^-)}{\Gamma(K^+ \to \mu^+ \nu)} \simeq \left\{ \frac{g_{PS}^2}{M_p^2} \frac{M_W^2}{g_2^2 \sin\theta_C} \right\}^2 4\left\{ \frac{m_K^2}{m_\mu m_s} \right\}^2 < (2.5 \times 10^{-5})^2 \tag{B.9}$$

which finally yields

$$\left(\frac{M_p}{g_{PS}} \right) > 310 \text{ TeV} . \tag{B.10}$$

Fig. 5. Production mechanisms for ρ_T^a.

Appendix C

CALCULATION OF THE DECAY RATES OF EQ. (4 8) [11]

In order to calculate the decay rate of a flavor singlet, color octet technirho we use standard vector meson dominance ideas. Let J_μ^a be the gauge current which acts as a source for ρ_T^a. In general we have

$$J_\mu^a \equiv \bar{\psi}\gamma_\mu t^a \psi \tag{C.1}$$

[where t^a are defined in eq. (2.4)] and in addition

$$\langle 0|J_\mu^a|\rho_T^b\rangle \equiv iF_{\rho_T}m_\rho\varepsilon_\mu\delta_{ab} . \tag{C.2}$$

The decay amplitude for the processes $\rho_T^a \to \bar{u}u$, $\bar{d}d$, GG (see figs. 5a, b) are as follows:

$$T_{\rho_T^a \to \bar{u}u} = \bar{u}\tfrac{1}{2}\lambda_a\gamma_\mu u \frac{g_3^2}{m_{\rho_T}^2}\sqrt{2}M_{\rho_T}F_{\rho_T}\varepsilon^\mu(q) ,$$

$$T_{\rho_T^a \to \bar{d}d} = \bar{d}\tfrac{1}{2}\lambda_a\gamma_\mu d \frac{g_3^2}{m_{\rho_T}^2}\sqrt{2}m_{\rho_T}F_{\rho_T}\varepsilon^\mu(q) , \tag{C.3}$$

$$T_{\rho_T^a \to GG} = \varepsilon^\star(p_1)\varepsilon^\star(p_2)(p_1-p_2)_\mu f_{abc}\frac{g_3^2}{m_{\rho_T}^2}\sqrt{2}M_{\rho_T}F_{\rho_T}\varepsilon^\mu(q) .$$

We then obtain the decay rates

$$\Gamma_{\rho_T^a \to \bar{u}u} \equiv \Gamma_{\rho_T \to \bar{d}d} = \frac{4\pi}{3}\alpha_3^2\frac{F_{\rho_T}^2}{m_{\rho_T}} ,$$

$$\Gamma_{\rho_T^a \to GG} \equiv 2\pi\alpha_3^2\frac{F_{\rho_T}^2}{M_{\rho_T}} . \tag{C.4}$$

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