## **QUASI-SUPER-RENORMALIZABLE MODELS**

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We study softly broken supersymmetric N=4 Yang-Mills theories. In the absence of finiteness constraints among the soft-breaking terms, we term these quasi-super-renormalizable. We show that all the constraints leading to finiteness constitute an infrared fixed hypersurface. We argue that an ultimate finite globally supersymmetric model requires fine-tuning; however, for the purpose of model building at some pseudo-infrared scale, finite theories may be useful. Thus, quasi-super-renormalizable N=4 can have a nearly finite theory as its low-energy effective field theory. We conclude with some further observations on scale-invariant theories.

### 1. Introduction

The electroweak and strong interactions are well described by an effective field theory below 100 GeV. The theory is, however, only a phenomenological model as it contains a large number of parameters whose origin is unclear, and hence the Higgs sector is beset by naturalness problems which suggest some new phenomena may emerge at a scale around 1 TeV. There are quite a few suggestions as to what these new phenomena might be, all of which have very attractive features but none of which is free of phenomenological problems. It happens already in massless, asymptotically-free, renormalizable field theories that all dimensionless ratios are in principle calculable; however, the physics described is always at energies below some cut-off  $\Lambda$ . In massless finite theories no such constraint seems to exist; a finite theory, although still dependent on one parameter, may describe physics up to an arbitrarily small distance provided the "beta" function for the gauge coupling remains zero non-perturbatively. Supersymmetric theories which are finite to all orders in perturbation theory have indeed been constructed in both two and four dimensions; theories which are finite in the large-N limit exist also in three dimensions. These theories have no ultraviolet divergences in perturbation theory; however, the absence of any scale makes the study of the infrared properties of

massless matter interacting with massless non-abelian gauge particles rather difficult. This difficulty may be resolved if non-singlet scalar fields develop non-zero vacuum expectation values, thus breaking the gauge symmetry and some global symmetries induced by the extended supersymmetry. This was studied by Fayet for N=4 supersymmetric theories [1]. However, although generating masses, this mechanism always leaves the massless sector finite. In fact, a successive set of these spontaneous symmetry breakings will reduce the SU(N) gauge symmetry to U(1) factors describing perturbatively non-interacting massless particles and heavier interacting particles. Needless to say, this does not bear any resemblance to the known low-energy behavior of particles. It has also not been shown that the new candidate vacuum is indeed stable despite the flat directions in the tree approximation. A more radical suggestion, following an idea originated by Fubini, is that Lorentz invariance may be broken in scale-invariant finite theories [2].

Another way to study finite theories was suggested by several authors in N = 4 [3], N = 2 [4] and N = 1 (which can be made at least two-loop finite) [5] supersymmetric theories. Dimensional parameters have been added to the lagrangian which break scale invariance and softly break supersymmetry. It turns out that if these mass parameters obey certain relations among themselves, the theory remains finite. In these finite but scale-non-invariant theories, the infrared problems of the gauge field is resolved. For scales smaller than these masses, physics is described by an effective non-abelian gauge theory, which, in the absence of the heavy particles, is asymptotically free. Such a theory would presumably resolve its infrared difficulties the same way QCD does. The embarrassment resulting from having re-introduced mass scales has turned into a discovery that these mass parameters are related among themselves and to the gauge couplings. Thus, although some parameters have been introduced, they are interconnected [3-5]. Moreover, if a finite, globally supersymmetric (up to soft breakings) theory is a descendent of a yet-to-be-discovered finite theory of supergravity, these mass parameters are even less arbitrary. We find this very attractive for reasons we shall discuss later. In such a context the relations assuring finiteness are both natural and very important.

Consider the possibility that finite, globally supersymmetric models are prototypes of an ultimate theory at short distances, and perhaps even gravity can be derived from it as a low-energy theory (such ideas have been discussed in [6]). In such a case one must pose the question: Are the relations leading to finiteness natural? In other words, suppose some hypothetical experimentalist provides us with the values of the dimensionful parameters at a certain distance scale. Suppose, in addition, that these numbers obey, within the experimental accuracy, the relations leading to finiteness. Can we conclude from these measurements that there is a strong indication for the existence of an ultimate finite theory?

We analyze these possibilities by considering N=4 Yang-Mills theory, softly broken by dimension-two and -three operators but *not* so arranged to cancel ultraviolet divergences. Since these soft breakings leave the gauge coupling renormal-

ization finite\*, the resulting theory is very much like a super-renormalizable theory, except that we have not established that there are only a finite number of primitively divergent graphs. Allowing that there may in fact be divergences in the dimensionful couplings and masses to all orders in the gauge coupling, we will call this a quasi-super-renormalizable theory. We analyze the scale dependence of the dimensionful couplings and masses, by calculating the one-loop beta function for each. It turns out that the relations among effective couplings tend to those relations ensuring finiteness in the infrared, i.e. at long distances, and, inversely, the deviations from finiteness increase as the distance scale decreases. Thus, from a fundamental point of view, these finite but softly broken theories do not seem natural. On the other hand, suppose that supergravity or some other fundamental theory yields an effective field theory on some scale (presumably below the Planck mass) which is a finite, globally supersymmetric theory except for masses (i.e. a quasi-super-renormalizable theory). Then our result that in the infrared limit the masses run toward the finite theory suggests the possibility that the finiteness relations would be approximately satisfied for masses on a much lower scale. Just how closely the finiteness relations were obeyed would depend on details of the model, such as the gauge group and the size of the gauge coupling. However, since these relations become natural in the infrared, it encourages their use in model-building.

In this paper we concern ourselves with the naturality of non-invariant finite models. In a companion paper, we study the symmetry breaking properties of such quasi-super-renormalizable theories. The structure of the paper is as follows: in sect. 2 we discuss super-renormalizable broken N=4 theories and review those relations which render them finite. In sect. 3 we describe the calculation of the running dimensionful coupling constants, and assess the naturality of finiteness relations. We conclude in sect. 5 with a discussion of results and with some remarks on SUSY breaking in finite theories.

# 2. Quasi-super-renormalizable broken N = 4 supersymmetric models

N=4 super-Yang-Mills theory [6] in the Wess-Zumino gauge is given by the following lagrangian:

$$L = \text{Tr} \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (D_{\mu} A_{i}) (D^{\mu} A_{i}) + \frac{1}{2} (D_{\mu} B_{j}) (D^{\mu} B_{j}) + \frac{1}{4} g^{2} ([A_{i}, A_{j}]^{2} + [B_{i}, B_{j}]^{2} + 2[A_{i}, B_{j}]^{2}) + \frac{1}{2} i \overline{\lambda}_{K} \not D \lambda_{K} + \frac{i}{2} g \overline{\lambda}_{K} [\alpha_{KL}^{i} A_{i} + \beta_{KL}^{j} B_{j} i \gamma^{5}, \lambda_{L}] \right\}.$$
(2.1)

<sup>\*</sup> We do not entertain breaking the supersymmetric relations among dimension-four operators because this introduces quadratic divergences with their attendant naturalness difficulties.

Throughout the paper we shall use component notation. All fields are in adjoint representations of some internal group, e.g.  $(A_i)_{ab} = -if_{abc}A_ic$ , where  $f_{abc}$  are the structure constants. The trace is over the internal indices, all repeated indices (i, j, K, L) are summed over. For each internal degree of freedom there are six spin-zero fields (three scalars  $A_i$  and three pseudoscalars  $B_j$ ), four Majorana fermions  $\lambda_K$ , and one massless gauge particle. The lagrangian is invariant under a global SU(4)  $\cong$  O(6) transformation on these indices. The  $\alpha^i$  and  $\beta^i$  (i = 1, 2, 3) are real antisymmetric matrices satisfying the algebra

$$\{\alpha^{i}, \alpha^{j}\} = \{\beta^{i}, \beta^{j}\} = -2\delta^{ij}, \qquad [\alpha^{i}, \alpha^{j}] = -2\varepsilon^{ijk}\alpha^{k},$$
$$[\beta_{i}, \beta_{i}] = 2\varepsilon_{ijk}\beta_{k}, \qquad [\alpha^{i}, \beta^{j}] = 0.$$
(2.2)

A convenient representation is

$$\alpha^{1} = \begin{pmatrix} 0 & \sigma^{1} \\ -\sigma^{1} & 0 \end{pmatrix}, \qquad \alpha^{2} = \begin{pmatrix} 0 & -\sigma^{3} \\ \sigma^{3} & 0 \end{pmatrix}, \qquad \alpha^{3} = \begin{pmatrix} i\sigma_{2} & 0 \\ 0 & i\sigma_{2} \end{pmatrix},$$

$$\beta^{1} = \begin{pmatrix} 0 & i\sigma^{2} \\ i\sigma^{2} & 0 \end{pmatrix}, \qquad \beta^{2} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \beta^{3} = \begin{pmatrix} -i\sigma^{2} & 0 \\ 0 & i\sigma^{2} \end{pmatrix}. \tag{2.3}$$

It has been shown [8] that the beta function for the gauge coupling is zero to all orders in perturbation theory. By Symanzik's theorem [9], this beta function will remain zero even if one adds to the lagrangian lower dimension operators. The modified theory is in general super-renormalizable and the dimensional parameters will run as the renormalization scale is changed. The soft-breaking terms are of the form

$$\delta \mathcal{L} = -\frac{1}{2} a_{ij}^2 A_i^a A_j^a - \frac{1}{2} b_{ij}^2 B_i^a B_j^a - \frac{1}{2} c_{ij}^2 A_i^a B_j^a - \frac{1}{2} m_p \tilde{\lambda}_p^a \lambda_p^a$$

$$-\frac{1}{2} p_{jk}^i f_{abc} A_i^a B_j^b B_k^c - \frac{1}{2} q_{jk}^i f_{abc} B_i^a A_j^b A_k^c - \frac{1}{6} r \varepsilon_{ijk} f_{abc} A_i^a A_j^b A_k^c$$

$$-\frac{1}{6} s \varepsilon_{ijk} f_{abc} B_i^a B_j^b B_k^c. \tag{2.4}$$

Without loss of generality, it is always possible to choose the fermion mass matrix to be real and diagonal, and we have done so above. The remaining couplings are all real for hermiticity. In this basis, the  $A_i^a$  fields remain scalars;  $B_i^a$ , pseudoscalars. The couplings  $c_{ij}^2$ ,  $q_{jk}^i$  and s thus are all parity violating. The soft-breaking terms break both scale invariance and supersymmetry but not the internal gauge symmetry. For certain relations among these parameters (which we will review subsequently), it is believed that the theory will be finite. This has been investigated for parity-conserving models by the authors cited in ref. [3]. Some relations have been

established to all orders of perturbation theory, and this belief seems plausible if not proved\*.

Our interest, as discussed previously, is in the naturality of the finiteness relations. To investigate this, we must calculate the one-loop beta functions for all the massive parameters to determine whether the finiteness relations are infrared or ultraviolet stable, to which we now turn.

## 3. Renormalization group analysis

In a general quasi-super-renormalizable theory, all dimensional couplings can be expected to be scale dependent. As conventionally defined, the "running" masses and couplings vary according to their engineering dimensions as well as because of an "anomalous" dependence determined by the dynamical renormalizations of the theory. Our concern, however, is with the naturalness and stability of relations among these, so our definition of running masses and couplings will omit the naive variation or engineering dimension. Let m denote the  $4 \times 4$ , symmetric fermion mass matrix in a basis in which it is real (i.e. parity conserving) but not necessarily diagonal. Then the non-zero one-loop beta functions are given by

$$\beta_{a_{ij}^{2}} = \frac{C_{2}(G)}{16\pi^{2}} \left[ 2g^{2} \operatorname{tr}(a^{2} + b^{2}) \delta_{ij} - 8g^{2} \left( \delta_{ij} \operatorname{Tr} m^{2} - \frac{1}{2} \operatorname{Tr}(\alpha^{i} m \alpha^{j} m) \right) \right. \\ \left. + 2r^{2} \delta_{ij} + p_{kl}^{i} p_{kl}^{j} + 2q_{ik}^{l} q_{jk}^{l} \right],$$

$$\beta_{b_{ij}^{2}} = \frac{C_{2}(G)}{16\pi^{2}} \left[ 2g^{2} \operatorname{tr}(a^{2} + b^{2}) \delta_{ij} - 8g^{2} \left( \delta_{ij} \operatorname{Tr} m^{2} + \frac{1}{2} \operatorname{Tr}(\beta^{i} m \beta^{j} m) \right) \right. \\ \left. + 2s^{2} \delta_{ij} + 2p_{ik}^{l} p_{jk}^{l} + q_{kl}^{i} q_{kl}^{j} \right],$$

$$\beta_{c_{ij}^{2}} = \frac{C_{2}(G)}{16\pi^{2}} \left[ r \varepsilon_{ikl} q_{jk}^{j} + s \varepsilon_{jkl} q_{ik}^{l} \right],$$

$$\beta_{p_{jk}^{\prime}} = \frac{C_{2}(G)}{16\pi^{2}} \left[ 12g^{2} p_{jk}^{i} - 12g^{3} \varepsilon_{ljk} \operatorname{Tr}(m \alpha^{i} \beta^{l}) \right].$$

$$\beta_{q_{jk}^{\prime}} = \frac{C_{2}(G)}{16\pi^{2}} \left[ 12g^{2} q_{jk}^{i} \right],$$

$$\beta_{s} = \frac{C_{2}(G)}{16\pi^{2}} \left[ 12g^{2} r - 12g^{3} \operatorname{Tr} m \right],$$

$$\beta_{s} = \frac{C_{2}(G)}{16\pi^{2}} \left[ 12g^{2} s \right]. \tag{3.1}$$

<sup>\*</sup> The lack of rigor stems from technical difficulties associated with the absence of a superspace formalism in gauges other than the light-cone gauge, and, in that gauge, power counting and the treatment of gauge singularities beyond one loop are problematic.

The  $\beta$ -function for the gauge coupling g, of course, remains zero, and it turns out that the  $\beta$ -functions vanish for the fermion masses  $m_{pq}$  as well. As mentioned earlier, it is convenient to choose a basis in which m is diagonal, in which case we can evaluate the fermion traces in terms of the four eigenvalues of m:

$$\operatorname{Tr} m = \sum m_{p},$$

$$\operatorname{Tr} m^{2} = \sum m_{p}^{2},$$

$$\operatorname{Tr} (\alpha^{i} m \alpha^{j} m) = \operatorname{Tr} (\beta^{i} m \beta^{j} m) = -2 \delta^{ij} (m_{4} m_{j} + m_{k} m_{l}),$$

$$\operatorname{Tr} (m \alpha^{i} \beta^{j}) = -\delta^{ij} (m_{4} + m_{i} - m_{k} - m_{l}),$$

where the indices j, k, l take the values 1, 2, 3, cyclicly. Finally, the trace over scalar boson masses is denoted as  $tr(a^2 + b^2) = a_{ij}a_{ji} + b_{ij}b_{ji}$ .

Before proceeding further, we pause to emphasize that these formulae are relevant only at energy scales above all thresholds associated with the masses of the theory. At energies below some threshold, the heavy particles decouple and a more useful description is provided in terms of an effective lagrangian involving only the light fields and their interactions. Subsequently, when we speak of the infrared limit of these equations, we tacitly assume that we remain at an energy scale above all thresholds.

After a bit of algebra, one can show that all beta functions vanish if and only if the following relations prevail:

$$p_{jk}^{i} = g\varepsilon_{ljk}\operatorname{Tr}(m\alpha^{i}\alpha^{l}),$$

$$q_{jk}^{i} = 0,$$

$$r = g\operatorname{Tr}m,$$

$$s = 0,$$

$$\operatorname{tr}(a^{2} + b^{2}) = 2\operatorname{Tr}m^{2},$$

$$c_{ij}^{2} = C_{ij}^{2}.$$
(3.2)

Except for the inclusion of parity-violating terms, these are precisely the finiteness conditions derived previously [3]. This establishes that these are the only "fixed points" (actually a hypersurface) of the renormalization group.

We now wish to show that these relations are infrared stable. This could be done by linearizing about this hypersurface. However, it is straightforward to integrate the equations exactly. Letting  $t = \ln \mu$ , the running cubic couplings are

$$\tilde{p}_{jk}^{i} = P_{jk}^{i} e^{bt} + g \varepsilon_{ljk} \operatorname{Tr}(m \alpha^{i} \beta^{l}),$$

$$\tilde{q}_{jk}^{i} = Q_{jk}^{i} e^{bt},$$

$$\tilde{r} = R e^{bt} + g \operatorname{Tr} m,$$

$$\tilde{s} = S e^{bt},$$
(3.3)

where  $b = 12g^2C_2(G)/16\pi^2$  and all capital letters  $(P_{jk}^i, Q_{jk}^i, R, S)$  denote constants determined by initial conditions. Apparently, they are measures of the deviations from finiteness. Already we see that, in the infrared limit  $(t \to -\infty)$ , the couplings tend to the finiteness conditions. In particular, all parity violations entering via cubic couplings vanish in the infrared.

The exact formulas for the running masses are a bit tedious and we will spare the reader the gory details. Since  $\beta_{c_{ij}^2}$  involves only cubic couplings, we see immediately from the preceding that, as  $t \to -\infty$ ,  $\beta_{c_{ij}^2} \to 0$ , since  $\tilde{q}_{jk}^i$  and  $\tilde{s}$  vanish in that limit. Thus the parity-violating mass  $c_{ij}^2 \to C_{ij}^2$ , a constant in the infrared. This is the only low-energy residue of parity violation unless, as we remarked earlier, one crosses a mass threshold before the parity-violating cubic couplings become negligible. The parity-conserving running masses satisfy the following relation:

$$\operatorname{Str} \tilde{M}^{2} = \operatorname{tr}(\tilde{a}^{2} + \tilde{b}^{2}) - 2\operatorname{Tr} \tilde{m}^{2} = Ne^{bt} + \frac{bt}{g} e^{bt} \left( R\operatorname{Tr} m + P_{jk}^{i} \varepsilon_{jkl} \operatorname{Tr}(m\alpha^{i}\beta^{l}) \right) + e^{2bt} \left( R^{2} + S^{2} + \frac{1}{2} \left( P_{jk}^{i} P_{jk}^{i} + Q_{jk}^{i} Q_{jk}^{i} \right) \right), \quad (3.4)$$

where N is another integration constant. Thus, in the infrared limit, the remaining condition for finiteness is satisfied. From the preceding, one can show that  $\beta_{a_{ij}^2}$  and  $\beta_{b_{ij}^2}$  vanish in the infrared, so there are no further finiteness constraints.

It can also be seen from eqs. (3.3) and (3.4) that, in the ultraviolet regime  $(t \to +\infty)$ , there is no memory whatever of the finiteness relations, the asymptotic behavior being set by the integration constants representing deviations from finiteness.

We will next discuss the interpretation and significance of these results.

## 4. Discussion

The nature of the fixed point of the effective couplings has two types of consequences. (i) First, the fact that it is infrared attractive suggests that, in the present context, an ultimate globally scale-non-invariant but finite theory of nature

is unnatural since (a) the finiteness conditions, eq. (3.2), do not correspond to any known symmetry and (b) an arbitrarily small deviation from these conditions blows up in the high-energy limit. Although this is only a logarithmic growth (in contrast to the quadratic blow-up characteristic of the unnaturalness associated with scalar masses in non-supersymmetric theories), it makes these models unattractive candidates for ultimate unification. One might think so in any case because gravity has not been explicitly included, but it has been speculated that gravity might be dynamically generated [6]. In any case, a hypothetical experimental colleague would be unable to resolve the question of whether the ultimate theory is finite. (ii) On the other hand, since the fixed point is infrared attractive, the finiteness conditions emerge at low energy regardless of the values of the input couplings and masses at short distances. By low energy, we mean an energy scale below the initial input scale but above the scale of any of the masses themselves, a regime we might term pseudo-infrared. Below these mass scales, the effective field theory will appear to be renormalizable in any case and the effective gauge, Yukawa, and scalar self-couplings all begin to run (as mentioned in the sect. 1).

Thus we regard a quasi-super-renormalizable model as an effective field theory relevant to an energy regime below a scale of some new unknown physics (the cutoff) and above the mass scales of soft breaking of supersymmetry. We have found that an arbitrary set of dimensional parameters characterizing the soft breakings evolves at lower energy scales in such a way that the pseudo-infrared regime may approximate a finite, scale-non-invariant theory. This is reminiscent of the idea that infrared fixed points may endow an effective theory with *more* symmetry than the underlying theory [10]. In our case, however, the effective theory has no apparent higher symmetry but nevertheless relations among dimensionful couplings emerge!

We note that Veltman has suggested that the low-energy effective field theories describing non-renormalizable interactions are themselves renormalizable [11]. Another type of theory is an asymptotically free renormalizable one (such as QCD) which, because of confinement may be described at low energies by a super-renormalizable field theory of the bound states (hadrons). We have shown that for N=4 softly broken supersymmetric Yang-Mills, a quasi-super-renormalizable theory evolves in the pseudo-infrared regime toward a low-energy finite theory. It would be interesting to establish whether this property holds in other softly-broken finite field theories (N=2 [3] and, possibly, N=1 [4]).

We have argued that softly-broken finite theories are more naturally regarded as quasi-super-renormalizable models. To what extent might such effective field theories arise as low-energy approximations arising from spontaneous breakdown of a truly scale-invariant, underlying globally supersymmetry theory? The answer is that such a scenario is impossible\*! The effective potential is non-negative and, since it

<sup>\*</sup> These observations arose in discussion with P.C. West. This also has been noted by E. Witten (private communication). We suppose in this discussion that there exists a translationally invariant ground state. It has been speculated that spontaneous breakdown of translational invariance might occur in certain theories. See Fubini [2,12].

vanishes when all fields vanish, any potential ground state other than the origin must also have zero energy. (In fact, by scale invariance, such a state lies on a zero-energy ray from the origin.) Thus, regardless of whether scale invariance is spontaneously broken, global supersymmetry remains unbroken. Since the spectrum remains manifestly supersymmetric, any low-energy effective field theory, obtained by "integrating out" the heavy fields, will also be manifestly supersymmetric. Thus, finite globally scale-invariant theories could not possibly explain the origin of masses in a world (such as ours) in which supersymmetry is broken. It seems possible, however, that a locally supersymmetric, scale-invariant (superconformal) theory could spontaneously break supersymmetry and yet retain a zero value for the vacuum energy (cosmological constant). So we regard these observations as another strike against finite, globally supersymmetric models.

In a subsequent paper, we study the effective potential in this class of theories. It has been noted\* that, in finite versions of this type, the constraints on masses coming from the desire for spontaneous breakdown of gauge symmetries, on the one hand, and the boundedness of the energy from below, on the other, are incompatible. The best one can do is postulate that a field develops a vacuum expectation value in a flat direction. We will show that, in quasi-super-renormalizable models, scalar masses-squared can change sign with changing scale. Thus, just as in softly broken renormalizable models, the incompatibility of these conflicting constraints can be resolved.

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#### References

- [1] P. Fayet, Nucl. Phys. B149 (1979) 137
- [2] S. Fubini, Nuovo Cim. 34A (1976) 521
- [3] M.A. Namazie, A. Salam and J. Strathdee, Phys. Rev. D28 (1983) 1481;
  - A.J. Parkes and P.C. West, Nucl. Phys. B222 (1983) 269; Phys. Lett. 138B (1984) 99;
  - S. Thomas and P.C. West, Nucl. Phys. B245 (1984) 45;
  - J.J. van der Bij and Y.-P. Yao, Phys. Lett. 125B (1983) 171
- [4] A.J. Parkes and P.C. West, Nucl. Phys. B127B (1983) 353;
  - J.M. Frère, L. Mezincescu and Y.-P. Yao, Phys. Rev. D29 (1984) 1196;
  - University of Michigan preprint UM-Th-84-4 (May 1984)
- [5] D.R.T. Jones, L. Mezincescu and Y.-P. Yao, Univ. of Colorado preprint COLO-HEP-73 (June 1984)
- [6] M.A. Namazie, A. Salam and J. Strathdee, ref. [3];
  - D.R.T. Jones and S. Raby, Phys. Lett. 143B (1984) 137
- [7] L. Brink, J.H. Schwarz and J. Scherk, Nucl. Phys. B121 (1977) 77;
  - F. Gliozzi, J. Scherk and D. Olive, Nucl. Phys. B122 (1977) 253

<sup>\*</sup> This is discussed in Namazie, Salam and Strathdee, ref. [3].

- [8] S. Mandelstam, Nucl. Phys. B213 (1983) 149
- [9] K. Symanzik, Comm. Math. Phys. 23 (1971) 49
- [10] D. Foerster, H.B. Nielsen and M. Ninomiya, Phys. Lett. 94B (1980) 135;
  - S. Chadha and H.B. Nielsen, Nucl. Phys. B217 (1983) 125;
  - I. Antoniadis, J. Iliopoulos and T. Tomaras, Nucl. Phys. B227 (1983) 447;
  - H.B. Nielsen and N. Brene, Nucl. Phys. B224 (1983) 396
- [11] M. Veltman, Acta, Phys. Polonica B12 (1981) 437
- [12] S. Fubini and E. Rabinovici, Nucl. Phys. B245 (1984) 17