# TECHNICOLOR GIMNASTYCS

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We propose a Glashow–Iliopoulos–Maiani mechanism for extended technicolor theories and illustrate it in an explicit model. We find that flavor changing neutral current effects are adequately suppressed if $m_t \ll 50$ GeV.

*1. Technigim.* Technicolor [1] (TC) is a very attractive idea, which might address several of the fundamental puzzles of modern particle theory, but for the fact that it seems to lead to a phenomenological disaster. The SU(2) × U(1) breaking induced by TC must be communicated to the light quarks and leptons by some interaction which breaks the global chiral flavor symmetries. Fundamental scalars can do this communication, but such scalars are no more attractive than fundamental Higgs mesons. The only other possibilities would seem to be extended technicolor (ETC) gauge interactions which cause transitions between light fermions and technifermions or a dynamical model in which light fermions and technifermions are both built out of the same constituents. To date, all such schemes have been plagued by flavor changing neutral current (FCNC) effects which are too large.

In this paper, we suggest a solution to the FCNC problem in TC theories. We will describe the solution in the language of ETC and then exemplify it in a specific ETC model. However, we believe that our mechanism is more general and applies to constituent models as well.

The problem with ETC theories is associated with the structure of an ETC gauge group. The ETC gauge generators and the corresponding gauge bosons are of three types: flavor (F), symmetry generators associated with transitions between flavors; TC generators; and generators associated with transitions from ordinary fermions to technifermions. This last type is required to generate light fermion masses. Henceforth, we will reserve the name ETC for these transition generators and gauge bosons. The flavor and TC generators must exist because of they are produced by communication of ETC generators with their adjoints. It is the flavor generators that cause the trouble. Typically, flavor gauge boson exchange contributes to $\Delta S = 2$ or $\Delta C = 2$ processes, or to both. These effects cannot be suppressed by simply increasing the flavor gauge boson masses, because anything which increases the flavor gauge boson masses also increases the ETC gauge boson masses, which in turn decreases the light fermion masses.

To evade this snare, we turn to a generalization of the Glashow–Iliopoulos–Maiani (GIM) mechanism which banished FCNC effects from the standard model. In the standard model, there are no FCNC effects in lowest order because the gauge interactions have a very large flavor symmetry, SU($n$) × SU($n$) × SU($n$) (× irrelevant U(1)'s) for flavors. This flavor symmetry

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allows us to move the mixing angles in the quark sector from the charge 2/3 (U) quarks to the charge −1/3 (D) quarks or back at our convenience. FCNC effects can appear only when both U and D quarks are involved, as in the usual box diagram.

It is clear that this flavor symmetry argument cannot be trivially generalized to ETC. For one thing, in ETC theories, there are more gauge interactions. For the flavor interactions to have a flavor symmetry, the flavor gauge bosons must be degenerate. What is worse, the quark mass matrix cannot have the flavor symmetry, because the quarks are not degenerate. But the quark mass matrix comes from the gauge interactions. Thus the gauge interactions cannot have the flavor symmetry either.

The solution is simple:

**Break the flavor symmetry where there is no mixing.**
**Introduce mixing only where there is flavor symmetry.**

We will illustrate this mechanism in a toy model of quarks, in which the ETC group is a semisimple group, $SU(N)_L \times SU(N)_U \times SU(N)_D$. We ignore leptons and put in the ETC breaking by hand with fundamental scalar fields, in order to simplify the discussion and concentrate on our TECHNIGIM mechanism.

In the remainder of this section, we describe the model in words. In section 2, we describe it in technical detail. The casual reader may want to skim (or skip) section 2 and proceed to section 3 which contains our conclusions and speculations.

LH SU(2) doublets of quarks and techniquarks transform like N's of $SU(N)_L$, denoted $\psi_{\mathbf{A}L}$, where **A** is an $SU(N)_L$ index which runs from 1 to $N$ [and SU(2) indices are suppressed].

RH singlets of charge 2/3 [−1/3] quarks and techniquarks transform like N's of $SU(N)_U$ [$SU(N)_D$], denoted $U_{\mathcal{A}R}$ [$D_{AR}$] where $\mathcal{A}$ [$A$] is an $SU(N)_U$ [$SU(N)_D$] index. In all of these SU($N$)'s, the first three values of the index refer to flavor, the last $N-3$ to a TC $SU(N-3)$. We will use lower case letters (in the appropriate type face) at the beginning of the alphabet to denote SU($N$) flavor indices (1, ..., 3), lower case letters at the end of the alphabet to denote TC indices (4, ..., $N$), and capital letters (as above) to denote generic indices.

We introduce symmetry breaking which treats the L ETC very differently from the U and D ETC. The $SU(N)_L$ is broken directly down to $SU(N-3)$, preserving an SU(3) global flavor symmetry of the $SU(N)_L$ flavor interactions. $SU(N)_U$ and $SU(N)_D$ are broken down to $SU(N-3)$ in stages, preserving global U(1) symmetries but completely breaking the nonabelian flavor symmetries.

Finally, we must couple the various ETC's together. We break the three independent TCs down to a single diagonal TC. This produces a TC interaction which breaks SU(2) × U(1), but it is not enough to generate quark masses.

All of the above symmetry breakings scales are of the order of 1 TeV or larger. We introduce mixing between the flavor subgroups of $SU(N)_L$ and $SU(N)_{U\text{ and }D}$ at a lower scale $\mu$. This produces quark masses of order $\mu$. At this point we introduce nontrivial flavor mixing by inputting different mixings for L with U and L with D. For the simplest form of the mixing, we find that to lowest order in $\mu$, only the LH quarks get mixed. Then because of the SU(3) flavor symmetry of the LH gauge interactions, there is a GIM mechanism which eliminates FCNC's in lowest order in $\mu$. Just as in the standard model, the mixing can be moved from $U_L$ to $D_L$ without changing the gauge interactions. The FCNC's from processes which involve both U's and D's are suppressed by extra powers of the small scale $\mu$. If $\mu$ (which sets the scale of all the quark masses) is small enough, the FCNC's will not cause phenomenological problems. This leads to a bound on the t quark mass, $m_t \lesssim 20$ GeV.

*2. $SU(N)_L \times SU(N)_U \times SU(N)_D$.* We begin by listing the scalar fields and vacuum expectation values (VEV's) which break the ETC symmetries.

$SU(N)_L$ is broken by the VEV of a field $\phi_{\mathbf{A}\alpha}$, where $\alpha = 1, ..., 3$ is a global label and **A** is an $SU(N)_L$ index.

$$\langle\phi_{\mathbf{a}\alpha}\rangle = M\delta_{\mathbf{a}\alpha}\,, \quad \langle\phi_{\mathbf{x}\alpha}\rangle = 0\,. \tag{1}$$

The global SU(3) of the $SU(N)_L$ gauge interactions is generated by the diagonal sum of the generators of the flavor SU(3) subgroup of $SU(N)_L$ and the generators of the SU(3) acting on the global label $\alpha$.

$SU(N)_U$ is broken by the VEV of a field $\rho_{\mathcal{A}j}$, where $j = 1, ..., 3$ is a global label and $\mathcal{A}$ is an $SU(N)_U$ index.

$$\langle\rho_{aj}\rangle = \rho_a\delta_{aj}\,, \quad \langle\rho_{xj}\rangle = 0\,. \tag{2}$$

We will see that $a$ or $j$ = 1, 2, 3 refer to U quark mass eigenstates, u, c, t; and that $\rho_a^2 \propto 1/m_a$. Thus $\rho_1^2 \gg \rho_2^2 \gg \rho_3^2$ because $1/m_u \gg 1/m_c \gg 1/m_t$.

$SU(N)_D$ is broken by the VEV of $\lambda_{Ar}$ where $r = 1, ..., 3$ and $A$ is an $SU(N)_D$ index

$$\langle\lambda_{ar}\rangle = \lambda_a \delta_{ar} , \quad \langle\lambda_{xr}\rangle = 0 . \tag{3}$$

Here, $a, r = 1, ..., 3$ refer to D quark mass eigenstates d, s, b. Hence $\lambda_1^2 \gg \lambda_2^2 \gg \lambda_3^2$.

The mixing between $SU(N)_L$ and $SU(N)_U$ is due to the VEV of $\xi_{\mathbf{A}\mathscr{A}}$, which transforms like an $(N, \bar{N})$ of $SU(N)_L \times SU(N)_U$.

$$\langle\xi_{\mathbf{a}a}\rangle = \mu_U V^U_{\mathbf{a}a} , \quad \langle\xi_{\mathbf{a}x}\rangle = 0 ,$$

$$\langle\xi_{\mathbf{x}a}\rangle = 0 , \quad \langle\xi_{\mathbf{x}x}\rangle = \mu'_U \delta_{\mathbf{x}x} , \tag{4}$$

where $V^U$ is a unitary $3 \times 3$ matrix.

The mixing between $SU(N)_L$ and $SU(N)_D$ is due to the VEV of $\chi_{\mathbf{A}A}$, which transforms like an $(N, \bar{N})$ of $SU(N)_L \times SU(N)_D$.

$$\langle\chi_{\mathbf{a}a}\rangle = \mu_D V^D_{\mathbf{a}a} , \quad \langle\chi_{\mathbf{a}x}\rangle = 0 ,$$

$$\langle\chi_{\mathbf{x}a}\rangle = 0 , \quad \langle\chi_{\mathbf{x}x}\rangle = \mu'_D \delta_{\mathbf{x}x} , \tag{5}$$

where $V^D$ is a unitary $3 \times 3$ matrix.

The VEV's (4) and (5) break the three independent TC's down to the diagonal TC group at the scales $\mu'_U$ and $\mu'_D$ and mix the (already broken) flavor subgroups at the smaller scales $\mu_U$ and $\mu_D$.

To analyze the quark masses and FCNC's, we need to know something about the gauge boson masses. The processes in which we are interested all occur at momenta small compared to the flavor and ETC masses, so the ETC gauge coupling constants are irrelevant. The factors of couplings in the vertices cancel those from the masses in the propagators. Thus we will suppress the coupling constant dependence and show only the dependence on the scales. We will exhibit the masses for $\mu_U = \mu_D = 0$ and treat the mixings induced at the $\mu$ scales as perturbations. The approximate mass eigenstates are shown in table 1. The W's are coupled to corresponding generators. For example, $W^U{}_{\mathscr{A}\mathscr{B}}$ couples to $T^U{}_{\mathscr{B}\mathscr{A}}$ where

$$(T^U_{\mathscr{A}\mathscr{B}})_{\mathscr{C}\mathscr{D}} = \delta_{\mathscr{A}\mathscr{D}}\delta_{\mathscr{B}\mathscr{C}} \quad \text{for} \quad \mathscr{A} \neq \mathscr{B} , \tag{6}$$

$$(T^U_{\mathscr{A}\mathscr{A}})_{\mathscr{B}\mathscr{C}} = [(N-\mathscr{A})(N-\mathscr{A}+1)]^{-1/2}$$

$$\times \left((N-\mathscr{A})\delta_{\mathscr{A}\mathscr{B}}\delta_{\mathscr{A}\mathscr{C}} - \sum_{\mathscr{D}=\mathscr{A}+1}^{N} \delta_{\mathscr{D}\mathscr{B}}\delta_{\mathscr{D}\mathscr{C}}\right) . \tag{7}$$

Table 1

| W | $\text{Mass}^2 \propto$ |
|---|---|
| $SU(3)_L$ | $2M^2$ |
| $U(1)_L$ | $2[2(N-3)M^2 + 3\mu'^2_U + 3\mu'^2_D]/N$ |
| $W^L_{\mathbf{ax}}, W^L_{\mathbf{xa}}$ | $M^2 + \mu'^2_U + \mu'^2_D$ |
| $W^U_{ab}\ a \neq b$ | $\rho_a^2 + \rho_b^2$ |
| $W^U_{aa}$ | $[2/(N-a)(N-a+1)] \times \left((N-a)^2\rho_a^2 + \sum_{b=a+1}^{3} \rho_b^2 + (N-3)\mu'^2_U\right)$ |
| $W^U_{ax}, W^U_{xa}$ | $\rho_a^2 + \mu'^2_U$ |
| $W^U \to W^D$ | $\rho \to \lambda \quad \mu'_U \to \mu'_D$ |

The basis (7) for the diagonal generators of $SU(N)_U$ and $SU(N)_D$ is convenient because it almost diagonalizes the gauge boson mass matrix for $\rho_1^2 \gg \rho_2^2 \gg \rho_3^2$, $\lambda_1^2 \gg \lambda_2^2 \gg \lambda_3^2$. Even in this limit, the fields $W^U_{33}$ and $W^D_{33}$ mix with the $SU(N)_L$ U(1) gauge boson (whose coupling commutes with flavor and TC). However, this mixing does nothing interesting [because the U(1) couplings are completely flavor symmetric] and we will ignore it.

The VEV (4) induces mixing between $W^L_{\mathbf{ax}}$ and $W^U_{ax}$ proportional to

$$2\mu_U \mu'_U \delta_{\mathbf{x}x} V^U_{\mathbf{a}a} , \tag{8}$$

while (5) induces mixing between $W^L_{\mathbf{ax}}$ and $W^D_{ax}$,

$$2\mu_D \mu'_D \delta_{\mathbf{x}x} V^D_{\mathbf{a}a} . \tag{9}$$

These mixings produce quark masses through the diagram shown in fig. 1, giving a mass term

$$-\bar{\psi}_{\mathbf{a}L} M^U_{\mathbf{a}a} U_{aR} - \bar{\psi}_{\mathbf{a}L} M^D_{\mathbf{a}a} D_{aR} + \text{h.c.} , \tag{10}$$

where

$$M^U_{\mathbf{a}a} = [2\mu_U\mu'_U\Lambda^3/(M^2 + \mu'^2_U + \mu'^2_D)]\, V^U_{\mathbf{a}a}/(\rho_a^2 + \mu'^2_U) , \tag{11}$$

$$M^D_{\mathbf{a}a} = [2\mu_D\mu'_D\Lambda^3/(M^2 + \mu'_U + \mu'_D)]\, V^D_{\mathbf{a}a}/(\lambda_a^2 + \mu'^2_D) . \tag{12}$$

$\Lambda^3$ is the TC condensate, appropriately scaled to make (10) and (11) true. We expect $\Lambda \sim 1$ TeV.

Because $V^U$ and $V^D$ are assumed to be unitary, (11) and (12) can be diagonalized trivially and as promised, the mixing is only on the LH fields. In terms of

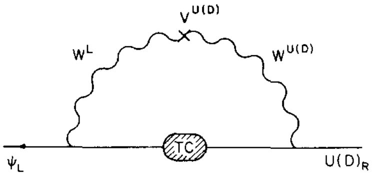


Fig. 1. Feynman diagram which produces the U(D) quark mass matrix.

mass eigenstates $U_{aL}$ and $D_{aL}$,

$$\psi^{U}_{aL} = V^{U}_{aa} U_{aL} , \quad \psi^{D}_{aL} = V^{D}_{aa} D_{aL} . \tag{13}$$

Thus the KM matrix is

$$V = V^{U\dagger} V^{D} . \tag{14}$$

The masses are

$$m^{U}_{a} = [2\mu'_{U}\mu_{U}\Lambda^3/(M^2 + \mu'^2_{U} + \mu'^2_{D})](\rho^2_a + \mu'^2_{U})^{-1} , \tag{15}$$

$$m^{D}_{a} = [2\mu_{D}\mu'_{D}\Lambda^3/(M^2 + \mu'^2_{U} + \mu'^2_{D})](\lambda^2_a + \mu'^2_{D})^{-1} . \tag{16}$$

Now that the model is completely explicit, we can study FCNC. Obviously no FCNC's arise from the exchange of L, U or D flavor bosons to lowest order in $\mu$ ($\mu_U = \mu_D = 0$). The U and D flavor boson masses and couplings to the RH quark fields conserve flavor number. The L flavor couplings do not conserve flavor because $V^U$ and $V^D$ are different, but because of the flavor symmetry of the L flavor boson masses, flavor changing can only occur in processes in which both $V^U$ and $V^D$ are relevant. All of the FCNC effects involving the flavor or ETC gauge bosons are of order $\mu^4$. The leading contribution to a $\Delta S = 2$ process comes from the one-flavor–boson-exchange diagram shown in fig. 2, where we have used the flavor symmetry to go to a basis in which $V^D = I$ and picked out the diagram which gives the dominant contribution in

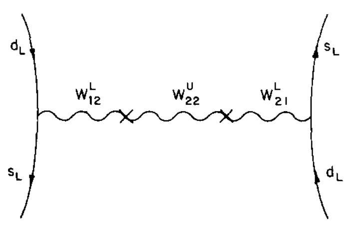


Fig. 2. Feynman diagram which produces the leading $\Delta S = 2$ effect.

the limit that the KM angles $\theta_2$ and $\theta_3$ are small. This gives a contribution to the $\Delta S = 2$ hamiltonian of the form

$$(\bar{d}s)^2 (s^2_1 c^2_1 / 2\rho^2_2)\mu^4_U / M^4 , \tag{17}$$

where we have approximated $[(N-2)/(N-1)]^{1/2} \sim 1$ and ignored terms of order $m_c/m_t$ and $m_u/m_c$. To satisfy the phenomenological constraints, we must have

$$\Delta S_2 \equiv (\mu^4_U / 2\rho^2_2 M^4) \lesssim 10^{-11}\ \mathrm{GeV}^{-2} . \tag{18}$$

We can put the constraint (18) in a more useful form by eliminating $\mu_U$ and $\rho_2$ in flavor of $m_c$ and $m_t$, using (15). This gives

$$\Delta S_2 = m_c m^3_t (M^2 + \mu'^2_U + \mu'^2_D)^4 (\rho^2_3 + \mu'^2_U)^3 / 32 M^4 \Lambda^{12} \mu'^4_U \tag{19}$$

To make $\Delta S_2$ as small as possible, we should choose $\rho_3 = 0$ and take $\mu'_U$ and $\mu'_D$ as small as possible. But the $\mu$'s are the scales at which the TC interaction coalesces from the separate TC subgroups of the three ETC groups. They cannot be smaller than the TC scale, or the mechanism we have used in generating the quark masses breaks down. So we take

$$\mu'_U = \mu'_D = \Lambda . \tag{20}$$

Now we find that the minimum of $\Delta S_2$ as a function of $M$ occurs for $M^2 = 2\Lambda^2$, which gives $\Delta S_2 = 2 m_c m^3_t / \Lambda^6$ or

$$m_t \lesssim 20\ \mathrm{GeV}\ (\Lambda / 350\ \mathrm{GeV})^2 (1\ \mathrm{GeV}/m_c)^{1/3} . \tag{21}$$

This may be barely acceptable.

Before going on to general conclusions in section 3, we make a few technical comments. The global SU(3) symmetry of (1) and the global U(1) symmetries of (2) and (3) cannot be exact global symmetries of the theory because they are spontaneously broken by (4) and (5). That is no problem. Only the VEV's are required to have the symmetries. For example, the VEV, (1), can arise naturally if the $\phi$ interactions have a permutation symmetry, but not an SU(3) symmetry. Thus we need not worry about Goldstone bosons produced by spontaneous breaking of the flavor symmetries. As in the usual GIM mechanism, they are only symmetries of the gauge interactions, not of the full theory.

The constraint (21) that we found from considering the $\Delta S = 2$ processes is the strongest bound on the parameters of the theory. $\Delta B = 2$ processes mediated by the flavor bosons are much smaller than the corresponding effect from the box diagram. Similarly, if we minimize the strength of $\Delta C = 2$ (and $\Delta T = 2$) processes by taking $\lambda_3 = 0$, these flavor boson contributions are much smaller than the box diagram.

*3. ... and all that.* It would be overstating the case to claim that this model is beautiful. But it works, for light quarks. Can we include leptons? Not without further enlarging the ETC group. We have not eliminated the flavor interactions. If leptons are included in the same ETC groups as the quarks, we get a very large $K \to \mu e$ decay. Thus we need still more factors in our ETC gauge group. Explicit models will be discussed elsewhere.

More interesting, it seems to us, is the possibility that a mechanism like ours could arise dynamically in a subconstituent model. In a model of this kind, both quarks and techniquarks are built out of the same subconstituents, and they are light because the dynamics of the binding forces leaves some chiral symmetries unbroken. These are in turn broken by weaker gauge interactions which produce the analogs of the ETC interactions. Because our mechanism is essentially group theoretical, involving the flavor symmetry properties of the ETC and flavor interactions, we can hope to find a dynamical model with the same structure. Perhaps, in this way, we can avoid unwanted inflation of the ETC group.

*References*

[1] E. Farhi and L. Susskind, Phys. Rep. 74 (1981) 277, and references therein.