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SPONTANEOUS SYMMETRY BREAKING IN QUASI-SUPER-RENORMALIZABLE MODELS

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By studying the effective potential and renormalization group for quasi-super-renormalizable models it is demonstrated the models can undergo spontaneous symmetry breaking in a manner consistent with stability A relation among ratios of masses is seen to hold at the minimum of the potential

1. Introduction

In a recent paper [1] we introduced the notion of quasi-super-renormalizable theories as those models obtained from finite, globally supersymmetric theories by adding arbitrary soft-breaking terms (scalar masses, fermion masses, and cubic scalar couplings) The gauge coupling constant is not renormalized in such models, while the massive parameters in general require infinite counterterms, the soft breakings can be chosen to leave the theories finite, but as these finiteness relations are not enforced by any known symmetry, such a choice, we argued, would involve an unnatural fine tuning of parameters For example, one could take *any* field theory which has a fixed point and choose the coupling constant to take on precisely the value at that point, whereupon the theory has a zero beta function, but one would not imagine one had a sensible finite theory, as a slight deviation from this metastable value would cause the parameter to run with the scale In ref [1] we showed that the finiteness relations among the soft breakings correspond to an *infrared* attractive hypersurface in the space of the coupling constants and masses, and that a small deviation from finiteness grows with energy so that in the ultraviolet limit there is no memory of these relations Such globally supersymmetric theories are therefore unappealing as prototypes of a fundamental theory that would be expected to be finite in the high energy regime But they do have an interesting feature since an arbitrary set of soft breakings tends to converge toward the fixed hypersurface in the infrared, relations among masses – in particular, between boson and fermion masses – may emerge naturally in the low-energy effective field theory

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[2] Thus quasi-super-renormalizable models offer intriguing possibilities for model building

In this paper we discuss the effective potential for quasi-super-renormalizable models Spontaneous symmetry breaking (SSB) does not occur in the models which are arranged to be finite [6] – in certain directions the potential due to cubic and quartic terms is flat, so broken symmetry states can be degenerate with the unbroken phase Adding negative masses-squared, the usual signature of spontaneous symmetry breaking, causes the potential to be unbounded from below, destabilizing the theory But in non-finite models the mass parameters run with the scale, and a scalar mass-squared may change sign, engendering symmetry breaking without destabilization As in softly-broken supergravity models [3] the mass could be positive in the ultraviolet (stabilizing the ground state) but negative in the infrared (driving spontaneous breaking of the gauge symmetry)

We first discuss, in the next section, the one-loop contribution to the effective potential As with the Coleman-Weinberg analysis of SSB by radiative corrections [4], this perturbative analysis suffices to establish the phenomenon when the normalization scale is chosen appropriately In sect 3 we consider the renormalization-group-improved effective potential and demonstrate that SSB is compatible with stability, in fact, the effective potential grows at large scales Finally, in sect 4, we conclude with a summary of the possible role for quasi-super-renormalizable models

2. The effective potential

The model we are studying is the softly broken $N = 4$ supersymmetric Yang-Mills theory

$$L = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}(D_\mu A_i)(D^\mu A^i) + \tfrac{1}{2}(D_\mu B_j)(D^\mu B^j) + \tfrac{1}{2}\bar{\lambda}_k \not{D}\lambda_k$$

$$-\tfrac{1}{2}\bar{\lambda}_K (m_f)_{KI}\lambda_I + \tfrac{1}{2} i g \bar{\lambda}_K \left[\Gamma^i_{KI}\phi^i\ \lambda_I\right] - V_0 \qquad (2\,1)$$

A trace over gauge indices is implied The theory contains six spin-zero fields, four Majorana fermions, and one massless gauge boson All fields are in the adjoint representation of the arbitrary gauge group $A^i = A^{ia}T^a$, $[T^a, T^b] = i f^{abc}T^c$, $\mathrm{Tr}\, T^a T^b = C_2 \delta^{ab}$ We can write the potential as

$$V_0 = \tfrac{1}{2}m^2_{ij}\phi_i\phi_j + \frac{i}{3!}c_{kjk}\phi_i\left[\phi_j, \phi_k\right] - \tfrac{1}{4}g^2\left[\phi_i, \phi_j\right]^2 \qquad (2\,2)$$

Here, $i, j, k = 1, 2, \ldots, 6$ and $K, L = 1, \ldots, 4$, are indices for an erstwhile SU(4) or O(6) global symmetry* imposed by supersymmetry but broken by eq (2.2)

The effective potential is, to one-loop order and in the Landau gauge,

$$V_{\text{eff}} = V_0 + V_1,$$

where

$$V_1 = \frac{1}{64\pi^2} \operatorname{Str} M^4 \ln \frac{M^2}{\mu^2} \tag{2 3}$$

The supertrace, Str, is the sum over bosons minus the sum over fermions, weighted by the number of helicity states $2J+1$ This form of the effective potential assumes a renormalization scheme where polynomial terms are absorbed into the counter-terms, and one should subtract from it the value it takes when all coupling constants vanish (so V_1 is zero for a free theory) The vector, fermion, and scalar mass matrices are

$$\left(M_{\text{V}}^2\right)^{ab} = g^2 f^{ade} f^{bce} \phi_k^d \phi_k^c,$$

$$\left(M_{\text{F}}^{+} M_{\text{F}}\right)^{ab}_{KL\ \alpha\beta} = \left(m_{\text{f}}^2\right)_{KL} \delta^{ab} \delta_{\alpha\beta} - g f^{abc} \phi_i^c \left(m_{\text{f}} \Gamma^i + \Gamma^{i+} m_{\text{f}}\right)_{KL\ \alpha\beta}$$

$$+ g^2 f^{ace} f^{bde} \phi_i^c \phi_j^d \left(\Gamma^{i+} \Gamma^j\right)_{KL\ \alpha\beta},$$

$$\left(M_{\text{S}}^2\right)^{ab}_{ij} = m_{ij}^2 \delta^{ab} - c_{ijk} f^{abc} \phi_k^c + g^2 \left\{ f^{ace} f^{bde} \left[\phi_k^c \phi_k^d \delta_{ij} - \phi_i^c \phi_j^d \right] + f^{abe} f^{cde} \phi_i^c \phi_j^d \right\} \tag{2 4}$$

(In eq (2 3) the trace over Dirac indices has already been factored out)

We are interested in the flat directions of the tree potential, in the absence of soft breakings, $m^2 = m_f = c_{ijk} = 0$, the potential V_0 is zero only when $[\phi_i, \phi_j] = 0$, i e when the fields are in the Cartan subalgebra of the group In a flat direction, therefore, the fields ϕ_i can be taken as diagonal in the above mass formulas Even so, the terms linear in the fields in the scalar and fermion mass matrices will not be diagonal, making the effective potential very difficult to evaluate in general The QSR nature of the theory, however, provides us with a convenient technique for approximating the effective potential, because in the absence of soft breakings V_1

* This notation is related to that of ref [1] as follows If $x, y, z = 1, 2, 3$ then $\phi_x = A_x$, $\phi_{x+3} = B_x$, $c_{xyz} = r\varepsilon_{xyz}$, $c_{x+3\,y+3\,z+3} = s\varepsilon_{xyz}$, $c_{x\,y+3\,z+3} = p^x_{yz}$, $c_{x+3\,y\,z} = q^x_{yz}$, $m^2_{xy} = a^2_{xy}$, $m^2_{x+3\,y+3} = b^2_{xy}$, $m^2_{x\,y+3} + m^2_{y+3\,x} = c_{xy}$ $\Gamma^x_{KI} = \alpha^x_{KL}\delta_{\alpha\beta}$, $\Gamma^{x+3}_{KI} = \beta^x_{KI}(i\gamma 5)_{\alpha\beta}$ Here α and β are the six matrices that span the space of real traceless antisymmetric 4×4 matrices, forming a homomorphism between the fundamental representations of O(6) and SU(4) and have an algebra similar to that of the Pauli matrices $[\alpha^x, \alpha^y] = -2\varepsilon^{xyz}\alpha^z$, $[\beta^x, \beta^y] = -2\varepsilon^{xyz}\beta^z$ $[\alpha^x, \beta^y] = 0$

vanishes, suggesting we expand the supertrace about the symmetric theory – an expansion that will be valid when the fields are large compared to the massive parameters in the theory

The analysis of the effective potential will demonstrate that for appropriate, but not very restrictive, choices of the parameters, the theory is stable [5] and allows for spontaneous symmetry breaking via a Coleman-Weinberg-like mechanism

Symbolically, writing (cf eq (2 5))

$$M^2 = m^2 + T + M^{2\prime} \tag{2 5}$$

for either the fermion or scalar mass matrix, with $M^{2\prime}$ the matrix for the supersymmetric theory and T the linear term, we expand in the breaking terms

$$\begin{aligned} \operatorname{Str} M^4 \ln M^2 &= \operatorname{Str} M^4 \ln\left[M^{2\prime}\left(1 + \frac{m^2 + T}{M^{2\prime}}\right)\right] \\ &= \operatorname{Str} M^4 \ln M^{2\prime} + \operatorname{Str} M^4 \ln\left(1 + \frac{m^2+T}{M^{2\prime}}\right) \\ &= \operatorname{Str} M^4 \ln M^{2\prime} + \operatorname{Str}\left(m^2 M^{2\prime} + \tfrac{3}{2} T^2\right) + \mathrm{O}(\phi) \end{aligned} \tag{2 6}$$

(Note $\operatorname{Tr} M^{2\prime} T = 0 = \operatorname{Tr} T M^{2\prime} \ln M^{2\prime}$, by symmetry of the matrices)

The second line follows if $[m^2 + T, M^{2\prime}] = 0$, which we will now demonstrate Consider eq (2 4) Because we are working in a flat direction the $M^{2\prime}$ matrices are diagonal in the group space indices in the adjoint representation, $(T^a)^{bc} = -if^{abc}$, so $f^{ace} f^{bde} \phi^c \phi^d = (T^c T^d)_{ab} \phi^c \phi^d$, lying in the Cartan subalgebra of diagonal matrices By the same reasoning the linear (T) terms are diagonal in group space Then one checks that $[T, M^{2\prime}] = 0$ in the remaining indices, for the scalar matrix, the totally antisymmetric objects ϵ_{ijk}, contracted with ϕ_k^c, will either commute past δ_{ij} or give zero when contracted with another field in $M_S^{2\prime}$, and for the fermions one finds, using $\alpha^i \alpha^j = \beta^i \beta^j = -\delta^{ij}$ and the commutation relations for α and β matrices (see previous footnote), that $M_F^{+\prime} M_F^{\prime}$ is proportional to the unit matrix in the K, L indices This last statement also gives $[m_f^2, M_F^{+\prime} M_F^{\prime}] = 0$ It is not true in general that $[m^2, M_s^{2\prime}] = 0$, but under the trace the only order ϕ^2 contribution to (2 6) from this commutator is $\operatorname{Tr} M^{4\prime}[\ln M^{2\prime}, (M^{2\prime})^{-1} m^2]$, which is zero

Then using $\operatorname{Str} M^{4\prime} \ln M^{2\prime} = 0$,

$$V_{\text{eff}} = V_0 + \frac{1}{64\pi^2} \operatorname{Str}\left[\left(\{m^2, M^{2\prime}\} + T^2\right) \ln \frac{M^{2\prime}}{\mu^2} + \tfrac{3}{2} T^2 + m^2 M^{2\prime}\right] \tag{2 7}$$

This form can also be obtained from evaluating directly the sum of one-loop graphs, inserting up to two breaking vertices in each graph, (but if one absorbs the polynomials from the expansion into the counterterms one will be using a different renormalization prescription from eq (2 3)) The supertrace is over fermions and scalars only An important result of this approximation is that the leading behavior of the effective potential is of order ϕ^2, so the theory appears stable in the directions where the cubic and quartic terms vanish, as long as the signs of the masses are chosen to be positive when the theory is renormalized at large scales The full justification behind this point involves the renormalization group, as described in sect 3

We now search for a minimum of the potential (other than the origin) For simplicity let us examine only the ϕ_{a3} direction, i e $\phi_{ai} = \phi_a \delta_{i3}$ – a choice which, because of the explicit breaking of the original 0(6) symmetry by the soft-breaking terms, provides a loss of generality, yet which will demonstrate that a global minimum away from the origin does exist somewhere in field space, for as $V_{\rm eff}(0) = 0$ the fact that $V_{\rm eff} < 0$ for some ϕ indicates the existence of such a minimum, provided $V_{\rm eff} \geqslant 0$ as ϕ goes to infinity

In the (flat) $\phi_a \delta_{i3}$ direction, take the directional derivative of eq (2 7) by multiplying ϕ by a scale factor κ and finding $\kappa\, \partial V / \partial \kappa$ Because of homogeneity the derivative on all terms except the logarithm gives us twice the original expression, so if we choose the renormalization point μ by requiring that $V_1 = 0$ at the minimum, $\phi_{\rm min}$, of the potential, these terms vanish, leaving to

$$\kappa \frac{\partial V_{\rm eff}}{\partial \kappa}(\phi_{\rm min}) = (m^2)_{33}(\kappa\phi^a)^2 + \left(\frac{1}{64\pi^2}\right) 2\,{\rm Str}[2m^2 M^{2\prime} + T^2], \qquad (2\ 8)$$

where we have used the fact that $M^{2\prime}$ is diagonal in the chosen direction, and have assumed for simplicity that the m^2 masses are diagonal Employing eq (2 4),

$$\begin{aligned}{\rm Str}[2m^2M^{2\prime} + T^2] = C_2(G) g^2 \Big[2\,{\rm Tr}\, m^2 - 2(m^2)_{33} - 8\big({\rm Tr}\, m_{\rm f}^2 + m_{\rm f4} m_{\rm f3} \\ + m_{\rm f2} m_{\rm f1}\big) + \frac{1}{g^2}(c_{ij3})^2 \Big] (\kappa\phi^a)^2 \qquad (2\ 9)\end{aligned}$$

Recalling that ${\rm Str}\, M^4$ is related to the beta-functions through the renormalization group equation (see eq (3 1)), it is not surprising that the above factor, which is ${\rm Str}\, M^4 - {\rm Str}\, m^2$, can be re-expressed in terms of the beta function for the scalar mass $(m^2)_{33}$ From ref [1] (in the present notation)

$$\beta_{m^2_{33}} = \frac{g^2 C_2}{16\pi^2}\left[2\,{\rm Tr}\, m^2 - 8\big({\rm Tr}\, m_{\rm f}^2 + m_{{\rm f}_4} m_{{\rm f}_3} + m_{{\rm f}_1} m_{{\rm f}_2}\big) + \frac{1}{g^2} c_{ij3} c_{ij3} \right] \qquad (2\ 10)$$

and therefore

$$\frac{1}{64\pi^2}\mathrm{Str}[2m^2M^{2\prime} + T^2] = \tfrac{1}{4}\left[\beta_{m^2_{33}} - \gamma(m^2)_{33}\right](\kappa\phi^a)^2, \qquad (2\,11)$$

with $\gamma = 2g^2C_2(G)/16\pi^2$ The minimum condition (2 8) becomes

$$(2-\gamma)(m^2)_{33} + \beta_{m^2_{33}} = 0, \qquad (2\,12)$$

which is the promised relation among masses The term proportional to γ is of higher order at the minimum (where $(m^2)_{33}$ is of order g^2, so $\gamma(m^2)_{33}$ is of order g^4) and will be dropped

To see that this is indeed a relative minimum in the direction being considered, we calculate

$$V''_{\mathrm{eff}}(\phi_{\mathrm{min}}) = \left[\beta_{m^2_{33}}\right](\phi_{\mathrm{min}})^2,$$

$$V_{\mathrm{eff}}(\phi_{\mathrm{min}}) = -\tfrac{1}{4}\left[\beta_{m^2_{33}}\right](\phi_{\mathrm{min}})^2 \qquad (2\,13)$$

There is a minimum if $\beta > 0$ (whereupon $m^2_{33} < 0$, as expected) As the relation (2 12) is a restriction only on *ratios* of masses, and not absolute masses, the scale of the minimum can be chosen larger than any values of the parameters, as required for the approximation to be valid We therefore have a Coleman-Weinberg-like mechanism of dimensional transmutation, with *ratios* of masses and massive couplings playing the part of dimensionless couplings The relation at the minimum trades in one of the free ratios of masses for one less free ratio and an arbitrary scale, ϕ_{min}

3. The improved effective potential

In order to know if the relation (2 12) is consistent with a stable theory we need to compare a calculation of the effective potential made at two widely separate scales – the scale of the minimum and an arbitrarily large scale representing the behavior at infinity The large logarithms which then occur in eq (2 3) when the fields become large, apparently causing a breakdown of perturbation theory, are controlled by redefining the renormalization scale, μ, leading to a new and renormalization-group-improved effective potential In particular we require that the scalar mass $(m^2)_{33}$ be able to run with scale so that it is positive at large ϕ^2 but becomes negative as it approaches the chosen scale of the minimum

The renormalization group equation obeyed by the exact effective potential is

$$\left(\mu\frac{\partial}{\partial\mu} + \beta_p\frac{\partial}{\partial\lambda_p} - \gamma\phi^{ai}\frac{\partial}{\partial\phi^{ai}}\right)V(\phi^{ai}, \lambda_p, \mu) = 0 \qquad (3\,1)$$

(there is no gauge parameter in Landau gauge), where λ_p is the set of masses and massive coupling constants The anomalous dimension, which is the same for all the fields because of the O(6) symmetry of the unbroken theory, depends only on the gauge coupling g, to one loop we have $\gamma = 2g^2C_2/16\pi^2$ The solution to this equation is therefore simplified because of the QSR feature $\beta_g = 0$, so the anomalous dimension is a constant

The equation is solved using Cauchy's method of characteristics The solution can be written as

$$V(\phi^{ai}, \lambda_p, \mu) = \hat{V}\left[\phi^{ai}e^{-\gamma t}, \bar{\lambda}_q(\lambda, t)\right],$$

with

$$t = -\ln\frac{\mu}{\mu_0},$$

$$\frac{\partial}{\partial t}\bar{\lambda}_p(\lambda_q, t) = \beta_p\left[\bar{\lambda}_q(\lambda, t)\right], \tag{3 2}$$

where $\hat{V}$ is any function and μ_0 is an arbitrary value at which the initial conditions are defined

$$\bar{\lambda}_p(\lambda, 0) = \lambda_p,$$

$$\hat{V}\left[\phi_{ai}, \bar{\lambda}_p(\lambda, 0)\right] = \hat{V}(\phi_{ai}, \lambda_p) = V(\phi_{ai}, \lambda_p, \mu_0) \tag{3 3}$$

The solution is checked by inserting it into (3 1) and using the deducible equation

$$\beta_p(\lambda)\frac{\partial\bar{\lambda}_q}{\partial\lambda_p} - \frac{\partial\bar{\lambda}_q}{\partial t} = 0$$

For field magnitudes on the order of μ_0 we already have a one-loop approximation to V_{eff}. Let ϕ_0^{ai} be a value of the field at which V_{eff} is equal to the tree potential

$$V(\phi_0^{ai}, \lambda_p, \mu_0) = V_0(\phi_0^{ai}) \tag{3 4}$$

The choice of ϕ_0^{ai} selects out a direction in field space We find an expression for the potential at all scales in this particular direction by using dimensional analysis

$$V(\kappa\phi_0^{ai}, \lambda_p, \mu_0) = \kappa^4 V(\phi_0^{ai}, \lambda_p/\kappa, g, \mu_0/\kappa),$$

$$\lambda_p/\kappa \equiv (m_{ij}^2/\kappa^2, m_f/\kappa, c_{ijk}/\kappa), \tag{3 5}$$

and then employing the solution (3 2) at these values of ϕ, λ, and μ, and using the fact that g does not run

$$V\left(\phi_0^{ai}, \lambda_p/\kappa, g, \mu_0/\kappa\right) = \hat{V}\left[\phi_0^{ai} e^{-\gamma t}, \bar{\lambda}_p(\lambda/\kappa, t), g\right]$$

$$t = -\ln(\mu_0/\kappa)/\mu_0 = \ln\kappa \qquad (3\,6)$$

So as we scale along this direction the result is

$$V\left(\kappa\phi_0^{ai}, \lambda, \mu_0\right) = e^{4t} V_0\left[\phi_0^{ai} e^{-\gamma t}, \bar{\lambda}_q(\lambda e^{-t}, t), g\right],$$

$$t = \ln\kappa \qquad (3\,7)$$

As the running couplings are homogeneous in the massive parameters [1] we have,

$$\bar{\lambda}_p(\lambda, \kappa, t) = k^{-\delta_p}\bar{\lambda}_p(\lambda, t), \qquad \delta_p = \text{dimension of } \lambda_p = 1 \text{ or } 2$$

If ϕ_0^{ai} is chosen to be a flat direction, the tree potential reduces only to the mass term

$$V\left(\kappa\phi_0^{ai}, \lambda, \mu_0\right) = \tfrac{1}{2} e^{(2-2\gamma)t} \overline{m}_{ij}^2(\lambda, t)\phi_0^{ai}\phi_0^{aj} \qquad (3\,8)$$

We now seek a minimum along this direction by differentiating with respect to κ (equivalently, t), if we choose a single O(6) direction, such as $\phi_0^{ai} = \phi_0^a\delta^{i3}$ the arbitrary point ϕ_0 drops out, leaving

$$2(1-\gamma)\overline{m}_{33}^2(\lambda, t) + \beta_{m_{33}^2}\left[\bar{\lambda}(\lambda, t)\right] = 0, \qquad (3\,9)$$

which at $t = 0$ is identical to the previous perturbative result (2 12) if we choose $\phi_0 = \phi_{\min}$ (and again drop the higher-order γ term) Our interest here is in the behavior of (3 8) for large fields, $t = \ln\kappa \to \infty$ Scaling up from the minimum to large fields one finds the leading behavior of the running masses to be, e g ,

$$\overline{m}_{33}^2(t) \sim \frac{1}{48g^2}\left[(c_{ijk})^2 + (c_{ij3})^2\right] e^{2bt}, \qquad b = \frac{12g^2C_2}{16\pi^2} = 6\gamma$$

and similar equations for the other scalar masses, the coefficient is positive, indicating that for large enough fields all masses will be positive, insuring stability The converse is not true, i e , not all values of positive masses at large initial scale $t = 0$ will run so as to satisfy the minimum condition at some $t < 0$ The condition

(3 9), in terms of these initial values, is

$$m_{33}^2 = \left[\tfrac{5}{6} + \left(\gamma + \tfrac{1}{6}\right)e^{bt}\right]^{-1}$$

$$\times\Bigg\{\left[\left(\gamma + \tfrac{1}{6}\right)e^{bt} - \tfrac{1}{6}\right]\mathrm{Str}'m^2 + \left[\left(b + \tfrac{1}{2}\right)e^{2bt} + \tfrac{1}{2} - (b+1)e^{bt}\right]\left(\frac{1}{36g^2}\right)\left(C_{ijk}\right)^2$$

$$+\left[\left(b + \tfrac{1}{2}\right)e^{2bt} - \tfrac{1}{2}\right]\left(\frac{1}{12g^2}\right)\left(C_{3jk}\right)^2$$

$$+\left[\tfrac{1}{6} - \tfrac{1}{6}e^{bt} + (1+\gamma)bte^{bt}\right]\left(\frac{1}{6g^2}\right)\left(C_{ijk}c(f)_{ijk}\right)\Bigg\}, \qquad (3\ 10)$$

where $\mathrm{Str}'m^2 = \mathrm{Str}\,m^2 - m_{33}^2$, and C_{ijk} is related to the initial values of the cubic couplings, c_{ijk}, by $C_{ijk} = c_{ijk} - c(f)_{ijk}$, with $c(f)$ the finiteness-condition values [1] At $t = 0$ any initial values for the parameters other than $(m^2)_{33}$ may be chosen, with the allowed values of $(m^2)_{33}$ that will result in a minimum being given by letting t on the right-hand side of (3 10) range over all values $-\infty < t < 0$ – with everything subject to the further restriction that the scalar masses be positive at $t = 0$, and if $\overline{m}_{33}^2(t)$ is less than zero at that value of t (see eq (2 13)) These are rather broad restrictions for example, if one chose $C_{ijk} = 0$ (finiteness condition for cubic couplings), these conditions can be satisfied for any value of $\mathrm{Str}'m^2$ and a range of values of m_{33}^2 (And we remark again that the necessity that the calculated minimum be at a scale higher than any mass threshold is easily achieved for, roughly, $t = \ln(\phi/\phi_0)$, so if the minimum occurs at $t_{\min}$ we simply select ϕ_0 to be large enough so that $\phi_{\min}$ is larger than any mass at that value of t)

4. Discussion

The awkwardness of softly-broken finite models [7] stem from their artificiality in arranging breaking parameters to preserve finiteness – this is an unjustifiable extension of the valid concept of arranging fermionic and bosonic *particle* content to obtain cancellations of graphs The *natural* minimal extensions of finite globally-supersymmetric theories are QSR models Because in QSR models the finiteness relations are approached in the infrared regime, the beauty of finite theories, their predictions for mass relations, need not be completely lost The excitement of finite superstring theories has diminished interest in these kinds of models, since it is assumed that, well below the Planck scale, there remains a softly-broken, $N = 1$ globally supersymmetric theory. The primary motivation for insisting on $N = 1$ is the desire for manifestly chiral theories However, should interest turn to left-right symmetric models with soft or spontaneous breaking of parity, it is much less clear which, if any, type of low-energy supersymmetry will survive In such a context quasi-super-renormalizable models of the kind considered in this and our previous paper may become phenomenologically more attractive

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