GAUGE AND MATTER FIELDS COUPLED TO $N = 2$ SUPERGRAVITY

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We consider the potential of a general matter system of $N = 2$ vector and scalar multiplets coupled to supergravity. For lagrangians that are initially quadratic in the matter fields we prove that the potential is either positive or unbounded from below. Our results have been obtained in the framework of a superconformal multiplet calculus, and we have verified that they can be derived from each of the three off-shell representations. As an example we consider SO(6) Yang–Mills theory coupled to scalar multiplets in the $\mathbf{10} + \overline{\mathbf{10}}$ representation, which, for suitably chosen parameters, leads to the potential of gauged $N = 8$ supergravity. We discuss possibilities for residual nonabelian symmetry groups after breaking of $N = 8$ supersymmetry to $N = 1$ or 2.

Extended supergravity offers a unique framework for understanding the unification of elementary particles and their interactions. Assuming that supergravity is a consistent quantum theory of gravity, and thus free of ultraviolet divergences, supersymmetry breaking will set the scale for the cutoff of all conventional particle interactions. At lower mass scales the hope is that the theory exhibits $N = 1$ supersymmetry, so that it is effectively described by a supersymmetric version of a grand-unified theory. This hope has motivated recent studies of $N = 1$ supergravity coupled to matter [1], and of specific unification scenarios based on these theories (see e.g. ref. [2]).

It is rather obvious that $N = 1$ supergravity is an incomplete theory. The introduction of supersymmetric matter leads to a multitude of unrelated coupling constants, which limits its predictive power. But more importantly, $N = 1$ supergravity coupled to matter is known to be inconsistent at the quantum level. This makes it impossible to establish the existence of an effective low-energy theory, or to discuss the emergence of a hierarchy of mass scales associated with the fundamental forces in a rigorous fashion.

Extended supergravity offers a solution to these problems, but its phenomenological relevance is much harder to grasp. In order to make further progress it is therefore important to devote more attention to models of matter coupled to extended supergravity. Although these models will still share some of the shortcomings with the simple supergravity theories, they may lead to important clues in the understanding of

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the relation between extended supersymmetry and the phenomenology of elementary particles. So far a systematic understanding of the off-shell structure of extended supersymmetry is limited to $N = 2$. For that case a systematic calculus for the construction of invariant actions has been developed, which employs superconformal techniques [3] (for an alternative approach, see ref. [4]). Of course, it is always possible to cast higher N theories in a form in which only two supersymmetries are manifest. Such an approach has been used to derive important results on the structure of quantum divergences [5].

In this letter we discuss the invariant actions for a large variety of matter multiplets coupled to $N = 2$ supergravity (general $N = 2$ matter systems without supergravity were recently discussed in ref. [6]). Since one of the primary objectives is to establish the existence of a so-called super-BEH (Brout–Englert–Higgs) effect, we will concentrate mainly on the potential. One of our results is that spontaneously broken realizations of supersymmetry do only occur for potentials that are not bounded from below. This is proven for lagrangians which are quadratic in the matter fields before the coupling to supergravity, but we suspect that this is a universal feature of all $N = 2$ theories. This is an important deviation from the situation in $N = 1$ supergravity, where it is rather easy to ensure that the potential has an absolute minimum at which supersymmetry breaking may take place [1]. The tendency for unbounded potentials for $N = 2$ was already noticed in ref. [3] for a more restricted class of models. In fact this phenomenon is familiar from higher-N models as well, where the gauging of the SO(N) symmetry leads to a scalar-field potential that is not bounded from below [7,8]. It is important to realize that in the context of gravity an unbounded potential does not imply that the theory is unstable; under certain conditions one can demonstrate the stability for small fluctuations about an anti-de Sitter background [9].

We consider $N = 2$ vector multiplets, which describe a gauge theory associated with a gauge group G, scalar multiplets that transform under G according to a certain representation, and tensor multiplets. The fields of the *vector multiplets* are all in the adjoint representation of G [or in the singlet representation for an abelian (sub)group]. Therefore we will use a notation in which these fields are written as matrices that can be decomposed in terms of the generators of G in some representation. These generators t_A will be defined such that the group transformations are of the form $\exp(\xi^A t_A)$ with *real* transformation parameters ξ^A. For instance, the t_A are antihermitean for unitary groups, and real and antisymmetric for orthogonal groups; for a (non)compact generator t_A we have $\mathrm{tr}(t_A^2) < 0$ (>0). The fields of the vector multiplet are complex scalars $X_\alpha{}^\beta$, a doublet of spinors $\Omega_{i\alpha}{}^\beta$, the gauge fields $W_{\mu\alpha}{}^\beta$ and auxiliary scalar fields $Y_{ij\alpha}{}^\beta$. The indices α, β refer to the fact that these fields take their values in the Lie algebra of G; the indices i refer to the behaviour under chiral SU(2) which is associated with $N = 2$ conformal supergravity. The gauge fields $W_{\mu\alpha}{}^\beta$ are real, and the auxiliary fields satisfy a reality condition

$$(W_{\mu\alpha}{}^\beta)^* = W_\mu{}^\alpha{}_\beta\,, \quad (Y_{ij\alpha}{}^\beta)^* \equiv Y^{ij\alpha}{}_\beta = \epsilon^{ik}\epsilon^{jl}Y_{kl}{}^\alpha{}_\beta\,. \quad (1)$$

We note that complex conjugation is always effected by raising and lowering of indices.

The *scalar multiplets* consist of scalar fields $A_i{}^\alpha$, Majorana spinors ζ^α and auxiliary scalars $A_i{}^{\alpha(z)}$. The superscript (z) indicates that $A_i{}^{\alpha(z)}$ follows from $A_i{}^\alpha$ by application of an infinitesimal central charge transformation. Similarly we have $\zeta^{\alpha(z)}$, $A_i{}^{\alpha(zz)}$, etc., but unlike $A_i{}^{\alpha(z)}$ these do not correspond to new degrees of freedom, because they are subject to certain constraints [10]. Again we distinguish SU(2) indices and indices α associated with the gauge group G, and as suggested by this notation it is convenient to write the Lie-algebra valued fields of the vector multiplet in the (not necessarily irreducible) representation that is relevant for the scalar multiplet. The scalar fields satisfy the condition

$$(A_i{}^\alpha)^* \equiv A^i{}_\alpha = \epsilon^{ij}\rho_{\alpha\beta}A_j{}^\beta\,. \quad (2)$$

Consistency requires that $(\rho_{\alpha\beta})^* \equiv \rho^{\alpha\beta}$ satisfies

$$\rho_{\alpha\beta}\rho^{\beta\gamma} = -\delta_\alpha{}^\gamma\,. \quad (3)$$

By taking the determinant of both sides of (3) one deduces that the indices α run over an even number of values ($\alpha = 1, ..., 2M$). Furthermore, the scalar multiplet must transform according to a real representation of G, such that (2) is covariant; more precisely, one must have

$$(t_{A\alpha}{}^\beta)^* \equiv t_A{}^\alpha{}_\beta = -\rho^{\alpha\gamma}t_{A\gamma}{}^\delta\rho_{\delta\beta}\,. \quad (4)$$

The fields of the *tensor multiplet* are scalars L^{ij}, satisfying

$$L_{ij} = L_{ji} = \epsilon_{ik}\epsilon_{jl}L^{kl}\,, \tag{5}$$

Majorana spinors φ^i, a tensor gauge field $E_{\mu\nu}$, and an auxiliary complex scalar field G. The tensor multiplet must be inert under the gauge transformations of the group G.

Vector and tensor multiplets can be realized in the presence of the fields of $N = 2$ conformal supergravity. These fields are the vierbein $e_\mu{}^a$, the gravitino fields $\psi_\mu{}^i$, the gauge fields of chiral SU(2) × U(1) denoted by $\mathcal{V}_\mu{}^i{}_j$ and A_μ, an antisymmetric tensor $T_{ab}{}^{ij}$, a real scalar D and a spinor χ^i. Scalar multiplets require an extension of the background configuration in order to locally realize the central charge transformations. This is achieved by introducing an extra vector multiplet, with bosonic fields a, $B_\mu{}^{ij}$ and S^{ij}, and a Majorana spinor ξ^i. The gauge field $B_\mu{}^{ij} = -B_\mu{}^{ji}$ is now associated with local central charge transformations [10]. The combination of this vector multiplet with the conformal supergravity fields is called the minimal field representation, which comprises 32 + 32 field degrees of freedom.

The construction of $N = 2$ supergravity theories is based on a small variety of invariant actions, which we will now discuss one by one. We only exhibit those terms that are relevant for the computation of the potential.

(a) For vector multiplets there is a lagrangian quadratic in the fields. The relevant terms are

$$e^{-1}L_{\text{vector}} = \mathrm{tr}(\partial_\mu X\,\partial^\mu X^*) + \mathrm{tr}(XX^*)(-\tfrac{1}{6}R + D)$$

$$-\tfrac{1}{8}\mathrm{tr}(Y_{ij}Y^{ij}) - g^2\,\mathrm{tr}([X, X^*]^2)\,, \tag{6}$$

where R denotes the standard curvature scalar. We note that complex conjugation is performed in the context of a given basis of the Lie algebra associated with the gauge group G, so that X and X^*, or Y_{ij} and Y^{ij} are decomposed into the same set of generators. In other words, complex conjugation is not implied on the generators $t_{A\alpha}{}^\beta$ themselves. The lagrangian (6) is based on the quadratic invariant $f(X) = \mathrm{tr}(XX)$, but it can be generalized to any invariant function of the Lie-algebra valued fields. We will return to this aspect shortly.

(b) For scalar multiplets there exists a lagrangian which contains the standard kinetic terms. The relevant terms read [‡1]

$$e^{-1}L_{\text{kin}} = -|\partial_\mu A|^2 + |A|^2(\tfrac{1}{6}R + \tfrac{1}{2}D) + |a|^2|A^{(z)}|^2$$

$$+ gA^i{}_\alpha Y^{jk\alpha}{}_\beta A_k{}^\beta\epsilon_{ij} + 4g^2A^i{}_\alpha X^\alpha{}_\beta X^{*\beta}{}_\gamma A_i{}^\gamma\,, \tag{7}$$

where $|A|^2 \equiv A^i{}_\alpha A_i{}^\alpha$. There is also an invariant mass term for scalar multiplets which contains the mass terms

$$e^{-1}L_{\text{mass}} = im_\alpha{}^\beta(A_i{}^\alpha A^k{}_\beta\epsilon_{jk}S^{ij} + 2|a|^2A_i{}^\alpha A^i{}_\beta{}^{(z)})$$

$$+ 2igm_\alpha{}^\beta(A^i{}_\beta a^*X^\alpha{}_\gamma A_i{}^\gamma - A_i{}^\alpha aX^*{}_\beta{}^\gamma A^i{}_\gamma)\,. \tag{8}$$

(c) For the tensor multiplet the relevant terms of the lagrangian read

$$e^{-1}L_{\text{tensor}} = -\tfrac{1}{2}|\partial_\mu L_{ij}|^2L^{-1} + L(\tfrac{1}{3}R + D)$$

$$+ L^{-1}|G|^2\,. \tag{9}$$

Furthermore, there is a second lagrangian in which the tensor multiplet couples linearly to an abelian vector multiplet. For instance, if one chooses the vector multiplet associated with central charge transformations, the relevant terms read

$$e^{-1}L_{\text{lin}} = m_0(aG + \tfrac{1}{2}S^{ij}L_{ij})\,. \tag{10}$$

‡1 Lagrangians for scalar multiplets are based on an invariant tensor $\eta_{\alpha\beta}$, which has been used to define a real gauge invariant expression quadratic in the scalar multiplet fields, $(A_i, A_j') = \eta_{\alpha\beta}A_i{}^\alpha A_j'{}^\beta$. The tensor $\eta_{\alpha\beta}$ thus satisfies

$$t_{A\gamma}{}^\alpha\eta^{\gamma\beta} + \eta^{\alpha\gamma}t_{A\gamma}{}^\beta = 0 \qquad \text{(G invariance)}\,,$$

$$\eta^{\alpha\beta} \equiv (\eta_{\alpha\beta})^* = \rho^{\gamma\alpha}\rho^{\delta\beta}\eta_{\gamma\delta} \qquad \text{(reality)}\,.$$

From an (anti)symmetric tensor η one can construct a hermitean tensor $i\eta^{\alpha\gamma}\rho_{\gamma\beta}$ $(\eta^{\alpha\gamma}\rho_{\gamma\beta})$ which can always be diagonalized by a suitable field redefinition. In order to have kinetic terms one must choose η antisymmetric and since we insist on kinetic terms that are positive definite we must be able to redefine the fields such that $\eta^{\alpha\gamma}\rho_{\gamma\beta} = \delta^\alpha{}_\beta$, or $\eta_{\alpha\beta} = -\rho_{\alpha\beta}$ [cf. eq. (3)]. In that case ρ is antisymmetric after the field redefinition, and the group G must be unitary. The antisymmetry of ρ enables one to prove

$$A^i{}_\alpha A_j{}^\alpha = \tfrac{1}{2}\delta^i{}_j|A|^2\,.$$

A mass term for scalar multiplets can be constructed from a symmetric tensor η, and the corresponding hermitean mass matrix, which is not necessarily diagonal, takes the form $m^\alpha{}_\beta = i\eta^{\alpha\gamma}\rho_{\gamma\beta}$. The mass matrix is invariant under the transformation of G, i.e.

$$t_{A\gamma}{}^\alpha m^\gamma{}_\beta + m^\alpha{}_\gamma t_A{}^\gamma{}_\beta = 0\,.$$

Supersymmetric matter systems described by vector, scalar and tensor multiplets can now be coupled to $N = 2$ supergravity by using the full actions corresponding to (6), (7), (8), (9) and (10). The lagrangian of $N = 2$ supergravity itself follows from applying (6) to the vector multiplet associated with the central charge. This leads to

$$e^{-1}L_{\text{comp}} = \tfrac{1}{4}|\partial_\mu a|^2 + \tfrac{1}{4}|a|^2(-\tfrac{1}{6}R + D) - \tfrac{1}{8}|S_{ij}|^2. \quad (11)$$

The reason for a "wrong" overall sign in (11) is that we are in the context of a full superconformal theory. Therefore the lagrangians are still invariant under local scale transformations, and this invariance may be exploited to adjust the field a to a constant, or some function of the other fields. If a is adjusted to a constant, (11) contains Einstein's lagrangian with the correct sign, and because the complete version of (11) is supersymmetric we find the full supergravity lagrangian (apart from a subtlety that we will discuss below). Fields such as a play the role of a compensator [11], because they allow one to compensate for any apparent lack of scale invariance. Obviously, such a field does not correspond to physical degrees of freedom, so that the nonconventional sign of the kinetic term in (11) does not lead to physical consequences. The reason that the compensating multiplet is an *abelian* vector multiplet is that the central charge is abelian. In the absence of a central charge it is possible to use a nonabelian vector multiplet associated with a noncompact local gauge group; this option will be discussed elsewhere [12].

One can now write down the lagrangian for the desired configuration of vector, scalar and tensor multiplets combined with (14). The curvature scalar is then multiplied by a function, $-\frac{1}{6}\{\frac{1}{4}|a|^2 + \mathrm{tr}(XX^*) - |A|^2 - 2L\}$, which may now be adjusted at will by using the local scale invariance of the theory. We emphasize that $-\mathrm{tr}(XX^*) + |A|^2 + 2L$ is positive, so that for sufficiently large values of a the function can be scaled to a *negative* constant (if a is not taken large enough one usually ends up with lagrangians that describe negative metric states). The most convenient gauge choice is to restrict a as follows

$$a = 2[1 - \mathrm{tr}(XX^*)]^{1/2}, \quad (12)$$

where we have also adjusted the phase of a by exploiting the chiral U(1) invariance of the theory.

The last obstacle is that the field D appears linearly and after imposing (12) multiplies $1 + \frac{1}{2}|A|^2 + L$. Therefore, D acts as a Lagrange multiplier and this leads to a constraint $1 + \frac{1}{2}|A|^2 + L = 0$ that cannot be satisfied. The model is therefore inconsistent. To avoid this problem one must introduce an extra supermultiplet, and it is here that three auxiliary field formulations of $N = 2$ supergravity become possible [13]. We have explicitly verified that our results can be obtained in all three formulations, with the exception of an SU(2) Fayet–Iliopoulos term, which seems impossible to derive in the third auxiliary field formulation. It therefore suffices to present our work in the context of one of these formulations, and we choose the one which makes use of the nonlinear multiplet [3]. In that case one may simply substitute (we only give the relevant bosonic terms)

$$D = -\tfrac{1}{3}R - \tfrac{1}{4}|M^{ij}|^2 - g(Y_{ik})^k{}_j\epsilon^{ij} - 4g^2\mathrm{tr}(XX^*), \quad (13)$$

where M^{ij} is an auxiliary field that is contained in the nonlinear multiplet. The reason why the α indices of $(Y_{ij})^\alpha{}_\beta$ have been replaced by SU(2) indices, is that the nonlinear multiplet can *at most* couple to an SU(2) or SO(2). An alternative option is to let the gauge transformations associated with the compensating vector multiplet act on the nonlinear multiplet or on the scalar multiplets. This would amount to replacing $(Y^{ij})_\alpha{}^\beta$ by $-S^{ij}$ times the appropriate abelian generator, and X by $\frac{1}{2}a$ times the same generator. Because of the gauge choice (12), the factor multiplying the curvature scalar R now has the canonical value $-\frac{1}{2}$.

If we now analyze whether the combined lagrangian leads to a potential that is bounded from below one can immediately deduce that the contributions proportional to g in (13) must be absent, i.e. a gauging of the nonlinear multiplet will always lead to an unbounded potential [disregarding the trivial case where no vector multiplets other than (11) are present]. This is shown by restricting the fields $A_i{}^\alpha$ to zero and considering the behaviour of the potential for large values of the fields X and X^*. After eliminating the auxiliary fields M^{ij}, $A_i{}^{\alpha(z)}$, G, $Y^{ij}{}_\alpha{}^\beta$ and S^{ij}, we find the potential

$$\begin{aligned} V = {} & g^2\,\mathrm{tr}([X, X^*]^2) \\ & + 4|\{[1 - \mathrm{tr}(XX^*)]^{1/2}m^\alpha{}_\beta - \mathrm{i}gX^\alpha{}_\beta\}A_i{}^\beta|^2 \\ & - 2|\mathrm{i}A_i{}^\alpha m_\alpha{}^\beta A^k{}_\beta\epsilon_{kj} - \tfrac{1}{2}m_0 L_{ij}|^2 \\ & + \tfrac{1}{2}g^2|t_{A\alpha}{}^\beta A^k{}_\beta(A_j{}^\alpha\epsilon_{ik} + A_i{}^\alpha\epsilon_{jk})|^2 \\ & + m_0^2 L[1 - \mathrm{tr}(XX^*)], \end{aligned} \quad (14)$$

where we have used generators normalized to $\mathrm{tr}(t_A t_B) = -\delta_{AB}$ in the elimination of the fields Y_{ij}. We may disregard the option in which the gauge transformations of the compensating vector multiplet act on the scalars other than by a central charge transformation, because this will simply amount to a redefinition of the mass matrix $m^\alpha{}_\beta$.

The requirement that (14) is bounded from below now leads to the conditions

$$m_0 = 0\,,$$

$$g^2 |t_{A\alpha}{}^\beta A^k{}_\beta (A_j{}^\alpha \epsilon_{ik} + A_i{}^\alpha \epsilon_{jk})|^2 \geqslant 4 |A_i{}^\alpha m^\beta A^k{}_\beta \epsilon_{kj}|^2\,. \tag{15}$$

The first condition follows by considering the behaviour of (13) for large values of the fields L_{ij}. The second condition is necessary, because if there were fields $A_i{}^\alpha$ for which the condition is not satisfied, those may be scaled to arbitrary large values; since the $|A|^4$ terms would then dominate the potential, there would be no lower bound. Because of the second condition we now find that the potential satisfies $V \geqslant 0$, which implies the existence of a minimum for $X = X^* = A_i{}^\alpha = 0$, where supersymmetry is preserved. However, the potential has more zeros, such as for $A_i{}^\alpha = 0$, $[X, X^*] = 0, X \neq 0$, which do not break supersymmetry either. These zeroes always form valleys which are connected to the origin. Therefore we conclude that unless one accepts potentials that are not bounded from below, the situation that is familiar from rigid supersymmetry with a positive potential repeats itself.

We have already mentioned that the lagrangian (6) for the vector multiplets is not the most general one, and that it is possible to construct lagrangians for any invariant (complex) function $f(X)$ of the Lie algebra valued field X [‡2]. The action for vector multiplets is based on $N = 2$ chiral multiplets. If F_G is the (reduced) chiral superfield that describes the G-covariant field strength, then the lagrangian (6) is based on the highest component of the chiral superfield $\mathrm{tr}(F_G^2)$. This can now be extended to any G-invariant function of the Lie-algebra valued fields if one introduces the field strength F_ξ associated with the compensating vector multiplet in such a way that scale invariance is preserved. This leads to $f(F_G F_\xi^{-1})\, F_\xi^2$, which can now encompass both lagrangians (6) and (11). In this case, after the substitution (13) has been performed, the curvature scalar in the lagrangian is multiplied by a complicated expression, viz.

$$|a|^2 \{f(Xa^{-1})_\beta{}^\alpha [a^{-1} X_\alpha{}^\beta - a^{*-1} X^*{}_\alpha{}^\beta] - 2f(Xa^{-1}) + \mathrm{h.c.}\}\,, \tag{16}$$

where $f(X)^\alpha{}_\beta = \partial f(X)/\partial X^\beta{}_\alpha$. This factor can be adjusted to a constant by exploiting the local scale invariance. In this case it is convenient to first rescale the fields X to $Z = X/a$ and then adjust a to

$$a = \{4f(Z) + 4f^*(Z^*) + 2[f(Z)_\beta{}^\alpha - f^*(Z^*)_\beta{}^\alpha](Z^*{}_\alpha{}^\beta - Z_\alpha{}^\beta)\}^{-1/2}\,. \tag{17}$$

The derivation of the potential now proceeds exactly as before, but it is much more complicated to verify its behaviour as a function of the fields Z and Z^*. Concerning the kinetic term for the scalars it can be shown that the scalar fields Z and Z^* parametrize a

‡2 The previous matter lagrangians are all quadratic in the fields *before* the coupling to supergravity. A crucial difference with the coupling to $N = 1$ supergravity is seen in the kinetic terms for the gauge fields, which already at this stage acquire nonpolynomial modifications through the elimination of the auxiliary tensor $T_{ab}{}^{ij}$ [14]. The kinetic terms for the scalar fields are naturally described as a nonlinear sigma model. For the quadratic case the scalar fields of the vector multiplets can be parametrized as a noncompact version of a complex projective space $\mathrm{U}(1,n)/\mathrm{U}(n) \times \mathrm{U}(1)$, where n is the dimension of the group G (this observation was already made some time ago by E. Cremmer and J. Scherk). For general lagrangians one is dealing with a Kähler manifold. If G is nonabelian, the sigma model is gauged in the same sense as $N \geqslant 4$ supergravity can be gauged [7,8]. For the scalar multiplets one has the structure of a noncompact version of a quaternionic projective space $\mathrm{Sp}(1,M)/\mathrm{Sp}(M) \times \mathrm{Sp}(1)$. In this case there are further options by coupling the scalar multiplets to vector multiplets without adding a corresponding kinetic term. The auxiliary fields Y of these vector multiplets then act as Lagrange multipliers, which impose restrictions on the scalar manifold [15]. A classification for scalar multiplets has been given in ref. [16] based on Noether coupling techniques, and it is not known whether all possible cases can be constructed by exploiting the methods of this paper. In the presence of tensor multiplets the above result takes a different form, which may still be equivalent through a duality transformation on the tensor gauge fields. This remark is also relevant if one wants to derive some of these results in the context of the third auxiliary field formulation of $N = 2$ supergravity [13].

Kähler manifold, with Kähler potential proportional to the logarithm of (17). Further aspects of the general vector multiplet lagrangian will be discussed elsewhere.

As a specific example let us present the potential for SO(6) Yang–Mills and scalar multiplets in the $\mathbf{10} + \overline{\mathbf{10}}$ representation of SO(6) coupled to $N = 2$ supergravity. This case is of interest because for a specific choice of the parameters it corresponds to the potential of gauged $N = 8$ supergravity [8]. The vector multiplets thus transform in the 15 representation of SO(6). Hence they can be written as antisymmetric matrices in SO(6) indices $a, b = 1, ..., 6$. The scalar multiplets have indices α which are represented by antisymmetric index triples $[abc]$. The reality matrix $\rho_{\alpha\beta}$ is then $\rho_{[abc][def]} = \frac{1}{6}\epsilon_{abcdef}$ and an SO(6) invariant mass matrix is $m_{[abc]}{}^{[def]} = \frac{1}{2}\mathrm{i}m\rho_{[abc][def]}$. SO(6) covariant derivatives take the form

$$D_\mu X^{ab} = \partial_\mu X^{ab} - 2g W_\mu{}^{c[a} X^{b]c} ,$$

$$D_\mu A_i{}^{abc} = \partial_\mu A_i{}^{abc} + 3g W_\mu{}^{d[a} A_i{}^{bc]d} . \tag{18}$$

If we base the Yang–Mills lagrangian on the quadratic invariant $f(X) = \mathrm{tr}(XX)$, then there are only three independent parameters, namely the SO(6) and SO(2) gauge coupling constants g and g', and the mass parameter m. The resulting potential is

$$\begin{aligned}
V = g^2 \{ & -8X^{ab}X^{bc}X^{*cd}X^{*da} + 8X^{ab}X^{*bc}X^{cd}X^{*da} \\
& + 2|A_i{}^{abc}|^2|X^{de}|^2 - 24A^i{}_{abc}X^{ad}X^{*be}A_i{}^{cde} \\
& + \tfrac{3}{8}(|A_i{}^{abc}|^2)^2 + \tfrac{9}{2}A^i{}_{abc}A_i{}^{cde}A^j{}_{def}A_j{}^{abf} \\
& + 18A^i{}_{abc}A_i{}^{cde}A^j{}_{aef}A_j{}^{bdf}\} - 8g'^2(1 + \tfrac{1}{2}|A_i{}^{abc}|^2) \\
& \times (3 + \tfrac{1}{2}|A_j{}^{def}|^2 + 2|X^{de}|^2) \\
& + m^2\{(1 + |X^{de}|^2)|A_i{}^{abc}|^2 + \tfrac{1}{8}(|A_i{}^{abc}|^2)^2 \\
& - \tfrac{9}{4}A^i{}_{abe}A_i{}^{cde}A^j{}_{abf}A_j{}^{cdf}\} \\
& + 6mg(1 + |X^{df}|^2)^{1/2}\epsilon^{ij}A_i{}^{abc}(X + X^*)^{ae}A_j{}^{bce} \\
& + 2\sqrt{2}mg'(1 + \tfrac{1}{2}|A_i{}^{abc}|^2)(A_j{}^{def})^2 .
\end{aligned} \tag{19}$$

In order to make contact with gauged $N = 8$ supergravity the parameters g, g' and m must be proportional to the SO(8) gauge coupling constant.

If one accepts potentials that are not bounded from below then one may discuss supersymmetry breaking. In view of phenomenological applications it is then of interest to investigate the possibility for residual supersymmetry in conjunction with a certain residual gauge symmetry. In first instance, this is a group theoretical question which can be answered independently of the specific form of the potential. The crucial observation is that the massive gravitinos associated with the broken supersymmetries are still contained in supermultiplets of the residual supersymmetry [17]. Since massive supermultiplets always consist of a combination of massless multiplets, it is a nontrivial requirement to realize the necessary combination of massless multiplets transforming in identical representations of the residual Yang–Mills group. For instance, to break $N = 2$ supersymmetry to $N = 1$, the $(\frac{3}{2}, 1)$ multiplet that contains the gravitino of the second supersymmetry must be combined with a massless $(1, \frac{1}{2})$ and $(\frac{1}{2}, 0, 0)$ multiplet to give a massive $N = 1$ multiplet. Hence the Yang–Mills group must be broken such that the latter two multiplets are indeed available in a singlet of the residual Yang–Mills group. The above argument becomes considerably more restrictive if one starts from a higher-N theory. For instance, the maximal subgroup of SO(8) that can be realized for an $N = 1$ supersymmetric realization of gauged $N = 8$ supergravity is G_2. However, it is known that this configuration is not a solution of the $N = 8$ potential [18]. Under G_2 the 8 supersymmetries decompose according to $\mathbf{8} \to \mathbf{7} + \mathbf{1}$, so $N = 2$ solutions with G_2 are obviously excluded. The maximal subgroup of SO(8) that can coexist with $N = 2$ supersymmetry is $SU(3) \times U(1) \times U(1)$. The $N = 2$ multiplets that contain the massive gravitini decompose into massless multiplets of maximal spin $\frac{3}{2}$ and $\frac{1}{2}$ transforming in the $\mathbf{3} + \overline{\mathbf{3}}$ representation of SU(3). The supersymmetry algebra imposes further restrictions here, because the anticommutator of two $N = 2$ supersymmetries leads to one or two field-independent U(1) transformations depending on which fields have acquired a vacuum expectation value [‡3]. Therefore the residual gauge symmetry

[‡3] In general, if $N = 8$ supergravity breaks to $N < 8$ supersymmetry then the groundstate must at least be invariant under the SO(8) subgroup characterized by $\Lambda^{IJ} = \mathrm{Re}\{(u^{IJ}{}_{ij} + v_{IJij})z^{ij}\}$, where I and J are SO(8) indices, u and v the submatrices of the 56-bein [8] and z^{ij} arbitrary complex coefficients with i and j indices corresponding to the unbroken supersymmetries.

should contain at least one U(1) as a factor. According to ref. [18] the SU(3) $\times$ U(1) option is actually realized. In the context of the theory that underlies (19) this implies that at least one of the scalar fields $A_i{}^{[abc]}$ acquires a vacuum expectation value. Group-theoretically this solution can be realized with $N = 1$ supersymmetry, but in that case no U(1) group will survive. However, this realization seems not allowed on the basis of the $N = 8$ potential [18].

These group-theoretical considerations show that breaking to lower supersymmetry is severely restricted, irrespective of the precise form of the potential. In fact we have also considered a variety of other subgroups and we have found for example, that an SO(4) symmetric ground state does not allow residual supersymmetry. We should add that most of these solutions will have a cosmological term. Strictly speaking this analysis should therefore be done in the context of an anti-de Sitter space where supermultiplets may occur which differ in some crucial aspects from their Minkowski counterparts [19]. This requires more study, but we do not believe that it will affect our conclusions here. Finally we should mention that the results of this letter can also be used to study appropriate truncations of Kaluza–Klein realizations of $d = 11$ supergravity, in view of the recently found $N = 2$ supersymmetric compactification with SU(3) $\times$ SU(2) $\times$ U(1) symmetry [20]. The massless sector of this theory cannot be interpreted within the framework of $N = 8$ supergravity.

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