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CROSS SECTIONS FOR DARK MATTER PARTICLES AND IMPLICATIONS FOR ALLOWED MASSES, INTERACTIONS, AND DETECTION

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We present scattering and annihilation cross sections for a number of candidates for dark matter: heavy neutrinos, photinos, scalar-neutrinos, and higgsinos. In the mass region below about 10 GeV, annihilation cross sections can be enhanced at resonance masses, so some masses are allowed below the generally quoted continuum values of a few GeV. Use of conventional neutral current couplings of a heavy neutrino L^0 to the Z^0 implies $m_{L^0} > 3.3$ GeV (rather than the usually quoted 2 GeV). Cross sections are generally expected to be small (of order 10^{-38} cm^2, often smaller than might be naively expected) which affects possible implications involving concentration mechanisms, energy transfer, and detectability via energy transfer or annihilation products. Current limits on supersymmetric partners are incorporated.

1. Introduction

Particle physics is in the remarkable situation of having a theory, the "standard model" of quarks and leptons interacting via gauge bosons, which describes all known data and experiments. The Higgs sector of the theory is not understood, and a Higgs particle must be found, but no experiment to data has had the sensitivity to detect one. In spite of this satisfying state of science, theorists have pushed for more. We do not understand yet why the theory takes the form it does (a lagrangian invariant under $SU(3) \times SU(2) \times U(1)$ transformations in various internal spaces), why the SU(2) symmetry is broken while the others are exact, so that photons and gluons are massless but W's are very heavy, why fermion masses and couplings have the values they do, why the weak scale is so light compared to other scales that might be present such as the Planck scale or the grand unification scale, and more.

As a result, it is expected that new physics is to be found, and a variety of arguments suggest it will appear at energies not far above presently accessible ones. Most ways of thinking about these questions suggest that a set of new particles will be found. Often the lightest of them is stable and has a mass in the few GeV range. It would have been in equilibrium with other particles in the early universe. Its interactions with normal particles are usually mediated either by Z^0's and W's, or

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by other heavy particles, so they are often quite small. Consequently, a large number of such particles would have survived from the early universe, as non-luminous matter which constitutes a significant fraction of the mass of the universe.

It should be emphasized that such dark matter would generally be hypothesized to exist on the basis of extensions of particle physics beyond the standard model. We approach the subject from the particle physics point of view, studying how such objects interact, what effects they might have, and how they might be detected. Even if there had been no hint of dark matter from astronomy experiments, the subject would have emerged from the present direction of particle physics; it is even more exciting that indications [1] of the existence of some form(s) of dark matter come from astronomy and cosmology.

The kinds of effects one can hope for depend significantly on numbers. Consequently, generic analyses or cross sections given by dimensional arguments are not precise enough, especially since cancellations often occur (e.g. in the photino-nucleon cross section). Further, strong new lower limits exist on masses of supersymmetric partners that participate in some processes, so some cross sections are smaller than they might have been.

In this paper we give detailed results for two types of particles:

(i) heavy neutrinos coupled as normal sequential ones, and

(ii) possibilities in a supersymmetric theory, where the lightest supersymmetric particle (LSP) is taken to be stable. The supersymmetric candidates for LSP are photino, scalar-neutrino (sneutrino), and higgsino, the appropriate partner of the Higgs boson. Some references which consider supersymmetric partners as dark matter include refs. [2–10].

We briefly consider effects such dark matter might have when concentrated in the galaxy or the sun, and possible experimental approaches to directly detect dark matter with GeV masses. We do not consider dark matter in the form of axions, or any of the relevant information from the field of galaxy formation.

2. Annihilation cross sections

The basic annihilation mechanisms are of course characteristic of the theory. Heavy neutrinos that are assumed to be in $SU(2)_L$ doublets can annihilate through a Z^0 (fig. 1a), or via a (presumably Cabibbo suppressed) $W^{\pm}$ exchange (fig. 1b) to a lighter charged lepton. We consider only the case where the latter mechanism is numerically negligible, as would be expected in most models.

The LSP annihilation depends strongly on what the LSP is. Detailed cross sections are given in appendix A. For photinos the sfermion exchange dominates, with $f =$ any quark or lepton, as in fig. 2. For higgsinos there are two contributions, fig. 3a and 3b. Fig. 3a in general dominates since $\tilde{h}$ carries electroweak quantum number $T_3 \neq 0$; the $\tilde{h}\tilde{h}Z^0$ coupling is proportional to $1 - v_2/v_1$, where v_1 and v_2 are the two vacuum expectation values required in a supersymmetric theory, so in

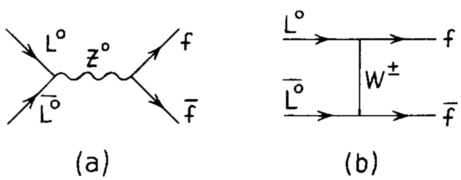

Fig. 1. Massive neutrinos can annihilate via these processes. Diagram (b) is probably suppressed by mixing angle factors.

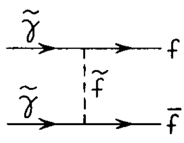

Fig. 2. Photinos annihilate mainly by this process.

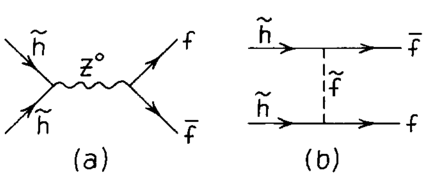

Fig. 3. Higgsinos annihilate by these processes. The cross section for (a) is proportional to $(1 - v_2/v_1)^2$ and the cross section for (b) is proportional to fermion masses to the fourth power.

models where $v_2 \approx v_1$ this mechanism can be quite small. Fig. 3b involves $\tilde{h}\tilde{f}f$ couplings with factors $m_{\mathrm{f}}/2m_{\mathrm{W}}$, so the rates are suppressed by $(m_{\mathrm{f}}/2m_{\mathrm{W}})^4$ and are usually quite small; we neglect contributions of fig. 3b. Sneutrinos do have normal electroweak quantum numbers and can annihilate through a Z^0, but the dominant mechanism is the exchange of a Majorana zino as in fig. 4; note the annihilation is $\tilde{\nu}\tilde{\nu} \to \nu\nu$, not $\tilde{\nu}\bar{\tilde{\nu}} \to \nu\bar{\nu}$. This is because the Majorana nature of $\tilde{\mathrm{Z}}$ gives a factor of $1/m_{\tilde{\mathrm{Z}}}$ in the amplitude (rather than $1/m_{\tilde{\mathrm{Z}}}^2$), so the cross section only has one G_{F} rather than the normal G_{F}^2. Lepton number is not violated since $\tilde{\nu}$ and ν both carry the same lepton number, presumably a different one for each flavor. Fermion number does change, which is all right since the Majorana nature of Z allows that to happen, but that is not observable if equal numbers of $\tilde{\nu}$ and $\bar{\tilde{\nu}}$ exist since $\bar{\tilde{\nu}}\bar{\tilde{\nu}} \to \bar{\nu}\bar{\nu}$ occurs equally often.

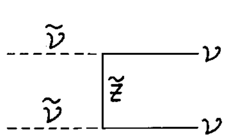

Fig. 4. Sneutrinos annihilate mainly via Majorana zino excharge if it is not suppressed, giving a rate independent of $\tilde{m}_{\nu}$.

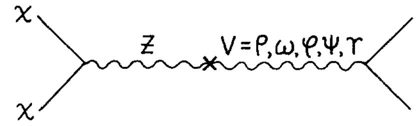

Fig. 5. When $2m_\chi \simeq m_X$, any dark matter particle χ which couples to Z^0 can have enhanced annihilation into a vector meson X.

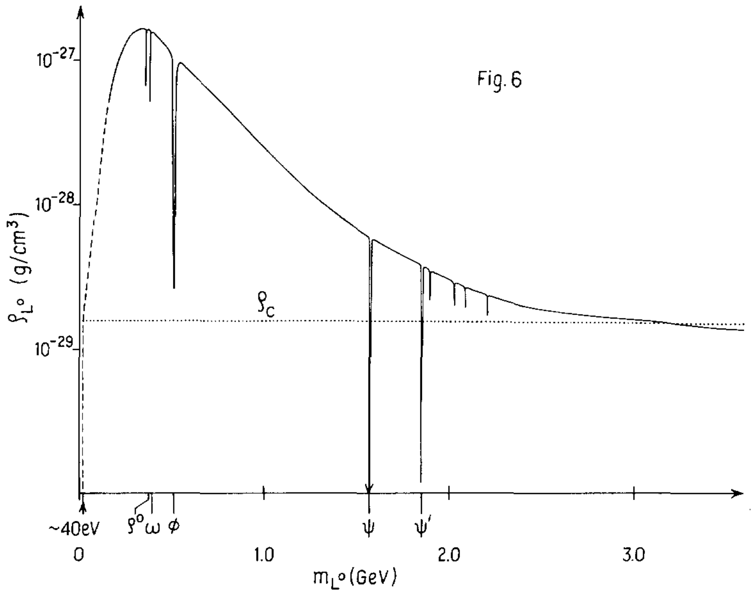

Fig. 6. The mass density for heavy neutrinos L^0 is shown as a function of m_{L^0}, with the critical density corresponding to $\Omega = 1$ shown for comparison. Any value of m_{L^0} where $\rho_{L^0} < \rho_c$ is an allowed mass for L^0. Note that the continuum part, which is calculated with a conventional neutral current coupling of L^0 to Z^0, requires $M_{L^0} > 3.3$ GeV.

To encompass all the above cases, and to present some techniques generally, we denote a general stable dark matter particle by χ. Whenever necessary, a $\bar{\chi}$ will be understood if needed.

An additional annihilation effect* is present for the masses of interest, in the GeV region. Whenever $2m_\chi = m_X$, where X is a vector meson resonance ($X = \rho, \omega, \rho, \psi, \Upsilon$), the rate for $\chi\chi \to Z \to f\bar{f}$ is enhanced, e.g. for fig. 1a, fig. 3a, as shown in fig. 5 Then an analysis following Lee and Weinberg [10] gives a density of dark matter candidates which is shown for heavy neutrinos in fig. 6 and for higgsinos in fig. 7. We see that the usual result is obtained that masses above some limit are allowed, but *isolated masses below that limit are also allowed*. For example, heavy neutrinos of mass 1.55 GeV are allowed, as are photinos of mass 275 MeV. The

* This was independently suggested by A. de Rujula, ref. [11].

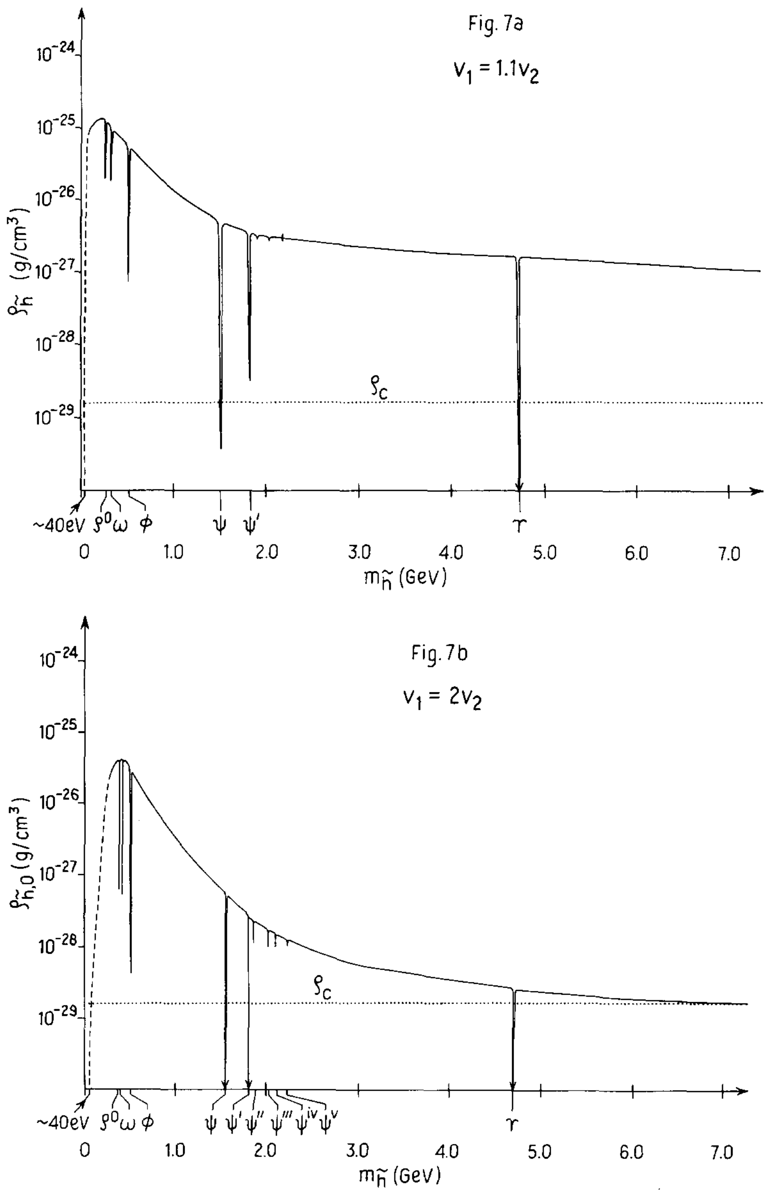

Fig. 7. Similar to fig. 6, for higgsinos. Any value of $m_{\tilde{h}}$ where $\rho_{\tilde{h}} < \rho_c$ is an allowed higgsino mass.

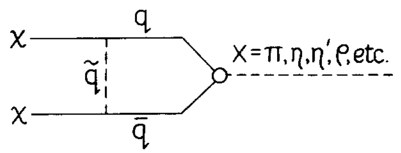

Fig. 8. When annihilation is through sfermion exchange the strongest coupling is to pseudoscalar resonances, which for some states enhance the annihilation and allow masses below the continuum.

details of this calculation are given in appendix C. The vector meson resonances decay dominantly into pions and kaons.

A similar effect occurs whenever the annihilation is into final state quarks, e.g. in fig. 1b, or fig. 2, as shown in fig. 8. Then the quarks can resonate. The rate gets suppressed by a factor of the wave function of the X resonance at the origin, but enhanced by M_X/Γ_X, and gives a large enough annihilation cross section to reduce the χ density below the critical density, so again certain isolated masses are allowed. The results are shown in fig. 9 for photinos.

For heavy neutrinos, note that $m_{\nu_H} > 3.3$ GeV is required from the continuum curve rather than the "2 GeV" usually quoted for the Lee-Weinberg limit [12]. That is because we use the neutral current annihilation that a sequential ν_H would naively be expected to have, rather than the generic charged current cross section of ref. [10].

The annihilation cross sections are useful in (at least) two situations: (i) They allow a calculation of the density of any given dark matter candidate as we have seen. (ii) As discussed in refs. [6, 7] the annihilation products include photons, antiprotons, positrons, and neutrinos that may be detectable under appropriate conditions. A further point that should be emphasized is that a given dark matter candidate leads to characteristic numbers of the various detectable particles. Most obviously, for example, sneutrinos give essentially only monoenergetic neutrinos in their annihilation; if the $\bar{p}$'s of ref. [13] are cosmological, then sneutrinos cannot be the dark matter. For other candidates the $\bar{p}$ fluxes differ, and the ratios of positron to $\bar{p}$ fluxes differ. So if any signal is detected, it will be possible to decide experimentally what is being observed.

3. Scattering cross sections

A similar situation holds for energy transfer by scattering. For every dark matter candidate, the cross section on quarks and leptons is calculable. It may depend on basic (not yet measured or calculated) parameters of the theory, such as the ratio of the two vacuum expectation values in a supersymmetric theory. For practical consequences it is necessary to calculate the cross sections on a proton and a neutron. That is somewhat subtle since cancellations occur, and the naive quark model does not work well here. Our calculations are given in appendix B.

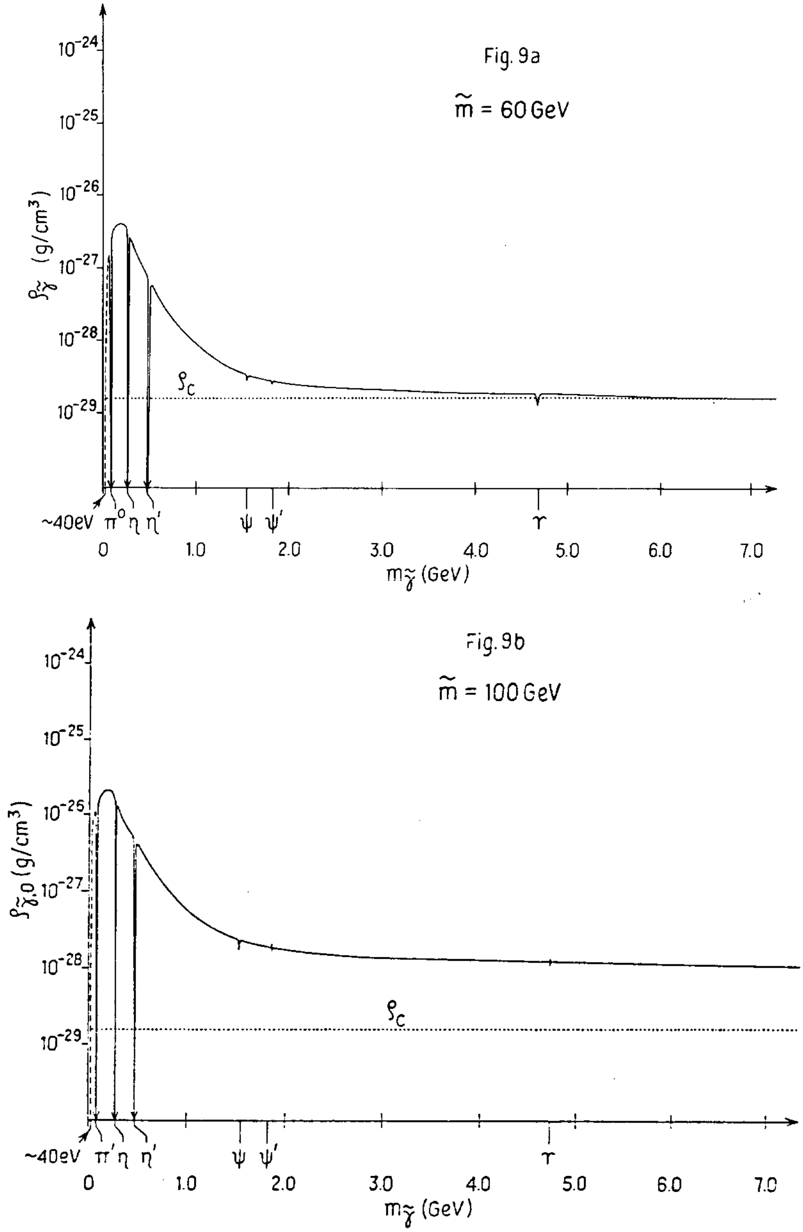

Fig. 9. Similar to fig. 6, for photinos. Any value of $m_{\tilde{\gamma}}$ where $\rho_{\tilde{\gamma}} < \rho_c$ is an allowed photino mass.

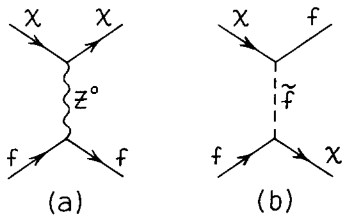

Fig. 10. These are the dominant mechanisms for scattering of dark matter candidates on quarks and leptons.

Goodman and Witten [8] have already discussed the basic ideas about using energy transferred in a scattering process to detect the presence of dark matter. We add various considerations, including an additional cross section, for higgsinos, and a more realistic calculation of all dark matter cross sections on protons and neutrons (which, unfortunately, suppresses the cross sections by a factor which can be as much as five).

Any dark matter candidate with a mass $M_{\rm DM}$ on the GeV scale can transfer an energy of the order of $(2M_{\rm DM}v)^2/2M_{\rm N}$ in a collision with a nucleus N (which can be a proton or a neutron or a heavier nucleus) at relative velocity v. In the simplest view one can imagine we are moving with a velocity $v \simeq 230$ km/sec with respect to the galaxy, while the dark matter, of mass $M_{\rm DM}$, is at rest. In practice there will be a spectrum of velocities, and directional considerations, which have not yet been discussed. If the above energy transfer can be of order 100 eV, in principle detectors should exist to observe the collisions.

Next it is necessary to calculate the cross sections to determine whether the probability of a collision is large enough. The collisions on a quark or lepton (fermion, f) are as shown in fig. 10; they can either proceed via a neutral current interaction, exchanging a Z^0, or exchange of a new object such as a scalar fermion in supersymmetry. The calculation of the basic rates on quarks or leptons is a standard procedure, for which we give some details in appendix B.

More subtle is the way cross sections on quarks are combined to give those on protons or neutrons (which can then be combined further to give the cross section on any nucleus). Again, we give the details in the appendices. There is one subtlety [14]. Using quark model arguments does not work well for the axial vector isosinglet. For example, the matrix element $\langle p|\bar{u}\gamma\gamma_5 u + \bar{d}\gamma\gamma_5 d|p\rangle$ is given by $2\langle p|s|p\rangle$ in the naive quark mode. If the D and F SU(3) coefficients are explicitly written, the coefficient of $2\langle p|s|p\rangle$ is $3F-D$. The naive quark model sets $3F-D = 1$, while experimentally it is about 0.45, which enters squared in rates. Keeping track of this effect, we arrive at the cross sections in table 1, which we believe can be used as a reliable basis for considering experiments. The resulting cross sections are small, but may be large enough to permit workable detectors to be constructed.

TABLE 1
Scattering cross sections for dark matter candidates on protons (p) and neutrons (n) are given

	p	n
ν_H	1.7	1.7
$\tilde{\gamma}$	0.7	0.05
$\tilde{\nu}$	0.01	0.7
$\tilde{h}$	3.4Δ	3.4Δ

The table gives the coefficient λ in

$$\sigma = \lambda G_F^2 M_{DM}^2 M_p^2 / 1.45\pi \left(M_{DM} + M_p \right)^2 \simeq 1.16\lambda 10^{-38} M_{DM}^2 / \left(M_{DM} + M_p \right)^2 \ \mathrm{cm}^2 .$$

The numbers assume all heavy superpartner masses are given by M_W. Larger cross sections will not be likely for supersymmetric partners since lower limits of about 70 GeV already exist for squarks. The quantity Δ in the higgsino entry depends on the presently unknown ratio of vacuum expectation values in a supersymmetric theory, $\Delta = (1 - v_1^2/v_2^2)^2/(1 + v_1^2/v_2^2)^2 = \cos^2(2\beta)$; Δ can range from a rather small number of order 0.01–0.02, up to about 0.5.

The event rate is discussed in ref. [8] and we repeat it for completeness; it is given by $R = \text{flux} \times \sigma \times K$, $K =$ number of nuclei in the target. The flux is $\rho v / M_{DM}$. The cross section σ is that on a nucleus, which presumably scales as the nucleon number A. Then the resulting rate is

$$R \simeq 6 \left(\frac{2\ \mathrm{GeV}}{M_{DM}} \right) \left(\frac{\rho}{10^{-24}\ \mathrm{gm/cm^3}} \right) \left(\frac{v}{230\ \mathrm{km/sec}} \right) \left(\frac{\sigma}{A \times 10^{-38}\ \mathrm{cm}^2} \right) \text{events/kg/day}.$$

This may be a large enough rate to be encouraging.

One very important thing to note from table 1 is how different the ratio of cross section on neutrons and cross section on protons is, and how the ratio is characteristic of different dark matter candidates. As a result, it is probable that if a signal is ever found for dark matter it will be possible to determine whether it can be any of the proposed candidates, and if not what kind of particle is required.

4. Current particle physics and the LSP

As discussed in the introduction, if nature were supersymmetric on the scale of electroweak interactions, the lightest supersymmetric particle (LSP) may be stable and would then necessarily provide some dark matter. As shown in figs. 6, 7 9, for large ranges of masses the LSP contribution to dark matter gives $\Omega_{LSP} \sim 1$.

If evidence for supersymmetry is found in particle physics experiments, presumably the mass of the LSP will be directly measured, and Ω_{LSP} can be calculated. When particle theories are better understood perhaps the masses of superpartners (or the equivalent particles in a different theory) can be calculated and therefore Ω_{LSP} can be predicted.

In fact, it is important to note that particle theories are likely to naturally contain more than one form of dark matter. For example, a supersymmetric grand unified theory will probably contain a LSP with mass of a few GeV, it will probably contain (3) massive neutrinos with masses in the eV or keV range, and Goldstone bosons (axions) that get some mass from non-perturbative effects and are needed because of the strong *CP* problem. While the masses and the fractions of Ω_{cr} in each kind of dark matter might seem arbitrary, presumably they are determined by the theory and the relative amounts will be in principle calculable. Most likely the fraction of Ω that is baryonic is equally calculable and related to the rest, as it should be determined by the same underlying theory.

Of course, we are not quite at that stage yet. Good models exist where $\tilde{\gamma}$, $\tilde{\mathrm{h}}$, or $\tilde{\nu}$ is the LSP. There are some restrictions from data. If $\tilde{\gamma}$ or $\tilde{\nu}$ = LSP, the UA1 data [15] implies [16] $m_{\tilde{q}} \gtrsim 70$ GeV and $m_{\tilde{g}} \gtrsim 65$ GeV. The ASP data implies [17] a lower limit on $m_{\tilde{e}}$ of about 50 GeV for $m_{\tilde{\gamma}} = 0$, decreasing to no limit as $m_{\tilde{\gamma}}$ increases to 12 GeV. If $\tilde{\mathrm{h}}$ = LSP, photinos decay to $\gamma\tilde{\mathrm{h}}$, so the above limits do not hold; data on $e^+e^- \to \gamma\gamma$ + missing momentum imply [18] $m_{\tilde{e}} \gtrsim 100$ GeV, and the UA1 data allows [16] a signal to be present with $m_{\tilde{g}} \simeq 5$ GeV, $m_{\tilde{q}} \simeq 110$ GeV, or requires [16] $m_{\tilde{q}} \gtrsim 50$ GeV, $m_{\tilde{g}} \gtrsim 45$ GeV.

These numbers already have some consequences for dark matter candidates. If, for example, the $\bar{\mathrm{p}}/\mathrm{p}$ ratio of ref. [13] turns out to be cosmological in origin, and therefore presumably due to dark matter, then

(i) obviously $\tilde{\nu} \neq$ LSP since $\tilde{\nu}$ mainly annihilates to ν;

(ii) it cannot be arranged that $\tilde{\gamma}$ = LSP and that $\Omega_{\tilde{\gamma}} \simeq 1$ since having $m_{\tilde{q}} \gtrsim 70$ GeV reduces the $\tilde{\gamma}$ annihilation cross section enough that $m_{\tilde{\gamma}} \gtrsim m_{\mathrm{b}}$, and the $\bar{\mathrm{p}}$ flux from $\tilde{\gamma}$ annihilation is not sufficient to give the 10^{-4} ratio.

Whatever the dark matter, it can be seen from table 1 that the cross sections for WIMP's concentrated in the sun to transfer energy away from the core and lower its temperature are about 10^{-38} cm^2, which are about two orders of magnitude less than those remarked on as optimal in ref. [19]. Whether sufficiently large concentrations can be achieved to overcome these small cross sections is not clear to us at present.

We appreciate helpful conversations with and suggestions from D. Hegyi, D. Sciama, G. Tarlé, and H. Haber.

Appendix A

SELF-ANNIHILATION PROCESSES

A.1. SELF-ANNIHILATION OF THE NEUTRAL GAUGE AND HIGGS FERMIONS

The low-energy effective lagrangian density relevant to the annihilation $\tilde{\chi}^0\tilde{\chi}^0 \to \mathrm{f}\bar{\mathrm{f}}$ is given by (refs. [3,4,20]):

$$L = \sum_{\mathrm{f}} \bar{\tilde{\chi}}^0 \gamma^\mu \gamma_5 \tilde{\chi}^0 \bar{f} \gamma_\mu (A_{\mathrm{f}} P_{\mathrm{L}} + B_{\mathrm{f}} P_{\mathrm{R}}) f, \tag{A.1}$$

where

$$A_{\mathrm{f}} \equiv A_{\mathrm{f}}^{\mathrm{z}} + A_{\mathrm{f}}^{\tilde{\mathrm{f}}}, \qquad B_{\mathrm{f}} \equiv B_{\mathrm{f}}^{\mathrm{z}} + B_{\mathrm{f}}^{\tilde{\mathrm{f}}} \tag{A.2}$$

and

$$A_{\mathrm{f}}^{\mathrm{z}} = (\gamma^2 - \delta^2)\frac{g'\sin\theta_{\mathrm{w}} + g\cos\theta_{\mathrm{w}}}{4m_{\mathrm{Z}}^2}\left(\tfrac{1}{2}Y_{\mathrm{f}}^{\mathrm{L}}g'\sin\theta_{\mathrm{w}} - T_{3\mathrm{f}}^{\mathrm{L}}g\cos\theta_{\mathrm{w}}\right), \tag{A.3}$$

$$B_{\mathrm{f}}^{\mathrm{z}} = (\gamma^2 - \delta^2)\frac{g'\sin\theta_{\mathrm{w}} + g\cos\theta_{\mathrm{w}}}{8m_{\mathrm{Z}}^2}Y_{\mathrm{f}}^{\mathrm{R}}g'\sin\theta_{\mathrm{w}}, \tag{A.4}$$

$$A_{\mathrm{f}}^{\tilde{\mathrm{f}}} = \frac{\left(T_{3\mathrm{f}}^{\mathrm{L}}\alpha g + \frac{1}{2}Y_{\mathrm{f}}^{\mathrm{L}}\beta g'\right)^2}{2m_{\tilde{\mathrm{f}}_{\mathrm{L}}}^2} - \begin{cases}\gamma^2 m_{\mathrm{f}}^2/4v_1^2 m_{\tilde{\mathrm{f}}_{\mathrm{R}}}^2 & \mathrm{u,c,t}\\ \delta^2 m_{\mathrm{f}}^2/4v_2^2 m_{\tilde{\mathrm{f}}_{\mathrm{R}}}^2 & \mathrm{e},\mu,\tau,\mathrm{d,s,b}\end{cases}, \tag{A.5}$$

$$B_{\mathrm{f}}^{\tilde{\mathrm{f}}} = -\frac{\left(\frac{1}{2}Y_{\mathrm{f}}^{\mathrm{R}}\beta g'\right)^2}{2m_{\tilde{\mathrm{f}}_{\mathrm{R}}}^2} + \begin{cases}\gamma^2 m_{\mathrm{f}}^2/4v_1^2 m_{\tilde{\mathrm{f}}_{\mathrm{L}}}^2 & \mathrm{u,c,t}\\ \delta^2 m_{\mathrm{f}}^2/4v_2^2 m_{\tilde{\mathrm{f}}_{\mathrm{L}}}^2 & \mathrm{e},\mu,\tau,\mathrm{d,s,b}\end{cases}, \tag{A.6}$$

$$v_{1,2} \equiv \langle 0|H_{1,2}^0|0\rangle, \qquad Q = T_3 + \tfrac{1}{2}Y.$$

$\tilde{\mathrm{f}}_{\mathrm{L(R)}}$ are the scalar partners of the left-handed (right-handed) conventional fermions. It is understood from (A.2) that in general neutralinos interact with conventional fermions (f) via conventional Z^0 boson exchange and also exchange of sfermions $\tilde{\mathrm{f}}_{\mathrm{L,R}}$. The effective low-energy lagrangian density (A.1) in the case $\tilde{\chi}^0 = \tilde{\gamma}^0$ is given by:

$$L = \sum_{\mathrm{f}}\bar{\tilde{\gamma}}^0\gamma^\mu\gamma_5\tilde{\gamma}^0\bar{f}\gamma_\mu(A_{\mathrm{f}}P_{\mathrm{L}} + B_{\mathrm{f}}P_{\mathrm{R}})f, \tag{A.7}$$

when

$$A_{\mathrm{f}} \equiv A_{\mathrm{f}}^{\mathrm{z}} + A_{\mathrm{f}}^{\tilde{\mathrm{f}}}, \qquad B_{\mathrm{f}} \equiv B_{\mathrm{f}}^{\mathrm{z}} + B_{\mathrm{f}}^{\tilde{\mathrm{f}}}, \tag{A.8}$$

$$A_{\mathrm{f}}^{\mathrm{z}} = B_{\mathrm{f}}^{\mathrm{z}} = 0, \qquad A_{\mathrm{f}}^{\tilde{\mathrm{f}}} = \frac{(ee_{\mathrm{f}})^2}{2m_{\tilde{\mathrm{f}}_{\mathrm{L}}}^2}, \qquad B_{\mathrm{f}}^{\tilde{\mathrm{f}}} = -\frac{(ee_{\mathrm{f}})^2}{2m_{\tilde{\mathrm{f}}_{\mathrm{R}}}^2}. \tag{A.9}$$

The lowest order diagram contributing to photino annihilation $\tilde{\gamma}^0\tilde{\gamma}^0 \to \mathrm{f}\bar{\mathrm{f}}$ is given by fig. 2.

The total low-energy cross section for $\tilde{\gamma}^0\tilde{\gamma}^0 \to \mathrm{f}\bar{\mathrm{f}}$ corresponding to fig. 2 is given by [5]: (here v_{rel} is the relative velocity of annihilating particles)

$$\begin{aligned} v_{\mathrm{rel}}\sigma(\tilde{\gamma}^0\tilde{\gamma}^0 \to \mathrm{f}\bar{\mathrm{f}}) &= \frac{1}{2\pi}\left(1 - \frac{m_{\mathrm{f}}^2}{m_{\tilde{\gamma}}^2}\right)^{1/2}\Big[\left(A_{\mathrm{f}}^2 + B_{\mathrm{f}}^2\right)\left\{\tfrac{1}{6}\left(4m_{\tilde{\gamma}}^2 - m_{\mathrm{f}}^2\right)v_{\mathrm{rel}}^2 + m_{\mathrm{f}}^2\right\} \\ &\qquad - 2A_{\mathrm{f}}B_{\mathrm{f}}m_{\mathrm{f}}^2\left(1 - \tfrac{1}{2}v_{\mathrm{rel}}^2\right)\Big] \\ &\simeq \frac{1}{2\pi}\left[\tfrac{2}{3}m_{\tilde{\gamma}}^2 v_{\mathrm{rel}}^2\left(A_{\mathrm{f}}^2 + B_{\mathrm{f}}^2\right) + m_{\mathrm{f}}^2\left(A_{\mathrm{f}} - B_{\mathrm{f}}\right)^2\right], \end{aligned} \tag{A.10}$$

where A_f and B_f are given by eqs. (A.8), (A.9) and the second expression on the r.h.s. assumes $m_f \ll m_{\tilde{\gamma}}$. In deriving (A.10) one should be careful to include an additional factor of 2 in the matrix element to account for the fact that photinos are Majorana fermions.

Next, we consider the case $\tilde{\chi}^0 = \tilde{h}^0$ (see fig. 3). The effective lagrangian density in this case is given by:

$$L = \sum_f \bar{\tilde{h}}^0 \gamma^\mu \gamma_5 \tilde{h}^0 \bar{f} \gamma^\mu (A_f P_L + B_f P_R) f, \tag{A.11}$$

where

$$A_f \equiv A_f^z + A_f^{\tilde{f}}, \qquad B_f \equiv B_f^z + B_f^{\tilde{f}} \tag{A.12}$$

and

$$A_f^z = \frac{g^2}{4m_Z^2 \cos^2\theta_w} \left(T_{3f}^L - e_f \sin^2\theta_w\right) \cos(2\beta), \tag{A.13}$$

$$B_f^z = \frac{-g^2}{4m_Z^2 \cos^2\theta_w} e_f \sin^2\theta_w \cos(2\beta), \tag{A.14}$$

$$A_f^{\tilde{f}} = -\begin{cases} \left(\dfrac{v_2}{v_1}\right)^2 m_f^2/4v^2 m_{\tilde{f}_R}^2 & \mathrm{u, c, t} \\ \left(\dfrac{v_1}{v_2}\right)^2 m_f^2/4v^2 m_{\tilde{f}_R}^2 & \mathrm{e}, \mu, \tau, \mathrm{d, s, b} \end{cases}, \tag{A.15}$$

$$B_f^{\tilde{f}} = +\begin{cases} \left(\dfrac{v_2}{v_1}\right)^2 m_f^2/4v^2 m_{\tilde{f}_L}^2 & \mathrm{u}, c, \mathrm{t} \\ \left(\dfrac{v_1}{v_2}\right)^2 m_f^2/4v^2 m_{\tilde{f}_L}^2 & \mathrm{e}, \mu, \tau, \mathrm{d, s, b} \end{cases}, \tag{A.16}$$

$$\tan\beta = \frac{v_2}{v_1}, \qquad v = \left(v_1^2 + v_2^2\right)^{1/2}. \tag{A.17}$$

The total low-energy cross section $\sigma(\tilde{h}^0\tilde{h}^0 \to f\bar{f})$ is the same as for the photino and is given by eq. (A.10) with A_f and B_f now given by (A.12)–(A.16), and $m_{\tilde{\gamma}} \to m_{\tilde{h}}$.

A.2. SELF-ANNIHILATION OF THE SCALAR NEUTRINOS

Following the notation of ref. [5], the total nonrelativistic cross section arises approximately from fig. 4 and is given by:

$$(\sigma v_{\mathrm{rel}}) = \frac{1}{16\pi} \sum_{i=1}^{4} \left[\frac{(\alpha_i g - \beta_i g')^2}{M_{\tilde{\chi}_i}^0}\right]^2. \tag{A.18}$$

TABLE 2
The annihilation cross section $\sigma(\chi\bar{\chi} \to c\bar{c}, \tau\bar{\tau})$ are numerically computed using $m_\chi = 5$ GeV; $\chi = \gamma^0, h^0, \tilde{\nu}, L^0$

$f \backslash \chi$	$\tilde{\gamma}^0$	$\tilde{h}^0$	$\tilde{\nu}$	L^0
c	1.66×10^{-38}	$2.84 \times 10^{-38}\cos^2(2\beta)$	2.71×10^{-42}	1.71×10^{-37}
τ	3.54×10^{-38}	$1.22 \times 10^{-38}\cos^2(2\beta)$	1.15×10^{-42}	4.75×10^{-38}

Other choices are $m_{\tilde{c}} = m_{\tilde{\tau}} = m_W$, $m_c = 1.5$ GeV, $m_\tau = 1.7$ GeV, $v_{rel} = 5 \times 10^{-3}$, and $\beta = \tan^{-1}(v_2/v_1)$.

A.3. SELF-ANNIHILATION OF HEAVY-STABLE-DIRAC NEUTRINOS

We assume that heavy-Dirac neutrinos (L^0) have the usual neutral-weak interactions. Then the low-energy effective lagrangian density is give by (fig. 1a):

$$L = \sqrt{\tfrac{1}{2}}\, G_F \sum_f \bar{L}^0 \gamma^\mu (1 - \gamma_5) L^0 \bar{f} \gamma_\mu (a_f + b_f \gamma_5) f, \tag{A.19}$$

where

$$a_f = T_{3f}^L - 2e_f \sin^2\theta_w, \qquad b_f = -T_{3f}^L. \tag{A.20}$$

The low-energy total cross section for the annihilation $L^0\bar{L}^0 \to f\bar{f}$ is given by:

$$\sigma v_{rel} = \frac{G_F^2}{4\pi}\left(1 - \frac{m_f^2}{m_L^2}\right)^{1/2} \left[(a_f^2 + b_f^2)\left\{2m_L^2 + \tfrac{1}{6}(7m_L^2 - m_f^2)v_{rel}^2\right\} + (a_f^2 - b_f^2)m_f^2(1 + \tfrac{1}{2}v_{rel}^2)\right], \tag{A.21}$$

where m_L is the mass of heavy neutrino L^0 and m_f is the mass of final state fermions. As before, we have not kept higher orders in v_{rel}. Note that in the limit where $m_L \gg m_f$ and $v_{rel} \to 0$, eq. (A.21) becomes more familiar:

$$\sigma(L^0\bar{L}^0 \to f\bar{f})v_{rel} = \frac{G_F^2 m_L^2}{2\pi}(a_f^2 + b_f^2). \tag{A.22}$$

Some numerical estimates for $\sigma(\chi\bar{\chi} \to c\bar{c})$ and $\sigma(\chi\bar{\chi} \to \tau\bar{\tau})$, $\chi = \tilde{\gamma}^0, \tilde{h}^0, \tilde{\nu}, L^0$ are given in table 2.

Appendix B

ELASTIC SCATTERING FROM QUARKS AND NUCLEONS

Following the notations and conventions employed in the appendix A, we now proceed to systematically study, for each type of particle considered ($\tilde{\gamma}^0, \tilde{h}^0, \tilde{\nu}, L^0$), its scattering from individual quarks. Then we add these appropriately to obtain the scattering from nucleons.

B.1. ELASTIC SCATTERING FROM QUARKS

The effective low-energy lagrangian (A.1) describing the effective coupling of neutralinos to conventional fermions can be directly used to describe the scattering process: $\tilde{\chi}^0 q \to \tilde{\chi}^0 q$. In the non-relativistic regime, one obtains for the cross section:

$$\sigma(\tilde{\chi}^0 q \to \tilde{\chi}^0 q) = 3\frac{m_{\tilde{\chi}^0}^2 m_q^2}{\pi(m_{\tilde{\chi}^0}+m_q)^2}\left[\left(A_q^2+B_q^2\right)\left\{1+\tfrac{1}{3}v_{\mathrm{rel}}^2\left(2-m_{\tilde{\chi}^0}\frac{2m_q-m_{\tilde{\chi}^0}}{(m_q+m_{\tilde{\chi}^0})^2}\right)\right\}\right.$$
$$\left.-2A_qB_q\left\{1+\tfrac{1}{3}v_{\mathrm{rel}}^2\frac{m_q^2}{(m_q+m_{\tilde{\chi}^0})^2}\right\}\right], \quad \text{(B.1)}$$

where in the case $\tilde{\chi}^0=\tilde{\gamma}^0$, the quantities A_q and B_q are given by (A8,9) and in the case $\tilde{\chi}^0=\tilde{h}^0$ by (A.12)–(A.16). Note that v_{rel} in this case is defined by:

$$v_{\mathrm{rel}} \equiv \frac{p(m_{\tilde{\chi}^0}+m_q)}{m_{\tilde{\chi}^0}m_q},$$

where p is the magnitude of the 3-momentum of the scattering particles in the center of mass frame. Furthermore in the limit $v_{\mathrm{rel}} \ll 1$ (B.1) reduces to a simple form: (see also ref. [8])

$$\sigma(\tilde{\chi}^0 q \to \tilde{\chi}^0 q) \underset{v_{\mathrm{rel}}\to 0}{\to} 3\frac{m_{\tilde{\chi}^0}^2 m_q^2}{\pi(m_{\tilde{\chi}^0}+m_q)^2}(A_q-B_q)^2. \quad \text{(B.2)}$$

A color factor of 1 has been included for averaging over initial and summing three colc over final colors. These processes are shown in fig. 10.

The scalar neutrinos scatter from quarks only through the Z^0 boson exchange, (see fig. 10a). The low-energy elastic scattering cross section in this case is given by:

$$\sigma(\tilde{\nu}_\ell q \to \tilde{\nu}_\ell q) = \frac{3G_F^2}{2\pi}\frac{m_{\tilde{\nu}}^2 m_q^2}{(m_{\tilde{\nu}}+m_q)^2}\left[\left(a_q^2+b_q^2\right)\left\{2+v_{\mathrm{rel}}^2\left(3+m_{\tilde{\nu}}\frac{4m_q-m_{\tilde{\nu}}}{(m_{\tilde{\nu}}+m_q)^2}\right)\right\}\right.$$
$$\left.+\left(a_q^2-b_q^2\right)\left(2+v_{\mathrm{rel}}^2\frac{m_q^2}{(m_{\tilde{\nu}}+m_q)^2}\right)\right], \quad \text{(B.3)}$$

where a_q and b_q are given by eq. (A.20) and v_{rel} is defined (with $m_{\tilde{\chi}^0}$ replaced by $m_{\tilde{\nu}}$) as below eq. (B.1). Note that in the limit $v_{rel} \ll 1$ (B.3) reduces to a simple form: (see also ref. [8])

$$\sigma(\tilde{\nu}_\ell q \to \tilde{\nu}_\ell q) = \frac{6G_F^2 m_{\tilde{\nu}}^2 m_q^2}{\pi(m_{\tilde{\nu}} + m_q)^2} a_q^2. \tag{B.4}$$

Finally, consider the scattering of heavy Dirac neutrinos (L^0) from quarks. Using the low-energy effective lagrangian density (A.19) one simply finds:

$$\sigma(L^0 q \to L^0 q) = \frac{6G_F^2}{\pi} \frac{m_L^2 m_q^2}{(m_{L^0} + m_q)^2} \left[c_q^2(1 + v_{rel}^2) + d_q^2 \left\{ 1 + v_{rel}^2 \left[1 - \frac{2m_{L^0} m_q}{(m_{L^0} + m_q)^2} \right] \right\} \right.$$
$$\left. - c_q d_q \left\{ 1 + \frac{m_q^2}{(m_{L^0} + m_q)^2} v_{rel}^2 \right\} \right], \tag{B.5}$$

where $c_q \equiv T_{3q}^L - e_q \sin^2\theta_w$, $d_q = -e_q \sin^2\theta_w$ and v_{rel} is defined below (B.1) with $m_{\tilde{\chi}^0}$ replaced by m_L.

Once again in the limiting case $v_{rel} \ll 1$, eq. (B.5) reduces to a simple form:

$$\sigma(L^0 q \to L^0 q) \underset{v_{rel} \to 0}{\to} \frac{6G_F^2}{\pi} \frac{m_{L^0}^2 m_q^2}{(m_{L^0} + m_q)^2} \left[c_q^2 + d_q^2 - c_q d_q \right]. \tag{B.6}$$

B.2. ELASTIC SCATTERING FROM NUCLEONS

Assuming strong SU(3)-flavor symmetry and the CVC hypothesis, it is simple to obtain the cross sections on nucleons n, p. Following the notation of ref. [14], this process can be summarized as follows. The matrix element inside a proton of the vector and axial quark currents are given by:

$$\langle \text{proton}, p', s' | \bar{q}(0) \gamma_\mu q(0) | \text{proton}, p, s \rangle = \bar{u}_p^{(s')}(p') \gamma_\mu u_p^{(s)}(p) N_q^V,$$

$$\langle \text{proton}, p', s' | \bar{q}(0) \gamma_\mu \gamma_5 q(0) | \text{proton}, p, s \rangle = \bar{u}_p^{(s')}(p') \gamma_\mu \gamma_5 u_p^{(s)}(p) N_q^A, \tag{B.7}$$

where $p(p')$ and $s(s')$ denote the initial (final) four momentum and spin of the proton, and N_q^V and N_q^A are (respectively) the vector and axial charges associated with the constituent quark (q). Similarly the matrix elements inside a neutron are given by:

$$\langle n, p', s' | \bar{q}(0) \gamma_\mu q(0) | n, p, s \rangle = \bar{u}_n^{(s')}(p') \gamma_\mu u_n^{(s)}(p) M_q^V, \tag{B.8}$$

$$\langle n, p', s' | q(0) \gamma_\mu \gamma_5 q(0) | n, p, s \rangle = \bar{u}_n^{(s')}(p') \gamma_\mu \gamma_5 u_n^{(s)}(p) M_q^A, \tag{B.9}$$

where $M_{\mathrm{q}}^{\mathrm{V}}$ and $M_{\mathrm{q}}^{\mathrm{A}}$ are again vector and axial charges associated with constituent quark (q) of the neutron.

Let us first derive the expression for the total cross sections $\sigma(\chi\mathrm{p} \to \chi\mathrm{p})$ and $\sigma(\chi\mathrm{n} \to \chi\mathrm{n})$; $\chi = \tilde{\gamma}^0, \tilde{\mathrm{h}}^0, \tilde{\nu}_\ell, \mathrm{L}^0$.

The low-energy effective lagrangian (A.1) can be used to arrive at the matrix element for elastic scattering $\tilde{\gamma}^0\mathrm{p} \to \tilde{\gamma}^0\mathrm{p}$:

$$M(\tilde{\gamma}^0\mathrm{p} \to \tilde{\gamma}^0\mathrm{p}) = 2 \sum_{\mathrm{q}=\mathrm{u,d}} \bar{u}(\tilde{\gamma}^0)\gamma^\mu\gamma_5 u(\tilde{\gamma}^0)\bar{u}(p)\gamma_\mu \times \left\{ \tfrac{1}{2}(A_{\mathrm{q}} + B_{\mathrm{q}}) N_{\mathrm{q}}^{\mathrm{V}} + \tfrac{1}{2}(B_{\mathrm{q}} - A_{\mathrm{q}}) N_{\mathrm{q}}^{\mathrm{A}}\gamma_5 \right\} u(p), \quad \text{(B.10)}$$

where the 4-momenta and spins are explicit and quantities A_{q} and B_{q} are given by eqs. (A.2)–(A.6). Let us assume for convenience that $\tilde{\mathrm{q}}_{\mathrm{L}}$ and $\tilde{\mathrm{q}}_{\mathrm{R}}$ are degenerate in mass $m_{\tilde{\mathrm{q}}_{\mathrm{L}}} = m_{\tilde{\mathrm{q}}_{\mathrm{R}}} = \tilde{m}_{\mathrm{q}}$. Then one arrives at a simple expression for the cross sections in the limit $v_{\mathrm{rel}} \to 0$:

$$\sigma(\tilde{\gamma}^0\mathrm{p} \to \tilde{\gamma}^0\mathrm{p}) = 3\frac{m_{\tilde{\gamma}}^2 m_{\mathrm{p}}^2}{\pi(m_{\tilde{\gamma}} + m_{\mathrm{p}})^2}\left[\sum_{\mathrm{q}=\mathrm{u,d}} \frac{(ee_{\mathrm{q}})^2}{\tilde{m}_{\mathrm{q}}^2} N_{\mathrm{q}}^{\mathrm{A}}\right]^2. \quad \text{(B.11)}$$

Similarly

$$\sigma(\tilde{\gamma}^0\mathrm{n} \to \tilde{\gamma}^0\mathrm{n}) = 3\frac{m_{\tilde{\gamma}}^2 m_{\mathrm{n}}^2}{\pi(m_{\tilde{\gamma}} + m_{\mathrm{n}})^2}\left[\sum_{\mathrm{q,u,d}} \frac{(ee_{\mathrm{q}})^2}{\tilde{m}_{\mathrm{q}}^2} M_{\mathrm{q}}^{\mathrm{A}}\right]^2. \quad \text{(B.12)}$$

In this case since $A_{\mathrm{q}} = -B_{\mathrm{q}}$, these equations are obtained from eq. (B.2) by simple substitutions:

$$A_{\mathrm{q}} \to \sum_{\mathrm{q}=\mathrm{u,d}} A_{\mathrm{q}} N_{\mathrm{q}}^{\mathrm{A}}$$

etc., except for a factor of 3 which corresponds to the fact that nucleons are color singlet states whereas quarks are color-triplet states. The presence of axial charges in (B.11), (B.12) follows from the spin structure of the effective lagrangian.

Proceeding similarly, the low-energy scattering cross sections for processes $\tilde{\mathrm{h}}^0\mathrm{p} \to \tilde{\mathrm{h}}^0\mathrm{p}$ and $\tilde{\mathrm{h}}^0\mathrm{n} \to \tilde{\mathrm{h}}^0\mathrm{n}$ can be computed and are given by:

$$\sigma(\tilde{\mathrm{h}}^0\mathrm{p} \to \tilde{\mathrm{h}}^0\mathrm{p}) = 6\frac{m_{\tilde{\mathrm{h}}^0}^2 m_{\mathrm{p}}^2}{\pi(m_{\tilde{\mathrm{h}}^0} + m_{\mathrm{n}})^2} G_{\mathrm{F}}^2 \cos^2(2\beta)\left[\sum_{\mathrm{q}=\mathrm{u,d}} T_{3\mathrm{q}}^{\mathrm{L}} N_{\mathrm{q}}^{\mathrm{A}}\right]^2 \quad \text{(B.13)}$$

and similarly:

$$\sigma(\tilde{\mathrm{h}}^0\mathrm{n} \to \tilde{\mathrm{h}}^0\mathrm{n}) = 6\frac{m_{\tilde{\mathrm{h}}^0}^2 m_{\mathrm{n}}^2}{\pi(m_{\tilde{\mathrm{h}}^0} + m_{\mathrm{n}})^2} G_{\mathrm{F}}^2 \cos^2(2\beta)\left[\sum_{\mathrm{q}=\mathrm{u,d}} T_{3\mathrm{q}}^{\mathrm{L}} M_{\mathrm{q}}^{\mathrm{A}}\right]^2, \quad \text{(B.14)}$$

where β is given by (A.17).

Similar arguments apply in cases $\chi = \tilde{\nu}_\ell$ and $\chi = L^0$ and we simply given the final results below:

$$\sigma(\tilde{\nu}_\ell p \to \tilde{\nu}_\ell p) = 2G_F^2 \frac{m_{\tilde{\nu}^0}^2 m_p^2}{\pi(m_{\tilde{\nu}^0} + m_p)^2} \left[\sum_{q=u,d} \left(T_{3q}^L - 2e_q \sin^2\theta_w \right) N_q^V \right]^2, \quad (B.15)$$

$$\sigma(\tilde{\nu}_\ell n \to \tilde{\nu}_\ell n) = 2G_F^2 \frac{m_{\tilde{\nu}^0}^2 m_n^2}{\pi(m_{\tilde{\nu}^0} + m_n)^2} \left[\sum_{q=u,d} \left(T_{3q}^L - 2e_q \sin^2\theta_w \right) M_q^V \right]^2, \quad (B.16)$$

$$\sigma(L^0 p \to L^0 p) = \tfrac{3}{2} G_F^2 \frac{m_{L^0}^2 m_p^2}{\pi(m_{L^0} + m_p)^2} \left[N_V^2 + 3N_A^2 \right], \quad (B.17)$$

$$\sigma(L^0 n \to L^0 n) = \tfrac{3}{2} G_F^2 \frac{m_{L^0}^2 m_p^2}{\pi(m_{L^0} + m_p)^2} \left[M_V^2 + 3M_A^2 \right]. \quad (B.18)$$

$$N_V = \sum_{q=u,d} \left(T_{3q}^L - 2e_q \sin^2\theta_w \right) N_q^V, \qquad M_V = \sum_{q=u,d} \left(T_{3q}^L - 2e_q \sin^2\theta_w \right) M_q^V,$$

$$N_A = \sum_{q=u,d} T_{3q}^L N_q^A, \qquad M_A = \sum_{q=u,d} T_{3q}^L M_q^A.$$

It is easy to see that N_q^V and M_q^V are simply given by the naive quark content, i.e.

$$N_u^V = 2, \qquad N_d^V = 1, \qquad M_u^V = 1, \qquad M_d^V = 2. \quad (B.19)$$

The situation for axial charges, N_q^A and M_q^A, is more complex since there are two isoscalar charges, originating from the symmetric and antisymmetric octets. The best way to proceed [14] is to express the results in terms of the measured F and D coefficients. They can be related to the charges N_q^A, M_q^A by considering formally the decays $n \to pe^- \bar{\nu}_e$ and $\Xi^- \to \Xi^0 e^0 \bar{\nu}_e$. Assuming flavor SU(3) one finds:

$$F + D = N_u^A - N_d^A, \qquad F - D = N_d^A - N_s^A, \quad (B.20)$$

where F and D are the usual coupling parameters with approximate measured values $F = 0.425$ and $D = 0.825$ ($g_A = F + D$). We can further assume that N_s^A can be ignored since the strange and anti-strange content of the proton is assumed to be very small (and in addition it is expected not to be very polarized). With this assumption, plus the result that the u content of a neutron is the d content of a proton, etc. one has:

$$N_u^A = 2F \simeq 0.85, \qquad N_d^A = F - D \simeq -0.4, \qquad M_u^A = N_d^A, \qquad M_d^A = N_u^A. \quad (B.21)$$

Using these and (B.19) in (B.11)–(B.18) give the results in table 1.

Appendix C

Here we given the details for analyzing the situation of fig. 5 and fig. 8, when annihilation can happen via a resonant state because $m_{\text{res}} = 2m_X$.

C.1. PHOTINO ANNIHILATION THROUGH RESONANCES

Let R^0 be any quarkonium state which can decay to two photinos. Since $\tilde{\gamma}^0$ is an eigenstate of charge conjugation with odd C, the system of two photinos is even under C. Therefore only C-even resonances can decay into two photinos. However, if $m_{\tilde{\mathrm{q}}_\mathrm{L}} \neq m_{\tilde{\mathrm{q}}_\mathrm{R}}$, the theory is obviously no larger invariant under parity, while it is clearly CP invariant, so it is no longer invariant under C. Then C-odd resonances can decay into two photinos, with amplitude proportional to $\Delta\tilde{m}_\mathrm{q}^2 = m_{\tilde{\mathrm{q}}_\mathrm{L}}^2 - m_{\tilde{\mathrm{q}}_\mathrm{R}}^2$

Using the standard formula:

$$\Gamma(\mathrm{R}^0 \to \tilde{\gamma}^0\tilde{\gamma}^0) = \frac{4}{2J+1}\frac{|R(0)|^2}{4\pi}\lim_{s\to 4m_\mathrm{q}^2}\frac{\left(s-4M_\mathrm{q}^2\right)^{1/2}}{m_\mathrm{q}}\sigma(\mathrm{q}\bar{\mathrm{q}} \to \tilde{\gamma}^0\tilde{\gamma}^0), \quad (\mathrm{C.1})$$

where J is the spin of R^0, $R(0)$ is the quarkonium radial wave function evaluated at the origin, and R^0 has the form $\mathrm{R}^0 = \Sigma_\mathrm{q} C_\mathrm{q}|\bar{\mathrm{q}}\mathrm{q}\rangle$ and is properly normalized, one finds:

$$\Gamma(\mathrm{R}^0_{C\text{-even}} \to \tilde{\gamma}^0\tilde{\gamma}^0) \simeq \frac{12\alpha^2 m_{\tilde{\gamma}}^2|R(0)|^2}{2J+1}\left(1-\frac{4m_{\tilde{\gamma}}^2}{m_{\mathrm{R}^0}^2}\right)^{1/2}\left[\sum_\mathrm{q} C_\mathrm{q} e_\mathrm{q}^2 \frac{1}{\tilde{m}_{\tilde{\mathrm{q}}_\mathrm{L}}^2}\right]^2 \quad (\mathrm{C.2})$$

and

$$\Gamma(\mathrm{R}^0_{C\text{-odd}} \to \tilde{\gamma}^0\tilde{\gamma}^0) \simeq \frac{3\alpha^2 m_{\tilde{\gamma}}^2|R(0)|^2}{2J+1}\left(1-\frac{4m_{\tilde{\gamma}}^2}{m_{\mathrm{R}^0}^2}\right)^{1/2}\left[\sum C_\mathrm{q} e_\mathrm{q}^2 \frac{1}{m_{\tilde{\mathrm{q}}_\mathrm{L}}^2}\left(\frac{\Delta\tilde{m}_\mathrm{q}^2}{m_{\tilde{\mathrm{q}}_\mathrm{L}}^2}\right)\right]^2. \quad (\mathrm{C.3})$$

The quantities $|R(0)|^2$ can be empirically derived using known rates for $\Gamma(\mathrm{R}^0_{C\text{-even}} \to \gamma\gamma)$ and $\Gamma(\mathrm{R}^0_{C\text{-odd}} \to \mathrm{e}^+\mathrm{e}^-)$. Letting

$$\sigma_{\text{res}} v_{\text{rel}} \equiv \sigma(\tilde{\gamma}^0\tilde{\gamma}^0 \to \mathrm{R}^0 \to \mathrm{X}) v_{\text{rel}}\big|_{m_{\tilde{\gamma}} = \frac{1}{2}m_{\mathrm{R}^0}},$$

one finally obtains

$$\sigma_{\text{res}}(\tilde{\gamma}^0\tilde{\gamma}^0 \to \mathrm{R}^0_{C\text{-even}} \to \mathrm{X}) v_{\text{rel}} = 192\pi\frac{m_{\tilde{\gamma}^0}^2}{\tilde{m}^4}\mathrm{BR}(\mathrm{R}^0_{c\text{-even}} \to \gamma\gamma) \quad (\mathrm{C.4})$$

and

$$\sigma_{\mathrm{res}}\left(\tilde{\gamma}^0\tilde{\gamma}^0 \to \mathrm{R}^0_{C\text{-odd}} \to \mathrm{X}\right)v_{\mathrm{rel}} = 27\pi \frac{m^2_{\tilde{\gamma}^0}}{\tilde{m}^4}\left(\frac{\Delta\tilde{m}^2}{\tilde{m}^2_{\mathrm{q}}}\right)^2 \left(\frac{\sum\limits_{\mathrm{q}} C_{\mathrm{q}} e^2_{\mathrm{q}}}{\sum\limits_{\mathrm{q}} C_{\mathrm{q}} e_{\mathrm{q}}}\right)^2$$

$$\times \mathrm{BR}\left(\mathrm{R}^0_{C\text{-odd}} \to \mathrm{e}^+\mathrm{e}^-\right), \qquad \text{(C.5)}$$

where $\mathrm{R}^0 \equiv \Sigma_{\mathrm{q}} C_{\mathrm{q}} |\bar{q}q\rangle$ is a normalized meson state and we have taken $\tilde{m} = m_{\tilde{\mathrm{q}}_{\mathrm{L}}} = m_{\tilde{\mathrm{q}}_{\mathrm{R}}}$ for all quarks (q).

The present mass density of photinos is given by:

$$\rho_{\tilde{\gamma}^0} = 0.8\left(\frac{T_{\tilde{\gamma}}}{T_{\gamma}}\right)^3 T^3_{\gamma} m_{\tilde{\gamma}^0} \frac{1}{a x_{\mathrm{f}} + \frac{1}{2} b x^2_{\mathrm{f}}}, \qquad \text{(C.6)}$$

where a and b are defined by:

$$a = \frac{m_{\tilde{\gamma}^0}}{K^3}\left(\frac{8\pi^3 N_{\mathrm{F}} G}{45}\right)^{-1/2} \tilde{a}, \qquad b = \frac{m_{\tilde{\gamma}^0}}{K^3}\left(\frac{8\pi^3 N_{\mathrm{F}} G}{45}\right)^{-1/2} \tilde{b} \qquad \text{(C.7)}$$

and $\langle \sigma v_{\mathrm{rel}} \rangle \equiv \tilde{a} + \tilde{b}x$. Furthermore $x_{\mathrm{f}} \equiv KT_{\mathrm{f}}/m_{\tilde{\gamma}^0}$ is known to satisfy the equation:

$$x_{\mathrm{f}}^{-1/2} \exp(1/x_{\mathrm{f}}) = 2(2\pi)^{-3/2} K^3 (a + b x_{\mathrm{f}}). \qquad \text{(C.8)}$$

Using $\sigma v_{\mathrm{rel}} \equiv (\sigma_{\mathrm{st}} + \sigma_{\mathrm{res}}) v_{\mathrm{rel}}$, with $\sigma_{\mathrm{st}} v_{\mathrm{rel}} \equiv \Sigma\sigma(\chi\chi \to \mathrm{ff})$ the conventional annihilation cross section, and making uses of eqs. (C.1)–(C.8), one arrives at results for the density of photinos of mass $m_{\tilde{\gamma}} = \frac{1}{2} m_{\mathrm{R}^0}$. They are shown in figs. 9a, b for $\tilde{m} = 60$ GeV and 100 GeV.

C.2. HIGGSINO ANNIHILATION THROUGH RESONANCES

The rate equation for Majorona higgsinos is the same as that of photinos.

The contributions to annihilation are summarized in appendix A for the effective low-energy lagrangian coupling higgsinos to the conventional fermions. Here we obtain the contribution from fig. 2a.

Consider the neutral-current interaction hamiltonian density given by:

$$H^{\mathrm{NC}} = \frac{g}{2\cos\theta_{\mathrm{w}}} \bar{f}\gamma^{\mu}(a_{\mathrm{f}} + b_{\mathrm{f}}\gamma_5) f Z^0_{\mu} + e_{\mathrm{Z}}^{(\mathrm{V}^0)} \frac{m^2_{\mathrm{V}^0}}{g_{\mathrm{V}^0}} V^{0\mu} Z^0_{\mu}. \qquad \text{(C.9)}$$

The first term on the r.h.s. is the usual coupling of Dirac fermions to the Z^0 boson with a_f and b_f given by (A.12)–(A.16), and the second term is the phenomenological interaction hamiltonian density coupling vector mesons (V^0) to the Z^0 boson. Here m_{V^0} is the mass of V^0 and, assuming a normalized expression, $V^0 = \Sigma_q C_q |\bar{q}q\rangle$, $e_Z^{(V^0)}$ is defined by:

$$e_Z^{(V^0)} \equiv \frac{g}{2\cos\theta_w} \sum_q C_q \left[\left(T_{3q}^L - e_q \sin^2\theta_w \right) + \left(T_{3q}^R - e_q \sin^2\theta_w \right) \right]. \tag{C.10}$$

This quantity is a measure of the effective "neutral-weak-charge" of the meson V^0. Other appropriate constants are then absorbed systematically into the factor g_{V^0} which further accounts for strong interaction effects and is measured from the rate $(V^0 \to e^+e^-)$.

Using these one arrives at:

$$\sigma_{res} v_{rel} \equiv \sigma(\tilde{h}^0\tilde{h}^0 \to V^0 \to X)\, v_{rel}\big|_{m_{\tilde{h}^0} = \frac{1}{2}m_{V^0}} = \frac{4G_F^2 m_{V^0}^3}{g_{V^0}^2 \Gamma_{V^0}} \cos^2(2\beta)\, \eta_{V^0}^2, \tag{C.11}$$

where $\eta_{V^0} \equiv e_Z^{V^0}/(g/2\cos\theta_w)$ and Γ_{V^0} is the full width of V^0.

Using $\sigma v_{rel} \equiv (\sigma_{st} + \sigma_{res}) v_{rel}$ and noting that $\langle v_{rel}^2 \rangle = 6x$, one arrives at the numerical values for $\rho_{\tilde{h}^0}$ ($m_{\tilde{h}^0} = \frac{1}{2} m_{V^0}$).

We have displayed the behavior of present mass density $\rho_{\tilde{h}^0}$ as a function of the mass of higgsino ($m_{\tilde{h}^0}$) in fig. 7a,b corresponding to two special choices of parameter β.

Our result in the case $v_1 = 2v_2$ are consistent with those of ref. [4], except for the effects of resonances.

C.3. HEAVY DIRAC NEUTRINOS

The case of heavy stable Dirac neutrinos as a dark matter candidate has usually been studied assuming a generic "V − A" charged current interaction. The rate equation for Dirac fermions is same as that for Majorona fermions. The neutral-current interaction hamiltonian density relevant to our consideration is also presented in eq. (C.7) (see also (A.19)). The contribution from fig. 1a to the annihilation of heavy Dirac neutrinos was worked out in appendix A. We denote this contribution by:

$$\sigma_{st} v_{rel} \equiv \sum_f \sigma(L^0\bar{L}^0 \to f\bar{f})\, v_{rel}. \tag{C.12}$$

The contribution from fig. 5 can be computed using the steps outlined previously and the final result is given by:

$$\sigma_{res} v_{rel} \equiv \sigma(\bar{L}^0 L^0 \to V^0 \to X)\big|_{m_{L^0} = \frac{1}{2}m_{V^0}} = \frac{2G_F^2 m_{V^0}^3}{g_{V^0}^2 \Gamma_{V^0}} \eta_{V^0}^2. \tag{C.13}$$

Using $\sigma v_{\mathrm{rel}} \equiv (\sigma_{\mathrm{st}} + \sigma_{\mathrm{res}}) v_{\mathrm{rel}}$ one arrives at the values $\rho_{\mathrm{L}^0}(m_{\mathrm{L}^0} = \frac{1}{2} m_{\mathrm{V}^0})$ which are shown for some vector mesons in fig. 6.

C.4. SCALAR-NEUTRINOS

The rate equation in this case is same as that for Majorana particles; and the present mass density in this case is also the same as in the case for Majorona fermions and is given by eq. (C.6) with $T_{\tilde{\nu}}$ replacing $T_{\tilde{\gamma}}$ and $m_{\tilde{\nu}}$ replacing $m_{\tilde{\gamma}^0}$.

The dominant contribution to the annihilation cross section is given by fig. 3, which leads to eq. (A.18). Since σv_{rel} is independent of $m_{\tilde{\nu}}$, $\rho_{\tilde{\nu}}$ is approximately independent of $m_{\tilde{\nu}}$. This is to be contrasted with the cases previously considered in which the annihilation cross section essentially varies as m_{χ}^2 $(\chi = \tilde{\gamma}^0, \tilde{\mathrm{h}}^0, \mathrm{L}^0)$ or as final state (mass)^2. In such cases, the cosmological mass density grows at least as $1/m_{\chi}^2$ for small m_{χ} which lead to a lower bound on m_{χ}. In the present case, however, there is no lower bound on $m_{\tilde{\nu}}$ provided the contribution to the cross section from fig. 3 is non-negligible.

In order for the contribution to $\langle \sigma v_{\mathrm{rel}} \rangle$ from fig. 3 to be non-negligible, it is necessary that the gaugino or higgsino mass (M_2 or ε) in the lagrangian [4,20] be non-negligible. In the limit M_2, $\epsilon \to 0$, besides two light neutralino eigenstates (pure photino and pure higgsino), there are two eigenstates $\tilde{\mathrm{Z}}_{\pm}$ which are nearly degenerate, having masses $M_{\tilde{\mathrm{Z}}_{\pm}} \simeq m_{\mathrm{Z}}$. Neither photino nor higgsino contribute to sneutrino annihilation, while contributions from two degenerate eigenstates $\tilde{\mathrm{Z}}_{\pm}$ are equal and opposite and hence cancel. However if the higgsino mass parameter ε is > O(1 GeV), this splits the degeneracy between the two $\tilde{\mathrm{Z}}_{\pm}$ enough so that sneutrinos annihilate easily, in which case there is no lower bound on $m_{\tilde{\nu}}$ from cosmology. In contrast to fig. 3 which is "semi-weak", other diagrams give weak contributions to the $\tilde{\nu}$ annihilation cross section, and also are helicity-and/or p-wave suppressed. In the absence of fig. 3, there would be a lower bound on $m_{\tilde{\nu}}$ which is comparable to those derived from photinos or heavy neutrinos. Therefore it follows that by varying the parameters in the lagrangian, particularly ε, one can vary the strength of fig. 3 and thereby adjust the present mass density of $\tilde{\nu}$ to any desirable value up to or exceeding closure density.

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