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RGM STUDY OF THE NN INTERACTION IN AN EXTENDED QUARK MODEL*

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Abstract: The extended quark model study of the NN interaction, in which the $(q\bar{q})$ excitations inherent in the quark-gluon interaction are explicitly incorporated into the model space, is completed with the inclusion of the $(q\bar{q})(q\bar{q})$ excitations generated by RPA-type terms of the color Breit interaction. The new coupling kernels connecting the dominant (3q)-(3q) components of the NN system to the (3q)-(3q)$(q\bar{q})(q\bar{q})$ components lead to potentials with the characteristics of conventional σ and δ meson exchange potentials and furnish the additional medium-range attraction needed to bind the deuteron. The full model is subjected to a quantitative test through a solution of the RGM equations in a coupled channel formalism. With one improvement of the model, to yield an $N\pi$ tensor force with OPEP strength and long-range characteristics, this model leads to a prediction of the low-energy NN scattering data and deuteron bound state characteristics which is in semiquantitative agreement with the experimental data and is free of parameter adjustments.

1. Introduction

In the simplest quark models the nucleon is assumed to be a pure three-quark (3q) system. Since studies of the NN interaction based on simple (3q)-(3q) models [1]) could elucidate only the extreme short range parts of this interaction, an extended quark model has recently been built [2]) in which the $(q\bar{q})$ excitations inherent in the quark-gluon interaction lagrangian are explicitly incorporated into the model space. By studying the NN interaction within the framework of the resonating group method very explicit coordinate space results can be attained. These make it possible to isolate the interaction arising from the exchange of a $(q\bar{q})$ pair between two nucleons, thus leading to a unified picture in which baryons and meson fields are treated on an equal footing in terms of a pure quark model.

In ref. [3]) it was shown that this model makes good contact with the conventional meson exchange potentials by isolating from the many more complicated exchange terms those contributions to the exchange kernels which correspond to an exchange between two nucleons of a $(q\bar{q})$ pair with the color singlet character of a real pseudoscalar or vector meson. The effective potentials arising from these simple $(q\bar{q})$ exchanges are in remarkably good agreement with the corresponding OBEP's for $r \geq 1.2$ fm and have the same qualitative radial features over an even wider range. The simple $(q\bar{q})$ exchange potentials also have all the characteristics of conventional

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OBEP's in their dependence on nucleon $(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$ and $(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$ factors and the relative importance and signs of the spin-spin, spin-independent central, *LS*, and tensor terms. The major quantitative failure of the simple $(q\bar{q})$ exchange potential involves the pion tensor term which is too weak by a factor of ~ 3 in agreement with the predicted [2]) pion-nucleon coupling constant, $g^2_{NN\pi}$. Since a simple $(q\bar{q})$ cluster with the quantum numbers of a pion cannot be expected to give a realistic picture of the pseudoscalar meson with the mass of a real pion, a quantitative fit of the OPEP (including its long-range Yukawa tail) was not expected. However, we consider it an advantage of the model that it can pinpoint such specific components of the NN interaction in terms of the corresponding pieces of the quark-exchange kernel. These can therefore be isolated and improved to give a realistic picture of the NN interaction. It was shown in ref. [4]) that, with such an adjustment of the pion tensor term, both tensor and *LS* forces of the extended quark model are in remarkably good agreement with the experimental facts.

In refs. [2-4]) the quark exchange kernels for the two-nucleon system are calculated with improved nucleon wave functions in which the (3q) components of the single-nucleon wave functions are augmented by the $(3q)(q\bar{q})$ components generated by the off-shell terms of the Fermi-Breit quark-gluon interaction. The coupling kernels which connect the (3q)-(3q) components to the $(3q)$-$(3q)(q\bar{q})$ components of the two-nucleon system lead to a medium range attractive part in the effective NN potential and a greatly reduced repulsive core with a strong energy dependence. However, the attractive part was too weak [2]) to bind the deuteron or fit the low-energy phase shifts. Since the $(q\bar{q})$ excitations inherent in the quark-gluon interaction lagrangian cannot carry the quantum numbers of a scalar σ or δ meson, the counterparts of the σ or δ exchange potentials of conventional meson theory treatments of the NN interaction were missing from the quark models of refs. [2-4]). However, it was pointed out [3,5]) that excitations with the quantum numbers of a σ or δ meson can be incorporated into the extended quark model of the NN interaction through the $(q\bar{q})(q\bar{q})$ excitations generated by RPA-type off-shell terms which are also a natural part of the full Fermi-Breit quark-gluon interaction. In ref. [5]) it was shown that the inclusion of such excitations in the improved single-nucleon wave functions leads to coupling kernels which give a satisfying semiquantitative fit of the experimental NN scattering data. It is the purpose of the present contribution to show how the $(q\bar{q})(q\bar{q})$ excitations are incorporated into the extended quark model of the NN system and how the associated RGM formalism is developed to lead to the phase-shift calculations including the important effects of channel coupling.

Sect. 2 shows how the improved single-nucleon wave functions are modified by the inclusion of $(3q)(q\bar{q})(q\bar{q})$ components of σ or δ type. These lead to only minimal changes in the $(3q)(q\bar{q})$ amplitudes and themselves have amplitudes which are small enough so that the full quark-exchange kernels are given to good approximation by the inclusion of the new coupling kernels connecting the dominant (3q)-(3q)

components of the two-nucleon system to the (3q)-(3q)(qq̄)(qq̄) components. The construction of these coupling kernels is described in sect. 3. As for the earlier kernels, it is achieved by a transformation from complex GCM to standard RGM form which is simple so that the Wigner transforms of these coupling kernels can again be given in complete analytic form. The single channel S-wave equivalent local potentials are discussed in sect. 4, where it is shown that the coupling kernels of (3q)-(3q)(qq̄)(qq̄) type are vital to gain sufficient medium-range attraction, whereas the strong energy dependence of the short-range parts of the potential arises mainly through the coupling kernels of (3q)-(3q)(qq̄) type. The addition of both contributions, however, lead to potentials in odd partial waves with central terms which are too repulsive. Since channel-coupling effects are vital for the binding and low-energy phase shift analysis of the $^3S_1$ channel, a full RGM analysis is vital for a detailed comparison with experiment. The partial wave decomposition of the RGM kernels and the calculation of the RGM matrix elements is given in sect. 5, (with some of the details given in appendices). The analysis of the experimental NN scattering data in terms of the extended quark model is given in sect. 6. The model gives a satisfying semi-quantitative fit of the experimental facts in terms of a unified picture which not only leads to an understanding of the extreme short-range part of the NN interaction in terms of quark exchanges, but, through the exchange of (qq̄) pairs between nucleons, also contains the main features of the conventional meson exchange description.

2. Improved single nucleon wave functions

The extended quark model of refs. [2-4]) uses a quark interaction in which a phenomenological quark-confining potential is combined with a gluon exchange interaction of the general form of a one-gluon exchange potential through the color analog of the Fermi-Breit approximation. Since the quark-gluon interaction lagrangian includes qq̄ creation and annihilation terms the full Breit interaction includes the five types of terms shown in fig. 1. The detailed form of the off-shell (qq̄)-pair creation interaction, (4a), was given in ref. [2]), together with the Breit qq interaction and the annihilation diagram contribution, (3b). Off-shell contributions of type (5) can lead to additional interactions in a multi-quark system through the

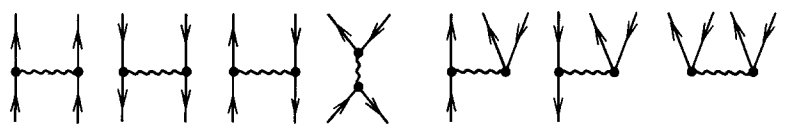

Fig. 1. The full Breit interaction hamiltonian.

quark exchange mechanism. Using the methods and notations of ref. [2]) terms of type (5) lead to a $(q\bar{q})^2$-creation interaction, which in coordinate representation has the form

$$H_{\to(q\bar{q})^2} = \tfrac{1}{2}\int \mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3\,\mathrm{d}x_4\; U_{\to(q\bar{q})^2}(x_1x_3;\, x_2x_4)a^\dagger(x_1)a^\dagger(x_2)b^\dagger(x_4)b^\dagger(x_3) \quad (1a)$$

with

$$U_{\to(q\bar{q})^2}(x_1x_3;\, x_2x_4) = -\alpha_s\,\hbar c 4\sqrt{6}\delta(x_1-x_3)\delta(x_2-x_4) \times F(x_1-x_2)[\,V^{10(11)}(1,3)\times V^{10(11)}(2,4)]^{00(00)}\,, \quad (1b)$$

where the $V^{ST(\lambda\mu)}(i,j)$ are spin, isospin, color-coupled $(q\bar{q})$-pair spin, isospin, color functions which carry the color octet, $(\lambda\mu)=(11)$, $ST=10$ quantum numbers of a gluon [see eq. (8) of ref. [2])]. The spatial function has the form

$$F(\boldsymbol{r}) = -\frac{1}{\pi}\frac{\hbar}{mc}\frac{1}{r^2} + \pi\left(\frac{\hbar}{mc}\right)^2\delta(\boldsymbol{r})\,, \qquad \boldsymbol{r}=\boldsymbol{x}_1-\boldsymbol{x}_2\,, \quad (2)$$

where, following the philosophy of ref. [2]), only terms to second order have been retained in the $\hbar/mc$ expansion. In this interaction the four-particle functions appear naturally in a $(q\bar{q})^2$ basis

$$\xi^{a,ST}_{(q\bar{q})^2}(12;34) = [\,V^{S_1'T_1'(\lambda'\lambda')}_{(q\bar{q})}(1,3)\times V^{S_2'T_2'(\lambda'\lambda')}_{(q\bar{q})}(2,4)]^{ST(00)}\,, \quad (3a)$$

with

$$a \equiv S_1'T_1'S_2'T_2'(\lambda'\lambda')\,, \quad (3b)$$

and where the $(q\bar{q})^2$-creation interaction singles out the term with

$$a_0 = 1010(11) \qquad \text{and} \qquad ST = 00\,. \quad (3c)$$

Since the antisymmetrized form of these $(q\bar{q})^2$ functions do not lead to an orthonormal set, it is more convenient to make a transformation to a $q^2\bar{q}^2$ basis

$$\zeta^{b,ST}_{(q^2\bar{q}^2)}(12;34) = [\,V^{S_1T_1(\lambda\mu)}_{q^2}(1,2)\times V^{S_2T_2(\mu\lambda)}_{\bar{q}^2}(3,4)]^{ST(00)} \quad (4a)$$

with

$$b \equiv S_1T_1S_2T_2(\lambda\mu)\,. \quad (4b)$$

Properly antisymmetric functions are restricted to the set b' with

$$b'\begin{cases} S_1T_1(S_2T_2) = 10 \text{ or } 01 \text{ only for } (\lambda\mu)=(20) \\ S_1T_1(S_2T_2) = 11 \text{ or } 00 \text{ only for } (\lambda\mu)=(01)\,. \end{cases} \quad (4c)$$

The transformation coefficients from the $(q\bar{q})^2$ to the $q^2\bar{q}^2$ basis

$$\xi^{a,ST} = \sum_b \zeta^{b,ST}M^{ST}_{ba} \quad (5)$$

are given by simple recoupling coefficients

$$M^{ST}_{ba} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & S_1' \\ \frac{1}{2} & \frac{1}{2} & S_2' \\ S_1 & S_2 & S \end{bmatrix}\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & T_1' \\ \frac{1}{2} & \frac{1}{2} & T_2' \\ T_1 & T_2 & T \end{bmatrix}\begin{bmatrix} (10) & (01) & (\lambda'\lambda') \\ (10) & (01) & (\lambda'\lambda') \\ (\lambda\mu) & (\mu\lambda) & (00) \end{bmatrix}, \quad (6)$$

where the coefficients in square brackets are the unitary form of the 9-j(SU_2) or 9-($\lambda\mu$)(SU_3) recoupling coefficients. Since the $(q\bar{q})^2$-creation interaction preserves antisymmetry among quarks and among antiquarks, only terms of type b' survive in the transformation to the preferred $q^2\bar{q}^2$ form of the $\xi^{a_0,00}$ which are created within the interaction of eqs. (1a) and (1b).

As in ref. [2]) the first step in the construction of the quark-exchange kernels involves the calculation of improved single-nucleon wave functions in which the dominant (3q) component is now augmented not only by the (3q)($q\bar{q}$) components generated by the ($q\bar{q}$)-pair creation interaction (4a) of fig. 1 but also by the (3q)($q^2\bar{q}^2$) components generated by the $(q\bar{q})^2$-creation interaction, part (5) of fig. 1. The improved single-nucleon wave function then has the form

$$\Psi_N = c_0(\boldsymbol{K})\, e^{i\boldsymbol{K}\cdot\boldsymbol{X}_3}\phi_0(3q) + \sum_{\alpha=1}^{24} c_\alpha(\boldsymbol{K})\, e^{i\boldsymbol{K}\cdot\boldsymbol{X}_5}\phi_\alpha((3q)(q\bar{q}))$$

$$+ \sum_{\beta=1}^{6} c_\beta(\boldsymbol{K})\, e^{i\boldsymbol{K}\cdot\boldsymbol{X}_7}\phi_\beta((3q)(q^2\bar{q}^2))\,. \tag{7}$$

The ϕ_α with the 24 possible spin, isospin, color combinations, including 15 hidden color states, are defined through cluster RGM wave functions by eqs. (10)-(12) of ref. [2]). The additional ϕ_β are expressed in a (3q)($q^2\bar{q}^2$) cluster basis through

$$\phi_\beta = N_{\beta\beta}^{-1/2}\mathcal{A}'\chi_{0s}(\boldsymbol{r},\gamma)[\phi_{(3q)}^{S_1T_1(00)}(123)\times\phi_{(q^2\bar{q}^2)}^{b'S_{23}T_{23}(00)}(45;67)]_{M_SM_T}^{ST(00)}\,, \tag{8}$$

where the spin, isospin color function of the (3q) cluster with $S_1T_1(00)$ is coupled to a $q^2\bar{q}^2$ spin, isospin, color function, $\zeta^{b',S_{23}T_{23}}$, of type (4a) with $b = S_2T_2S_3T_3(\lambda\mu)$ restricted to the antisymmetric combinations b' of eq. (4c), and β is specified by $S_1T_1b'S_{23}T_{23}$; ST. The orbital factors of $\phi_{(3q)}$ and $\phi_{(q^2\bar{q}^2)}$, are assumed to be 0s oscillator functions in the internal variables with a common oscillator length parameter, b. The cluster relative motion function, $\chi(\boldsymbol{r},\gamma)$, is also taken to be a 0s oscillator function with this same b, $\gamma = (\mu/2b^2)$ with reduced mass factor $\mu = \frac{12}{7}$, and $\boldsymbol{r} = \boldsymbol{X}_4 - \boldsymbol{X}_3$, where the $\boldsymbol{X}_n$ are n-particle c.m. vectors. The antisymmetrizer, $\mathcal{A}'$, which antisymmetrizes between the 3- and 2-quark clusters can be given in terms of a double-coset generator expansion in a cluster-function matrix element,

$$\mathcal{A}' \equiv \mathcal{A}'_{32} = \left[\binom{5}{2}\right]^{-1/2}(1 - 6P_{34} + 3P_{14}P_{25}) \equiv \sqrt{\tfrac{1}{10}}\sum_P C_P P. \tag{9}$$

The ϕ_β form a nonorthogonal set, a property shared with the ϕ_α. However, the ϕ_α, because of their required intrinsic p-wave excitation, are nearly orthogonal [2]) with very small overlaps $\langle\phi_\alpha|\phi_{\alpha'}\rangle$, with $\alpha \neq \alpha'$. Since the ϕ_β, with their 0s-wave relative-motion functions, span a smaller space, a smaller β-basis has been selected to avoid problems of overcompleteness. Hidden color components have been excluded; that is, the (3q) and ($q^2\bar{q}^2$) components are restricted to color singlet, (00), type. In addition the ($q^2\bar{q}^2$) components have been restricted to carry the quantum numbers

of scalar mesons only, with $S_{23}=0$, ($T_{23}=0$ for σ and $T_{23}=1$ for δ-type); and axial-vector terms with $S_{23}=1$ have been omitted from the basis. The single-nucleon basis of ref. [2]) is thus to be expanded by the inclusion of six ϕ_β's, with the b' values listed in table 1. It should be noted that these fully antisymmetrized ϕ_β of scalar (σ, δ) meson type effectively do carry some axial-vector meson contributions because of the fairly strong nonorthogonality of 0s cluster functions of type ϕ_β.

TABLE 1

Single nucleon $(3q)$-$(q^2\bar{q}^2)$ components

β	b' $S_2T_2S_3T_3(\lambda\mu)$	$S_{23}T_{23}$	$N^{-1}_{\beta\beta}$	$X^\beta_{(34)}$	$X^\beta_{(14)(25)}$	$H_{\beta 0}$(MeV)	c_β
$N\sigma_1$	0000(01)	00	1	$-\frac{1}{24\sqrt{2}}$	$-\frac{1}{12\sqrt{2}}$	0	−0.004
$N\sigma_2$	1111(01)	00	$\frac{9}{5}$	$\frac{1}{24\sqrt{2}}$	$\frac{1}{108\sqrt{2}}$	555	−0.100
$N\sigma_3$	0101(20)	00	2	$\frac{\sqrt{3}}{48}$	0	806	−0.195
$N\sigma_4$	1010(20)	00	2	$-\frac{1}{48\sqrt{3}}$	0	−269	0.043
$N\delta_1$	1111(01)	01	$\frac{3}{2}$	$\frac{1}{72}$	$\frac{1}{108}$	179	−0.053
$N\delta_2$	0101(20)	01	$\frac{3}{2}$	$\frac{1}{72}\sqrt{\frac{3}{2}}$	0	329	−0.138

To solve the improved single-nucleon problem and evaluate the c_0, c_α, c_β, new coupling matrix elements of type $H^{\boldsymbol{K}}_{\beta 0}$ and $H^{\boldsymbol{K}}_{\beta\beta'}$ have to be evaluated. Matrix elements of type $H^{\boldsymbol{K}}_{\beta\alpha}$ would make only negligible contributions to the c_α and c_β and are therefore set equal to zero. The simple coupling matrix elements of type $H^{\boldsymbol{K}}_{\beta 0}$ are given through the analog of eq. (9) of ref. [2]) by

$$H^{\boldsymbol{K}}_{\beta 0}=-\sqrt{10}\langle e^{i\boldsymbol{K}\cdot\boldsymbol{X}_7}\phi_\beta \,|\, U_{(\to q\bar{q})^2}(\boldsymbol{x}_4\boldsymbol{x}_6;\boldsymbol{x}_5\boldsymbol{x}_7)|\, e^{i\boldsymbol{K}\cdot\boldsymbol{X}_3}\phi_0\rangle\,. \tag{10}$$

These separate into space and spin, isospin, color parts through

$$H^{\boldsymbol{K}}_{\beta 0}=\alpha_s\hbar c 4\sqrt{6}N^{-1/2}_{\beta\beta}\sum_P C_P I^{\boldsymbol{K}}_P X^\beta_P\,, \tag{11}$$

where the weighting coefficients, C_P, are given in eq. (9). The spin, isospin, color factors are given by

$$\begin{aligned} X^\beta_P &\equiv X^{S_1T_1b'S_{23}T_{23};ST}_P \\ &= M^{00}_{b_0'a_0}\langle P^{\beta,ST}_{\zeta(3q)(q^2\bar{q}^2)}(1,\ldots,7)\,|\,\zeta^{STb_0'00;\,ST}_{(3q)(q^2\bar{q}^2)}(1,\ldots,7)\rangle\,, \end{aligned} \tag{12}$$

where the 4-particle spin, isospin, color function of the $(q\bar{q})^2$-creation interaction with a_0, $S_{23}T_{23}=00$, (see eq. (3)), has been transformed to $q^2\bar{q}^2$ β'-form when acting

on the (3q) ket with $S_1 T_1 = ST$. Since $b' \equiv S_2 T_2 S_3 T_3 (\lambda\mu)$ in the bra function, and since P is independent of the particle numbers 6 and 7 of the antisymmetric $\bar{q}^2$ function, the $\bar{q}^2$ overlap together with the ket value $S_{23} T_{23} = 00$ restricts the b'_0 value in the summation of the transformation (5) to the single value $b'_0 = S_3 T_3 S_3 T_3 (\lambda\mu)$. The overlaps are evaluated by the recoupling techniques of ref. [2]). The X_P^β are enumerated in table 1.

The spatial integrals I_P^K of eq. (11) are given by

$$I_P^K = \langle e^{i\boldsymbol{K}\cdot\boldsymbol{X}_7} P \chi_{0s}(\boldsymbol{r}, \gamma) \phi_{0s}^{(3q)}(123) \phi_{0s}^{(q^2\bar{q}^2)}(45; 67) | \delta(\boldsymbol{x}_4 - \boldsymbol{x}_6) \times \delta(\boldsymbol{x}_5 - \boldsymbol{x}_7) F(\boldsymbol{x}_4 - \boldsymbol{x}_5) | e^{i\boldsymbol{K}\cdot\boldsymbol{X}_3} \phi_{0s}^{(3q)}(123) \rangle . \tag{13}$$

With $\boldsymbol{X}_3 = \boldsymbol{X}_7 - \frac{4}{7}\boldsymbol{r}$, the center-of-mass integral, (over the assumed unit volume, with value 1), is carried out first. Since the product of 0s oscillator functions in the bra is totally symmetric in the $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_7$, and hence independent of P, it can be evaluated for $P = 1$. The internal and $\boldsymbol{r}$ integrals can then be carried out through an expansion of $F(\boldsymbol{r})$ in terms of gaussian functions, using the techniques of refs. [2-4]), to yield

$$I_P^K = -\frac{x}{b}\frac{1}{\pi}\left(\frac{7}{3}\right)^{3/4} [1 - \tfrac{1}{2} x \sqrt{\tfrac{1}{2}\pi}] \exp[-\tfrac{2}{21}(b\boldsymbol{K})^2] , \tag{14}$$

with $x = \hbar / mcb$. Only the exchange terms, (with $P = (34)$ and $P = (14)(25)$), are retained. Although there would be a direct term for coupling to states β with $S_{23} = T_{23} = 0$, this direct term, arising through a disconnected diagram, would give a contribution even at large separation of the (3q) and $(q^2\bar{q}^2)$ clusters. The contributions of such disconnected diagrams should in principle involve a renormalization of the vacuum expectation value, and should also yield self-energy contributions to the constituent-quark mass in quark exchange diagrams, particularly in simple $(q\bar{q})^2$-exchange diagrams for the NN system. Since we do not expect to be able to predict absolute values of baryon or meson masses with our model and our aim instead is to investigate the NN interaction through the quark-exchange kernels, only the exchange terms will be retained. These lead to the coupling matrix elements

$$H_{\beta 0}^K = \alpha_s x^2 mc^2 \frac{12\sqrt{6}}{\pi}\left(\frac{7}{3}\right)^{3/4} N_{\beta\beta}^{-1/2} [1 - \tfrac{1}{2} x \sqrt{\tfrac{1}{2}\pi}] \times [2X_{(34)}^\beta - X_{(14)(25)}^\beta] \exp[-\tfrac{2}{21}(b\boldsymbol{K})^2] . \tag{15}$$

As in ref. [2]) the secular problem for the c_0, c_α, c_β is solved for $\boldsymbol{K} = 0$, since the interest is in low-energy NN scattering so that the $c(\boldsymbol{K})$ can be replaced by $c(0)$. Thus the rest frame $\boldsymbol{K} = 0$ values for the single-nucleon matrix elements are sufficient. The numerical values for these new coupling matrix elements are shown in table 1.

Single-nucleon matrix elements diagonal in the $(3q)(q^2\bar{q}^2)$ space can be expressed through

$$H_{\beta\beta'}^K = \left(7mc^2 + \frac{\hbar^2 \boldsymbol{K}^2}{2(7m)}\right) N_{\beta\beta}^{-1/2} N_{\beta'\beta'}^{-1/2} N_{\beta\beta'} + N_{\beta\beta}^{-1/2} N_{\beta'\beta'}^{-1/2} \hat{H}_{\beta\beta'} . \tag{16}$$

The $\hat{H}_{\beta\beta'}$ gain contributions from the kinetic energy (motion relative to the c.m.) and the quark interactions of types (1), (2), (3a) and (3b) of fig. 1,

$$\hat{H}_{\beta\beta'} = \sum_P C_P \langle \hat{\phi}_\beta | \sum_{i=1}^{7} t_i - T_{\text{c.m.}} + \sum_{i<j}^{5} U_{\text{qq}}(ij) + U_{\bar{\text{q}}\bar{\text{q}}}(67) + \sum_{i=1}^{5}\sum_{j=6}^{7} [U_{\text{q}\bar{\text{q}}}(ij) + U_{\text{q}\bar{\text{q}}}^{\text{ann.}}(ij)] | P\hat{\phi}_{\beta'} \rangle , \tag{17}$$

where $\hat{\phi}_\beta$ are defined through eq. (8) by

$$\phi_\beta = N_{\beta\beta}^{-1/2} \mathscr{A}' \hat{\phi}_\beta . \tag{18}$$

In terms of the four parameters of our interaction α_s, m, b, a_c (with $x = \hbar/mcb$), the $\hat{H}_{\beta\beta'}$ can be reduced to spin, isospin, color matrix elements; and with $\boldsymbol{K} = 0$, yield

$$H_{\beta\beta'} = N_{\beta\beta}^{-1/2} N_{\beta'\beta'}^{-1/2} N_{\beta\beta'} \Big[56 a_c b^2 + mc^2 \Big\{ 7 + \tfrac{9}{2}x^2 + \sqrt{\frac{2}{\pi}}\alpha_s x [-\tfrac{14}{3} + x^2(\tfrac{7}{6} - \tfrac{1}{24}F_{\beta\beta'}^{\text{SC}} + \tfrac{1}{24}F_{\beta\beta'}^{\text{ann.}})] \Big\} \Big] , \tag{19}$$

where the norm, spin-color (SC), and annihilation diagram (ann.) spin, isospin, color factors are given by

$$N_{\beta\beta'} \begin{Bmatrix} 1 \\ F_{\beta\beta'}^{\text{SC}} \\ F_{\beta\beta'}^{\text{ann.}} \end{Bmatrix} = \sum_P C_P \langle \zeta^{\beta,ST} | \begin{Bmatrix} 1 \\ \sum_{i<j}^{7} (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)(\lambda_i \cdot \lambda_j) \\ \sum_{i=1}^{5}\sum_{j=6}^{7} \left(\frac{1 - \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j}{2}\right)\left(\frac{\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j}{2}\right)^2 \left(\frac{\lambda_i + \lambda_j}{2}\right)^2 \end{Bmatrix} | P\zeta^{\beta',ST} \rangle . \tag{20}$$

The diagonal terms ($\beta' = \beta$), in particular, can be given by

$$H_{\beta\beta} = E^{S_1 T_1}(3\text{q}) + E_{b'b'}^{S_{23} T_{23}}(\text{q}^2\bar{\text{q}}^2) + mc^2 \Big\{ \tfrac{3}{4}x^2 + \sqrt{\frac{2}{\pi}}\alpha_s x^3 [\tfrac{1}{2} - \tfrac{2}{9}S_1(S_1+1) - \tfrac{1}{24}(F_{\beta\beta}^{\text{SC}} - f_{b'b'}^{\text{SC};S_{23}T_{23}}) + \tfrac{1}{24}(F_{\beta\beta}^{\text{ann.}} - f_{b'b'}^{\text{ann.};S_{23}T_{23}})] \Big\} \tag{21}$$

with $\beta = S_1 T_1 b' S_{23} T_{23}$; ST and $b' = S_2 T_2 S_3 T_3 (\lambda\mu)$ following the restrictions of eq. (4c). The color singlet (3q) internal energy $E^{S_1 T_1}(3\text{q})$ can be read from eq. (17) of ref. [2]), (with $(\lambda\lambda) = (00)$), while the $\text{q}^2\bar{\text{q}}^2$ internal energy is given by

$$E_{b'b'}^{ST}(\text{q}^2\bar{\text{q}}^2) = 32 a_c b^2 + mc^2 \Big[4 + \tfrac{9}{4}x^2 + \sqrt{\frac{2}{\pi}}\alpha_s x(-\tfrac{8}{3} + \tfrac{2}{3}x^2) \Big] + \sqrt{\frac{2}{\pi}}\alpha_s x^3 mc^2 (-\tfrac{1}{24} f_{b'b'}^{\text{SC};ST} + \tfrac{1}{24} f_{b'b'}^{\text{ann.};ST}) . \tag{22}$$

The $q^2\bar{q}^2$ spin, isospin, color factors are given by

$$f^{SC;ST}_{b'b''} = \delta_{b'b''}[S_2(S_2+1)+S_3(S_3+1)-3][\langle\lambda^2\rangle_{(\lambda\mu)}-\tfrac{32}{3}] + 4\sum_a M^{ST}_{b'a} M^{ST}_{b''a}[S^a_1(S^a_1+1)-\tfrac{3}{2}][\langle\lambda^2\rangle_{(\lambda^a\lambda^a)}-\tfrac{32}{3}], \quad (23)$$

$$f^{ann.;ST}_{b'b''} = \sum_a M^{ST}_{b'a} M^{ST}_{b''a} S^a_1(S^a_1+1)[2-T^a_1(T^a_1+1)]\langle\lambda^2\rangle_{(\lambda^a\lambda^a)}, \quad (24)$$

where

$$\langle\lambda^2\rangle_{(\lambda\mu)} = \tfrac{4}{3}(\lambda^2+\lambda\mu+\mu^2+3\lambda+3\mu), \quad (25)$$

and the $M^{ST}_{b'a}$ are the transformation coefficients from the $q^2\bar{q}^2$ basis with b'; $ST(= S_2\,T_2\,S_3\,T_3(\lambda\mu); S_{23}\,T_{23})$, to the $(q\bar{q})(q\bar{q})$ basis specified by a; $ST = S^a_1 T^a_1 S^a_2 T^a_2(\lambda^a\lambda^a)$; ST, (cf. eqs. (3)-(5)). The $N_{\beta\beta'}F^{SC}_{\beta\beta'}$, $N_{\beta\beta'}F^{ann.}_{\beta\beta'}$ have been evaluated by the recoupling techniques used in ref. [2]).

With the $H_{\beta 0}$ and $H_{\beta\beta'}$ of eqs. (15) and (19), diagonalization in the full 31-dimensional space of the single-nucleon wave function of eq. (7) leads to a new set of amplitudes c_0, c_α, c_β. Since the ϕ_β do not contribute directly to the single-nucleon electromagnetic properties or the nucleon-vector meson coupling constants, two of the properties used in fixing the parameters of the model, no reevaluation of these parameters was undertaken. The values of ref. [2]) were retained for α_s, m, b, and a_c. Although the new improved single-nucleon wave functions no longer give an exact fit to the N and Δ masses, it was shown in ref. [2]) that the NN interation was given by the strengths of the c's but was not sensitive to the details of the parameter fits. In particular, it was shown to be very insensitive to the value of a_c, (confinement potential constant) which was chosen to fit the N mass in the previous calculation of refs. [2,3]). The new c_α and c_0 are almost identical to those quoted in table 1 of ref. [2]). (The new c_0 is 0.849 compared with 0.857 of ref. [2]); $c_{N\rho 3/2}$, e.g., is changed to -0.239 from -0.255). The final values of the six additional c_β are listed in table 1. It can be seen that some of these have strengths comparable to those of the important c_α, but still smaller than the biggest value $c_{\Delta\rho 3/2} = -0.241$.

3. The $(3q)(q\bar{q})^2$ coupling kernels

The improved single-nucleon functions of sect. 2 are used to evaluate the quark-exchange kernels for the two-nucleon system

$$G(\boldsymbol{R},\boldsymbol{R}') = c_0^4 G'_0(\boldsymbol{R},\boldsymbol{R}') + c_0^3 \sum_{\alpha=1}^{24} c_\alpha G^{\mathscr{S}}_\alpha(\boldsymbol{R},\boldsymbol{R}') + c_0^3 \sum_{\beta=1}^{6} c_\beta G^{\mathscr{S}}_\beta(\boldsymbol{R},\boldsymbol{R}') + \cdots, \quad (26a)$$

where the symmetrized form of the coupling kernels, e.g.,

$$G^{\mathscr{S}}_\beta(\boldsymbol{R},\boldsymbol{R}') = G_\beta(\boldsymbol{R},\boldsymbol{R}') + G_\beta(\boldsymbol{R}',\boldsymbol{R})^\dagger \quad (26b)$$

are needed since the $(3q)(q^2\bar{q}^2)$ admixtures in the single-nucleon internal wave functions appear symmetrically in both bra and ket. As in refs. [2-4]) terms of second and higher order in the c_α, c_β are neglected. The new coupling kernels G_β connecting the (3q)-(3q) components to the $(3q)$-$(3q)(q^2\bar{q}^2)$ components now have the form

$$G_\beta(\boldsymbol{R}, \boldsymbol{R}') = -\left[\binom{8}{2}\right]^{1/2} \langle \mathscr{A}'_{53}\, \phi_0(123)\phi_\beta(4\ldots10)\delta(\boldsymbol{r}_{0\beta}-\boldsymbol{R})| \times U_{\to(q\bar{q})^2}(x_7 x_9; x_8 x_{10})|\mathscr{A}'_{33}\, \phi_0(123)\phi_0(456)\delta(\boldsymbol{r}_{00}-\boldsymbol{R}')\rangle\,, \tag{27}$$

where we have again used the analog of eq. (9) of ref. [2]), and where

$$\mathscr{A}'_{53} = \left[\binom{8}{3}\right]^{-1/2} \sum_P{}' \delta_P P, \qquad \mathscr{A}'_{33} = \left[\binom{6}{3}\right]^{-1/2} \sum_P{}' \delta_P P \tag{28}$$

antisymmetrize quarks between the 5- and 3-quark clusters and the 3- and 3-quark clusters of bra and ket, respectively (with $\delta_P = \pm 1$ for P even/odd). Note that the ϕ_β in the $(3q)(q^2\bar{q}^2)$ basis are automatically antisymmetric in the antiquarks labelled by 9 and 10. These antisymmetrizers can be converted to a single one acting on a 3+3+2-quark (3q)-(3q)-$(q^2\bar{q}^2)$ three-cluster function in the bra, by expanding ϕ_β through the $\mathscr{A}'_{32}$ of eq. (9), transferring $\mathscr{A}'_{33}$ to the bra side, and using the identity

$$\left[\binom{8}{2}\right]^{1/2} \mathscr{A}'_{33}\mathscr{A}'_{53}\mathscr{A}'_{32} = \mathscr{A}'' \equiv \sum_P{}'' \delta_P P. \tag{29}$$

Note that $\mathscr{A}''$ is normalized such that the $(8!/3!3!2!) = 560$ terms in the sum of the 3+3+2-particle three-cluster expansion are all weighted with ± 1, *without* additional normalization factors. Moreover, in the matrix element (27) the expansion of $\mathscr{A}''$ can be reduced to a sum over 35 double-coset generators [4,6]) which can be further simplified to a sum of 18 terms with the use of the exchange operator P_0, where $P_0 = P_{14}P_{25}P_{36}$ exchanges the three quarks of the two 3q-clusters. Thus

$$\begin{aligned}\mathscr{A}'' = (1-P_0)[&1 - 6P_{67} + 3P_{47}P_{58} - 6P_{37} + 3P_{17}P_{28} - 9P_{36} \\ &+ 18(P_{38}P_{67} + P_{36}P_{37} + P_{37}P_{36}) + 36(P_{36}P_{57} + P_{36}P_{27}) \\ &- 9(P_{36}P_{47}P_{58} + P_{36}P_{17}P_{28}) - 18(P_{57}P_{37}P_{48} + P_{27}P_{67}P_{18} \\ &+ P_{48}P_{37}P_{57} + P_{18}P_{67}P_{27})] - 72P_{14}P_{38}P_{67}\,.\end{aligned} \tag{30}$$

Since the combination of permutations

$$(1 - 6P_{67} + 3P_{47}P_{58})$$

contribute to the single-nucleon internal energies through the $\phi_\beta(4\cdots10)$ components of the improved single-nucleon wave functions, these must be eliminated from $\mathscr{A}''$ in order to convert the G_β of eq. (27) into a true two-nucleon exchange kernel. The factor $(1-P_0)$ on the left-hand side of this expression for $\mathscr{A}''$ can be replaced by the simple factor 2. In the product of antisymmetrizers in the bra side

of G_β the hermitean conjugation of such products brings the factor $(1-P_0)$ to the right-hand side where it gives the factor 2 when acting on a two-nucleon function of good parity in the ket; (see the discussion in connection with eqs. (33) and (35) of ref. [2])). In the two-nucleon exchange kernel, G_β, the antisymmetrizer can thus be replaced by

$$\begin{aligned}\mathcal{A}'' \to 2\sum_P C_P P \\ = 2[&-6P_{37}+3P_{17}P_{28}-9P_{36}+18(P_{38}P_{67}+P_{36}P_{37}+P_{37}P_{36}) \\ &+36(P_{36}P_{57}+P_{36}P_{27})-9(P_{36}P_{47}P_{58}+P_{36}P_{17}P_{28}) \\ &-18(P_{57}P_{37}P_{48}+P_{27}P_{67}P_{18}+P_{48}P_{37}P_{57}+P_{18}P_{67}P_{27})-36P_{14}P_{38}P_{67}]\end{aligned} \tag{31}$$

leading to 15 distinct types of exchange terms.

For the evaluation of the kernels it is useful to characterize the double coset generators for the 3+3+2 three-cluster decomposition of the 8 quarks in the $(3q)$-$(3q)(q^2\bar{q}^2)$ cluster function by the double-coset symbol [6]), d_{ab}, which gives the sum of the unit matrix elements in the abth submatrix of the 8×8 matrix representation of P. This can again be specified by four integers $xyuv$ (cf. eq. (8) of ref. [4]))

$$d_{ab}=\begin{pmatrix} 3-x & u & x-u \\ v & 3-y & y-v \\ x-v & y-u & 2-x-y+u+v \end{pmatrix}, \tag{32}$$

where the $xyuv$ are listed in table 2 for the 15 needed double-coset generators of eq. (31). The exchange kernels can be separated into space and spin, isospin, color factors through

$$G_\beta(\boldsymbol{R},\boldsymbol{R}')=N_{\beta\beta}^{-1/2}\sum_P C_P G_P^\beta(\boldsymbol{R},\boldsymbol{R}')\,, \tag{33a}$$

$$G_P^\beta(\boldsymbol{R},\boldsymbol{R}')=\alpha_s\,\hbar c 8\sqrt{6}I_P(\boldsymbol{R},\boldsymbol{R}')X_P^\beta\,, \tag{33b}$$

where the spin, isospin, color factors

$$\begin{aligned}X_P^\beta=\langle P[\zeta^{\frac{11}{22}(00)}(123)\times\zeta^{\beta,\frac{11}{22}}(4\cdots 10)]^{ST}|[V^{10(11)}(79) \\ \times V^{10(11)}(8\,10)]^{00(00)}|[\zeta^{\frac{11}{22}(00)}(123)\times\zeta^{\frac{11}{22}(00)}(456)]^{ST}\rangle\end{aligned} \tag{34}$$

TABLE 2

The basic exchange types

P	$xyuv$	P	$xyuv$	P	$xyuv$
P_{37}	1000	$P_{37}P_{36}$	1110	$P_{57}P_{37}P_{48}$	1210
$P_{17}P_{28}$	2000	$P_{36}P_{57}$	1211	$P_{27}P_{67}P_{18}$	2101
P_{36}	1111	$P_{36}P_{27}$	2111	$P_{48}P_{37}P_{57}$	1201
$P_{38}P_{67}$	1100	$P_{36}P_{47}P_{58}$	1311	$P_{18}P_{67}P_{27}$	2110
$P_{36}P_{37}$	1101	$P_{36}P_{17}P_{28}$	3111	$P_{14}P_{38}P_{67}$	2211

are evaluated by the generalization of the methods used to evaluate the single-nucleon spin, isospin, color factor of eq. (11). The spatial integrals

$$I_P(\boldsymbol{R}, \boldsymbol{R}') = \langle P\delta(\boldsymbol{r}_{0\beta} - \boldsymbol{R})\chi_{0s}(\boldsymbol{\rho}, \gamma)\phi_{0s}(3q)\phi_{0s}(3q)\phi_{0s}(q^2\bar{q}^2)| \times \delta(\boldsymbol{x}_7 - \boldsymbol{x}_9)\delta(\boldsymbol{x}_8 - \boldsymbol{x}_{10})F(\boldsymbol{x}_7 - \boldsymbol{x}_8)|\delta(\boldsymbol{r}_{00} - \boldsymbol{R}')\phi_{0s}(3q)\phi_{0s}(3q)\rangle \quad (35)$$

are calculated, as in refs. [2-4]), through their Bargmann transforms, (or complex GCM form [7-10]), and subsequent transformation from complex GCM to RGM form, by expanding the two parts of $F(\boldsymbol{r})$ in terms of gaussian radial functions. With the replacement

$$F(\boldsymbol{r}) \to \exp[-\chi^2 \boldsymbol{r}^2],$$

the gaussian complex GCM kernels are given by

$$I_P^{\mathrm{GCM}}(z, z') = \langle PA_{\gamma_2}(\boldsymbol{r}_{0\beta}, z)\chi_{0s}(\boldsymbol{\rho}, \gamma)\phi_{0s}(3q)\phi_{0s}(3q)\phi_{0s}(q^2\bar{q}^2)| \times \delta(\boldsymbol{x}_7 - \boldsymbol{x}_9)\delta(\boldsymbol{x}_8 - \boldsymbol{x}_{10})\, e^{-\chi^2(\boldsymbol{x}_7 - \boldsymbol{x}_8)^2}|A_{\gamma_0}(\boldsymbol{r}_{00}, z')\phi_{0s}(3q)\phi_{0s}(3q)\rangle . \quad (36)$$

The RGM form is obtained through the Bargmann transformation

$$I_P(\boldsymbol{R}, \boldsymbol{R}') = \int \mathrm{d}\mu(z)\, \mathrm{d}\mu(z') A_{\gamma_2}(\boldsymbol{R}, z) A_{\gamma_0}(\boldsymbol{R}', z')^* I_P^{\mathrm{GCM}}(z, z') . \quad (37)$$

Here, z, z' are 3-dimensional complex Bargmann space variables, and the 6-dimensional $\mathrm{d}\mu(z)$ contains the Bargmann weighting factor

$$\mathrm{d}\mu(z) = \pi^{-3} \exp(-z^* \cdot z)\, \mathrm{d}^3(\mathrm{Re}\, z)\, \mathrm{d}^3(\mathrm{Im}\, z) . \quad (38a)$$

The Bargmann kernel functions are given by

$$A_\gamma(\boldsymbol{R}, z) = \left(\frac{2\gamma}{\pi}\right)^{3/4} \exp[-\gamma(\boldsymbol{R} - z/\sqrt{\gamma})^2 + \tfrac{1}{2}z^2] . \quad (38b)$$

The relative motion vectors are given in terms of n-particle c.m. vectors, $\boldsymbol{X}_n$, by $\boldsymbol{\rho} = \boldsymbol{X}_4 - \boldsymbol{X}_3^{(2)}$, $\boldsymbol{r}_{00} = \boldsymbol{X}_3^{(1)} - \boldsymbol{X}_3^{(2)}$, $\boldsymbol{r}_{0\beta} = \boldsymbol{X}_3^{(1)} - \frac{1}{7}(4\boldsymbol{X}_4 + 3\boldsymbol{X}_3^{(2)})$, with associated oscillator parameters $\gamma = (6/7b^2)$, $\gamma_0 = (3/4b^2)$, $\gamma_2 = (21/20b^2)$. Since the coupling kernels of eqs. (35) and (36) involve a 3+5-quark cluster decomposition in the bra with internal wave functions symmetric in the 5-particle variables, whereas the $(q\bar{q})^2$-creation interaction acting on the (3q)-(3q) functions generates a 3+3+2-quark decomposition, the kernels depend on exchange type only through the double coset parameters x and u. Under reduction to a $3+5 \to 3+3+2$ cluster decomposition, the full $3+3+2 \to 3+3+2$ DC symbol of eq. (32) reduces (through addition of the 2nd and 3rd 3- and 2-particle rows) to a DC symbol independent of y and v,

$$d'_{ab} = \begin{pmatrix} 3-x & u & x-u \\ x & 3-u & 2-x+u \end{pmatrix} . \quad (39)$$

The gaussian kernels of eq. (36) are of simple exponential form [7,8]) and are given by

$$I_P^{\mathrm{GCM}}(z, z') = \left(\sqrt{\frac{5}{3}}\frac{1}{1+2b^2\chi^2}\right)^{3/2} \exp\left[-\tfrac{1}{2}fz^{*2} + h(z^* \cdot z')\right] \tag{40a}$$

with

$$f = -\tfrac{2}{7}\left[1 - \tfrac{5}{3}(x-u) + \tfrac{5}{18}(x-u)^2 - \delta_{x-u,1}\tfrac{5}{3}\kappa\right],$$
$$h = \sqrt{\tfrac{5}{7}}\left[1 - \tfrac{1}{3}(x+u)\right], \tag{40b}$$

where $\kappa = \chi^2 b^2/(1+2\chi^2 b^2)$. The relationship between exchange type, P, and the parameters x, u can be read from table 2. Note that the χ-dependence sits exclusively in terms with $x-u=1$. For a pure gaussian kernel of this form the transformation to RGM form is carried out by standard formulae (see, e.g., appendix B of ref. [9]) or table 1 of ref. [10])) and yields

$$I_P(\boldsymbol{R}, \boldsymbol{R}') = \left(\sqrt{\frac{5}{3}}\frac{1}{1+2b^2\chi^2}\frac{2\sqrt{\gamma_2\gamma_0}}{\pi}\frac{1}{d}\right)^{3/2} \exp\left[-\left(\frac{2}{d}-1\right)\gamma_2 \boldsymbol{R}^2 - \left(\frac{2(1-f)}{d}-1\right)\gamma_0 \boldsymbol{R}'^2 + \frac{4h}{d}\sqrt{\gamma_2\gamma_0}(\boldsymbol{R}\cdot\boldsymbol{R}')\right], \tag{41}$$

with $d = 1-f-h^2$.

As in refs. [2-4]) the Wigner transform of the RGM kernel will be used to construct an equivalent local potential. Since the coupling kernels appear in the symmetrized form of eq. (26) it will be useful to construct the Wigner transforms of the symmetrized $G_P^\beta(\boldsymbol{R}, \boldsymbol{R}')$ of eq. (33)

$$G_P^{\beta,\mathscr{S}}(\boldsymbol{R}, \boldsymbol{R}') = G_P^\beta(\boldsymbol{R}, \boldsymbol{R}') + G_P^\beta(\boldsymbol{R}', \boldsymbol{R})^\dagger, \tag{42a}$$

$$G_{P\mathrm{W}}^\beta(\boldsymbol{R}, \boldsymbol{P}) = \int \mathrm{d}\boldsymbol{s} \exp\left[\frac{i}{\hbar}(\boldsymbol{s}\cdot\boldsymbol{P})\right]\left\{\begin{matrix} G_P^{\beta,\mathscr{S}}(\boldsymbol{R}-\frac{1}{2}\boldsymbol{s}, \boldsymbol{R}+\frac{1}{2}\boldsymbol{s}) \\ G_P^{\beta,\mathscr{S}}(\boldsymbol{R}-\frac{1}{2}\boldsymbol{s}, -\boldsymbol{R}-\frac{1}{2}\boldsymbol{s}) \end{matrix}\right\}, \tag{42b}$$

where the upper/lower case applies for $h \gtrless 0 / h < 0$ in the $(\boldsymbol{R}\cdot\boldsymbol{R}')$ term of the RGM kernel of eq. (41), (cf. refs. [2-4])). This leads to a Wigner transform

$$G_{P\mathrm{W}}^\beta(\boldsymbol{R}, \boldsymbol{P}) = -\alpha_{\mathrm{s}}(\hbar^2/mb^2)\frac{16\sqrt{6}}{\pi} f_{xu}^l G_{xu}^{\mathrm{W}}(\boldsymbol{R}, \boldsymbol{P}) X_P^\beta \tag{43a}$$

with P given by $xyuv$, and

$$f_{xu}^l = \begin{cases} 1 \\ \delta_{l0} \\ (-1)^l \end{cases} \quad \text{for} \quad \begin{cases} x+u<3 \\ x+u=3 \\ x+u>3\,. \end{cases} \tag{43b}$$

Here, l is the orbital angular momentum of the NN relative motion function, which satisfies $(-1)^{l+S+T} = -1$. Note that with $x+u=3$, ($h=0$ in eq. (40)), there is a

contribution only for S-wave scattering. (In this case the RGM kernel becomes separable). For the simple $1/r^2$ and $\delta(r)$ radial factors of the $(q\bar{q})^2$-creation interaction, (cf. eq. (2)), it is possible to give the full Wigner transform in analytic form, with

$$G^{W}_{xu}(\boldsymbol{R},\boldsymbol{P}) = D_0^{3/2}\exp\left[-\eta_0\left(\frac{\boldsymbol{R}}{b}\right)^2-\zeta_0\left(\frac{b\boldsymbol{P}}{\hbar}\right)^2\right]\cos\left[\xi_0\frac{(\boldsymbol{R}\cdot\boldsymbol{P})}{\hbar}\right]$$
$$\times\left[D_1^{1/2}\left\{C(\alpha,\beta)+S(\alpha,\beta)\tan\left[\xi_0\frac{(\boldsymbol{R}\cdot\boldsymbol{P})}{\hbar}\right]\right\}-\frac{\hbar}{2mcb}\sqrt{\frac{\pi}{2}}\right] \quad (44a)$$

with

$$\alpha=\eta_1\left(\frac{\boldsymbol{R}}{b}\right)^2-\zeta_1\left(\frac{b\boldsymbol{P}}{\hbar}\right)^2, \qquad \beta=\xi_1\frac{(\boldsymbol{R}\cdot\boldsymbol{P})}{\hbar} \quad (44b)$$

and with

$$C(\alpha,\beta)=\int_0^1 \mathrm{d}t\,\mathrm{e}^{\alpha t^2}\cos(\beta t^2),$$
$$S(\alpha,\beta)=\int_0^1 \mathrm{d}t\,\mathrm{e}^{\alpha t^2}\sin(\beta t^2). \quad (44c)$$

The coefficients D_i, η_i, ζ_i, ξ_i are given for the needed xu in table 3.

TABLE 3

The xu-dependent coefficients for eq. (44)

xu	00	10 32	20 31	11	21
D_0	$\sqrt{\frac{7}{3}}$	$\frac{63}{47}\sqrt{\frac{7}{3}}$	$\frac{63}{32}\sqrt{\frac{7}{3}}$	$\frac{21}{10}\sqrt{\frac{7}{3}}$	$\frac{63}{20}\sqrt{\frac{7}{3}}$
η_0	0	$\frac{13\cdot 21}{4\cdot 94}$	$\frac{21}{16}$	$\frac{7}{10}$	$\frac{17\cdot 21}{4\cdot 40}$
ζ_0	$\frac{2}{21}$	$\frac{7}{94}$	$\frac{3}{16}$	$\frac{19}{45}$	$\frac{41}{3\cdot 40}$
ξ_0	0	$\frac{43}{94}$	$\frac{5}{8}$	$\frac{2}{15}$	$\frac{39}{40}$
D_1	1	$\frac{4\cdot 47}{5\cdot 37}$	1	1	$\frac{80}{77}$
η_1	0	$\frac{5\cdot 21\cdot 21}{8\cdot 37\cdot 47}$	0	0	$\frac{21\cdot 27}{40\cdot 44}$
ζ_1	0	$\frac{31\cdot 31}{10\cdot 37\cdot 47}$	0	0	$\frac{3\cdot 21}{11\cdot 40}$
ξ_1	0	$\frac{21\cdot 31}{2\cdot 37\cdot 47}$	0	0	$\frac{7\cdot 27}{11\cdot 40}$

4. Equivalent local potentials

In the single-channel approximation equivalent local potentials can be calculated from the Wigner transforms $G_W(R^2; P^2; (\boldsymbol{R}\cdot\boldsymbol{P})^2)$ of the quark-exchange kernels via the self-consistent equation

$$U^{\text{eff}}(R) = G_W(R^2; M_N[E_{\text{c.m.}} - U^{\text{eff}}(R)]; M_N R^2[E_{\text{c.m.}} - U^{\text{eff}}(R) - \hbar^2(l+\tfrac{1}{2})^2/M_N R^2]) . \tag{45}$$

Here, the Wigner transform, G_W, is that for the full kernel of eq. (26), where the (3q)-(3q) piece, G_0', gains a contribution both from an interaction kernel and the norm kernel

$$G = c_0^4[G^{(E)}(3q) - (2E_N + E_{\text{c.m.}})N^{(E)}] + c_0^3 \sum_{\alpha=1}^{24} c_\alpha G_\alpha^{\mathcal{S}} + c_0^3 \sum_{\beta=1}^{6} c_\beta G_\beta^{\mathcal{S}} . \tag{46}$$

The total energy has been expressed in terms of the internal energy of the two nucleons and $E_{\text{c.m.}}$ for the NN relative motion. Since the pair creation interactions contribute to the full G not only through the coupling kernels, $G_\alpha^{\mathcal{S}}$, $G_\beta^{\mathcal{S}}$, but also through their contributions to the internal energy, it may be useful to exhibit this dependence explicitly, particularly since it may be interesting to isolate the pure (3q) contributions to the potentials from those for the (3q)(q$\bar{\text{q}}$) and (3q)(q$^2\bar{\text{q}}^2$) cross terms. If γ designates one of the 31 components $\gamma = 0, \alpha, \beta$, the single-nucleon internal energy is obtained from

$$\sum_{\gamma'=0}^{30} [H_{\gamma\gamma'} - E_N N_{\gamma\gamma'}] c_{\gamma'} = 0 . \tag{47}$$

Setting $\gamma = 0$, this leads at once to

$$E_N = E_0 + \frac{1}{c_0}\left(\sum_\alpha c_\alpha H_{0\alpha} + \sum_\beta c_\beta H_{0\beta}\right), \tag{48}$$

where $E_0 = H_{00}$. Note that $N_{0\alpha} = N_{0\beta} = 0$, $N_{00} = 1$; and the single-nucleon matrix elements, such as $H_{0\beta} = H_{\beta 0}$, can be read from eq. (15). This leads to a separation of G into (3q) and cross-term contributions

$$G = c_0^4[G_0 - E_{\text{c.m.}} N^{(E)}] + \sum_{\gamma=\alpha,\beta}{}' c_0^3 c_\gamma [G_\gamma^{\mathcal{S}} - 2H_{0\gamma}N^{(E)}] \tag{49}$$

with

$$G_0 = G^{(E)}(3q) - 2E_0 N^{(E)} ,$$

where the γ-sum is over the terms $\alpha = 1, \ldots, 24$, $\beta = 1, \ldots, 6$ only. If the Wigner transform for this form of the kernel is substituted into eq. (45), it is convenient

also to make the substitutions of eq. (45) and set

$$E_{\text{c.m.}} = P^2/M_{\text{N}} + U^{\text{eff}},$$

leading to

$$U^{\text{eff}} = c_0^4[G_{0\text{W}} - (P^2/M_{\text{N}} + U^{\text{eff}})N_{\text{W}}^{(\text{E})}] + \sum_{\gamma=\alpha,\beta}{}' c_0^3 c_\gamma [G_{\gamma\text{W}}^{\mathscr{S}} - 2H_{0\gamma}N_{\text{W}}^{(\text{E})}], \quad (50)$$

or

$$U^{\text{eff}} = [c_0^4/(1+c_0^4 N_{\text{W}}^{(\text{E})})][G_{0\text{W}} - (P^2/M_{\text{N}})N_{\text{W}}^{(\text{E})}] + [1/(1+c_0^4 N_{\text{W}}^{(\text{E})})] \sum_{\gamma=\alpha,\beta}{}' c_0^3 c_\gamma [G_{\gamma\text{W}}^{\mathscr{S}} - 2H_{0\gamma}N_{\text{W}}^{(\text{E})}], \quad (51)$$

where functions of P^2 and $(\boldsymbol{R}\cdot\boldsymbol{P})^2$ in the Wigner transforms of the right-hand side are to be replaced with their local values, as exhibited explicitly in eq. (45), leading to the final form for the transcendental equation for the determination of U^{eff}. Although there is no uniqueness about this particular decomposition of U^{eff} into a pure (3q) and the two types of coupling-kernel contributions, the inclusion of the appropriate internal-energy contributions with each type of term does give a more realistic measure of the relative importance of each term. The use of the pure Wigner transforms of the coupling kernels by themselves, as used in fig. 12 of ref. [2]) as a measure of the coupling-kernel potentials, tends to overemphasize the magnitude of the coupling-kernel attraction as well as the pure (3q) repulsion.

Fig. 2 shows the equivalent local potentials for the ^{1}S channel, at $E_{\text{c.m.}} = 0$ and 350 MeV, as calculated from eq. (51). It can be seen that the overall attraction for the ^{1}S potential in the low-energy limit, $E_{\text{c.m.}} = 0$, requires the combined effects of the (3q)(q$\bar{\text{q}}$) and (3q)(q$^2\bar{\text{q}}^2$) coupling kernels to overcome a pure (3q) repulsive core of 669 MeV. The (3q)(q$^2\bar{\text{q}}^2$) coupling kernel, in particular, is vital for the needed medium and short-range attraction. It is also interesting to note that the $E_{\text{c.m.}} = 0$ ^{1}S potential is roughly similar to the simple square-well potentials of historical interest for the NN interaction. (The square-well binding rule, $M_{\text{N}} U_0 a^2/\hbar^2 \geqslant \frac{1}{4}\pi^2$, with a range parameter, a, of about 1.4 fm for $U_0 = 50$ MeV shows at once that the $^1S_0$ state is unbound). The 350 MeV potential shows that the strong energy dependence of the short-range parts of these potentials arises mainly through the coupling kernels of (3q)(q$\bar{\text{q}}$)-type. Similar single channel $^3S_1$ equivalent potentials show that these potentials are somewhat more repulsive than the $^1S_0$ potentials. (At $R = 0$ and 350 MeV, e.g., the $^3S_1$ potential has a repulsive core of 196 MeV compared with 154 MeV for the $^1S_0$ potential; see also fig. 2 of ref. [5])). The weaker attraction for the low energy $^3S_1$ central potential is in accord with the experimental facts since the binding in the $^3S_1$ channel gets important contributions through coupling to the $^3D_1$ channel via tensor force terms.

Our original motivation for including the (3q)(q$^2\bar{\text{q}}^2$) coupling kernels was based on the fact that the counterpart of σ-meson attraction of conventional meson exchange models of the NN interaction was missing from our quark model. It is

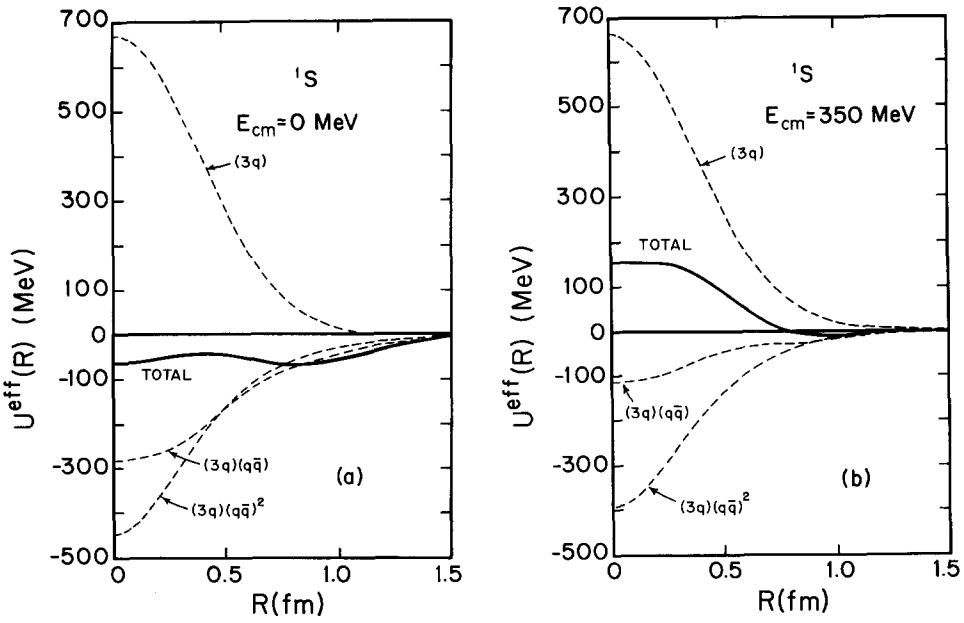

Fig. 2. The ^1S equivalent local potentials. The dashed curves give the contributions to U^{eff} from the pure (3q) part and the pure coupling terms of (3q)(qq̄) and (3q)(qq̄)^2 types, using the decomposition of eq. (51), including the c_0-dependent factors. The solid curve gives the solution for the full U^{eff} using all terms of eq. (51): (a) for $E_{c.m.} = 0$, (b) for $E_{c.m.} = 350$ MeV.

therefore interesting to compare the $(3q)(q^2\bar{q}^2)$ cross term contributions to the S-wave potentials with the conventional scalar meson exchange potentials of σ and δ type. These comparisons are made in figs. 3a and 3b. Since a $(q^2\bar{q}^2)$ cluster with $S_{23} = 0$, $T_{23} = 0$, (or $T_{23} = 1$), cf. table 1, carries the quantum numbers of a σ, (or δ), meson, the $(3q)(q^2\bar{q}^2)$ cross term contributions for $\beta = 1, \ldots, 4$ are compared with a σ-exchange potential in fig. 3a, those for $\beta = 5, 6$ with a δ-exchange potential in fig. 3b. (For the meson potentials, given by the dotted curves, the masses were taken as $m_\sigma = 550$ MeV, $m_\delta = 960$ MeV; the meson exchange potentials were regularized through the introduction of momentum-dependent form factors of the coupling constants, $g(k^2) = g(0)[\Lambda^2/(\Lambda^2 + k^2)]$, with $g_\sigma^2(0) = 6.22$, $g_\delta^2(0) = 1.83$, and a common cutoff parameter $\Lambda = 1530$ MeV; see refs. [3]) and [11])). Two types of curves are shown for the effective local potentials derived from the $(3q)(q^2\bar{q}^2)$ coupling kernels of Nσ and Nδ type. The solid curves give the full $U^{eff}(N\sigma)$ and $U^{eff}(N\delta)$, including the contributions of all 15 types of exchange terms generated through eq. (31). These curves involve the full solutions of the transcendental equations for U^{eff}. The dashed curves, on the other hand, are the bare $\boldsymbol{P} = 0$ Wigner transforms given only by the contribution of the two simple exchange terms arising from P_{37} and $P_{17}P_{28}$. The proper combination of these two terms, (see, e.g., fig. 13 of ref. [3])), leads to the simple exchange between the two nucleons of a $(q^2\bar{q}^2)$ cluster with the σ or δ quantum numbers. It is therefore satisfying to note that the dashed-curve potentials

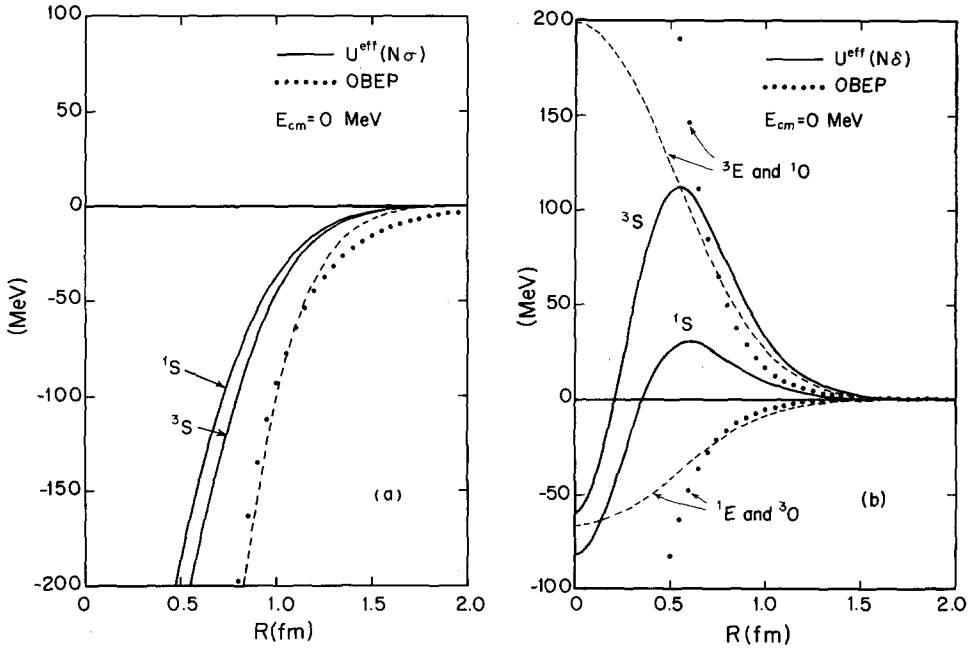

Fig. 3. The individual Nσ(*a*) and Nδ(b) potentials. The solid curves give the Nσ or Nδ contributions to the solution, U^{eff}, of the transcendental equation, eq. (51), for $E_{c.m.}=0$, and all exchange terms of the coupling kernels of Nσ or Nδ type. The dashed curves give the bare $P=0$ Wigner transforms for the simple $(q\bar{q})^2$-exchange terms only. Dotted curves are conventional OBEP's (see text for σ- and δ-parameters).

most closely resemble the more conventional σ and δ OBEP's. They also share the following OBEP properties. The simple $(q^2\bar{q}^2)$-exchange potentials of σ-type are common to all partial waves, whereas those of δ-type are common for ^{3}E and ^{1}O channels on the one hand and ^{1}E and ^{3}O on the other. Although the full $U^{eff}(N\sigma)$ and $U^{eff}(N\delta)$ are only in qualitative agreement with the corresponding OBEP's, fig. 3 shows that our $(3q)(q^2\bar{q}^2)$ coupling terms do carry the contributions usually attributed to σ- and δ-meson exchange potentials.

Potentials in the odd partial waves have central parts which are generally too repulsive. In this case the coupling terms, particularly those of $(3q)(q\bar{q})$ type, make an additional repulsive contribution to the repulsive potentials of pure (3q) type. Due to the negative parity of the odd-l relative motion functions, the dominant (3q)-(3q) components of the NN system can now couple to additional excitations, e.g. a configuration in which a 0s $(3q)(q\bar{q})$ single-nucleon negative-parity excited state of one N is coupled with a 0s (3q) cluster of the other N in a 0s relative motion wave at extreme short range. Although it was hoped that distortion due to such closed-channel contributions might lower the odd partial wave central repulsive cores, a detailed analysis shows that this particular type of distortion effect is almost completely negligible.

Since channel coupling effects are vital for a detailed quantitative comparison with the NN scattering data, the RGM equations for the full exchange kernels of eq. (46) have also been solved directly. This also makes it possible to test the quality of the predicted equivalent local potentials. Table 4 makes a comparison between the S-wave phase shifts, δ_{ELP}, calculated with our equivalent local potentials and those obtained from a full solution of the RGM equation, δ_{RGM}. For the ^3S case the equivalent local potentials are derived through eq. (51) for pure central-force kernels only and are compared with the corresponding single-channel RGM result, since our aim here is a comparison of the two phase-shift calculations rather than a comparison with the experimental results. Four types of comparison are made in table 4. (I) For the pure (3q)-(3q) model of the NN system, the ELP are derived by retaining only the first term in eq. (51), and the RGM equation is solved with a kernel including only the first term of eq. (49). Note, however, that the c_0^4 dependent factors are retained in both cases. This leads to (3q) S-wave phase shifts somewhat less repulsive than those of ref. [12]); (for comparison of ELP and RGM results see

TABLE 4

Comparison of RGM and equivalent local potential phase shifts (in deg.)

		^1S		^3S	
Model space	$E_{c.m.}$(MeV)	δ_{RGM}	δ_{ELP}	δ_{RGM}	δ_{ELP}
(3q)	5	−8.1	−8.1	−6.4	−6.6
	10	−11.4	−11.5	−9.1	−9.3
	25	−17.9	−18.0	−14.3	−14.6
	50	−24.9	−25.1	−19.9	−20.4
	100	−34.1	−34.4	−27.5	−28.1
	150	−40.3	−40.7	−32.6	−33.3
(3q)	5	7.2	7.3	2.1	2.6
(3q)(qq̄)	10	8.7	8.6	2.3	2.8
	25	8.9	7.9	1.3	1.4
	50	5.3	3.2	−2.1	−2.8
	100	−3.5	−6.8	−9.8	−11.4
	150	−11.3	−15.2	−16.5	−18.7
(3q)	5	−1.6	−1.0	−2.4	−1.9
$(3q)(q\bar{q})^2$	10	−2.5	−1.7	−3.5	−2.8
	25	−4.6	−3.9	−5.8	−5.2
	50	−7.7	−7.5	−8.9	−8.4
	100	−13.1	−13.8	−13.5	−13.6
	150	−17.2	−18.5	−16.8	−17.4
(3q)	5	69.2	49.1	14.6	13.0
(3q)(qq̄)	10	65.0	49.2	17.8	15.5
$(3q)(q\bar{q})^2$	25	54.9	42.8	19.3	15.9
	50	43.0	32.5	15.9	11.6
	100	27.0	17.5	7.3	2.0
	150	15.8	6.9	−0.1	−5.9

also fig. 4 of ref. [12])). For entries (II) and (III) of table 4 the phase shifts are calculated from the combinations of the pure (3q) terms of eqs. (49) or (51), as in (I), with the coupling kernel terms of pure α- or β-type. Finally, entry (IV) is calculated for the full model through all terms of eqs. (49) or (51) including both types of coupling kernels together. It is interesting to note that the coupling terms of $(3q)(q\bar{q})$ type by themselves are more effective in making the phase shifts more attractive than the coupling terms of $(3q)(q^2\bar{q}^2)$ type even though the latter lead to stronger attraction at $R=0$. (Note, however, the reversal of the $(3q)(q\bar{q})$ and $(3q)(q^2\bar{q}^2)$ ^{1}S-potentials of fig. 2a for $R>0.5$ fm). Clearly also the combination of both types of coupling terms is more than additive in making the S-wave phase shifts sufficiently attractive at very low energy. (Note again that important tensor force terms and channel-coupling effects are missing in the ^{3}S entries). For the simpler models the agreement between the δ_{ELP} and the exact δ_{RGM} is remarkably good. For the full model the agreement is at best semiquantitative. This is undoubtedly related to the near resonant characteristics of NN scattering around $E_{\mathrm{c.m.}}=0$, and shows again that full RGM solutions are needed for a detailed quantitative comparison with the NN scattering data. Nevertheless, the agreement is good enough so that the predicted equivalent local potentials can be trusted to give a reasonable picture of NN scattering. Further, it shows that the Wigner transform-WKB technique can be used to evaluate equivalent local potentials even in a relatively sophisticated quark model.

5. RGM formulation

The RGM equations

$$(E_{\mathrm{c.m.}}-T_R)\chi(\boldsymbol{R})=\int \mathrm{d}\boldsymbol{R}'\, G(\boldsymbol{R},\boldsymbol{R}')\chi(\boldsymbol{R}') \tag{52a}$$

for functions of good parity

$$\chi(\boldsymbol{R})=\tfrac{1}{2}[\chi(\boldsymbol{R})-(-1)^{S+T}\chi(-\boldsymbol{R})]\,, \tag{52b}$$

and with

$$T_R=-(\hbar^2/M_{\mathrm{N}})[\mathrm{d}^2/\mathrm{d}R^2-l(l+1)/R^2] \tag{52c}$$

in a partial wave expansion, has been solved for the NN scattering problem and the kernel $G(\boldsymbol{R},\boldsymbol{R}')$ of our quark model by the variational method of Kamimura [13]). The interior region ($R<R_{\mathrm{c}}$) trial function is expanded in terms of a set of nonorthogonal gaussian trial functions, $u_i^{(\mathrm{in})}$. The coefficients of these $u_i^{(\mathrm{in})}$ and their gaussian range parameters serve as the variational parameters. The interior trial functions are extended to $R>R_{\mathrm{c}}$ to match the proper scattering boundary conditions. The method requires the evaluation of the matrix elements

$$\mathcal{H}_{ij}=(u_i^{(\mathrm{in})},\mathcal{L}_l u_j^{(\mathrm{in})})-((u_i^{(\mathrm{in})},\mathcal{L}_l^{(0)}u_j^{(\mathrm{in})}))\,, \tag{53a}$$

where

$$\mathscr{L}_l = T_R + G - E_{\text{c.m.}}\,, \qquad \mathscr{L}_l^{(0)} = T_R - E_{\text{c.m.}} \tag{53b}$$

and the inner product ((,)) involves an R-integration from $R = R_c \to \infty$ only and is thus a small quantity, but essential for this particular variational method. The gaussian trial functions, $u_i^{(\text{in})} = R u_{l_i}(R, \eta_i)$, are chosen as

$$u_{lm}(\boldsymbol{R}, \eta) = u_l(R, \eta)\, Y_{lm}(\hat{\boldsymbol{R}})\,, \tag{54a}$$

with the radial functions

$$u_l(R, \eta) = \left[\frac{4\pi}{(2l+1)!!}\right]^{1/2} \left(\frac{2\eta}{\pi}\right)^{3/4} (2\sqrt{\eta}R)^l\, \mathrm{e}^{-\eta R^2}\,. \tag{54b}$$

For the matrix elements of the noncentral components of the kernel, the radial parts of these trial functions are to be combined with standard vector-coupled spin, angular functions

$$\mathscr{Y}_{lS}^{JM}(\hat{\boldsymbol{R}}; \text{spin}) = [\, Y_l(\hat{\boldsymbol{R}}) \times \chi_S]^{JM}\,, \tag{54c}$$

where χ_S is a two-nucleon spin function.

The direct terms of the RGM equation, given through $\mathscr{L}_l^{(0)} = T_R - E_{\text{c.m.}}$, involve the matrix elements

$$(u_i^{(\text{in})}, u_j^{(\text{in})}) - ((u_i^{(\text{in})}, u_j^{(\text{in})})) = I(x_c;\, l+\tfrac{3}{2}) e_{ij}\,, \tag{55}$$

$$(u_i^{(\text{in})}, T_R u_j^{(\text{in})}) - ((u_i^{(\text{in})}, T_R u_j^{(\text{in})})) = \frac{2\hbar^2}{M_{\text{N}}}(l+\tfrac{3}{2}) \left[\frac{2\eta\eta'}{\eta+\eta'} I(x_c;\, l+\tfrac{5}{2}) + 2\eta' \frac{\mathrm{e}^{-x_c} x_c^{l+3/2}}{\Gamma(l+\frac{5}{2})}\right] e_{ij}\,, \tag{56}$$

where $l_i = l_j = l$, and $\eta_i = \eta$, $\eta_j = \eta'$ are implied and

$$e_{ij} = \left(\frac{2\sqrt{\eta\eta'}}{\eta+\eta'}\right)^{l+3/2}\,. \tag{57}$$

In eqs. (55) and (56), $I(x;\, \nu)$ is the incomplete gamma function defined by

$$I(x;\, \nu) = \frac{1}{\Gamma(\nu)} \int_0^x \mathrm{e}^{-t} t^{\nu-1}\, \mathrm{d}t\,, \tag{58}$$

and x_c is given in terms of the channel radius R_c, by $x_c = (\eta + \eta') R_c^2$.

Since all parts of the exchange kernel $G(\boldsymbol{R}, \boldsymbol{R}')$ in the decomposition into (3q) and coupling kernels, as given by eq. (49), have been expressed through expansions in terms of gaussian kernels, the partial wave decomposition of these kernels and the evaluation of RGM matrix elements between the basic trial functions of eq. (54) can be carried out in a unified framework by the use of the general gaussian kernel formulae derived in appendix A. The RGM matrix elements of G in eq. (49) are separated into

$$G_{ij} \equiv (u_i^{(\text{in})}, G u_j^{(\text{in})}) = c_0^4 [G_{0_{ij}} - E_{\text{c.m.}} N_{ij}^{(\text{E})}] + \sum_{\gamma=\alpha,\beta}{}' c_0^3 c_\gamma [G_{\gamma ij}^{\mathscr{S}} - 2H_{0\gamma} N_{ij}^{(\text{E})}]\,. \tag{59}$$

For the (3q)(qq̄)-coupling kernels, $G^{\mathscr{S}}_{\alpha}$, the RGM matrix elements, $G^{\mathscr{S}}_{\alpha ij}$, are given by (with $l_i \equiv l$, $l_j \equiv l'$)

$$G^{\mathscr{S}}_{\alpha ij} = \alpha_s x^2 mc^2 \frac{4}{\sqrt{3\pi}} (\tfrac{4}{13}\sqrt{3})^{3/2} N^{-1/2}_{\alpha\alpha} \sum_{\mathscr{O}=\mathrm{M,D}} \sum_{P} C_P \sum_{i=1}^{6} \times \sum_{\Omega=\mathrm{C},LS,\mathrm{T}} [M^{\mathscr{O}\Omega,SJ}_{Pi,Il'}(\eta;\eta') + M^{\mathscr{O}\Omega,SJ}_{Pi,l'l}(\eta';\eta)] X^{\mathscr{O},\alpha\Omega}_{P,i}, \tag{60}$$

where we have used the expansion of G_α into interaction type, $\mathscr{O} = \mathrm{M}$ (momentum) and D (derivative)-type, exchange type, P, i, and into $\Omega = \mathrm{C}$ (central), LS, and T (tensor) parts. The spin, isospin, color factors, $X^{\mathscr{O},\alpha\Omega}_{P,i}$, are evaluated through eqs. (27) and (16) of ref. [3]). As is shown in appendix A, the width parameters $\eta = \eta_i$ and $\eta' = \eta_j$ of the trial functions always appear in the combination

$$\varepsilon = \frac{\gamma - \eta}{\gamma + \eta}, \qquad \varepsilon' = \frac{\gamma' - \eta'}{\gamma' + \eta'}, \tag{61}$$

where γ and γ' are the relative motion width parameters for the bra and ket sides, respectively, and $\gamma = \gamma_1 = (15/16b^2)$ and $\gamma' = \gamma_0 = (3/4b^2)$ in the present case of the (3q)(qq̄)-coupling kernels. Also, in the LS and tensor matrix elements, it is convenient to factor out the standard matrix elements of $(\boldsymbol{L}\cdot\boldsymbol{S})$ and $\boldsymbol{S}_{12}$ operators defined by

$$\langle \boldsymbol{L}\cdot\boldsymbol{S}\rangle = \tfrac{1}{2}[J(J+1) - l(l+1) - S(S+1)],$$

$$(S_{12})^J_{JJ} = 2, \qquad (S_{12})^J_{J-1,J+1} = (S_{12})^J_{J+1,J-1} = \frac{6\sqrt{J(J+1)}}{2J+1},$$

$$(S_{12})^J_{J-1,J-1} = \frac{2(1-J)}{2J+1}, \qquad (S_{12})^J_{J+1,J+1} = -\frac{2(J+2)}{2J+1}. \tag{62}$$

Thus, the matrix element $M^{\Omega}_{ll'}(\eta;\eta')$ in eq. (60) for a particular $\mathscr{O}Pi$ and SJ is expressed as (see also appendix B of ref. [14])),

$$M^{\Omega}_{ll'}(\eta;\eta') = (1-\varepsilon^2)^{(l+3/2)/2}(1-\varepsilon'^2)^{(l'+3/2)/2}(1/\tilde{D}^{(0)})^{3/2} \times \begin{Bmatrix} \delta_{ll'}[I_l(\tilde{C};\tilde{E}) + lI_{l-1}(\tilde{C};\tilde{V})] \\ -\delta_{S1}\delta_{ll'}\langle \boldsymbol{L}\cdot\boldsymbol{S}\rangle I_{l-1}(\tilde{C};\tilde{V}^{LS}) \\ \delta_{S1}(S_{12})^J_{ll'}[\delta_{ll'}(l+\frac{3}{2})I_{l-1}(\tilde{C};\tilde{V}) + \delta_{l,l'+2}\sqrt{(2l+1)(2l-1)} \\ \quad \times I_{l'}(\tilde{C};\tilde{F}) + \delta_{l+2,l'}\sqrt{(2l'+1)(2l'-1)}I_l(\tilde{C};\tilde{G})] \end{Bmatrix} \text{ for } \Omega = \begin{cases} \mathrm{C} \\ LS \\ \mathrm{T}. \end{cases} \tag{63}$$

The explicit expressions for $\tilde{D}^{(0)}$ and $I_l(\tilde{C};\tilde{\mathscr{D}})$ with $\tilde{\mathscr{D}} = \tilde{E}, \tilde{F}, \tilde{G}, \tilde{V}, \tilde{V}^{LS}$ are given in appendices A and B in terms of the coefficients of the GCM kernel, $f^{(0)}, \ldots, m$, which are explicitly given in eq. (11) of ref. [4]) for each $\mathscr{O}Pi$.

Since the $(3q)(q^2\bar{q}^2)$ coupling kernels of sect. 3 are purely central, the spin-independent RGM matrix elements $G^{\mathscr{S}}_{\beta ij}$ in eq. (59) have the simpler form, (with $l'=l$),

$$G^{\mathscr{S}}_{\beta ij} = -\alpha_s x^2 mc^2 \frac{8\sqrt{6}}{\pi} (\tfrac{5}{3})^{3/4} N^{-1/2}_{\beta\beta} \sum_P C_P [M_{P,l}(\eta;\eta') + M_{P,l}(\eta';\eta)] X^{\beta}_P , \quad (64)$$

where the decomposition into spatial and spin, isospin, color factors, X^{β}_P, follows that of eq. (33), and the dependence on exchange type $P=(xyuv)$ is given through the parameters x and u only. The matrix element $M_{P,l}(\eta;\eta')$ is also given by the $\Omega=\mathrm{C}$ case of eq. (63) with $I_{l-1}(\tilde{C};\tilde{V})=0$. For ε and ε', defined in eq. (61), one should now use $\gamma=\gamma_2=(21/20b^2)$ and $\gamma'=\gamma_0=(3/4b^2)$. The explicit expressions for $\tilde{D}^{(0)}$ and $I_l(\tilde{C};\tilde{E})$ are given in appendix B in terms of the parameters x and u.

Since the various components of the (3q)-(3q) RGM kernels have already been analyzed by many authors [1]), a very brief description of the classification scheme of the exchange kernel is given with the final expressions of the RGM matrix elements. For the quadratic-type confinement potential, the confinement potential contribution to the (3q)-(3q) RGM kernel $G_0 = G^{(\mathrm{E})}(3q) - 2E_0 N^{(\mathrm{E})}$ of eq. (49) disappears precisely so that the (3q)-(3q) RGM kernel G_0 is composed only of the following pieces: (K) for the exchange kinetic energy contribution, (CC) for the color-coulombic or $(\lambda_i\cdot\lambda_j)/r_{ij}$ piece, (GC) for the combined color-delta and color-magnetic or $(\lambda_i\cdot\lambda_j)(1+\frac{2}{3}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j))\delta(r_{ij})$ piece, (sLS) for symmetric LS, (aLS) for antisymmetric LS, and (T) for tensor kernels [12]) (In this series of studies the momentum-dependent Breit retardation (or Darwin-like) term in the central force has been omitted.) Therefore, the (3q) RGM matrix elements G_{0ij} in eq. (59) are separated into

$$G_{0ij} = G^{\mathrm{K}}_{0ij} + G^{\mathrm{CC}}_{0ij} + G^{\mathrm{GC}}_{0ij} + G^{\mathrm{s}LS}_{0ij} + G^{\mathrm{a}LS}_{0ij} + G^{\mathrm{T}}_{0ij} , \quad (65)$$

where the K, CC, and GC terms are the central matrix elements. For each interaction type, $\mathscr{O}\Omega=$ CC, GC, sLS, aLS, or T, quark exchange types (which will be denoted by $\mathscr{T}$, corresponding to Pi and P for $(3q)(q\bar{q})$ and $(3q)(q\bar{q})^2$ coupling kernels, respectively) are specified by symbols $\mathscr{T}=$ E, S, D_+, and D_-, which uniquely determine the structure of the exponential factors, $\exp[\frac{1}{3}(z^*\cdot z') - \frac{1}{2}\lambda(pz^*+qz')^2]$, of the gaussian GCM kernel by [9])

$$p=q=0 \text{ for } \mathscr{T}=\mathrm{E}, \qquad p=1, q=0 \text{ for } \mathscr{T}=\mathrm{S},$$

$$p=q=1 \text{ for } \mathscr{T}=\mathrm{D}_+, \quad p=1, q=-1 \text{ for } \mathscr{T}=\mathrm{D}_- . \quad (66)$$

The correspondence between $\mathscr{T}$ and the notation $P(ij)\equiv P_{36}(ij)$ of ref. [15]) is: $(ij)=$ (45) for E, (26) for S, (25) for D_+, (36) for D_-. (See, in particular, fig. 1 of ref. [15]), and note that the exchange type (56) is the hermitian conjugate of (26).) With these

definitions, $N_{ij}^{(\mathrm{E})}$ in eq. (59) and each piece of eq. (65) can be expressed by

$$
\begin{aligned}
N_{ij}^{(\mathrm{E})} &= X_{\mathrm{N}} M_{\mathrm{E},l}^{\mathrm{GC}}(\eta, \eta'),\\
G_{0ij}^{\mathrm{K}} &= \tfrac{3}{4} x^2 mc^2 X_{\mathrm{N}} M_l^{\mathrm{K}}(\eta; \eta'),\\
G_{0ij}^{\mathrm{CC}} &= \sqrt{\frac{2}{\pi}}\, \alpha_{\mathrm{s}} x mc^2 X_{\mathrm{N}} \tfrac{4}{3}\{2M_{\mathrm{E},l}^{\mathrm{CC}}(\eta; \eta') - 2[M_{\mathrm{S},l}^{\mathrm{CC}}(\eta; \eta')\\
&\quad + M_{\mathrm{S},l}^{\mathrm{CC}}(\eta'; \eta)] + M_{\mathrm{D}_+,l}^{\mathrm{CC}}(\eta; \eta') + M_{\mathrm{D}_-,l}^{\mathrm{CC}}(\eta; \eta')\},\\
G_{0ij}^{\mathrm{GC}} &= \sqrt{\frac{2}{\pi}}\, \alpha_{\mathrm{s}} x^3 mc^2 \{X_{\mathrm{E}}[M_{\mathrm{E},l}^{\mathrm{GC}}(\eta; \eta') - M_{\mathrm{S},l}^{\mathrm{GC}}(\eta; \eta')\\
&\quad - M_{\mathrm{S},l}^{\mathrm{GC}}(\eta'; \eta)] + X_{\mathrm{D}_+} M_{\mathrm{D}_+,l}^{\mathrm{GC}}(\eta; \eta') + X_{\mathrm{D}_-} M_{\mathrm{D}_-,l}^{\mathrm{GC}}(\eta; \eta')\},\\
G_{0ij}^{\mathrm{s}LS} &= \sqrt{\frac{2}{\pi}}\, \alpha_{\mathrm{s}} x^3 mc^2 [X_{\mathrm{D}_+}^{LS} M_{\mathrm{D}_+,l}^{LS}(\eta; \eta') + X_{\mathrm{D}_-}^{LS} M_{\mathrm{D}_-,l}^{LS}(\eta; \eta')],\\
G_{0ij}^{\mathrm{a}LS} &= \sqrt{\frac{2}{\pi}}\, \alpha_{\mathrm{s}} x^3 mc^2 X_{\mathrm{S}}^{LS} [M_{\mathrm{S},l}^{LS}(\eta; \eta') + M_{\mathrm{S},l}^{LS}(\eta'; \eta)],\\
G_{0ij}^{\mathrm{T}} &= \sqrt{\frac{2}{\pi}}\, \alpha_{\mathrm{s}} x^3 mc^2 \{X_{\mathrm{S}}^{\mathrm{T}}[M_{\mathrm{S},ll'}^{\mathrm{T}}(\eta; \eta') + M_{\mathrm{S},l'l}^{\mathrm{T}}(\eta'; \eta)]\\
&\quad + X_{\mathrm{D}_+}^{\mathrm{T}} M_{\mathrm{D}_+,ll'}^{\mathrm{T}}(\eta; \eta') + X_{\mathrm{D}_-}^{\mathrm{T}} M_{\mathrm{D}_-,ll'}^{\mathrm{T}}(\eta; \eta')\},
\end{aligned}
\tag{67}
$$

where the spin, isospin, color factors, X_{N}, $X_{\mathscr{T}}$, $X_{\mathscr{T}}^{LS}$, and $X_{\mathscr{T}}^{\mathrm{T}}$ are given in appendix B. The matrix element $M_{\mathscr{T},ll'}^{\mathscr{O}\Omega}(\eta; \eta')$ for a particular $\mathscr{O}\Omega\mathscr{T}$ is again obtained from eq. (63) with $I_{l-1}(\tilde{C}; \tilde{V}) = 0$ for $\Omega = \mathrm{C}$. This time one should use ε, ε' of eq. (61) obtained from $\gamma = \gamma' = \gamma_0 = (3/4\mathrm{b}^2)$. The explicit expressions for $\tilde{D}^{(0)}$ and $I_l(\tilde{C}; \tilde{\mathscr{D}})$ for each $\mathscr{O}\Omega$ are again given in appendix B in terms of the coefficients p and q, which are explicitly given in eq. (66) for each $\mathscr{T}$.

With these explicit, analytic expressions for the RGM matrix elements of the exchange kernels in the decomposition of eq. (49), the RGM equations have been solved through the coupled channel variational method of ref. [13]). Again, following the general format of ref. [13]), the η_i, have been parametrized as follows,

$$
\eta_i = \frac{25}{9}\left(\frac{1}{50}\right)^{(i-1)/14} (\mathrm{fm}^{-2}), \qquad \text{with } i = 1, \ldots, i_{\max}, \tag{68}
$$

and $R_c = 3$ fm, where $i_{\max} = 15$, 12, 11, and 10 for $l = 0$, 1, 2, and 3, respectively, were the final chosen values. A test of the stability of the solutions with respect to the choice of R_c and the trial functions indicates that the accuracy of the final calculated phase shifts is better than $\Delta\delta = 0.1°$, unless the long-range tensor force of the one-pion exchange potential is involved. For the coupled-channel calculations of the triplet states, using the long-range tensor force of appendix C, the accuracy deteriorates up to a few degrees in the low-energy region. For instance, the channel radius R_c should be increased to a value of 5 fm, with $\eta_i = 3(\frac{1}{150})^{(i-1)/14}$, in order to

obtain enough accuracy for the low-energy parameters of the $^3S_1+^3D_1$ coupling problem.

6. RGM results

With the method outlined in sect. 5 and the results of appendices A and B, the coupled channel RGM equations have been solved and phase shifts have been calculated for low-energy NN scattering. Before discussing these phase shifts it should be emphasized that the results of our calculations are zero parameter predictions. The four model parameters α_s, m, b, a_c were determined from single-nucleon data in ref. [2]) and were used without further adjustment. Since our single-nucleon data were sensitive essentially only to the final values of the c_α of the single-nucleon wave functions but otherwise insensitive to the details of the parameter fit [2]), the addition of the c_β does not make significant changes in the single-nucleon predictions. The one exception is the absolute value of the nucleon mass which was not considered to be a crucial parameter of our model for NN scattering [2]).

However, the predicted pion tensor force of our simple $(q\bar{q})$-exchange potential [3]) was too weak by a factor of ~3, (see fig. 3 of ref. [4])), in agreement with our predicted coupling constant [2]), $g^2_{NN\pi}$, which was also too weak by a factor of ~3. Since our method can pinpoint the exchange terms responsible for this piece of the NN interaction, these simple $(q\bar{q})$-exchange terms of $N\pi$ type were removed from the tensor part of the $(3q)(q\bar{q})$-coupling kernel and were replaced with a more realistic OPEP tensor force. This more realistic pion tensor term uses a form factor $g^2(\boldsymbol{k}) = 14.17 \exp[-0.0943k^2]$ with the experimental value for $g^2_{NN\pi}(0)$ and a gaussian $\boldsymbol{k}$-dependent factor which gives a very good approximation to the actual $\boldsymbol{k}$-dependence predicted by our quark model through eq. (69) of ref. [2]). The RGM matrix elements of this form of the pion tensor term are evaluated in appendix C. With the exception of this one adjustment the phase-shift predictions are free of parameter fitting, including the use of the unadjusted predicted internal energy of $2E_N = 2$ (693 MeV). This predicted quantity does make nonnegligible contributions to the exchange kernel of eq. (49) through the norm kernel term. We do not expect that our quark model can give reliable predictions for the absolute value of baryon masses; but the low-energy NN scattering results are relatively insensitive to this quantity. Although an exact fit of the N mass could have been attained through a further adjustment of the model parameters, in particular through a change in the value of the confinement-potential constant, the NN scattering predictions are very insensitive to such a change. A change in the potential constant, a_c, will make large changes in the internal energy, but the a_c-dependent contribution to the (3q)-(3q) kernal $G_0 = G^{(E)}(3q) - 2E_0 N^{(E)}$ disappears precisely for a purely quadratic confinement potential and has been shown to be very small for a linear potential [12]). Even a very large change in a_c has only a small indirect effect on the NN interaction through small changes in the c_α. It is our philosophy to trust only those predictions

of our model which are insensitive to the strength of the confinement potential. The phase-shift analysis can therefore be made in terms of the original parameters [2]) of our extended quark model.

Figs. 4 and 5 show the predicted RGM phase shifts for the S-waves. The results of the single-channel calculation for the $^1S_0$ phase shifts are analyzed in fig. 4 in terms of the decomposition of the quark exchange kernel, given by eq. (49). It can be seen that the repulsive phase shifts of the pure (3q) components of the kernel are changed to a very weak attraction at extreme low energy by the inclusion of the (3q)(q$\bar{q}$)-coupling terms. The much stronger needed attraction is gained through the cooperative effect of the combined coupling kernels of (3q)(q$\bar{q}$) and (3q)(q$^2\bar{q}^2$) type. Although the final values of the predicted $^1S_0$ phase shifts, calculated with the full kernel, are now somewhat too attractive, we consider this a satisfying fit for a zero-parameter calculation, particularly since energy-dependent distortion effects through a $\boldsymbol{K}$-dependence of the c_α have been neglected. Such effects may influence the phase shifts at higher energies where the disagreement is greatest. A similar analysis of the $^3S_1$ phase shifts in fig. 5 shows that the combined effects of all central terms, including both the (3q)(q$\bar{q}$) and (3q)(q$^2\bar{q}^2$) coupling kernels, do not give sufficient attraction in this case. However, when the important effects of channel coupling to the $^3D_1$ channel through the tensor force are included the full coupled-

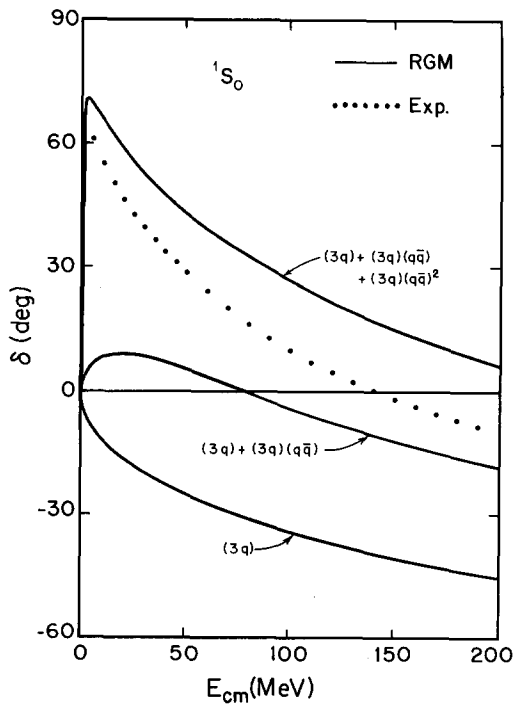

Fig. 4. Analysis of $^1S_0$ phase shifts. The (dotted) experimental points for figs. 4–7 are taken from ref. [16]).

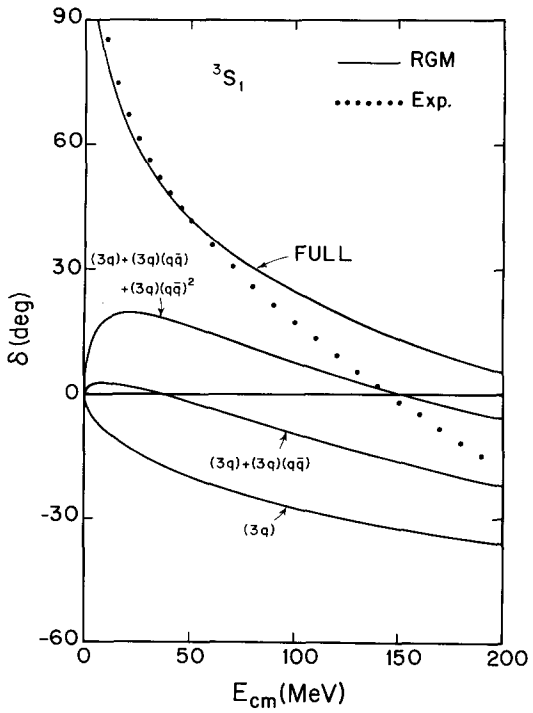

Fig. 5. Analysis of $^3S_1$ phase shifts, with improved Nπ tensor force.

channel calculation gives a good fit to the experimental $^3S_1$ phase shifts in the 0–100 MeV range. The comparable fits to the $^3D_1$ as well as the single-channel $^1D_2$ and $^3D_2$ phase shifts have already been shown in figs. 3 and 4 of ref. [5]). However, the mixing angle ϵ_1 in fig. 4 of ref. [5]) was inadvertently shown with the wrong sign. Using the conventional definition of ϵ_1 the extreme low-energy value of the predicted ϵ_1 is positive.

The predicted extreme low-energy S-wave phase shifts (in the 0–10 MeV range), have also been used to extract values for the singlet and triplet scattering length and effective-range parameters. The results of a standard low-k^2 analysis of the predicted S-wave phase shifts are shown in table 5. These low-energy parameters lead to a predicted deuteron binding energy of 0.68 MeV. Diagonalization of the interior region trial function with $\eta_i = 3(\frac{1}{150})^{(i-1)/14}$, on the other hand, led to a deuteron binding energy of 0.66 MeV. (The 0.5 MeV quoted in ref. [5]) came from diagonalization with eq. (68).) The small difference is due to the missing extreme long-range tail of the deuteron wave function, which is never described completely by the gaussian trial functions. The 0.68 MeV value from the low-k^2 analysis of the eigen-phase shifts is therefore the most accurate measure of our calculation. The low-energy analysis of the calculated phase shifts leads to a predicted $^1S_0$ virtual state at 57 keV. The analysis of the extreme low-energy values of the Blatt-Biedenharn

TABLE 5

Scattering length and effective-range parameters

	Calc.	Exp. [a])
a_s	−26.2	−23.75 ± 0.01
r_{es}	1.53	2.75 ± 0.05
a_t	8.75 [b])	5.42 ± 0.01
r_{et}	1.69 [b])	1.76 ± 0.01

[a]) np data taken from M. M. Nagels *et al.*, Nucl. Phys. **B147** (1979) 189.

[b]) Numbers obtained from $R_c = 5$ fm trial functions.

mixing parameter, ϵ_1, yields a positive value of the deuteron quadrupole moment, which is however too large by a factor of ~1.5. The results of figs. 4 and 5 and table 5 show that our extended quark model gives a good account of the low-energy scattering data and the deuteron bound-state characteristics. The difference between 0.68 MeV and the experimental value of the binding energy of 2.225 MeV must be considered insignificant in view of the remaining uncertainties of our quark-model potentials.

Figs. 6 and 7 show the calculated phase shifts for odd partial waves, again with the use of the unadjusted predicted internal energy and with no adjustments of the four model parameters. The ^{3}P predictions of fig. 6 show that our odd-L interaction is too repulsive. In odd partial waves the coupling kernel terms of $(3q)(q\bar{q})$ type lead to repulsive central terms which reinforce the repulsive central potential arising from the pure (3q)-(3q) kernels and are only partially compensated by a strong attractive contribution from the coupling kernels of $(3q)(q\bar{q})^2$ type. Since conventional analyses of the NN interaction lead to very weak triplet odd central terms in

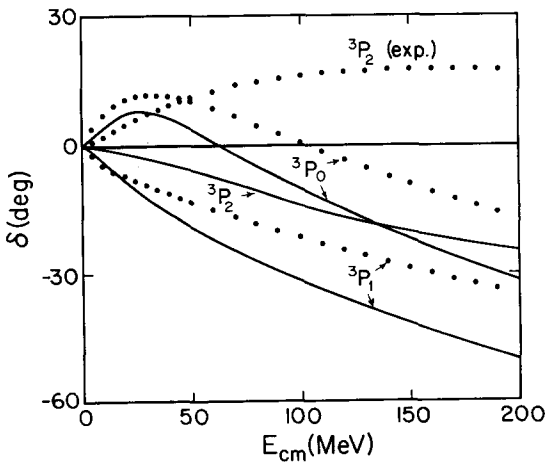

Fig. 6. The ^{3}P phase shifts, with improved Nπ tensor force, but with calculated E_N and no adjustment of model parameters.

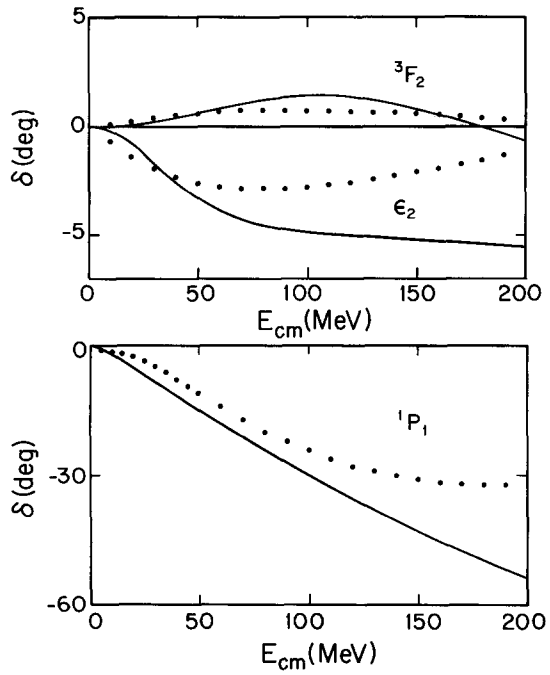

Fig. 7. The $^1P_1$ and $^3F_2$ phase shifts and mixing parameter ϵ_2.

the medium and long-range region, the ^3P low-energy phase shifts must be expected to arise almost exclusively through tensor and *LS* terms. It was shown in ref. [4]) that the *LS* and tensor terms of our quark model, (with the improved Nπ tensor force term), give a very good account of the observed ^3P phase shifts. With triplet odd central terms turned off, the RGM calculations give a good fit to the observed ^3P phase shifts (see fig. 11 of ref. [4])). Fig. 7 shows that the predicted $^1P_1$ phase shifts are also too repulsive. The quark exchange kernels of this investigation in general lead to odd-*L* central potentials which are too repulsive. Since the extreme short-range odd-*L* central potentials of our model result from partial cancellation of the strong repulsive (3q)(q$\bar{q}$) coupling terms with the attractive contributions of the (3q)(q$\bar{q}$)^2 coupling kernels, these central potentials are subject to considerable uncertainty. At extreme short range these central terms may also gain significant contributions from terms of second order in the c_α and c_β in the quark kernel expansions of eqs. (26) and (49). Such complicated terms have not been incorporated into the model.

7. Summary

Since simple (3q)-(3q) models of the NN interaction could elucidate only the extreme short-range part of this interaction and failed to make a natural connection with the meson exchange picture responsible for the long-range part, a number of

models have arisen [1]) in which the quark degrees of freedom of a nucleon interior are coupled to various meson fields. In such models quarks and meson fields are treated as separate entities. However, since quantum chromodynamics is believed to be the fundamental theory of the strong interaction a more satisfying picture should be based on a model built only from quarks and antiquarks. It is our philosophy that a model in which both baryons and mesons are described in terms of their common constituents is to be preferred over a model in which quarks and meson fields are treated as separate entities, particularly when quark exchange effects become important, since the Pauli principle among the quark constituents can be respected only in this way.

In this series of investigations [2-5]) an extended quark model has been built in which the $(q\bar{q})$ and $(q\bar{q})^2$ excitations inherent in the quark-gluon interaction lagrangian have been explicitly incorporated into the model space. By studying the NN interaction within the framework of the resonating group method very explicit coordinate space results have been attained which make it possible to isolate the interaction corresponding to the exchange of a $(q\bar{q})$ pair or a $(q\bar{q})^2$ cluster between the two nucleons. This has led to a unified picture in which both baryons and the exchanged mesons appear on an equal footing in a pure quark model.

Since low-energy hadron phenomena confront QCD with formidable problems, a QCD-inspired effective quark interaction must perforce form the starting point of any model. Following the successes of the 3q models of the nucleon, our quark interaction is built from a phenomenological quark confining potential which is combined with a gluon exchange interaction of the general form of a one-gluon exchange potential through the color analog of the Fermi-Breit interaction. However, the $(q\bar{q})$ and $(q\bar{q})^2$ creation terms which are part of the one-gluon exchange diagrams are explicitly included in the interaction in the hope that this improvement in the quark interaction is at least a first step toward the incorporation of meson field effects into the NN interaction. The final NN interaction has the fortunate property of being almost completely independent of the strength of the confining potential. The model therefore passes the crucial test of being insensitive to the details of the phenomenology. The quark mass and the gluon coupling constant are considered parameters of this effective interaction. Together with the confinement potential constant and the basic size parameter, b, these form the four parameters of our model. Since these parameters have been determined from single-nucleon properties [2]), and since no further adjustments of parameters are made, the model leads to a zero-parameter prediction of the NN scattering data.

The full quark exchange kernels have been analyzed to study the relative importance and the characteristics of various components. In particular, the pure (3q) contributions to the NN interaction have been isolated from the contributions arising from specific $(3q)(q\bar{q})$ and $(3q)(q^2\bar{q}^2)$ components of the single-nucleon wave functions. Due to the intrinsic p-wave character associated with the $(q\bar{q})$-pair creation process the fully antisymmetrized $(3q)(q\bar{q})$ components form a nearly orthonormal set so that real physical significance can be ascribed to the spin, isospin quantum

numbers of the $(q\bar{q})$ pairs. By isolating from the many more complicated exchange terms those contributions to the exchange kernels which correspond to an exchange between two nucleons of a $(q\bar{q})$ pair with the quantum numbers of a real pseudoscalar or vector meson, ref. [3]) showed that our model makes good contact with conventional one-boson exchange potentials. The present work completes the model with the inclusion of the $(3q)(q\bar{q})^2$ components which arise naturally through the RPA-type off-shell terms of the Breit interaction. Since the $(3q)(q\bar{q})^2$ components with their intrinsic 0s-wave relative-motion functions span a smaller part of the quark-model space, the significance of the $(q\bar{q})^2$ quantum numbers is partly washed out by quark antisymmetrization effects. The present work, however, shows again that those parts of the exchange kernel which correspond to the exchange between two nucleons of a $(q\bar{q})^2$ cluster with $S=0$, $T=0$ and 1 leads to interactions with the basic characteristics of conventional σ- and δ-meson exchange potentials. Moreover, these furnish the additional medium-range attraction needed to bind the deuteron.

The model has been subjected to a quantitative test of the scattering data through a solution of the RGM equations in a coupled-channel formalism. The partial wave decomposition of the RGM kernels has been carried out and the needed RGM matrix elements for the gaussian trial functions are given in analytic form. Since a simple $(q\bar{q})$-pair with the quantum numbers of a pion was not expected to give a realistic picture of a real pion, a fit to a realistic long range OPEP was not expected. In particular, the predicted $N\pi$ tensor force was too weak by a factor of ~ 3. Since the exchange terms responsible for this piece of the NN interaction can be identified, it is possible to improve this part of the interaction. The $(q\bar{q})$ exchange terms of $N\pi$ type were removed from the tensor part of the coupling kernel and replaced with a pion tensor term with the experimental value of $g^2_{NN\pi}$ and a gaussian form factor which gives a very good approximation to the form factor predicted by our quark model. With this one improvement of the interaction, the model gives a satisfying semiquantitative fit of the low-energy S-wave scattering data and the binding characteristics of the deuteron. Although the central parts of the odd-L potentials are still too repulsive, our model is close to a quantitative account of the experimental data.

It is noteworthy that our model gives a unified picture of the NN interaction which is versatile enough to lead to an understanding of the extreme short-range part of the NN interaction in terms of the quark exchange mechanism and at the same time contains much of the conventional meson exchange description through the exchange of $(q\bar{q})$ pairs between nucleons through the coupling kernels. The central parts of the NN interaction gain important contributions from all three major components of our exchange kernels, the pure $(3q)$-$(3q)$ kernels and the coupling kernels of $(3q)(q\bar{q})$ and $(3q)(q\bar{q})^2$-type. The pure $(3q)$-$(3q)$ kernels have strong central components which are repulsive in both even and odd partial waves and arise predominantly from the color-magnetic contact term of the quark-quark interaction. The coupling kernels of $(3q)(q\bar{q})$-type lead to central force terms which

are strongly energy dependent and show a weak attraction in the even partial waves but a very strong repulsion in the odd partial waves. The central forces arising from the $(3q)(q\bar{q})^2$ coupling kernels are attractive in both even and odd channels but not attractive enough to overcome the strong repulsive $(3q)(q\bar{q})$ potentials in the odd partial waves. Since momentum-dependent distortion effects have so far been neglected there is the possibility that such changes in the nucleon wave functions may decrease the repulsion of the strongly energy-dependent $(3q)(q\bar{q})$ coupling kernels. Since the central potentials result from a partial cancellation of strong attractive and repulsive components, contributions to the coupling kernels of second order in the single nucleon c_α and c_β may also have to be examined.

The tensor and spin-orbit forces were analyzed in ref. [4]). The contributions of the pure (3q)-(3q) kernels to the tensor force are completely negligible. The tensor force arises almost exclusively from the coupling kernels of $(3q)(q\bar{q})$ type and is dominated by simple $(q\bar{q})$-exchange terms of $N\pi$ and $N\rho$ type. It therefore has meson exchange characteristics. Since the ρ-exchange terms partially cancel the π-exchange terms and since the predicted $N\pi$ tensor force was too weak by a factor of ~ 3, the improvement of this term is vital to gain a tensor force sufficiently attractive in the even and repulsive in the odd-L channels. The LS force has the expected repulsive character in the even-L and attractive character in the odd-L channels. Its most important contributions come from simple $(q\bar{q})$ exchange terms, this time of $N\rho$ and $N\omega$ type. Although about 60–65% of the LS force in the most important medium-range region of the attractive odd-L potentials arises through the coupling kernels of $(3q)(q\bar{q})$ type, the pure (3q)-(3q) kernels do make important contributions. These arise from the combined effects of symmetric LS (70%) and antisymmetric LS (30%) terms in the quark-quark interaction.

Despite the successes of our extended quark model of the NN interaction important problems remain. The exchange of a simple $(q\bar{q})$ pair with the quantum numbers of a pion is clearly not a good model for a realistic OPEP with its long-range Yukawa tail. An improved quark model of the pion is needed for a truly quantitative description of the NN interaction. The good connection between our model and the conventional one-boson exchange picture is made through the RGM formalism and its explicit coordinate space representation which makes it possible to isolate the exchange of a $(q\bar{q})$ pair between nucleons. However, the RGM formalism is nonrelativistic. Although low-energy NN scattering is a nonrelativistic problem, relativistic effects may become very important at the level of quark exchange phenomena. Relativistic corrections should therefore be incorporated into the model, even if a truly relativistic formulation of the NN interaction is still too much of a challenge for this complicated many-body problem.

Appendix A

In this appendix, we will continue the mathematical discussion of appendix A of ref. [4]), and derive a convenient analytic formula for the RGM matrix elements

with respect to the gaussian trial functions of eq. (54). The class of the RGM kernels treated in this appendix is restricted to that of a gaussian two-cluster RGM kernel, but is otherwise quite general and involves the central, *LS*, and tensor kernels required not only for the (3q)-(3q) analysis but also for the (3q)-(3q)(q$\bar{q}$) and (3q)-(3q)(q$\bar{q}$)^2 coupling problems. The RGM matrix element is explicitly given by the coefficients of a particular term of the GCM kernel, which is assumed to be

$$I_{jk}^{\mathrm{GCM}}(z, z') = \exp\left[-\tfrac{1}{2}fz^{*2} - \tfrac{1}{2}gz'^2 + h(z^* \cdot z')\right] \times \left[(az^* + bz')_j (cz^* + dz')_k + e\delta_{jk}\right]. \tag{A.1}$$

The central (C), *LS*, and tensor (T) RGM kernels are defined through

$$\begin{aligned} \mathcal{M}^{\mathrm{C}}(\boldsymbol{R}, \boldsymbol{R}') &= \tfrac{1}{3}\sum_j I_{jj}(\boldsymbol{R}, \boldsymbol{R}'), \\ \mathcal{M}^{LS}(\boldsymbol{R}, \boldsymbol{R}') &= i\sum_{ijk} \varepsilon_{ijk} I_{jk}(\boldsymbol{R}, \boldsymbol{R}') S_i, \\ \mathcal{M}^{\mathrm{T}}(\boldsymbol{R}, \boldsymbol{R}') &= \tfrac{1}{2}\sum_{jk} \left[I_{jk}(\boldsymbol{R}, \boldsymbol{R}') + I_{kj}(\boldsymbol{R}, \boldsymbol{R}')\right]\sigma_{1j}\sigma_{2k} - \mathcal{M}^{C}(\boldsymbol{R}, \boldsymbol{R}')(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2), \end{aligned} \tag{A.2}$$

where $I_{jk}(\boldsymbol{R}, \boldsymbol{R}')$ are given by (A.1) through the Bargmann transformation (see eqs. (37) and (38))

$$I_{jk}(\boldsymbol{R}, \boldsymbol{R}') = \int \mathrm{d}\mu(z)\, \mathrm{d}\mu(z') A_\gamma(\boldsymbol{R}, z) A_{\gamma'}(\boldsymbol{R}', z')^* I_{jk}^{\mathrm{GCM}}(z, z'). \tag{A.3}$$

Let us express the RGM kernel of eq. (A.2) as

$$\mathcal{M}^{\Omega}(\boldsymbol{R}, \boldsymbol{R}') = \left(\frac{2\sqrt{\gamma\gamma'}}{\pi D}\right)^{3/2} \exp\left[-A\gamma R^2 - B\gamma' R'^2 + C\sqrt{\gamma\gamma'}(\boldsymbol{R} \cdot \boldsymbol{R}')\right] \times \begin{cases} E + F\gamma R^2 + G\gamma R'^2 + V\sqrt{\gamma\gamma'}(\boldsymbol{R} \cdot \boldsymbol{R}') \\ V^{LS}\sqrt{\gamma\gamma'}\, i([\boldsymbol{R} \times \boldsymbol{R}'] \cdot \boldsymbol{S}) \\ F\gamma S_{12}(\boldsymbol{R}, \boldsymbol{R}) + G\gamma' S_{12}(\boldsymbol{R}', \boldsymbol{R}') + V\sqrt{\gamma\gamma'} S_{12}(\boldsymbol{R}, \boldsymbol{R}') \end{cases} \quad \text{for } \Omega = \begin{cases} \mathrm{C} \\ LS \\ \mathrm{T}. \end{cases} \tag{A.4}$$

Here the tensor operators $S_{12}(\boldsymbol{U}, \boldsymbol{V})$ are defined by

$$S_{12}(\boldsymbol{U}, \boldsymbol{V}) = \tfrac{3}{2}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{U})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{V}) + \tfrac{3}{2}(\boldsymbol{\sigma}_2 \cdot \boldsymbol{U})(\boldsymbol{\sigma}_1 \cdot \boldsymbol{V}) - (\boldsymbol{U} \cdot \boldsymbol{V})(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2), \tag{A.5}$$

and the coefficients F, G, and V are introduced by

$$\begin{aligned} F &= \tfrac{4}{3}(F_{ab}/D)(F_{cd}/D), \\ G &= \tfrac{4}{3}(G_{ab}/D)(G_{cd}/D), \\ V &= \tfrac{4}{3}\left[(F_{ab}/D)(G_{cd}/D) + (G_{ab}/D)(F_{cd}/D)\right]. \end{aligned} \tag{A.6}$$

The coefficients $A, \ldots, E$, V^{LS}, $F_{ab}(F_{cd})$, and $G_{ab}(G_{cd})$ are explicitly given in appendix A of ref. [4]) in terms of the $a, \ldots, h$ defined by eq. (A.1). (See eqs. (A.6), (A.15), and (A.16) of ref. [4]).)

The first step in the derivation of the RGM matrix elements is the partial wave decomposition of the RGM kernel in eq. (A.4). Let us introduce the spherical Bessel function of an imaginary argument

$$\mathcal{I}_l(x) = 4\pi i^l j_l(-ix)$$
$$= \frac{4\pi}{(2l+1)!!} x^l + \mathrm{O}(x^{l+2}) \tag{A.7}$$

for the partial wave decomposition

$$\exp[C\sqrt{\gamma\gamma'}(\boldsymbol{R}\cdot\boldsymbol{R}')] = \sum_{lm} \mathcal{I}_l(C\sqrt{\gamma\gamma'}RR')\, Y_{lm}(\hat{\boldsymbol{R}})\, Y_{lm}(\hat{\boldsymbol{R}}')^* . \tag{A.8}$$

Then the spin, angular matrix element of eq. (A.4), with respect to the standard vector-coupled spin, angular functions of eq. (54c), yields

$$\mathcal{M}^{\Omega,SJ}_{ll'}(R, R')$$
$$\equiv \langle \mathcal{Y}^{JM}_{lS}(\hat{\boldsymbol{R}}; \text{spin}) | \mathcal{M}^{\Omega}(\boldsymbol{R}, \boldsymbol{R}') | \mathcal{Y}^{JM}_{l'S}(\hat{\boldsymbol{R}}'; \text{spin}) \rangle$$
$$= \left(\frac{2\sqrt{\gamma\gamma'}}{\pi D}\right)^{3/2} \exp[-A\gamma R^2 - B\gamma' R'^2]$$
$$\times \begin{cases} \delta_{ll'}\{[E + l(V/C) + F\gamma R^2 + G\gamma' R'^2]\mathcal{I}_l + V\sqrt{\gamma\gamma'}RR'\mathcal{I}_{l+1}\} \\ -\delta_{S1}\delta_{ll'}\langle \boldsymbol{L}\cdot\boldsymbol{S}\rangle (V^{LS}/C)\mathcal{I}_l \\ \delta_{S1}(S_{12})^J_{ll'}\{\delta_{ll'}(l+\frac{3}{2})(V/C)\mathcal{I}_l + F\gamma R^2\mathcal{I}_{l'} + G\gamma' R'^2\mathcal{I}_l \\ + V\sqrt{\gamma\gamma'}RR'[\delta_{ll'}\mathcal{I}_{l+1} + \delta_{l+2,l'}\mathcal{I}_{l+1} + \delta_{l,l'+2}\mathcal{I}_{l'+1}]\} \end{cases} \quad \text{for } \Omega = \begin{cases} \mathrm{C} \\ LS \\ \mathrm{T}. \end{cases} \tag{A.9}$$

In eq. (A.9), $\mathcal{I}_l$ is a shorthand notation for $\mathcal{I}_l = \mathcal{I}_l(C\sqrt{\gamma\gamma'}RR')$, and the standard matrix elements of eq. (62) for the $\boldsymbol{L}\cdot\boldsymbol{S}$ and the tensor operator $S_{12} = S_{12}(\hat{\boldsymbol{R}}, \hat{\boldsymbol{R}})$ are used.

The gaussian matrix element of eq. (A.9), in terms of the trial functions of eq. (54b) with the width parameters η and η', is most easily derived by the technique developed in appendix B of ref. [14]). By applying the formulae in eqs. (B.7) through (B.9) of ref. [14]) to the bra and ket states, one finds

$$M^{\Omega,SJ}_{ll'}(\eta;\eta') \equiv \int_0^\infty R^2\,\mathrm{d}R \int_0^\infty R'^2\,\mathrm{d}R'\, u_l(R,\eta) u_{l'}(R',\eta') \mathcal{M}^{\Omega SJ}_{ll'}(R,R')$$
$$= \left(\frac{\pi}{2\sqrt{\gamma\gamma'}}\right)^{3/2} \left[\frac{(2l+1)!!}{4\pi}\frac{(2l'+1)!!}{4\pi}(1-\varepsilon^2)^{l+3/2}(1-\varepsilon'^2)^{l'+3/2}\right]^{1/2}$$
$$\times \lim_{R\to 0}\lim_{R'\to 0} (\sqrt{-\varepsilon}^*2\sqrt{\gamma}R)^{-l}(\sqrt{-\varepsilon'}2\sqrt{\gamma'}R')^{-l'} \tilde{\mathcal{M}}^{\Omega,SJ}_{ll'}(R,R') , \tag{A.10}$$

where the η, η' dependence always appears in the fom of $\varepsilon, \varepsilon'$ of eq. (61), and $\tilde{\mathcal{M}}^{\Omega,SJ}_{ll'}(R, R')$ is the partial wave decomposition of the RGM kernel $\tilde{\mathcal{M}}^{\Omega}(\boldsymbol{R}, \boldsymbol{R}')$ which corresponds to the GCM kernel $\tilde{I}^{\mathrm{GCM}}_{jk}(z, z') = I^{\mathrm{GCM}}_{jk}(\sqrt{-\varepsilon}z, \sqrt{-\varepsilon'}z')$. Since we have

assumed the explicit form eq. (A.1) for the original GCM kernel, $\tilde{\mathcal{M}}^{\Omega,SJ}_{ll'}(R, R')$ is easily obtained from eqs. (A.4), (A.6), and eqs. (A.6), (A.15), (A.16) of ref. [4]) by a simple replacement

$$
\begin{aligned}
&f \to (-\varepsilon) f, \qquad g \to (-\varepsilon') g, \qquad h \to \sqrt{-\varepsilon}^{*}\sqrt{-\varepsilon'}\, h, \\
&a \to \sqrt{-\varepsilon}^{*} a, \qquad b \to \sqrt{-\varepsilon'}\, b, \\
&c \to \sqrt{-\varepsilon}^{*} c, \qquad d \to \sqrt{-\varepsilon'}\, d.
\end{aligned} \tag{A.11}
$$

The process of limiting $R, R' \to 0$ in eq. (A.10) allows only a restricted number of terms to survive in eq. (A.9); viz., due to the asymptotic behavior of the $\mathcal{J}_l(x)$ in (A.7), the F, G and V terms of the central kernel and the last V terms of the tensor part go to zero. As a result, one obtains the following simple expression for the RGM matrix element of eq. (A.10);

$$
\begin{aligned}
&M^{\Omega,SJ}_{ll'}(\eta;\eta') \\
&= (1-\varepsilon^2)^{(l+3/2)/2}(1-\varepsilon'^2)^{(l'+3/2)/2}(1/\tilde{D})^{3/2} \\
&\times \begin{cases}
\delta_{ll'}\tilde{C}^{l-1}(\tilde{C}\tilde{E} + l\tilde{V}) \\
-\delta_{S1}\delta_{ll'}\langle \boldsymbol{L}\cdot\boldsymbol{S}\rangle \tilde{C}^{l-1}\tilde{V}^{LS} \\
\delta_{S1}(S_{12})^{J}_{ll'}[\delta_{ll'}(l+\tfrac{3}{2})\tilde{C}^{l-1}\tilde{V} + \delta_{l,l'+2}\sqrt{(2l+1)(2l-1)} \\
\times \tilde{C}^{l'}\tilde{F} + \delta_{l+2,l'}\sqrt{(2l'+1)(2l'-1)}\tilde{C}^{l}\tilde{G}]
\end{cases}
\qquad \text{for } \Omega = \begin{cases} \mathrm{C} \\ LS \\ \mathrm{T}, \end{cases}
\end{aligned} \tag{A.12}
$$

where the coefficients $\tilde{C}, \ldots, \tilde{G}, \tilde{V}, \tilde{V}^{LS}$ are given by

$$
\begin{aligned}
&\tilde{D} = (1+\varepsilon f)(1+\varepsilon' g) - \varepsilon\varepsilon' h^2, \qquad \tilde{C} = (h/\tilde{D}), \\
&\tilde{E} = e + (\tilde{H}/\tilde{D}), \qquad \tilde{V}^{LS} = [(ad-bc)/\tilde{D}], \\
&\tilde{F} = \tfrac{1}{3}(\tilde{F}_{ab}/\tilde{D})(\tilde{F}_{cd}/\tilde{D}), \qquad \tilde{G} = \tfrac{1}{3}(\tilde{G}_{ab}/\tilde{D})(\tilde{G}_{cd}/\tilde{D}), \\
&\tilde{V} = \tfrac{1}{3}[(\tilde{F}_{ab}/\tilde{D})(\tilde{G}_{cd}/\tilde{D}) + (\tilde{G}_{ab}/\tilde{D})(\tilde{F}_{cd}/\tilde{D})], \\
&\tilde{F}_{ab} = (1+\varepsilon' g)a + \varepsilon' h b, \qquad \tilde{G}_{ab} = (1+\varepsilon f)b + \varepsilon h a, \\
&\tilde{H} = \varepsilon c\tilde{F}_{ab} + \varepsilon' d\tilde{G}_{ab} = \varepsilon a\tilde{F}_{cd} + \varepsilon' b\tilde{G}_{cd}.
\end{aligned} \tag{A.13}
$$

As for the transformation formula of the RGM kernel and for the Wigner transform in appendix A of ref. [4]), it is also convenient to let the coefficients of the GCM kernel adhere to the parameterization

$$
\begin{aligned}
&f = f^{(0)} + \lambda p^2, \qquad g = g^{(0)} + \lambda q^2, \qquad h = h^{(0)} - \lambda pq, \\
&a = a^{(0)} - \lambda pm, \qquad b = b^{(0)} - \lambda qm, \qquad c = c^{(0)} - \lambda pk, \\
&d = d^{(0)} - \lambda qk, \qquad e = e^{(0)} - \lambda mk,
\end{aligned} \tag{A.14}
$$

where the $\lambda = 0$ case generally corresponds to the normalization kernel. Let us again use the previous rule [4]) that the superscript (0) is always used to denote quantities evaluated with $\lambda = 0$; e.g.,

$$\tilde{F}^{(0)}_{ab} = (1+\varepsilon' g^{(0)})a + \varepsilon' h^{(0)} b\,, \qquad \tilde{G}^{(0)}_{ab} = (1+\varepsilon f^{(0)})b + \varepsilon h^{(0)} a\,. \tag{A.15}$$

Then the coefficient $\tilde{D}$ in eq. (A.13) is expressed by

$$\tilde{D} = \tilde{D}^{(0)}(1+\lambda\tilde{\alpha})\,, \qquad \tilde{\alpha} = (\tilde{A}^{(0)}_{pq}/\tilde{D}^{(0)})\,,$$
$$\tilde{A}^{(0)}_{ab} = \varepsilon a \tilde{F}^{(0)}_{pq} + \varepsilon' b \tilde{G}^{(0)}_{pq}\,. \tag{A.16}$$

For the other coefficients $\tilde{\mathscr{D}} = \tilde{C}, \tilde{E}, \tilde{F}, \tilde{G}, \tilde{V}$, and $\tilde{V}^{LS}$ in eq. (A.13), it is convenient to use a component notation $\tilde{\mathscr{D}}^{(0)}$, $\tilde{\mathscr{D}}^{(1)}$, and $\tilde{\mathscr{D}}^{(2)}$ through

$$\tilde{\mathscr{D}} = \tilde{\mathscr{D}}^{(0)} - \frac{\lambda}{1+\lambda\tilde{\alpha}}\tilde{\mathscr{D}}^{(1)} + \left(\frac{\lambda}{1+\lambda\tilde{\alpha}}\right)^2 \tilde{\mathscr{D}}^{(2)}\,, \tag{A.17}$$

where the $\tilde{\mathscr{D}}^{(0)}$ are obtained from eq. (A.13) by the simple replacement: $a, \ldots, h \to a^{(0)}, \ldots, h^{(0)}$. The non-zero higher order components $\tilde{\mathscr{D}}^{(1)}$ and $\tilde{\mathscr{D}}^{(2)}$ are given by

$$\tilde{C}^{(1)} = (\tilde{F}^{(0)}_{pq}/\tilde{D}^{(0)})(\tilde{G}^{(0)}_{pq}/\tilde{D}^{(0)})\,,$$
$$\tilde{E}^{(1)} = [\tilde{A}^{(0)}_{a^{(0)}b^{(0)}}/\tilde{D}^{(0)} + m][\tilde{A}^{(0)}_{c^{(0)}d^{(0)}}/\tilde{D}^{(0)} + k]\,,$$
$$\tilde{F}^{(1)} = \tfrac{1}{3}(\tilde{F}^{(0)}_{pq}/\tilde{D}^{(0)})\{(\tilde{F}^{(0)}_{a^{(0)}b^{(0)}}/\tilde{D}^{(0)})[\tilde{A}^{(0)}_{c^{(0)}d^{(0)}}/\tilde{D}^{(0)} + k]$$
$$+ (\tilde{F}^{(0)}_{c^{(0)}d^{(0)}}/\tilde{D}^{(0)})[\tilde{A}^{(0)}_{a^{(0)}b^{(0)}}/\tilde{D}^{(0)} + m]\}\,,$$
$$\tilde{F}^{(2)} = \tfrac{1}{3}(\tilde{F}^{(0)}_{pq}/\tilde{D}^{(0)})^2 \tilde{E}^{(1)}\,,$$
$$\tilde{V}^{(1)} = \tfrac{1}{3}\{[(\tilde{F}^{(0)}_{pq}/\tilde{D}^{(0)})(\tilde{G}^{(0)}_{c^{(0)}d^{(0)}}/\tilde{D}^{(0)}) + (\tilde{G}^{(0)}_{pq}/\tilde{D}^{(0)})(\tilde{F}^{(0)}_{c^{(0)}d^{(0)}}/\tilde{D}^{(0)})]$$
$$\times[\tilde{A}^{(0)}_{a^{(0)}b^{(0)}}/\tilde{D}^{(0)} + m]$$
$$+ [(\tilde{F}^{(0)}_{pq}/\tilde{D}^{(0)})(\tilde{G}^{(0)}_{a^{(0)}b^{(0)}}/\tilde{D}^{(0)}) + (\tilde{G}^{(0)}_{pq}/\tilde{D}^{(0)})(\tilde{F}^{(0)}_{a^{(0)}b^{(0)}}/\tilde{D}^{(0)})$$
$$\times[\tilde{A}^{(0)}_{c^{(0)}d^{(0)}}/\tilde{D}^{(0)} + k]\}\,,$$
$$\tilde{V}^{(2)} = \tfrac{2}{3}\tilde{C}^{(1)}\tilde{E}^{(1)}\,,$$
$$\tilde{V}^{LS(1)} = \tilde{\alpha}\tilde{V}^{LS(0)} - [(pb^{(0)} - qa^{(0)})k - (pd^{(0)} - qc^{(0)})m]/\tilde{D}^{(0)}\,. \tag{A.18}$$

The expression of $\tilde{G}^{(1)}(\tilde{G}^{(2)})$ is obtained from that of $\tilde{F}^{(1)}(\tilde{F}^{(2)})$ by a simple change $F \to G$. The explicit λ dependence in the coefficients in eqs. (A.16) and (A.17) is especially convenient for analytic evaluation of the integral over λ, which makes the formula given here applicable to the RGM kernels with arbitrary kinds of radial functions of the two-body interaction, as long as they can be represented by gaussian integrals.

Appendix B

In this appendix, the explicit expressions for $\tilde{D}^{(0)}$ and $I_l(\tilde{C}; \tilde{\mathscr{D}})$ of eq. (63) are given for each piece of the $(3q)(q\bar{q})$ and $(3q)(q\bar{q})^2$ coupling matrix elements and

of the pure (3q)-(3q) matrix elements. The spin, isospin, color factors of the (3q)-(3q) RGM kernel needed in eq. (67) are also given.

First, for the (3q)(qq̄) coupling matrix elements, the $I_l(\tilde{C};\tilde{\mathscr{D}})$ of eq. (63) for $\tilde{\mathscr{D}}=\tilde{E},\tilde{F},\tilde{G},\tilde{V}$, or $\tilde{V}^{LS}$ are given, through a common unified expression by

$$I_l(\tilde{C};\tilde{\mathscr{D}})=\frac{13}{6}\left(\frac{13}{13+\tilde{\alpha}}\right)^{1/2}\sum_{r=0}^{l}\binom{l}{r}\frac{(-1)^r}{2r+1}(\tilde{C}^{(0)})^{l-r}\left(\frac{\tilde{C}^{(1)}}{13+\tilde{\alpha}}\right)^r \times\left[\tilde{\mathscr{D}}^{(0)}-\frac{2r+1}{2r+3}\frac{\tilde{\mathscr{D}}^{(1)}}{13+\tilde{\alpha}}+\frac{2r+1}{2r+5}\frac{\tilde{\mathscr{D}}^{(2)}}{(13+\tilde{\alpha})^2}\right]. \tag{B.1}$$

The necessary coefficients, $\tilde{D}^{(0)}$, $\tilde{\alpha}$, $\tilde{\mathscr{D}}^{(0)}$, $\tilde{\mathscr{D}}^{(1)}$, $\tilde{\mathscr{D}}^{(2)}$, are given through eqs. (A.13), (A.16), (A.18) in terms of the coefficients $f^{(0)},\ldots,m$ given in eq. (11) of ref. [4]) as simple functions of $xyuv$ and $\alpha\beta$ for each $\mathcal{O}Pi$. In eq. (A.13) the expressions of the coefficients $\tilde{D}^{(0)}$ and $\tilde{\mathscr{D}}^{(0)}(=\tilde{C}^{(0)},\tilde{E}^{(0)},\tilde{F}^{(0)},\tilde{G}^{(0)},\tilde{V}^{(0)}$, or $\tilde{V}^{LS(0)})$ are obtained by the simple replacement: $a,\ldots,h\to a^{(0)},\ldots,h^{(0)}$.

For the $(3q)(q\bar{q})^2$ coupling matrix elements, $I_l(\tilde{C};\tilde{E})$ is given by

$$I_l(\tilde{C};\tilde{E})=(\tilde{C}^{(0)})^l\left(\frac{1}{1+\xi}\right)^{l+1}\left[\sum_{r=0}^{l}\binom{l}{r}\frac{1}{2r+1}\xi^r-\frac{\hbar}{2mcb}\sqrt{\frac{\pi}{2}}\left(\frac{1}{1+\xi}\right)^{1/2}\right], \tag{B.2}$$

where $\tilde{C}^{(0)}$, ξ, and $\tilde{D}^{(0)}$ are given by

$$\tilde{C}^{(0)}=\sqrt{\tfrac{5}{7}}[1-\tfrac{1}{3}(x+u)]/\tilde{D}^{(0)},\qquad \xi=\delta_{x-u,1}\tfrac{5}{21}(\varepsilon/\tilde{D}^{(0)}),$$
$$\tilde{D}^{(0)}=1-\varepsilon\tfrac{2}{7}[1-\tfrac{5}{3}(x-u)+\tfrac{5}{18}(x-u)^2]-\varepsilon\varepsilon'\tfrac{5}{7}[1-\tfrac{1}{3}(x+u)]^2, \tag{B.3}$$

for each $P=xyuv$.

For the (3q)-(3q) matrix elements, one should use

$$I_l(\tilde{C};\tilde{E})=\begin{cases}\tfrac{2}{3}(l+\tfrac{3}{2})(\tilde{C}^{(0)})^l[1-\varepsilon-\varepsilon'+\tfrac{1}{9}\varepsilon\varepsilon']/\tilde{D}^{(0)} & \\ \left(\dfrac{3}{3+\tilde{\alpha}}\right)^{3/2}\left[\tilde{C}^{(0)}-\dfrac{\tilde{C}^{(1)}}{3+\tilde{\alpha}}\right]^l & \text{for } \Omega=\begin{cases}\mathrm{K}\\ \mathrm{GC}\\ \mathrm{CC},\end{cases}\\ \left(\dfrac{3}{3+\tilde{\alpha}}\right)^{1/2}\displaystyle\sum_{r=0}^{l}\binom{l}{r}\frac{(-1)^r}{2r+1}(\tilde{C}^{(0)})^{l-r}\left(\frac{\tilde{C}^{(1)}}{3+\tilde{\alpha}}\right)^r & \end{cases}$$

$$I_l(\tilde{C};\tilde{V}^{LS})=(1/\tilde{D}^{(0)})\left(\frac{3}{3+\tilde{\alpha}}\right)^{3/2}\sum_{r=0}^{l}\binom{l}{r}\frac{(-1)^r}{2r+3}(\tilde{C}^{(0)})^{l-r}\left(\frac{\tilde{C}^{(1)}}{3+\tilde{\alpha}}\right)^r \quad \text{for } \Omega=LS,$$

$$I_l(\tilde{C};\tilde{\mathscr{D}})=\tilde{\mathscr{D}}\left(\frac{3}{3+\tilde{\alpha}}\right)^{5/2}\sum_{r=0}^{l}\binom{l}{r}\frac{(-1)^r}{2r+5}(\tilde{C}^{(0)})^{l-r}\left(\frac{\tilde{C}^{(1)}}{3+\tilde{\alpha}}\right)^r \quad \text{for } \Omega=\mathrm{T}, \tag{B.4}$$

where $\tilde{\mathscr{D}}=\tilde{V}$, $\tilde{F}$, or $\tilde{G}$ for the tensor term. The coefficients $\tilde{D}^{(0)}$, $\tilde{\alpha}$, $\tilde{C}^{(0)}$, $\tilde{C}^{(1)}$, and $\tilde{\mathscr{D}}$ are expressed by

$$\tilde{D}^{(0)}=1-\tfrac{1}{9}\varepsilon\varepsilon',\qquad \tilde{\alpha}=[\varepsilon p^2+\varepsilon' q^2+\tfrac{2}{3}\varepsilon\varepsilon' pq]/\tilde{D}^{(0)},$$
$$\tilde{C}^{(0)}=\tfrac{1}{3}(1/\tilde{D}^{(0)}),\qquad \tilde{C}^{(1)}=\tilde{C}^{(0)}(\tilde{\alpha}+3pq),$$
$$\tilde{V}=\tfrac{2}{3}\tilde{C}^{(1)},\qquad \tilde{F}=\tfrac{1}{3}[\tilde{C}^{(0)}(3p+\varepsilon' q)]^2,\qquad \tilde{G}=\tfrac{1}{3}[\tilde{C}^{(0)}(3q+\varepsilon p)]^2, \tag{B.5}$$

in terms of p and q given in eq. (66) for each exchange type $\mathcal{T}=$ E, S, D_+, or D_-. The necessary spin, isospin, color factors X_N, $X_{\mathcal{T}}$, $X_{\mathcal{T}}^{LS}$, and $X_{\mathcal{T}}^{T}$ in eq. (67) are given, in terms of the matrix elements of $(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$ $[2S(S+1)-3]$ and $(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)$ $[2T(T+1)-3]$, through [15,17])

$$
\begin{aligned}
X_N &= -\tfrac{3}{4}[1+\tfrac{1}{9}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)+\tfrac{1}{9}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)+\tfrac{25}{81}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)]\,,\\
X_E &= \tfrac{1}{6}[1+\tfrac{5}{9}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)+\tfrac{5}{9}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)+\tfrac{65}{81}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)]\,,\\
X_{D_+} &= \tfrac{11}{36}[1+\tfrac{5}{33}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)+\tfrac{1}{9}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)+\tfrac{7}{27}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)]\,,\\
X_{D_-} &= \tfrac{3}{4}[1+\tfrac{1}{81}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)+\tfrac{1}{9}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)+\tfrac{25}{729}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)]\,,\\
X_{D_+}^{LS} &= -\tfrac{1}{9}[1+\tfrac{1}{9}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)], \qquad X_{D_-}^{LS} = \tfrac{1}{3}[1+\tfrac{5}{9}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)]\,,\\
X_S^{LS} &= \tfrac{1}{27}[1+\tfrac{7}{9}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)]\,, \qquad X_S^{T} = -\tfrac{1}{54}[1-\tfrac{5}{9}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)]\,,\\
X_{D_+}^{T} &= \tfrac{1}{108}[1+\tfrac{1}{9}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)]\,, \qquad X_{D_-}^{T} = \tfrac{1}{54}[1+\tfrac{25}{9}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)]\,.
\end{aligned}
\tag{B.6}
$$

Appendix C

In this appendix, we will derive a compact expression for the gaussian matrix elements of the one pion tensor force when the form factor of the coupling constant is assumed to be gaussian, $g^2(\boldsymbol{k}) = g^2\exp(-\beta k^2)$. As is discussed in sect. 6, the RGM matrix element of the tensor component for the Nπ type (3q)(q$\bar{\text{q}}$) coupling kernels in the present quark model is replaced with this more realistic matrix element by assuming the experimental coupling constant $g^2_{NN\pi}=14.17$ and the (3q) value $\beta = 0.0943\ \text{fm}^2$; (see ref. [4]), in particular, the crossed curves of fig. 3 of ref. [4])).

The one-pion tensor force with the gaussian form factor is expressed by (leaving out a trivial isospin factor $(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)$)

$$
U^T = g^2\tfrac{1}{3}m_\pi c^2(m_\pi/2M_N)^2[-D_\alpha(x)+Z_\alpha(x)]S_{12}\,, \tag{C.1}
$$

where S_{12} is the tensor operator and $x=(m_\pi c/\hbar)R$ with a pion mass $m_\pi c^2 = 138$ MeV. The functions $D_\alpha(x)$ and $Z_\alpha(x)=[1-(3/x)(\partial/\partial x)]Y_\alpha(x)$ with $\alpha=(m_\pi c/\hbar)^2\beta$ in eq. (C.1) are defined through

$$
\begin{aligned}
\left.\begin{matrix} D_\alpha(x) \\ Y_\alpha(x) \end{matrix}\right\} &\equiv \frac{4\pi}{(2\pi)^3}\int \mathrm{d}\boldsymbol{q}\, \mathrm{e}^{i(\boldsymbol{q}\cdot\boldsymbol{x})}\left\{\begin{matrix} 1 \\ 1/(1+q^2)\end{matrix}\right\}\mathrm{e}^{-\alpha q^2}\\
&= \begin{cases} \dfrac{1}{\sqrt{4\pi}}\,\alpha^{-3/2}\exp[-(x^2/4\alpha)] \\[2ex] \dfrac{\mathrm{e}^\alpha}{2x}\left\{\mathrm{e}^{-x}\left[1+\operatorname{erf}\left(\dfrac{x}{2\sqrt{\alpha}}-\sqrt{\alpha}\right)\right]-\mathrm{e}^{x}\left[1-\operatorname{erf}\left(\dfrac{x}{2\sqrt{\alpha}}+\sqrt{\alpha}\right)\right]\right\}, \end{cases}
\end{aligned}
\tag{C.2}
$$

where erf is the standard error function. For $\beta\to 0$ these become the standard Yukawa functions; viz., $D_0(x)=4\pi\delta(x)$, $Y_0(x)=e^{-x}/x$, and $Z_0(x)=[1+(3/x)+(3/x^2)]Y_0(x)$.

The matrix element of U^{T} with respect to the gaussian trial functions of eq. (54) is given by

$$M^{SJ}_{ll'}(\eta;\eta') \equiv \langle u_l(R,\eta)\mathcal{Y}^{JM}_{lS}(\hat{\boldsymbol{R}};\text{spin})|U^{\mathrm{T}}|u_{l'}(R,\eta')\mathcal{Y}^{JM}_{l'S}(\hat{\boldsymbol{R}};\text{spin})\rangle$$
$$= \delta_{S1}(S_{12})^J_{ll'} g^2 \tfrac{1}{3} m_\pi c^2 (m_\pi/2M_{\mathrm{N}})^2 N_{ll'}(\eta;\eta')$$
$$\times\left[-\frac{1}{2\alpha\sqrt{\pi\alpha}}\left(\frac{4\alpha}{a^2+4\alpha}\right)^{(l+l'+3)/2} + \left(1-\frac{6}{a^2}\right)J_{(l+l')/2}(a;\alpha)\right.$$
$$\left. + \frac{6}{a^2}J_{(l+l')/2-1}(a;\alpha)\right], \tag{C.3}$$

where $a = (m_\pi c/\hbar)(\eta+\eta')^{-1/2}$ and

$$N_{ll'}(\eta;\eta') = \Gamma\left(\frac{l+l'}{2}+\frac{3}{2}\right)[\Gamma(l+\tfrac{3}{2})\Gamma(l'+\tfrac{3}{2})]^{-1/2}$$
$$\times\left(\frac{2\eta}{\eta+\eta'}\right)^{(l+3/2)/2}\left(\frac{2\eta'}{\eta+\eta'}\right)^{(l'+3/2)/2}. \tag{C.4}$$

The matrix elements $(S_{12})^J_{ll'}$ in eq. (C.3) are given in eq. (62). The function $J_p(a;\alpha)$ $(p=0,1,2,\ldots)$ in eq. (C.3) is defined by

$$J_p(a;\alpha) \equiv [2/\Gamma(p+\tfrac{3}{2})]\int_0^\infty \mathrm{d}x\, x^{2p+2}\,\mathrm{e}^{-x^2}Y_\alpha(ax)$$
$$= \frac{1}{a}\left(\frac{4a^2}{a^2+4\alpha}\right)^{p+1/2}\sum_{s=0}^{p}\frac{p!}{s!(2p-2s+1)!}\left(\frac{\alpha}{a^2}\right)^2 I_{2(p-s)+1}((a^2+4\alpha)^{1/2}), \tag{C.5}$$

through an integral

$$I_q(a) = \frac{2}{\sqrt{\pi}}\int_0^\infty \mathrm{d}t\, t^q\,\mathrm{e}^{-t^2-at}. \tag{C.6}$$

The numerical values of $\mathrm{I}_q(a)$ are most easily obtained through a recursion relation

$$I_0(a) = \mathrm{e}^{a^2/4}[1-\mathrm{erf}\,(\tfrac{1}{2}a)],$$
$$I_1(a) = (1/\sqrt{\pi}) - \tfrac{1}{2}aI_0(a),$$
$$I_{q+2}(a) = \tfrac{1}{2}(q+1)I_q(a) - \tfrac{1}{2}aI_{q+1}(a) \qquad (q=0,1,2,\ldots). \tag{C.7}$$

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