NEW QUANTUM OSCILLATIONS IN CURRENT DRIVEN SMALL JUNCTIONS

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We propose a framework of a quantum mechanical description of current driven tunnel junctions. Based on this description we predict several new effects. These effects can be observed for $k_BT < E_T < e^2/C$, where $E_T = \hbar/RC$ for a normal tunnel junction and $E_T = \hbar J_5/2e$ for a Josephson junction, C being the junction's capacitance.

Recent advances in the fabrication of small tunnel junctions [1] made it possible to attain the limit $k_BT \ll e^2/C$. In this letter we predict possible new observable effects in this limit. We base our prediction on a new quantum mechanical approach we propose for describing a current driven tunnel junction. Underlying our description is the observation that the response of an open system driven by a current source, $I(t)$, is equivalent to that of a closed system (i.e., open-ended junction) subject to an external time-dependent voltage bias $A(t)$, where $\dot{A}(t) = I(t)$.

We consider both a current biased Josephson junction (JJ) and a normal tunnel junction (NJT), and predict the following effects:

1. Voltage oscillations in dc current biased junctions (CBJ), both JJs and NTJs, with no dc voltage.
2. Steps of dc voltage in a CBJ in the presence of microwave radiation ("inverse Shapiro steps"), for a discrete set of values of the dc current such that $I/q = (n/m)f$, where q is the elementary charge that tunnels and f is the frequency of the radiation.
3. Voltage oscillation in both a JJ and a NTJ coupled by a capacitance to a CBJ.
4. The frequency of the voltage oscillations in the JJ is in most cases I/e and not $I/2e$.
5. The resistance of a current biased NTJ is unlike that of a voltage driven NTJ.

We first consider a JJ. The standard approach is to assume that the charging energy $2e^2/C$ is small, so that the probability of pairs tunneling across the junction is calculated using a degenerate perturbation theory [2]. The matrix elements of the tunneling hamiltonian in the basis of eigenstates of the operator $\hat{n}$ (that measures the number of transferred pairs are then given by

$$\langle n|H_T|m\rangle = \tfrac{1}{2}E_J(2\delta_{n,m} - \delta_{n,m+1} - \delta_{n,m-1}), \qquad (1)$$

where $E_J = \hbar I_J/2e$.

Next, the charging energy is added by

$$\langle n|H_C|m\rangle = (q^2/2C)n^2\delta_{n,m}, \qquad (2)$$

where $q = 2$. Another useful basis is the eigenstates $|\theta\rangle$ of the phase operator which satisfy $\langle\theta|n\rangle = (2\pi)^{-1/2}e^{i\theta n}$ (the phase operator $\hat{\theta}$ is the conjugate of $\hat{n}$). From eqs. (1) and (2) it is easy to show that the hamiltonian for the unbiased junction is given by

$$H = (e^2/2C)q^2\hat{n}^2 + E_J(1 - \cos\hat{\theta}), \qquad (3)$$

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with the following equations of motion for the operators

$$2e\dot{\hat{n}} = -I_J \sin\hat{\theta}\,, \quad (\hbar C/2e)\dot{\hat{\theta}} = 2e\hat{n}\,. \tag{4}$$

In the limit $E_J \gg 2e^2/C$ the standard approach is to replace $\hat{\theta}$ by its expectation value. Thus, for a current driven JJ in the absence of dissipation, one writes the following "classical" equation of motion [2]:

$$(\hbar C/2e)\ddot{\theta} + I_J \sin\theta = I_{dc}\,. \tag{5}$$

This equation of motion is usually understood to be derived from the "washboard hamiltonian" version of eq. (3), in which one describes the external current as an external field coupled to the phase operator $\hat{\theta}$, namely

$$H_\theta = H - (\hbar I_{dc}/2e)\hat{\theta}\,, \tag{6}$$

which yields the following

$$2e\dot{\hat{n}} = I_{dc} - I_J \sin\hat{\theta}\,, \quad (\hbar C/2e)\dot{\hat{\theta}} = 2e\hat{n}\,. \tag{7}$$

Evidently, this hamiltonian does not preserve the symmetry $\theta \to \theta + 2\pi$.

The crucial step in our analysis is writing down the alternative time-dependent hamiltonian

$$H_n = (e/c)(2e\hat{n} - I_{dc}t)^2 + E_J(1 - \cos\hat{\theta}) \tag{8}$$

and

$$2e\dot{\hat{n}} = -I_J \sin\hat{\theta}\,, \quad (\hbar C/2e)\dot{\hat{\theta}} = 2e\hat{n} + I_{dc}t\,. \tag{9}$$

This hamiltonian does preserve the symmetry $\theta \to \theta + 2\pi$ and yields the "classical" equation of motion (5). The action calculated from the hamiltonian of eq. (6) differs from that calculated from eq. (8) by the "surface terms" $[\theta I t]_i^f$. Our work implies that these terms are essential for the understanding of physical processes in the junction.

Within the framework of quantum mechanics, values of θ that differ by multiples of 2π (i.e., different winding numbers) are indistinguishable. This is not the case for a highly inductive rf-SQUID, for example, which is often used to simulate a CBJ. The magnetic flux confined within a superconducting ring breaks the $\theta \to \theta + 2\pi$ symmetry due to an additional inductive term that appears in the hamiltonian. In other words, one can distinguish between different winding numbers of the wavefunction along the ring. By contrast, in a CBJ, after a flux quantum has crossed the junction it can be removed to infinity. During this process the wavefunction may develop a transient and energy may be dissipated, but the final state of the system remains unchanged.

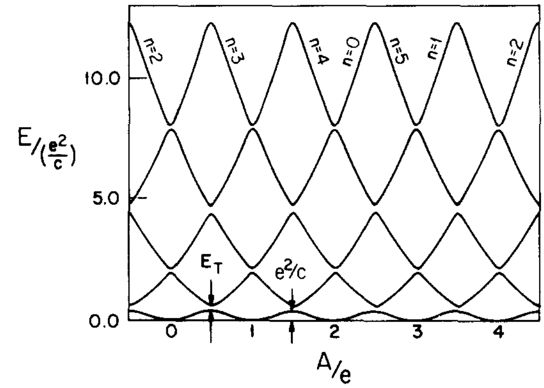

Fig. 1. Band structure of a tunnel junction. For a JJ $E_T = \hbar I_J/2e$, $E_C = 2e^2/C$. For a NTJ E_T is a characteristic energy associated with the tunneling ($\sim\hbar/RC$), and $E_C = e^2/2C$.

Next, we study the response of a JJ to an ideal source, beginning with the time-dependent hamiltonian. eq. (8). We first set $A \equiv I_{dc}t = 0$ and $\hat{n} \to -i\partial/\partial\theta$. The solutions of the Schrödinger equation of the problem are Mathieu's functions (periodic in 2π). For any A = const $\neq 0$, one can perform a gauge transformation and return to the $A = 0$ hamiltonian with solutions that are nonperiodic in 2π. Thus, ignoring the time dependence and treating A as a parameter, we obtain an eigenvalue spectrum which is periodic in A, with a period $2e$. This spectrum is shown schematically in fig. 1 for the "nearly free electron limit" (i.e., $E_J \ll e^2/C$). A dc current corresponds to $A = It$. Within the adiabatic limit this implies oscillations in the energy. Making the analogy between fig. 1 and the extended zone scheme of an electron in a lattice we note that the states of the system are characterized by two quantum numbers: the band number k and the analogue of the lattice momentum, $\tilde{n}$, which is defined externally by A. Moving within the same band from n to $n + 1$ corresponds to a pair transfer through the junction ‡1.

‡1 Büttiker, Imry, and Landauer [3] discussed oscillations in normal rings with magnetic flux (see also ref. [4]). However, the underlying physics of our oscillations is different: we allow for Coulomb interactions and do not require a well-defined phase relation between the wave functions on the two sides of the junction. Note also that electron transfer in "bursts" is discussed by Imry (ref. [5], section 3).

In order to understand the implication of the above description to the $I-V$ characteristics of a JJ, let us first ignore the possible effects of quasi-particles. Since the voltage is given by

$$\langle V\rangle = (1/2e)\,\partial\langle E\rangle/\partial A\,, \qquad (10)$$

we find that a JJ driven by a dc current I_{dc} exhibits voltage oscillations at a frequency $f = I/2e$. When, in addition to the dc current, the junction is subject to microwave radiation that acts as an ac current source ($I_{ac}\sin\omega_{ex}t$), we obtain nonvanishing dc components of the voltage ("inverse Shapiro steps") for values of I_{dc} that satisfy

$$I_{dc} = (n/m)2ef_{ex}\,, \qquad (11)$$

n and m being integers. The magnitude of the first harmonic is given by

$$\Delta V_1 \approx (e/c)\,[(I_{dc} - I_{ac})/2e\omega_{ex}]\,J_1(I_{ac}/2e\omega_{ex})\,, \qquad (12)$$

where J_1 is a Bessel function of the first kind.

Long ago Ivanchencko and Zilberman and recently Likharev and Zorin treated this problem starting with the "washboard hamiltonian" of eq. (6). This hamiltonian formally resembles that of an electron in a periodic 1D lattice under the influence of an external electric field. The band picture that follows yields oscillations in energy and thus oscillations in voltage. At first glance, this strategy appears to yield the same effect as ours. However, when the field is coupled to the phase operator $\hat{\theta}$, the effect is highly sensitive to the form of the tunneling hamiltonian H_T. We couple the field to the number operator $\hat{n}$, and H_T is then merely a perturbation whose exact form is irrelevant. Hence, our description is valid even in the limit $E_J \ll 2e^2/C$, when H_T is expected to change its form.

We emphasize also that our picture has implications on the theory of macroscopic quantum tunneling [8]. We argue that transitions of the system do not occur between minima of the washboard potential but rather correspond to interband transitions in our picture. Details on that will be published elsewhere.

To make the dissimilarity between the two approaches more obvious, one should consider a normal tunneling junction. In this element, the phase is not a macroscopically defined operator any longer (because of the inelastic processes in each electrode). Thus, according to the "washboard hamiltonian" approach there will be no oscillations in a NTJ, while our approach predicts oscillations with frequency I_{dc}/e.

In a previous work [9] it was shown that the response of a NTJ to an ideal voltage source (that has zero internal resistence) is related by the fluctuation–dissipation theorem to the current fluctuations S_I (measured by an ideal amperemeter with zero internal resistance). In these cases the capacitance of the junction is shortened, and the expression for the response of the junction contains the normal resistance,

$$R_N^{-1} \approx (\hbar/2\pi e^2)\,|T|^2 N_L(0) N_R(0)\,,$$

where $N_L(\epsilon)$ ($N_R(\epsilon)$) is the density of states on the left (right). By analogy, it is suggestive to relate the voltage fluctuations S_V (measured by an infinite resistance voltmeter) to the response of the junction to an ideal current source (that has an infinite internal resistance). In ref. [7], the response of a NTJ to an external bias (which, by our approach, is the response of a CBJ) as well as S_V were calculated. It was shown there that the resistance that appears in the case of a biased NTJ is not R_N. This effect becomes significant when the capacitance energy is larger than the tunneling energy.

The hamiltonian of an unbiased NTJ can be written as $H = H_L + H_R + H_T + H_C$. Here H_L (R) describes the noninteracting electrons in the left-hand (right-hand) electrode, and the tunneling across the junction is described by [7]

$$H_T = \int_L d^3x \int_R d^3x'\,T(x,x')\,\psi_{L\sigma}^\dagger(x)\,\psi_{R\sigma}(x') + \text{h.c.} \qquad (13)$$

H_C is given by $(e^2/2C)\hat{n}^2$ [eq. (2)] with $q = e$. For the current driven junction $H_C = (e/2C)(e\hat{n} - I_{dc}t)^2$. Thus, following ref. [9], the action in imaginary time for the CBJ becomes

$$A[\theta] = \int_0^\beta d\tau (tC/2e^2)(\dot{\theta} + I_{dc}/e)^2 + \int_0^\beta d\tau \int_0^\beta d\tau'\,\alpha(\tau-\tau')\sin^2\{[\theta(\tau)-\theta(\tau')]/2\}\,. \qquad (14)$$

When both electrodes are in thermal equilibrium

$$\alpha(\tau) = \frac{1}{2\pi}\frac{\hbar}{e^2 R_N}\frac{(\pi k_B T\hbar)^2}{\sin^2(\pi k_B T/\hbar)}\,.$$

For $\hbar/RC \gg k_B T$ the tunneling hamiltonian is a small perturbation which removes the degeneracies of the energy spectrum at $A = e(n + 1/2)$. Thus, we find a periodic band picture similar to that of the JJ case. This implies voltage oscillations at frequency I_{dc}/e and consequently "inverse Shapiro" steps.

These oscillations will survive when inelastic interactions within the junction are sufficiently weak, that is, interband transitions are negligible. We emphasize that interactions on both sides of the junction, which may destroy the phases of the electrons wavefunction, will not smear out this effect since the current is coupled here to the number operator.

From the expression for α [cf. eq. (14)] we find that the characteristic tunneling energy (and thus the minimal interband separation) is $\sim\hbar/\pi RC$. Hence, we expect to see the effects in the limit $k_B T < \hbar/RC < e^2/2C$.

We now turn back to the JJ and recall that both Cooper pairs and quasiparticles may contribute to the current through the junction. The amplitude of the oscillations of the energy bands, when we consider quasiparticles only, is lower than and of twice the frequency of the oscillations that correspond to the Cooper pairs. According to this phenomenological argument, within the adiabatic approximation, the system will follow the normal particle bands, and we obtain voltage oscillations of frequency $f = I/e$. Also, the remaining of the foregoing discussion concerning the inverse Shapiro steps remains valid, with $I/2e$ replaced by I/e.

An estimate of the experimental parameters needed to observe these effects in a JJ is $I_c \approx 1.5 \times 10^{-8}$ A, $C = 1 - 10^{-15}$ F, and $T \ll 0.4$ K. For $f \sim 10^{10}$ Hz, a current of the order of a few nA will yield inverse Shapiro steps of maximal voltage of the order of a few hundreds microvolts for NTJs one needs $C \sim 10^{-15}$ F and $R \approx 3$ kΩ.

Within our approach we can also consider two JJs coupled by a capacitance C, with one of the junctions being current driven. The hamiltonian of this system is

$$H = (e/C_1)(2e\hat{n}_1)^2 + (e/C_2)(2e\hat{n}_2 - A)^2 + (e/C)(2e\hat{n}_1 - 2e\hat{n}_2 + A)^2 + H_T\,, \qquad (15)$$

where n_1 and n_2 are the number operators of the junctions. One can plot the energy bands versus $\tilde{n}_1$ and $\tilde{n}_2$, and follow the path of the system in a three-dimensional space. Sending dc current through junction 2 results in voltage oscillations on junction 1 of frequency $I/2e$ and amplitude $\sim 2e(C^{-1} + \tilde{C}^{-1})$. One may now add ac current components on either junction or on both. The analysis of these cases and the resulting voltage spikes follow the approach outlined above.

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