NONCOMPACT N=2 SUPERGRAVITY

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A massive spin-one multiplet with central charge is coupled to N=2 supergravity. Compared to conventional gauge fields the anomalous magnetic moment of the spin-one particles is of the opposite sign. The construction of this theory is based on an N=2 supersymmetric gauge theory associated with the noncompact group SO(2, 1). As a byproduct we present a convenient expression for the N=2 Einstein–Yang–Mills lagrangian.

As is well-known, noncompact symmetry groups have quadratic invariants that are not positive definite. This aspect limits their applicability in realistic field theories, where one must insist on states with positive norm. The standard solution to this problem is to ensure that the noncompact transformations are realized nonlinearly. This can be achieved by introducing a nonlinear sigma model in which the fields parametrize the coset space G/H, where H is the maximal compact subgroup of the noncompact group G [1]. All other fields are assigned to representations of H and not of G. The compact group H acts then linearly on all the fields, whereas the noncompact transformations act nonlinearly. These nonlinear transformations take the form of an H transformation, but with parameters that depend on the fields of the nonlinear sigma model.

It is possible to formulate this theory in such a way that the full group G is realized linearly on the fields. In that case the group H is promoted to an independent local gauge group, so that the lagrangian is manifestly invariant under $G_{\text{rigid}} \times H_{\text{local}}$. Because G is noncompact the kinetic term for the scalars contains fields whose contribution is of the "wrong" sign, but those fields are precisely associated with the gauge degrees of freedom of H. The gauge invariance can thus be used to remove these negative-metric components, which have therefore no direct physical content [2].

A similar situation exists in gravity, where the scale factor of the gravitational field also occurs with the "wrong" sign. This is obvious if one rescales the metric in Einstein's lagrangian according to $g_{\mu\nu} \to \phi^2 g_{\mu\nu}$, after which this lagrangian assumes the form

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$$-\tfrac{1}{2}\sqrt{g}\,R \to -\tfrac{1}{2}\sqrt{g}\,R\phi^2 + 3\sqrt{g}\,g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi. \tag{1}$$

The right-hand side of (1) can be viewed as a conformally invariant Klein–Gordon lagrangian with the "wrong" sign. However, just as in the nonlinear sigma models this is deceptive; in fact ϕ does not correspond to a physical degree of freedom, because the right-hand side of (1) is invariant under local scale transformations. These transformations can be exploited to adjust ϕ to a constant. On the other hand, (1) shows that the gravitational field also contains negative metric components.

The field ϕ associated with the scale factor is called the compensating field. Such fields play a role in the context of conformally invariant formulations of gravity and supergravity [3]. The possibility that we will explore in this letter is that the compensating field is part of an entire multiplet of a noncompact group. The fields associated with the compact directions are used as gravitational compensators, whereas the remaining fields of the multiplet will correspond to physical degrees of freedom. In principle this will give rise to some kind of nonlinear sigma model coupled to gravity, but we will make the construction nontrivial by introducing a *local* noncompact group. At first sight, this leads to a gauge-field lagrangian of indefinite sign. However, this problem is avoided in $N = 2$ supergravity, where the compensating field is extended to a full supermultiplet which contains a massless gauge field [4,5] (for a different solution to this problem, see ref. [6]). Because of supersymmetry the kinetic term for this field initially occurs with the "wrong" sign as well, but the sign is reversed once the tensor auxiliary field of $N = 2$ supergravity has been eliminated. Thus one obtains the $N = 2$ supergravity lagrangian with the right Maxwell kinetic term for the vector field. The remaining fields of the compensating supermultiplet are auxiliary, so that the sign of their contribution is not relevant here.

Because N=2 supergravity is based on a compensating vector multiplet, we first consider the N=2 supersymmetric Yang–Mills theory coupled to conformal supergravity. For reasons explained above we will choose a gauge group with only one compact generator. The obvious candidate for this group is SO(2,1) or its covering group SU(1,1) [the latter is isomorphic to Sl(2) and Sp(1)]. However, it is easy to give the lagrangian for a general group G. The supermultiplet consists of gauge fields $W_\mu{}^A$, complex scalar fields X^A, Majorana spinors $\Omega_i{}^A$ and auxiliary fields $Y_{ij}{}^A$, where A labels the generators t_A of G, and indices $i,j,...,$ refer to the local SU(2) group of conformal supergravity. Our conventions are that complex conjugation is always effected by raising or lowering of indices (for notation, see e.g. ref. [7]). For instance, the SU(2) $\times$ G invariant constraint for the auxiliary fields Y takes the form

$$Y^{Aij} \equiv (Y_{ij}{}^A)^* = \epsilon^{ik}\epsilon^{jl}Y_{kl}{}^A. \tag{2}$$

The N=2 supersymmetric Yang–Mills lagrangian in a superconformal background is constructed by means of the superconformal multiplet calculus [7,4] (This calculus has been reviewed in ref. [8]). After various manipulations we find

$$\begin{aligned}
e^{-1}\mathcal{L}_{\rm YM} = \mathrm{tr}\{&\mathcal{D}_\mu X\mathcal{D}_\mu X^* + XX^*(D - \tfrac{1}{6}R) - \tfrac{1}{8}|Y_{ij}|^2 - g^2[X,X^*]^2\\
&+ [\tfrac{1}{8}\hat{F}^+_{\mu\nu}\hat{F}^{+\,\mu\nu} - \tfrac{1}{8}\hat{F}^{+\,\mu\nu}XT^+_{\mu\nu ij}\epsilon^{ij} + \tfrac{1}{64}X^2(T^-_{\mu\nu ij}\epsilon^{ij})^2 + \tfrac{1}{4}\bar{\Omega}^i\not{\mathcal{D}}\Omega_i + \tfrac{1}{2}g\epsilon^{ij}\bar{\Omega}_i[X^*,\Omega_j] - \bar{\chi}_i\Omega^iX\\
&+ \tfrac{2}{3}\bar{\Omega}_i\sigma^{\mu\nu}\mathcal{D}_\mu\psi_\nu{}^iX^* - \tfrac{1}{2}\bar{\psi}_\mu{}^i\not{\mathcal{D}}X^*\gamma^\mu\Omega_i + \tfrac{1}{2}\bar{\psi}_i\cdot\gamma\chi^iXX^* + \tfrac{1}{2}g\bar{\Omega}^i\gamma\cdot\psi^j\epsilon_{ij}[X,X^*] - \tfrac{1}{4}\bar{\Omega}^i\gamma_\mu\psi_\nu{}^j\epsilon_{ij}\tilde{\hat{F}}^{\mu\nu}\\
&+ \tfrac{1}{6}\bar{\Omega}^i\gamma_\mu\psi_\nu{}^jT^{+\,\mu\nu}{}_{ij}X + \tfrac{1}{6}e^{-1}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu i}\gamma_\nu\mathcal{D}_\rho\psi_\sigma{}^iXX^* + \tfrac{1}{4}e^{-1}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu i}\gamma_\nu\psi_\rho{}^i(\mathcal{D}_\sigma X - \tfrac{1}{2}\bar{\psi}_\sigma{}^j\Omega_j)X^* - \tfrac{1}{4}\bar{\psi}_{\mu i}\psi_{\nu j}\epsilon^{ij}\hat{F}^{\mu\nu}X\\
&- \tfrac{1}{12}\bar{\psi}_{\mu i}\psi_{\nu j}T^{-\,\mu\nu ij}XX^* + \tfrac{1}{8}\bar{\Omega}^i\gamma^\mu\gamma^\nu\psi_{\mu i}\bar{\psi}_\nu{}^j\Omega_j + \tfrac{1}{8}\bar{\psi}_{\mu i}\sigma^{\mu\nu}\psi_{\nu j}\epsilon^{ik}\epsilon^{jl}\bar{\Omega}_k\Omega_l - \tfrac{1}{16}\bar{\psi}_{\mu i}\psi_{\nu j}\epsilon^{ij}\epsilon^{kl}\bar{\Omega}_k\sigma^{\mu\nu}\Omega_l\\
&+ \tfrac{1}{8}e^{-1}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu i}\gamma_\nu\psi_\sigma{}^j(2\bar{\psi}_\rho{}^i\Omega_jX^* - \delta^i{}_j\bar{\psi}_\rho{}^k\Omega_kX^*) - \tfrac{1}{8}e^{-1}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu i}\psi_{\nu j}\epsilon^{ij}\epsilon^{kl}\bar{\psi}_{\rho k}(\gamma_\sigma\Omega_lX + \tfrac{1}{2}\psi_{\sigma l}X^2) + \text{h.c.}]\},
\end{aligned} \tag{3}$$

where g is the gauge coupling constant; the fields of the vector multiplet are written as Lie-algebra valued expressions, e.g.

$$X = X^A t_A, \quad X^* = X^{A*} t_A. \tag{4}$$

The overall sign adopted in (3) is such that for compact generators $[\mathrm{tr}(t_A t_A) < 0]$ the kinetic terms have the conventional sign. The superconformal fields are the vierbein and gravitino fields $e_\mu{}^a$ and $\psi_\mu{}^i$, the chiral U(1) and SU(2) gauge fields A_μ and $\mathcal{V}_\mu{}^i{}_j$, a Majorana spinor doublet χ^i, a tensor $T_{[ab]}{}^{[ij]}$ and a scalar field D. In (3) we have also used the definitions

$$\mathcal{D}_\mu X = \partial_\mu X - g[W_\mu, X] + \mathrm{i}A_\mu X,$$

$$\mathcal{D}_\mu \Omega_i = \partial_\mu \Omega_i - \tfrac{1}{2}\omega_\mu \cdot \sigma \Omega_i - g[W_\mu, \Omega_i] + \tfrac{1}{2}\mathrm{i}A_\mu \Omega_i - \tfrac{1}{2}\mathcal{V}_\mu{}^j{}_i \Omega_j,$$

$$\mathcal{D}_\mu \psi_{\nu i} = \partial_\mu \psi_{\nu i} - \tfrac{1}{2}\omega_\mu \cdot \sigma \psi_{\nu i} - \tfrac{1}{2}\mathrm{i}A_\mu \psi_{\nu i} - \tfrac{1}{2}\mathcal{V}_\mu{}^j{}_i \psi_{\nu j},$$

$$\hat{F}_{\mu\nu} = \hat{F}^+_{\mu\nu} + \hat{F}^-_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - g[W_\mu, W_\nu] + (-\tfrac{1}{2}\bar{\psi}_{\mu i}\gamma_\nu \Omega_j \epsilon^{ij} + \tfrac{1}{2}\bar{\psi}_{\nu i}\gamma_\mu \Omega_j \epsilon^{ij} - \bar{\psi}_{\mu i}\psi_{\nu j}\epsilon^{ij} X + \text{h.c.}). \tag{5}$$

The spin connection field $\omega_\mu{}^{ab}$ contains ψ-torsion, and R is its associated curvature scalar.

The action corresponding to (3) is invariant under all superconformal transformations, such as dilatations (D), chiral SU(2) and U(1) transformations, and Q and S supersymmetry. Some of these invariances may be exploited to remove some of the fields in (3), and to establish that such lagrangians are gauge equivalent to Poincaré supergravity, possibly coupled to some matter multiplets. An indication that such a phenomenon is in fact possible for (3) is indicated by the fact that if we adjust $\mathrm{tr}(XX^*)$ to a constant by means of a local scale transformation, the lagrangian (3) contains precisely the standard kinetic terms for the graviton and gravitini. Hence the degrees of freedom associated with $\mathrm{tr}(XX^*)$ may act as a compensating field for scale transformations in a way that we have explained in the introduction. Note again that a "correct" sign for graviton and gravitini kinetic terms is accompanied by a "wrong" sign for the compensating field.

At the moment it is not yet necessary to specify the gauge group, so we proceed to derive the general Einstein–Yang–Mills lagrangian. It turns out that the compensating field mechanism outlined above does not yet suffice in this case to derive the Einstein–Yang–Mills lagrangian, because the lagrangian (3) alone will lead to inconsistent field equations. This can be remedied in various ways [9] by introducing a second compensating multiplet. Here we choose the option of using the "nonlinear" multiplet, which contains the fields $(\Phi^i{}_\alpha, \lambda_i, M_{[ij]}, V_\mu)$. The scalar fields in $\Phi^i{}_\alpha$ parametrize elements of SU(2), and can therefore be used as compensating fields for the chiral SU(2) group. Hence Φ is restricted to the unit matrix, in which case there is no longer a distinction between indices α, β, .., and $i, j, \ldots$. Prior to this, one has the option of letting Φ transform under an invariant SU(2) or SO(2) subgroup of the full group acting on the indices $\alpha, \beta, \ldots$. This will lead to "gauged" N=2 supergravity or to a Fayet–Iliopoulos term, depending on whether the gauge field(s) associated with this subgroup belong(s) to supergravity or to the matter multiplets. We denote the corresponding multiplets by $(X^\alpha{}_\beta, \Omega_i{}^\alpha{}_\beta, W_\mu{}^\alpha{}_\beta, Y_{ij}{}^\alpha{}_\beta)$ and introduce a separate coupling constant g' to indicate the modifications in subsequent formulas. The final Einstein–Yang–Mills lagrangian will in general have a cosmological term of order g'^2.

Apart from these details the net effect of the introduction of the nonlinear multiplet is that the superconformal field D is expressed in tersm of the spinor λ_i, the complex scalar $M_{[ij]}$ and the vector field V_μ. To substitute the resulting expression in the lagrangian one needs

$$ef(\phi)D = -eV^\mu\partial_\mu f(\phi) + ef(\phi)\{-\tfrac{1}{2}V_\mu^2 - \tfrac{1}{4}|M_{ij}|^2 - \tfrac{1}{4}V_\mu{}^i{}_j V^{\mu j}{}_i - \tfrac{1}{3}R$$
$$+ [2\bar\lambda_i \not{\mathcal{D}}\lambda^i - \bar\lambda_i \not{V}^i{}_j\lambda^j - \tfrac{1}{2}\bar\lambda_i\sigma\cdot T^{-ij}\lambda_j + 2\bar\lambda_i\chi^i + \tfrac{8}{3}\bar\lambda_i\sigma^{\mu\nu}\mathcal{D}_\mu\psi_\nu{}^i - 2\bar\lambda_i\sigma^{\mu\nu}V_\mu{}^i{}_j\psi_\nu{}^j - \tfrac{1}{6}\bar\lambda_i\sigma\cdot T^{-ij}\gamma\cdot\psi_j$$
$$+ \bar\psi_\mu{}^i\lambda_i V^\mu - \tfrac{1}{2}\bar\psi^i\cdot\gamma\chi_i + \tfrac{1}{12}\bar\psi_{\mu i}\psi_{\nu j}T^{-\mu\nu ij} + \tfrac{1}{3}e^{-1}\epsilon^{\mu\nu\rho\sigma}\bar\psi_{\mu i}\gamma_\nu\mathcal{D}_\rho\psi_\sigma{}^i - \tfrac{1}{4}e^{-1}\epsilon^{\mu\nu\rho\sigma}\bar\psi_{\mu i}\gamma_\nu V_\rho{}^i{}_j\psi_\sigma{}^j$$
$$- 2\bar\psi_{\mu i}\sigma^{\mu\nu}\lambda^i(\bar\lambda^j\psi_{\nu j} + \bar\lambda_j\psi_\nu{}^j) - \tfrac{1}{2}e^{-1}\epsilon^{\mu\nu\rho\sigma}\bar\psi_{\mu i}\gamma_\nu\psi_\rho{}^i\bar\lambda^j\gamma_\sigma\lambda_j + 2\bar\psi_{\mu i}\psi_{\nu j}\bar\lambda^i\sigma^{\mu\nu}\lambda^j$$
$$+ (-2\bar\lambda_i\gamma^\rho\lambda^j + \tfrac{1}{2}e^{-1}\epsilon^{\mu\nu\rho\sigma}\bar\psi_{\mu i}\gamma_\nu\psi_\sigma{}^j)(2\bar\psi_\rho{}^i\lambda_j - \delta^i{}_j\bar\psi_\rho{}^k\lambda_k) + \tfrac{1}{2}g'\epsilon^{ij}Y_{kj}{}^k{}_i - 2g'^2X^{*i}{}_jX^j{}_i$$
$$- 4g'\epsilon^{jk}\bar\lambda_i(\Omega_k + \lambda_k X)^i{}_j + \text{h.c.}]\}, \tag{6}$$

where $f(\phi)$ is some arbitrary function of the fields involved, and

$$\mathcal{D}_\mu\lambda^i = \partial_\mu\lambda^i - \tfrac{1}{2}\omega_\mu\cdot\sigma\lambda^i + \tfrac{1}{2}\mathrm{i}A_\mu\lambda^i + \tfrac{1}{2}\mathcal{V}_\mu{}^i{}_j\lambda^j,$$
$$V_\mu{}^i{}_j = \mathcal{V}_\mu{}^i{}_j - g'W_\mu{}^i{}_j - 2\bar\psi_\mu{}^i\lambda_j\lambda^i + 2\bar\psi_{\mu j}\lambda^i + \tfrac{1}{2}\delta^i{}_j(\bar\psi_\mu{}^k\lambda_k - \bar\psi_{\mu k}\lambda^k). \tag{7}$$

In (6) we have dropped a total divergence.

Combining (3) and (6) with $f(\phi) = \mathrm{tr}(XX^*)$, we may now scale $\mathrm{tr}(XX^*)$ to a constant to obtain the lagrangian for Einstein–Yang–Mills supergravity. A second condition may be imposed on the spinors by exploiting S supersymmetry, and we remind the reader that we have already removed the local chiral SU(2) invariance by restricting the fields Φ of the nonlinear multiplet. Hence altogether we have the gauge conditions

$$\mathrm{tr}(XX^*) = 1 \quad (\mathrm{D}),\ \mathrm{tr}(X\Omega^i) = 0 \quad (\mathrm{S}),\ \Phi^i{}_\alpha = \delta^i{}_\alpha \quad (\mathrm{SU}(2)). \tag{9,10}$$

The second condition, which reduces the supersymmetry variation of the first one to zero, is convenient because it supresses the mixing between spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ fields in the kinetic terms of the final lagrangian (in the context of N=1 such gauge conditions have been studied in ref. [10]).

The field equations for the auxiliary fields can be substituted into the full lagrangian. Ignoring the optional SU(2) or SO(2) gauging of the nonlinear multiplet the relevant equations are

$$\mathrm{tr}(X^*\overleftrightarrow{\mathcal{D}}_\mu X) = \tfrac{1}{4}\mathrm{tr}(\bar\Omega^i\gamma_\mu\Omega_i),\quad V_\mu{}^i{}_j = -\tfrac{1}{2}\mathrm{tr}(\bar\Omega^i\gamma_\mu\Omega_j - \tfrac{1}{2}\delta^i{}_j\bar\Omega^k\gamma_\mu\Omega_k,\quad T^+_{\mu\nu ij} = 2\epsilon_{ij}[\mathrm{tr}(X^2)]^{-1}\mathrm{tr}(X\hat F^+_{\mu\nu}), \tag{11}$$

where we note that the first equation determines the chiral U(1) gauge field A_μ. However, none of these equations breaks the U(1) gauge invariance, so that at this stage this symmetry will remain preserved. Of course, we may always break it by imposing another gauge condition on the fields X.

Using (8)–(11) the lagrangian takes the form

$$e^{-1}\mathcal{L} = \mathrm{tr}(\hat\partial_\mu X\hat\partial^\mu X^*) + \tfrac{1}{4}[\mathrm{tr}(X^*\overleftrightarrow{\hat\partial}_\mu X)]^2 - \tfrac{1}{2}R - g^2\,\mathrm{tr}([X,X^*]^2) + \tfrac{1}{8}\mathrm{tr}(\hat F_{\mu\nu}\hat F^{\mu\nu}) - \tfrac{1}{4}\{[\mathrm{tr}(\hat F^+_{\mu\nu}X)]^2/\mathrm{tr}(X^2) + \text{h.c.}\}$$
$$+ \{\tfrac{1}{4}\mathrm{tr}(\bar\Omega^i\hat{\not\partial}\Omega_i) + \tfrac{1}{2}g\epsilon^{ij}\mathrm{tr}(\bar\Omega_i[X^*,\Omega_j]) - \tfrac{1}{16}\mathrm{tr}(X^*\overleftrightarrow{\hat\partial}_\mu X)\,\mathrm{tr}(\bar\Omega^i\gamma^\mu\Omega_i) + \tfrac{1}{32}\mathrm{tr}(\bar\Omega^i\gamma_\mu\Omega_j)\mathrm{tr}(\bar\Omega^j\gamma^\mu\Omega_i) - \tfrac{1}{64}[\mathrm{tr}(\bar\Omega^i\gamma_\mu\Omega_i)]^2 + \text{h.c}$$
$$+ \mathrm{tr}\{-\tfrac{1}{2}\bar\psi_\mu{}^i\hat{\not\partial}X^*\gamma^\mu\Omega_i + \tfrac{1}{2}g\bar\Omega^i\gamma\cdot\psi^j\epsilon_{ij}[X,X^*] - \tfrac{1}{4}\bar\Omega^i\gamma_\mu\psi_\nu{}^j\epsilon_{ij}\tilde{\hat F}^{\mu\nu} + \tfrac{1}{4}e^{-1}\epsilon^{\mu\nu\rho\sigma}\bar\psi_\mu{}^i\gamma_\sigma\psi_{\nu i}\hat\partial_\rho XX^* - \tfrac{1}{4}\epsilon^{ij}\bar\psi_{\mu i}\psi_{\nu j}\tilde{\hat F}^{\mu\nu}X$$
$$+ \tfrac{1}{8}\bar\Omega^i\gamma^\mu\gamma^\nu\psi_{\mu i}\bar\psi_\nu{}^j\Omega_j + \tfrac{1}{8}\bar\psi_{\mu i}\sigma^{\mu\nu}\psi_{\nu j}\epsilon^{ik}\epsilon^{jl}\bar\Omega_k\Omega_l - \tfrac{1}{16}\epsilon^{ij}\bar\psi_{\mu i}\psi_{\nu j}\epsilon^{kl}\bar\Omega_k\sigma^{\mu\nu}\Omega_l$$
$$- \tfrac{1}{8}e^{-1}\epsilon^{\mu\nu\rho\sigma}\bar\psi_{\mu i}\psi_{\nu j}\epsilon^{ij}\epsilon^{kl}\bar\psi_{\rho k}(\gamma_\sigma\Omega_l X + \tfrac{1}{2}\psi_{\sigma l}X^2) + \text{h.c.}\} + \tfrac{1}{2}e^{-1}(\epsilon^{\mu\nu\rho\sigma}\bar\psi_{\mu i}\gamma_\nu\hat\partial_\rho\psi_\sigma{}^i + \text{h.c.}), \tag{12}$$

where the derivatives $\hat\partial_\mu$ are equal to those given in (5) without the U(1) and SU(2) gauge fields A_μ and $\mathcal{V}_\mu{}^i{}_j$. We should add that the fields X and Ω are still subject to the conditions (8) and (9). This result now represents the general lagrangian for N=2 Einstein–Yang–Mills supergravity based on a quadratic form. As we have discussed recently, it is possible to generalize such lagrangians on the basis of arbitrary functions of chiral superfields [11], but this is somewhat outside the scope of this work.

In order to ensure that all kinetic terms have the correct signs, precisely one of the generators must have a different sign for $\mathrm{tr}(t_A{}^2)$. Usually this is achieved by introducing a single *abelian* multiplet where one can adjust the sign at will. The novel feature of the model that we are about to present is that this generator is part of a *nonabelian* group. Namely we choose the group SO(2,1) which has one compact and two noncompact generators. By changing the overall sign in the lagrangian we then obtain a compensating kinetic term associated with the compact generator with the "wrong" sign, and two kinetic terms associated with the noncompact ones with the "correct" sign. As explained in the introduction we should thus end up with a lagrangian that is of the "correct" sign for both the gravitational and the matter multiplets.

The SO(2,1) generators are defined by

$$[t_0, t_1] = -t_2, \quad [t_2, t_0] = -t_1, \quad [t_1, t_2] = t_0, \tag{13}$$

and to take care of the overall change of sign in the lagrangian we *define* the trace over these generators by

$$\mathrm{tr}(t_0{}^2) = 1, \quad \mathrm{tr}(t_1{}^2) = \mathrm{tr}(t_2{}^2) = -1, \tag{14}$$

where t_0 is now the compact generator. It is straightforward to write the Lie-algebra valued quantities on this basis, e.g.

$$X = X^0 t_0 + X^1 t_1 + X^2 t_2, \quad X^* = X^{0*} t_0 + X^{1*} t_1 + X^{2*} t_2, \tag{15}$$

so that (8) assumes the form

$$\mathrm{tr}(XX^*) = |X^0|^2 - |X^1|^2 - |X^2|^2 = 1. \tag{16}$$

A field configuration that satisfies (16) leaves only the compact SO(2) subgroup of SO(2,1) invariant. Therefore the gauge fields associated with the noncompact generators acquire a mass. It is thus convenient to impose an additional gauge condition on X with respect to the noncompact gauge transformation. For instance, X may be expressed in terms of a complex field a and two real fields A and B according to

$$X = a(t_0 + A t_1 + \mathrm{i}B t_2). \tag{17}$$

The field $Z = A - \mathrm{i}B$ now transforms under the SO(2) subgroup as

$$Z \to Z' = \exp(\mathrm{i}g\Lambda^0)\, Z\,, \tag{18}$$

but is inert under the scale and chiral U(1) transformations of the superconformal theory. The condition (16) now amounts to

$$|a|^2 = (1 - |Z|^2)^{-1}. \tag{19}$$

Note that (16) implies $|Z| < 1$.

It is not difficult to evaluate the scalar field potential

$$V(Z, Z^*) = 4g^2 |Z|^2 (1 - |Z|^2)^{-2}. \tag{20}$$

This potential has an absolute minimum at Z=0, with zero cosmological constant and no supersymmetry breaking.

As is shown in (12) the kinetic terms for the gauge fields come from two sources. At the minimum of the potential the second term contains only the "graviphoton" field strength F^0, and this term is responsible for reversing the sign of $(F^0{}_{\mu\nu})^2$. Let us redefine the gauge fields as

$$B_\mu = \tfrac{1}{2}\sqrt{2}\, W_\mu{}^0, \quad W_\mu{}^- = \tfrac{1}{2}(W_\mu{}^1 + \mathrm{i}W_\mu{}^2), \tag{21}$$

with corresponding field strengths

$$F_{\mu\nu}(B) = \partial_\mu B_\nu - \partial_\nu B_\mu - \sqrt{2}\,\mathrm{i}g(W_\mu{}^- W_\nu{}^+ - W_\nu{}^- W_\mu{}^+), \quad F_{\mu\nu}(W^-) = (\partial_\mu + \sqrt{2}\mathrm{i}gB_\mu)W_\nu{}^- - (\partial_\nu + \sqrt{2}\mathrm{i}gB_\nu)W_\mu{}^-, \tag{22}$$

where B_μ is the graviphoton of N=2 supergravity, and W_μ^- a complex (massive) gauge field. The terms in the lagrangian proportional to F^2 then read

$$e^{-1}\mathcal{L} = (1 + |Z|^2)^{-1} \{-\tfrac{1}{4}(1 - |Z|^2) F_{\mu\nu}(B)^2 - \tfrac{1}{2}|F_{\mu\nu}(W^-)|^2$$

$$+ \tfrac{1}{4}Z^2 F_{\mu\nu}(W^-)^2 + \tfrac{1}{4}Z^{*2}F_{\mu\nu}(W^+)^2 + \tfrac{1}{4}\mathrm{i}\sqrt{2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}(B)[ZF_{\rho\sigma}(W^-) + Z^*F_{\rho\sigma}(W^+)]\} . \tag{23}$$

The charge which measures the coupling of the graviphoton B to W^- (and by supersymmetry to all matter fields) is thus equal to $\sqrt{2}g$. A characteristic difference with charged vector bosons in the standard gauge theories is that the anomalous moment is of opposite sign. This originates from the concompact nature of the gauge group, in particular from the sign in the last commutator of (13).

To determine the masses for spin-0 and spin-1 fields we must also give the kinetic terms for Z with the corresponding interactions with the SO(2,1) gauge fields. The result takes the form

$$\mathcal{L} = -(1 - |Z|^2)^{-2}\{|\partial_\mu^0 Z|^2 + \tfrac{1}{4}(Z\overleftrightarrow{\partial}_\mu^0 Z^*)^2\}$$

$$- 4g^2(1 - |Z|^2)^{-1}\{W_\mu^+ W^{-\mu} - \tfrac{1}{4}[(3 + |Z|^2)/(1 - |Z|^2)](W_\mu^+ Z^* - W_\mu^- Z)^2\} , \tag{24}$$

where

$$\partial_\mu^0 Z = (\partial_\mu - \sqrt{2}\mathrm{i}gB_\mu)\, Z . \tag{25}$$

This theory describes the coupling of N=2 supergravity, with a graviton, two gravitini and the graviphoton, to a charged massive vector multiplet. The latter is based on the charged boson fields W_μ^+ and Z, and the chiral components of the charged Dirac fields $\Omega_i^+ \propto \Omega_i^1 - \mathrm{i}\Omega_i^2$, $\Omega^{+i} \propto \Omega^{1i} - \mathrm{i}\Omega^{2i}$ [remember that the spinors associated with t_0 are eliminated through the gauge condition (9)]. This represents precisely an N=2 massive spin-one multiplet with central charge, whose associated gauge field is the graviphoton. The central charge thus corresponds to the SO(2) generator t_0. In N=2 supersymmetry gauge transformations enter in the commutator of two supersymmetry transformations with parameter

$$\Lambda^A = -4X^A\bar{\epsilon}_{1i}\epsilon_{2j}\epsilon^{ij} - 4X^{A*}\bar{\epsilon}_1{}^i\epsilon_2{}^j\epsilon_{ij}, \tag{26}$$

where X is the scalar field of the corresponding guage multiplet. In the SO(2,1) theory it is the X^0 component that acquires a vacuum expectation value. Therefore Λ^0 will remain as the relevant central charge transformation for the states of the matter multiplet [12]. There is a typical relation between the mass and the charge which must hold for this matter multiplet, namely

$$M = 2g\kappa^{-1} , \tag{27}$$

where $\sqrt{2}g$ and M are the charge and the mass of the matter multiplet, and κ is the gravitational coupling constant which has been put to one in this paper. As is well-known, this relation gives rise to the phenomenon of "antigravity" [13].

We close with some comments regarding the coupling of supersymmetric matter to this theory. As was clearly indicated by our derivation of the Einstein–Yang–Mills lagrangian it is straightforward to introduce additional gauge field multiplets to this theory. Precisely as for the more conventional N=2 supergravity theories the gauge field lagrangian may be based on an arbitrary gauge invariant function of the Lie-algebra valued fields [11]. If the corresponding gauge group contains an invariant SU(2) or SO(2) subgroup one can introduce a Fayet–Iliopoulos term. This term will give rise to a cosmological term and corresponding SU(2) or SO(2) gauge field interaction with the gravitini. However, the graviphoton cannot have a minimal coupling to the gravitini. To introduce scalar multiplets is much harder. First of all a single scalar multiplet cannot couple to SO(2,1), so that it is not possible to introduce a coupling with the graviphoton. Even if this were the case, the corresponding kinetic terms would not be positive definite. However, it may be possible to use an independent gauge field in order to realize the local central charge transformations for the scalar multiplets separately.

In principle extended supersymmetry severely restricts the variety of invariant matter couplings or the nature of the scalar field potentials. Our results show that the techniques of multiplet calculus can reveal the possibility for new theories with an unusual structure. The theory at hand is one such example, which, unlike the standard $N=2$ lagrangian, cannot be viewed as a truncation of one of the known higher-extended supergravity theories.

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