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# AMPLITUDES AND CROSS SECTIONS FOR WW → $L\bar{L}$ AND HEAVY LEPTON PRODUCTION IN THE EFFECTIVE W APPROXIMATION

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We give the helicity amplitudes for the process WW → $L\bar{L}$, and examine the validity of the equivalence theorem by comparing longitudinal W scattering with the corresponding unphysical Higgs scattering. We also study the production rates of fourth-generation heavy leptons and neutrinos in the effective W approximation.

## 1. Introduction

The number of generations of quarks and leptons is still unknown within the framework of standard electroweak model. More generations may exist. Hence, it becomes important to investigate the production of possible new generations at future colliders.

As higher energies are reached so that the parton sub-energies are much greater than the $W^{\pm}$ and $Z^0$ masses, the $W^{\pm}$ and $Z^0$ bosons can be treated as constituents of the parton. This is named as the effective W approximation [1].

Recent studies have shown that vector-boson fusion plays an important role in the production of heavy Higgs bosons [1,2], vector-boson pairs [3] and heavy leptons [4]. Furthermore, the effective W approximation has been shown to be quite accurate [4]. Hence, the vector-boson fusion mechanism will be of great interest for future colliders.

In this paper, we present the scattering amplitudes for each helicity state for the processes

$$W^-W^+ \to L^-L^+ \quad \text{or} \quad N\bar{N},$$

$$Z^0Z^0 \to L^-L^+ \quad \text{or} \quad N\bar{N},$$

$$W^-Z^0 \to L^-\bar{N},$$

where $L^-$ and N are a sequential fourth-generation charged heavy lepton and its associated neutrino, respectively. We consider the case that N might be massive. The amplitudes, which have not previously appeared in the literature, can be used in future calculations. Cross sections are shown for some choices of $\sqrt{S}$ and possible lepton and Higgs boson masses. Then, we compare the results with those obtained using the equivalence theorem [5]. We conclude that caution is required when using the equivalence theorem if the Higgs width is not negligible. (W always includes $W^{\pm}$ and $Z^0$ unless the charge is specified.) In sect. 7, we compare our results with previous work [4] where possible.

## 2. The "extended" standard model

### 2.1. MODEL

In order to be specific about the couplings of a heavy lepton, we assume a sequential fourth generation, and extend the standard electroweak model, $SU(2)_L \times U(1)_Y$, by allowing the existence of a right-handed neutrino, which accordingly is massive. We also assume there are no $A_\mu \bar{L}\gamma^\mu\gamma^5 L$, $A_\mu \bar{N}\gamma^\mu\gamma^5 N$, or $A_\mu \bar{N}\gamma^\mu N$ couplings. Here $L^-$ is the fourth generation electron, and N is the fourth generation neutrino. They form an SU(2) doublet

$$\begin{pmatrix} N \\ L^- \end{pmatrix}_L . \tag{2.1}$$

We also assume there is no mixing between leptons. Then the complete lagrangian of this model will be the same as the standard model lagrangian [6], except for the $\mathscr{L}_{\text{fermion-Higgs}}$ part. Essentially, we are assuming an effective lagrangian for a fourth generation. As usual we have one complex doublet Higgs scalar

$$\phi = \begin{pmatrix} i\phi^+ \\ \dfrac{v + H - i\phi^0}{\sqrt{2}} \end{pmatrix}, \tag{2.2}$$

where $v$ is the vacuum expectation value, H is the Higgs scalar, $\phi^+$ and $\phi^0$ are unphysical Higgs scalars.

The Higgs lagrangian is

$$\mathscr{L}_{\text{Higgs}} = \left(D_\mu\phi\right)^+ \left(D^\mu\phi\right) - \mu^2\left(\phi^+\phi\right) - \lambda\left(\phi^+\phi\right)^2 . \tag{2.3}$$

The lepton part of $\mathscr{L}_{\text{fermion-Higgs}}$ is

$$\begin{aligned}\mathscr{L}_{\text{lepton-Higgs}} = & -\bar{\mathrm{N}} m_{\mathrm{N}} \mathrm{N} - \bar{\mathrm{L}} m_{\mathrm{L}} \mathrm{L} + \frac{i}{2\sqrt{2}} g \frac{m_{\mathrm{N}}}{M_{\mathrm{W}}} \phi^{+} \left(\bar{\mathrm{N}}(1-\gamma^5)\mathrm{L}\right) \\ & - \frac{i}{2\sqrt{2}} g \frac{m_{\mathrm{N}}}{M_{\mathrm{W}}} \phi^{-} \left(\bar{\mathrm{L}}(1+\gamma^5)\mathrm{N}\right) - \tfrac{1}{2} g \frac{m_{\mathrm{N}}}{M_{\mathrm{W}}} \mathrm{H}(\bar{\mathrm{N}}\mathrm{N}) - \tfrac{1}{2} i g \frac{m_{\mathrm{N}}}{M_{\mathrm{W}}} \phi^0 (\bar{\mathrm{N}}\gamma^5\mathrm{N}) \\ & - \frac{i}{2\sqrt{2}} g \frac{m_{\mathrm{L}}}{M_{\mathrm{W}}} \phi^{+} \left(\bar{\mathrm{N}}(1+\gamma^5)\mathrm{L}\right) + \frac{i}{2\sqrt{2}} g \frac{m_{\mathrm{L}}}{M_{\mathrm{W}}} \phi^{-} \left(\bar{\mathrm{L}}(1-\gamma^5)\mathrm{N}\right) \\ & - \tfrac{1}{2} g \frac{m_{\mathrm{L}}}{M_{\mathrm{W}}} \mathrm{H}(\bar{\mathrm{L}}\mathrm{L}) + \tfrac{1}{2} i g \frac{m_{\mathrm{L}}}{M_{\mathrm{W}}} \phi^0 (\bar{\mathrm{L}}\gamma^5\mathrm{L}) .\end{aligned} \tag{2.4}$$

Here $m_{\mathrm{N}}$ and $m_{\mathrm{L}}$ are the fourth-generation neutrino and electron masses, respectively, and $M_{\mathrm{W}}$ is the gauge–boson $\mathrm{W}^{\pm}$ mass.

### 2.2. NOTATION

We follow the notation of Bjorken and Drell, and work in the 't Hooft–Feynman gauge. Also we denote $\sin\theta_{\mathrm{w}}$ by $S_{\mathrm{w}}$, $\cos\theta_{\mathrm{w}}$ by $C_{\mathrm{w}}$, and

$$g = \frac{e}{S_{\mathrm{w}}} , \tag{2.5}$$

$$\alpha_{\mathrm{w}} = \frac{\alpha}{S_{\mathrm{w}}^2} . \tag{2.6}$$

$m_{\mathrm{H}}$ is the Higgs mass.

## 3. Unitarity

For a heavy-fermion pair production through WW fusion, the dominant mechanism is longitudinally polarized W-boson scattering [4]. Since our effective lagrangian is different from the lagrangian of the standard model only in its fourth-generation Higgs–fermion couplings, the Ward identity [7] should still hold. This can be proved by explicitly working out the calculation of 13 Feynman graphs involved in the Ward identity equation [7] corresponding to the process $\mathrm{W}^-\mathrm{W}^+ \to \mathrm{L}^-\mathrm{L}^+$. Instead, we will examine the high-energy behavior of $\mathrm{W}_{\mathrm{L}}^-\mathrm{W}_{\mathrm{L}}^+ \to \mathrm{L}^-\mathrm{L}^+$ as an example to verify the unitarity of this effective lagrangian.

In the high-energy limit, the polarization vector $\epsilon^{\mu}_{(0)}(k)$ of the longitudinally polarized W-boson is approximately

$$\epsilon^{\mu}_{(0)}(k) \to \frac{k^{\mu}}{M_{\mathrm{W}}} . \tag{3.1}$$

Using 4-momentum conservation, and the equations of motion

$$(\gamma \cdot P_2 + m_L)V(P_2) = 0,$$

$$\bar{U}(P_1)(\gamma \cdot P_1 - m_L) = 0, \tag{3.2}$$

we can get the following results corresponding to $t$- and $s$-channel diagrams.

$$(a) = \bar{U}(-2\gamma \cdot q)(1-\gamma^5)V \tag{3.3}$$

$$+ \bar{U}\left(-2m_L^2(1-\gamma^5)\frac{\gamma \cdot q}{q^2}\right)V \tag{3.4}$$

$$+ \bar{U}(-4m_L)V, \tag{3.5}$$

$$(b) = \bar{U}\left(-2\gamma \cdot q\left(4S_w^2 - 1 + \gamma^5\right)\right)V \tag{3.6}$$

$$+ \bar{U}\left(-2m_L\left(4S_w^2 - 1\right)\right)V, \tag{3.7}$$

$$(c) = \bar{U}\left(-2\gamma \cdot q\left(-4S_w^2\right)\right)V \tag{3.8}$$

$$+ \bar{U}\left(-2m_L\left(-4S_w^2\right)\right)V, \tag{3.9}$$

$$(d) = \bar{U}(2m_L)V\frac{S}{S - m_H^2}, \tag{3.10}$$

with

$$q = k_1 - P_1.$$

We have suppressed a factor $ig^2/(8M_W^2)$ for each term, and also taken $m_N = 0$ for simplicity. In eq. (3.10), $S$ is the square of the c.m. energy. Let us check the cancellations

$$(3.3) + (3.6) + (3.8) = 0, \tag{3.11}$$

$$(3.5) + (3.7) + (3.9) = \bar{U}(-2m_L)V, \tag{3.12}$$

$$(3.4) + (3.10) = \bar{U}\left(-2m_L^2(1-\gamma^5)\frac{\gamma \cdot q}{q^2} + 2m_L\frac{S}{S - m_H^2}\right)V. \tag{3.13}$$

We see that after the substitution

$$\frac{S}{S - m_H^2} = \frac{m_H^2}{S - m_H^2} + 1, \tag{3.14}$$

all the terms with bad behavior, which are proportional to $S$ or $\sqrt{S}$, cancel. (This can be seen explicitly from the exact amplitudes in the appendix.) For $S \gg M_{\mathrm{W}}^2$ and $S \gg m_{\mathrm{H}}^2$, the scattering amplitude becomes

$$M(\mathrm{W_L^- W_L^+} \to \mathrm{L^- L}) \cong \frac{ig^2}{8M_{\mathrm{W}}^2}(-2m_{\mathrm{L}}^2)\bar{U}(1-\gamma^5)\frac{\gamma \cdot q}{q^2}V. \tag{3.15}$$

Through straightforward algebra, eq. (3.15) gives the total cross section

$$\sigma(\mathrm{W_L^- W_L^+} \to \mathrm{L^- L^+}) \cong \frac{\pi\alpha_{\mathrm{w}}^2}{4S}\left(\frac{m_{\mathrm{L}}}{M_{\mathrm{W}}}\right)^4 \ln\left(\frac{1+\beta}{1-\beta}\right), \tag{3.16}$$

with

$$\beta = \sqrt{1 - \frac{4m_{\mathrm{L}}^2}{S}}\,. \tag{3.17}$$

Eq. (3.16) behaves like $(m_{\mathrm{L}}/M_{\mathrm{W}})^4 \ln(s)/s$, with $s = S/M_{\mathrm{W}}^2$; therefore unitarity is not violated. Hence, we observe the gauge cancellation and consequently good high-energy behavior of the process $\mathrm{W_L^- W_L^+} \to \mathrm{L^- L^+}$. There is one subtlety here. For light-fermion production, the dominant production channel of WW fusion is through transversely polarized W-boson fusion, not longitudinally polarized W's, because the luminosity of transverse W's is larger than longitudinal W's and because of the $(m_{\mathrm{L}}/M_{\mathrm{W}})^4$ factor.

From the equivalence theorem [5], we know that the asymptotic behavior of the longitudinally polarized W-bosons can be represented by their corresponding unphysical Higgs particles. As emphasized in ref. [5], the equivalence theorem does not imply that we can simply replace the $\mathrm{W}^{\pm}$-polarization vectors by eq. (3.1), if we demand the equivalence of the $S$-matrix to $\mathrm{O}(M_{\mathrm{W}}/E)$. Yet this statement is gauge dependent, and in 't Hooft–Feynman gauge the replacement (3.1) does display the main feature of the exact calculation results in the high-energy limit, as shown in sect. 5.

## 4. Helicity amplitudes for the reaction $\mathrm{V_1V_2} \to \mathrm{L_1\bar{L}_2}$

### 4.1. KINEMATICS AND POLARIZATION VECTORS

The 4-momentum assignment for the reaction $\mathrm{V_1V_2} \to \mathrm{L_1\bar{L}_2}$ is shown in fig. 1. $\mathrm{V_1}$ stands for vector-boson-1, and $\mathrm{L_1}$ for lepton-1. We shall work in the vector-boson

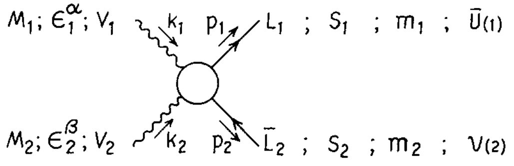


Fig. 1. Schematic view of the process $V_1V_2 \to L_1\bar{L}_2$.

$V_1V_2$ center-of-mass frame. The 4-momenta can be chosen to be

$$
\begin{aligned}
k_1^\mu &= (k_{10}, 0, 0, K),\\
k_2^\mu &= (k_{20}, 0, 0, -K),\\
P_1^\mu &= (P_{10}, PS_\theta, 0, PC_\theta),\\
P_2^\mu &= (P_{20}, -PS_\theta, 0, -PC_\theta).
\end{aligned} \tag{4.1}
$$

The angle $\theta$, as shown in fig. 2, is the angle between the 3-momenta $\boldsymbol{K}$ and $\boldsymbol{P}$, whose magnitudes are $K$ and $P$, separately. Also we define $C_\theta = \cos\theta$, $S_\theta = \sin\theta$. To establish conventions, the polarization vectors of the W-bosons are

$$
\begin{aligned}
(\epsilon_1)_0^\mu &= \frac{1}{M_1}(K, 0, 0, k_{10}),\\
(\epsilon_2)_0^\mu &= \frac{1}{M_2}(K, 0, 0, -k_{20}), \qquad \text{for} \quad \lambda = = 0,\\
(\epsilon_1)_+^\mu &= \sqrt{\tfrac{1}{2}}\,(0, -1, -i, 0),\\
(\epsilon_2)_+^\mu &= \sqrt{\tfrac{1}{2}}\,(0, 1, -i, 0), \qquad \text{for} \quad \lambda = = +1,\\
(\epsilon_1)_-^\mu &= \sqrt{\tfrac{1}{2}}\,(0, 1, -i, 0),\\
(\epsilon_2)_-^\mu &= \sqrt{\tfrac{1}{2}}\,(0, -1, -i, 0), \qquad \text{for} \quad \lambda = = -1,
\end{aligned}
$$

where $\lambda_1(\lambda_2)$ represents the helicity of vector-boson $V_1(V_2)$.

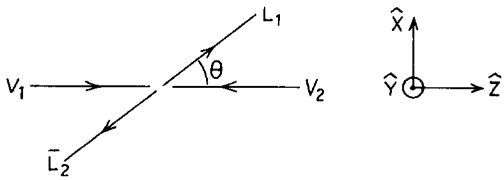


Fig. 2. The coordinate system for the process $V_1V_2 \to L_1\bar{L}_2$ in the center-of mass frame of $V_1V_2$.

### 4.2. THE METHOD OF CALCULATION

To get the scattering amplitude for each helicity state, we adopt the projection operator technique developed in ref. [8]. Since we are working in a different metric from that in ref. [8], we summarize the relevant identities as follows.

Let $U(P_1, s_1, h_1)$ denote the Dirac spinor for a spin-$\frac{1}{2}$ particle, mass $m_1$, moving with momentum $P_1^\mu$, polarization $s_1^\mu$, and helicity $h_1$. Similarly the Dirac spinor for an anti-fermion is denoted as $V(P_2, s_2, h_2)$. Dirac spinors are normalized so that

$$\bar{U}U = 2m_1\,,$$
$$\bar{V}V = -2m_2\,. \tag{4.2}$$

Define

$$U(1) \equiv U(P_1, s_1, h_1)\,,$$
$$V(2) \equiv V(P_2, s_2, h_2)\,,$$

then

$$U(1)\bar{U}(1) = \tfrac{1}{2}(\gamma\cdot P_1 + m_1)(1 + h_1\gamma_5\gamma\cdot s_1)\,,$$
$$V(2)\bar{V}(2) = \tfrac{1}{2}(\gamma\cdot P_2 - m_2)(1 + h_2\gamma_5\gamma\cdot s_2)\,, \tag{4.3}$$

$$V(2)\bar{U}(1) = \tfrac{1}{4}A^{-1}(\gamma\cdot P_2 - m_2)(1 + h_2\gamma_5\gamma\cdot s_2) \times(\gamma\cdot P_1 + m_1)(1 + h_1\gamma_5\gamma\cdot s_1)\,, \tag{4.4}$$

with

$$A^{-1} = e^{-i\Phi^+}\left[(-m_1m_2 + P_1\cdot P_2)(1 - s_1\cdot s_2) + P_1\cdot s_2 P_2\cdot s_1\right]^{-1/2}P_+(1,2) + e^{-i\Phi^-}\left[(-m_1m_2 + P_1\cdot P_2)(1 + s_1\cdot s_2) - P_1\cdot s_2 P_2\cdot s_1\right]^{-1/2}P_-(1,2)\,. \tag{4.5}$$

The projection operator is defined as

$$P_\pm(1,2) \equiv \tfrac{1}{2}(1 \pm h_1h_2)\,,$$

for the corresponding helicities $h_1$ and $h_2$. Furthermore, we will choose the phase factors $\Phi^+$ and $\Phi^-$ to be zero, because we are interested in the scattering amplitude squared for each helicity state for the process $WW \to L\bar{L}$. The same reasoning also applies to the choices of eqs. (4.1) and (4.6). The polarization vectors are

$$s_1^\mu = \left(\frac{P}{m_1}, \frac{P_{10}}{m_1}S_\theta - \varepsilon C_\theta, -i\varepsilon, \frac{P_{10}}{m_1}C_\theta + \varepsilon S_\theta\right),$$
$$s_2^\mu = \left(\frac{P}{m_2}, -\frac{P_{20}}{m_2}S_\theta + \varepsilon C_\theta, -i\varepsilon, -\frac{P_{20}}{m_2}C_\theta - \varepsilon S_\theta\right),$$
$$\varepsilon \ll 1\,. \tag{4.6}$$

$\theta$ is defined as in fig. 2. The reason a small transverse polarization vector is added is to make it possible to invert the corresponding projection operator [8].

Using the spin-projector operator method [8], we then have

$$\begin{aligned}\bar{U}_\alpha(1)\Gamma_{\alpha\beta}V_\beta(2) &= V_\beta(2)\bar{U}_\alpha(1)\Gamma_{\alpha\beta} \\ &= \mathrm{Tr}(V(2)\bar{U}(1)\Gamma)\end{aligned} \tag{4.7}$$

for any tensor $\Gamma_{\alpha\beta}$. Hence we can get the scattering amplitude $M(\lambda_1, \lambda_2, h_1, h_2)$, where $\lambda_1(\lambda_2)$ stands for the vector-boson $V_1(V_2)$ helicity, and $h_1(h_2)$ for the lepton $L_1(\bar{L}_2)$ helicity. The differential cross section in the vector-boson $V_1V_2$ c.m. frame is

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 S}\frac{P}{K}|M|^2. \tag{4.8}$$

### 4.3. SYMMETRY PROPERTIES

For the reactions

$$\begin{aligned}&W^-(\lambda_1) + W^+(\lambda_2) \to L^-(h_1) + L^+(h_2), \\ &W^-(\lambda_1) + W^+(\lambda_2) \to N(h_1) + \bar{N}(h_2), \\ &Z^0(\lambda_1) + Z^0(\lambda_2) \to L^-(h_1) + L^+(h_2), \\ &Z^0(\lambda_1) + Z^0(\lambda_2) \to N(h_1) + \bar{N}(h_2),\end{aligned} \tag{4.9}$$

the scattering amplitudes $M(\lambda_1, \lambda_2, h_1, h_2)$ for different helicity states have the symmetry

$$M(\lambda_1, \lambda_2, h_1, h_2) = (-1)^{\Delta\lambda} M(-\lambda_2, -\lambda_1, -h_2, -h_1),$$

with

$$\Delta\lambda = \lambda_1 - \lambda_2. \tag{4.10}$$

This is a consequence of $CP$ invariance. If the fourth-generation neutrino is massless, then

$$\begin{aligned}&M(\lambda_1, \lambda_2, +, +) \\ &= M(\lambda_1, \lambda_2, +, -) \\ &= M(\lambda_1, \lambda_2, -, -) \\ &= 0,\end{aligned} \tag{4.11}$$

for $N\bar{N}$ productions. These are just the standard model results: only left-handed neutrinos and right-handed anti-neutrinos interact. Obviously, the total cross sections of

$$e^+e^- \to W^+W^-, \qquad e^+e^- \to Z^0Z^0,$$

can be found by using the scattering amplitudes given in the appendix, with both $m_L = 0$ and $m_N = 0$. After taking into account the correct spin-averaging factor and redefining the angle $\theta$ as the angle between $e^+$ and $W^+$ momenta in $e^+e^-$ c.m. frame, we obtain a total cross section and differential cross section which agree with the results in ref. [9]. This is a check on our amplitudes for the case $m_L = m_N = 0$. In sects. 5 and 6, we will check our amplitudes in the regime $S \gg M_W^2$ using the equivalence theorem [5].

In the appendix, we work out the scattering amplitudes for each helicity state for the reaction $V_1V_2 \to L_1\bar{L}_2$ using the notation, method and symmetry properties stated in the previous subsections.

### 4.4. DISCUSSION

The scattering amplitudes are checked as follows.

(1) The sum of the square of all helicity amplitudes with the same vector-boson helicities $\lambda_1$ and $\lambda_2$ is checked to be the same as the results obtained using the conventional method of applying spinor-sum rules. The former is obtained by first taking the trace calculation of each helicity amplitude, as done in eq. (4.7), then summing over the square of those helicity amplitudes. The latter is obtained by first taking the square of the scattering amplitude, which includes Dirac spinors, then performing the trace calculation due to summing over the helicity states of fermions.

The trace calculations were done using SCHOONSCHIP, and have been partially checked by REDUCE.

(2) In the limit $m_L = 0$ and $m_N = 0$, the resultant differential cross sections for $e^-e^+ \to W^-W^+$ for fixed $W^-$ and $W^+$ helicity states are checked to agree algebraically with those in ref. [9]. The total cross section for $e^-e^+ \to Z^0Z^0$ is also checked to agree numerically with those in ref. [9].

(3) In the high-energy limit, the total cross sections for longitudinal W scattering are checked numerically by comparing the results with those obtained from both the equivalence theorem [5] and the effective lagrangian [10]. These will be shown in later sections. They agree very well except nearby the Higgs resonance.

## 5. Asymptotic behavior

Although it is straightforward to calculate the total cross section using the scattering amplitudes in the appendix, the result is too lengthy to be expressed here. Those who need the cross sections can obtain them most easily by putting the amplitudes directly in their own programs. Hence, in this section, we will present the asymptotic behavior of the total cross section for $W_L^- W_L^+ \to L^- L^+$. More precisely, we start from the exact total cross section, then take its limit as $S \gg M_W^2$, $S \gg m_L^2$ and $S \gg m_N^2$. Notice that we did not specify the Higgs mass here; it can be any number, as long as the above conditions are fulfilled.

We define the dimensionless variables

$$\mathscr{s} = S/M_W^2, \qquad y = m_L^2/M_W^2, \qquad x = m_N^2/M_W^2, \tag{5.1}$$

$$P_H = \frac{\left(-S + m_H^2\right) M_W^2}{\left(-S + m_H^2\right)^2 + m_H^2 \Gamma_H^2},$$

$$P_{HH} = \frac{M_W^4}{\left(-S + m_H^2\right)^2 + m_H^2 \Gamma_H^2},$$

$$\Omega_t = \ln\left|\frac{1 - \beta_P - \frac{2}{\mathscr{s}}(y - x)}{1 + \beta_P - \frac{2}{\mathscr{s}}(y - x)}\right|, \tag{5.2}$$

$$\beta_P = \sqrt{1 - \frac{4y}{\mathscr{s}}}\,. \tag{5.3}$$

In the high-energy limit, the total cross section for $W_L^- W_L^+ \to L^- L^+$ is

$$\sigma_{LL} \cong \frac{\pi\alpha_w^2}{S}\left[g_1\Omega_t + g_2\beta_P + g_3\mathscr{s}\beta_P + g_4\frac{\Omega_t}{\mathscr{s}}\right], \tag{5.4}$$

with

$$g_1 = -\tfrac{1}{4}(x - y)^2 + \tfrac{1}{2}xy\mathscr{s}P_H,$$

$$g_2 = -\tfrac{1}{4}\left[2y^2 + y - \tfrac{1}{6} - x(4y - 1 - 2x)\right]$$
$$\quad - \tfrac{1}{2}y(y + 2 - x)\mathscr{s}P_H - \tfrac{1}{4}y(2y + 3)\mathscr{s}^2 P_{HH},$$

$$g_3 = \tfrac{1}{8}y\left[1 + 2\mathscr{s}P_H + \mathscr{s}^2 P_{HH}\right],$$

$$g_4 = \left[\tfrac{1}{2}(y - x)^3 - y^2 + y + x(3y - 2x - 2)\right]$$
$$\quad + \tfrac{1}{2}\left[(y - x)^3 - 3y^2 + 2y + x\left((x - y)^2 - y\right)\right]\mathscr{s}P_H. \tag{5.5}$$

Suppose $S \gg m_{\mathrm{H}}^2$, then

$$s P_{\mathrm{H}} = -\left(1 + \frac{m_{\mathrm{H}}^2}{s M_{\mathrm{W}}^2}\right), \qquad s^2 P_{\mathrm{HH}} = 1 + \frac{2m_{\mathrm{H}}^2}{s M_{\mathrm{W}}^2}. \tag{5.6}$$

Thus $g_3$ vanishes in this case. There is accordingly no constant cross section in this regime. the total cross section falls like $\ln(s)/s$, as stated in sect. 3. Also, in eq. (5.4), the terms are arranged so that in the high-energy limit, the first term has a larger contribution than the second term, etc.

The asymptotic behavior for the process $\mathrm{W_L^- W_L^+} \to \mathrm{N\bar{N}}$ can be obtained by simply interchanging $x$ and $y$ in eqs. (5.3) through (5.6).

## 6. Equivalence theorem

From the equivalence theorem [5], we learned that in the high-energy limit, $E \gg M_{\mathrm{w}}$, the S-matrix for some longitudinally polarized vector-bosons and other physical particles is the same, up to a phase factor, as that obtained by replacing each longitudinally polarized vector boson by its corresponding unphysical Higgs scalar, i.e.

$$W_{\mathrm{L}}^{\pm} \to \phi^{\pm},$$

$$Z_{\mathrm{L}}^{0} \to \phi^{0}.$$

Motivated by the above theorem, we calculate the total cross section of the corresponding unphysical Higgs scalar scattering processes. It serves as a good check on our scattering amplitudes obtained in the appendix in the limit $S \gg M_{\mathrm{W}}^2$.

For this reaction, $\phi^- \phi^+ \to \mathrm{L^- L^+}$, we adopt the same notation as in eqs. (5.1) to (5.3). In the high energy limit, $S \gg M_{\mathrm{W}}^2$, the total cross section is expressed as

$$\sigma = \frac{\pi \alpha_{\mathrm{w}}^2}{S}\left[F_1 \Omega_{\mathrm{t}} + F_2 \beta_{\mathrm{P}} + F_3 P_{\mathrm{H}} + F_4 P_{\mathrm{HH}}\right], \tag{6.1}$$

with

$$F_1 = -\tfrac{1}{4}(x^2 + y^2) + \frac{1}{2s}\left[yx(1 + 2x - y) + y^2 - x^3\right],$$

$$F_2 = -\tfrac{1}{2}(x^2 - xy + y^2) - \tfrac{1}{4}x + \tfrac{1}{24},$$

$$F_3 = -\tfrac{1}{2}(m_{\mathrm{H}}/M_{\mathrm{W}})^2 y\left[-(x+y)\beta_{\mathrm{P}} + \left(\frac{(x+y) - (y-x)^2}{s} - x\right)\Omega_{\mathrm{t}}\right],$$

$$F_4 = \tfrac{1}{8}(m_{\mathrm{H}}/M_{\mathrm{W}})^4 s y \beta_{\mathrm{P}}^3. \tag{6.2}$$

This agrees with the results in ref. [4], where they start from an effective lagrangian [10] for a spontaneous symmetry-breaking Goldstone-boson sector. In eq. (6.1), we group terms in such a way as to see the effect of the Higgs propagator.

Comparing eqs. (5.4) with (6.1), we immediately find out that they do not agree for arbitrary Higgs mass if Higgs width is included as eq. (5.2). However, if $S \gg m_H^2$, they are indeed in good agreement. This can be easily shown by using eq. (5.6). Thus, caution is due when using the equivalence theorem unless the width of Higgs can be neglected.

As an example, consider the processes $W_L^- W_L^+ \to L_+^- L_+^+$ and $\phi^- \phi^+ \to L_+^- L_+^+$. Here $L_+^-$ stands for a right-handed $L^-$. The difference between these two amplitudes in the high-energy limit is

$$\left( \frac{ig^2}{M_W^2} \right) \frac{m_L P}{2} \frac{i m_H \Gamma_H}{-S + m_H^2 - i m_H \Gamma_H} . \tag{6.3}$$

It does not vanish for non-zero $m_L$ or $\Gamma_H$.

The correct way to include Higgs width is to consider one-loop corrections to the above processes. As long as only the tree-level results are considered, we can not tell which one of them should be taken in the region where they are different. Note the interesting point that if one puts $S = m_H^2$ in eq. (6.3), the difference between the two methods is independent of $\Gamma_H$.

## 7. Heavy lepton production via a proton–proton collider

From sect. 5, we see that the total cross section for heavy lepton pair production via WW fusion in the high-energy limit grows like $(m_L/M_W)^4$ at fixed c.m. energy of WW. This leads to a larger cross section for larger $m_L$. WW fusion then becomes an important mechanism to produce heavy leptons. We have used the effective W approximation [1] to obtain the cross section $\sigma(pp \to L_1 \bar{L}_2 X)$, here $L_1 \bar{L}_2 = L^- L^+$, $N\bar{N}$ or $L^- \bar{N}$. Since our numerical results agree with Dawson and Willenbrock [4] within a factor of 2, we will not show any figures here. Instead we remark that our analytical results agree with Dawson and Willenbrock [4], apart from differences due to our use of longitudinal W's versus their use of the equivalence theorem, as discussed in sect. 6, but disagree with 'Eboli et al. [4]. In order to make eq. (3.5) in Dawson and Willenbrock [4] valid for $m_N = 0$, one should replace $(m_L^2 - m_N^2)$ by $(m_L^2 - m_N^2 + 2M_W^2)$ in $\Lambda_1$, and $(m_N^2 - m_L^2)$ by $(m_N^2 - m_L^2 + 2M_W^2)$ in $\Lambda_2$.

As stated in sects. 4, 5 and 6, we checked our amplitudes in two limits. One limit is for $m_N = 0$ and $m_L = 0$, which gives us the correct cross sections for $e^- e^+ \to W^- W^+$ and $e^- e^+ \to Z^0 Z^0$ [9]. Another limit is checked by examining the asymptotic behavior of the total cross sections. In this limit our results agree with those obtained from both the equivalence theorem [5] and the effective lagrangian [4,10]

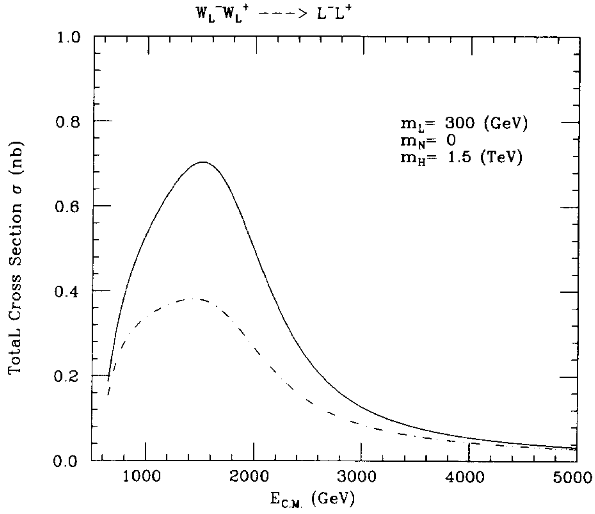


Fig. 3. Cross section $\sigma(W_L^- W_L^+ \to L^- L^+)$ versus $E_{\text{c.m.}}$ for $m_L = 300$ GeV, $m_N = 0$ and $m_H = 1.5$ TeV. The solid line is from the exact calculation. The dot-dash line is from the effective lagrangian [4, 10].

for heavy lepton production, if it is off the Higgs resonance. However, our constituent cross sections do not agree with theirs when near the Higgs resonance. We give some numerical comparisons for $m_H = 1.5$ TeV in fig. 3.

## 8. Heavy lepton production via a electron–positron collider

In this section, we wish to examine the heavy lepton production in an $e^- e^+$ collider, although it is not yet known whether TeV electron linear colliders can be built. We will use the effective W method to exmaine the cross section $\sigma(e^- e^+ \to \nu_e \bar{\nu}_e L^- L^+)$, and compare it with $\sigma(e^- e^+ \to L^- L^+)$.

The total cross section versus lepton mass at $e^- e^+$ c.m. energy $\sqrt{S} = 4$ TeV for both $m_H = 0.1$ and 1 TeV is shown in fig. 4. The cross section for this process is much smaller than $10^{-38}$ cm$^2$ for $\sqrt{S} = 2$ TeV. For instance, it is about $3 \times 10^{-3}$ pb at $m_L = 200$ GeV and $m_H = 1$ TeV, while the cross section of $e^- e^+ \to L^- L^+$ is about $4.4 \times 10^{-3}$ pb.

## 9. Comments

This paper has given the complete set of amplitudes for $WW \to L\bar{L}$. They were used to compute some cross sections in pp, $p\bar{p}$ and $e^- e^+$ collisions. Although the cross sections are large enough to give many events at some hadron colliders, we have not considered here the separate and model-dependent questions of how the heavy leptons decay and whether they can be detected. We leave these to future

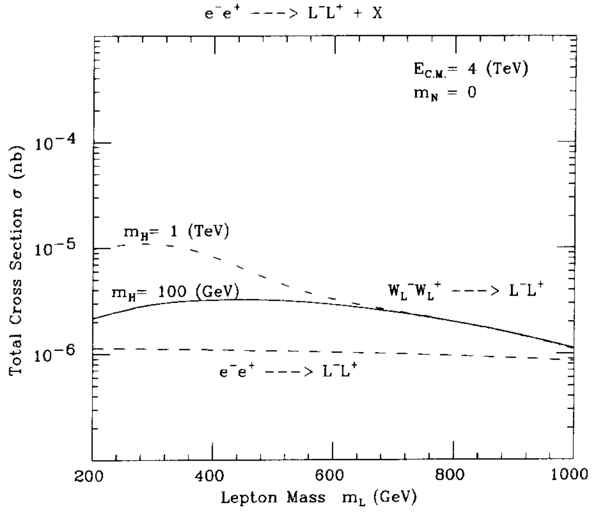


Fig. 4. Cross section versus the charged lepton mass at $E_{\mathrm{c.m.}} = 4$ TeV and $m_N = 0$. Dot-dash (solid) line is for $e^- e^+ (W_L^- W_L^+) \to L^- L^+ \nu_e \bar{\nu}_e$, with $m_H = 1$ TeV ($m_H = 100$ GeV). Dash line is for $e^- e^+ \to L^- L^+$.

studies. In addition, we found that the equivalence theorem does not give the same results as calculating with longitudinal W's near the Higgs resonance. Without including the Higgs width via a one-loop calculation, it is not possible to decide which is more appropriate for comparison with experiments.

The author would like to thank Professor G. Kane for his suggestions, advice and guidance. He also appreciates helpful conversations with Dr. G. Passarino. This work was supported in part by the US Department of Energy.

## Appendix
## The helicity amplitudes of $V_1V_2 \to L_1\bar{L}_2$

A.1. $W^- W^+ \to L^- L^+$

Following the notation in subsects. 4.1 and 4.2, we can write eq. (4.5) as

$$A^{-1} = \frac{1}{2P} P_+(1,2) + \frac{1}{2P\varepsilon} P_-(1,2), \quad \varepsilon \ll 1. \tag{A.1}$$

Our matrix element is defined so that

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{g^4}{32\pi S} \frac{P}{K} |M|^2. \tag{A.2}$$

Denote

$$v = -1 + 4\sin^2\theta_w ,$$

$$M_{h_1h_2}^{\lambda_1\lambda_2} = M(\lambda_1, \lambda_2, h_1, h_2) . \tag{A.3}$$

We define the following variables

$$a_1 = E + P , \quad a_2 = E - P , \quad a_3 = E + K , \quad a_4 = E - K , \quad a_5 = K + P ,$$

$$a_6 = K - P , \quad a_7 = P + KC_\theta , \quad a_8 = P - KC_\theta , \quad a_9 = K + PC_\theta , \quad a_{10} = K - PC_\theta ,$$

$$a_{11} = PS_\theta , \quad a_{12} = PS_\theta^2 , \quad a_{13} = KS_\theta , \quad a_{14} = KC_\theta , \quad a_{15} = P , \quad a_{16} = E ,$$

$$a_{17} = vE + P , \quad a_{18} = vE - P , \quad a_{19} = 2vE , \quad a_{20} = 2v^2ES_\theta , \quad a_{21} = 2vKS_\theta ,$$

$$a_{22} = 2v^2K , \quad a_{23} = 2vPS_\theta , \quad a_{24} = P + 2KC_\theta , \quad a_{25} = P - 2KC_\theta ,$$

$$a_{26} = K + EC_\theta , \quad a_{27} = K - EC_\theta , \quad a_{28} = K ,$$

$$b_1 = 2PE , \quad b_2 = PES_\theta , \quad b_3 = PES_\theta^2 , \quad b_4 = PEC_\theta S_\theta , \quad b_5 = 2KE ,$$

$$b_6 = 2KES_\theta , \quad b_7 = v^2KE , \quad b_8 = 2vPKS_\theta , \quad b_9 = E^2 - P^2 , \quad b_{10} = b_{14} - 4P^2 ,$$

$$b_{11} = b_{14} + 2PK , \quad b_{12} = b_{14} - 2PK , \quad b_{13} = 2v^2E^2 , \quad b_{14} = K^2 + E^2 ,$$

$$b_{15} = K^2 - E^2 , \quad b_{16} = K^2 - 3E^2 , \quad b_{17} = 2vP^2S_\theta ,$$

$$c_1 = KE^2 , \quad c_2 = 2PK^2 , \quad c_3 = 4vEP^2S_\theta^2 , \quad c_4 = PE^2C_\theta ,$$

$$c_5 = PE^2S_\theta^2 , \quad c_6 = PE^2C_\theta S_\theta ,$$

$$d_1 = 1 + C_\theta , \quad d_2 = 1 - C_\theta , \quad d_3 = S_\theta , \quad d_4 = 1 + v^2 ,$$

$$e_1 = 4vP^2E^2C_\theta S_\theta , \quad e_2 = 2PE^3C_\theta S_\theta ,$$

$$f_1 = \sqrt{2}\,M_W , \quad f_2 = M_W^2 , \quad f_3 = 4C_w^2 , \quad f_4 = \sqrt{2}\,f_3M_Z , \quad f_5 = f_3M_Z^2 ,$$

and

$$U_L = \frac{1}{-U + m_L^2} , \quad U_N = \frac{1}{-U + m_N^2} , \quad T_L = \frac{1}{-T + m_L^2} , \quad T_N = \frac{1}{-T + m_N^2} ,$$

$$S_A = \frac{4S_w^2}{-S} , \quad S_W = \frac{1}{-S + M_W^2} , \quad S_Z = \frac{1}{-S + M_Z^2} , \quad S_H = \frac{1}{-S + m_H^2 - im_H\Gamma_H} , \tag{A.4}$$

$S$, $T$ and $U$ are the Mandelstam variables. $\Gamma_H$ is the full width of the Higgs particle, $M_W$ ($M_Z$) is the vector-boson $W^{\pm}$( $Z^0$) mass, $E=\sqrt{S}/2$, and $\theta$ is the angle between $L^-$ and $W^-$ in WW center-of-mass frame.

21 of the 36 helicity amplitudes are shown below. The remaining 15 helicity amplitudes can be obtained from these using the symmetry properties, c.f. eq. (4.10).

$$M^{++}_{++}=m_L\left[a_{14}(S_A-vS_Z)-S_Ha_{15}+T_N(a_{12}+a_6d_1)/2\right],$$

$$M^{++}_{--}=m_L\left[a_{14}(S_A-vS_Z)-S_Ha_{15}+T_N(a_{12}-a_5d_2)/2\right],$$

$$M^{++}_{-+}=-S_Ab_6/2+S_Za_{13}a_{18}-T_Na_1a_{10}d_3/2,$$

$$M^{++}_{+-}=-S_Ab_6/2+S_Za_{13}a_{17}-T_Na_2a_{10}d_3/2,$$

$$M^{-+}_{++}=M^{+-}_{++}=-m_LT_Na_{12}/2,\quad M^{-+}_{-+}=-T_Na_{11}a_1d_1/2,\quad M^{-+}_{+-}=T_Na_{11}a_2d_2/2,$$

$$M^{+-}_{-+}=T_Na_{11}a_1d_2/2,\quad M^{+-}_{+-}=-T_Na_{11}a_2d_1/2,$$

$$f_1M^{0+}_{++}=m_L\left[b_6(S_A-vS_Z)+T_N(a_4a_{13}-a_3a_{11}-2b_4)/2\right],$$

$$f_1M^{0+}_{--}=m_L\left[b_6(S_A-vS_Z)+T_N(a_4a_{13}+a_3a_{11}-2b_4)/2\right],$$

$$f_1M^{0-}_{++}=m_L\left[-b_6(S_A-vS_Z)-T_N(a_3a_{13}+a_4a_{11}-2b_4)/2\right].$$

$$f_1M^{0-}_{--}=m_L\left[-b_6(S_A-vS_Z)-T_N(a_3a_{13}-a_4a_{11}-2b_4)/2\right],$$

$$f_1M^{0+}_{-+}=2S_Ac_1d_1-S_Zb_5a_{18}d_1+T_Na_1(a_4a_6d_1/2+b_3),$$

$$f_1M^{0+}_{+-}=-2S_Ac_1d_2+S_Zb_5a_{17}d_2+T_Na_2(-a_4a_5d_2/2+b_3),$$

$$f_1M^{0-}_{-+}=2S_Ac_1d_2-S_Zb_5a_{18}d_2+T_Na_1(a_3a_5d_2/2-b_3),$$

$$f_1M^{0-}_{+-}=-2S_Ac_1d_1+S_Zb_5a_{17}d_1+T_Na_2(-a_3a_6d_1/2-b_3),$$

$$f_2M^{00}_{++}=m_L\left[a_{14}b_{16}(S_A-vS_Z)-S_Ha_{15}b_{14}+T_N(-a_8b_{15}-2c_5)/2\right],$$

$$f_2M^{00}_{-+}=-S_Ab_6b_{16}/2+S_Za_{13}a_{18}b_{16}-T_Na_1\left[a_{13}(b_{15}-b_1)/2+c_6\right],$$

$$f_2M^{00}_{+-}=-S_Ab_6b_{16}/2+S_Za_{13}a_{17}b_{16}-T_Na_2\left[a_{13}(b_{15}+b_1)/2+c_6\right].$$

### A.2. $Z^0Z^0 \to L^-L^+$

Use the same definitions (A.2)–(A.4), and define $\theta$ to be the angle between $L^-$ and $Z^0$.

21 of the 36 helicity amplitudes are given below. The remaining 15 helicity amplitudes can be obtained from these using the symmetry properties, c.f. eq. (4.10).

$$2f_3 M^{++}_{++} = m_{\mathrm{L}}\big[U_{\mathrm{L}}\big(a_{28}d_4d_2 + a_{12}d_4 - 2a_{15}d_2\big) - 8S_{\mathrm{H}}a_{15} + T_{\mathrm{L}}\big(a_{28}d_4d_1 + a_{12}d_4 - 2a_{15}d_1\big)\big],$$

$$2f_3 M^{++}_{--} = m_{\mathrm{L}}\big[U_{\mathrm{L}}\big(-a_{28}d_4d_1 + a_{12}d_4 - 2a_{15}d_1\big) - 8S_{\mathrm{H}}a_{15} + T_{\mathrm{L}}\big(-a_{28}d_4d_2 + a_{12}d_4 - 2a_{15}d_2\big)\big],$$

$$2f_3 M^{++}_{-+} = U_{\mathrm{L}}\big(b_4d_4 + b_6d_4/2 - a_{23}a_9\big) + T_{\mathrm{L}}\big(b_4d_4 - b_6d_4/2 + a_{23}a_{10}\big),$$

$$2f_3 M^{++}_{+-} = U_{\mathrm{L}}\big(b_4d_4 + b_6d_4/2 + a_{23}a_9\big) + T_{\mathrm{L}}\big(b_4d_4 - b_6d_4/2 - a_{23}a_{10}\big),$$

$$2f_3 M^{-+}_{++} = 2f_3 M^{+-}_{++} = -m_{\mathrm{L}}\big(T_{\mathrm{L}} + U_{\mathrm{L}}\big)a_{12}d_4,$$

$$2f_3 M^{-+}_{-+} = \big(T_{\mathrm{L}} + U_{\mathrm{L}}\big)d_1\big(-b_2d_4 + b_{17}\big),$$

$$2f_3 M^{-+}_{+-} = \big(T_{\mathrm{L}} + U_{\mathrm{L}}\big)d_2\big(b_2d_4 + b_{17}\big),$$

$$2f_3 M^{+-}_{-+} = \big(T_{\mathrm{L}} + U_{\mathrm{L}}\big)d_2\big(b_2d_4 - b_{17}\big),$$

$$2f_3 M^{+-}_{+-} = \big(T_{\mathrm{L}} + U_{\mathrm{L}}\big)d_1\big(-b_2d_4 - b_{17}\big),$$

$$2f_4 M^{0+}_{++} = m_{\mathrm{L}}\big[U_{\mathrm{L}}(2b_2d_2 - a_{20}a_9 + a_{21}a_5) + T_{\mathrm{L}}(-2b_2d_1 + a_{20}a_{10} + a_{21}a_5)\big],$$

$$2f_4 M^{0+}_{--} = m_{\mathrm{L}}\big[U_{\mathrm{L}}(-2b_2d_1 - a_{20}a_9 + a_{21}a_6) + T_{\mathrm{L}}(2b_2d_2 + a_{20}a_{10} + a_{21}a_6)\big],$$

$$2f_4 M^{0-}_{++} = m_{\mathrm{L}}\big[U_{\mathrm{L}}(2b_2d_1 + a_{20}a_9 + a_{21}a_6) + T_{\mathrm{L}}(-2b_2d_2 - a_{20}a_{10} + a_{21}a_6)\big],$$

$$2f_4 M^{0-}_{--} = m_{\mathrm{L}}\big[U_{\mathrm{L}}(-2b_2d_2 + a_{20}a_9 + a_{21}a_5) + T_{\mathrm{L}}(2b_2d_1 - a_{20}a_{10} + a_{21}a_5)\big],$$

$$2f_4 M^{0+}_{-+} = U_{\mathrm{L}}\big(-a_{15}b_{11}d_4d_1 - a_{22}b_9d_1 + 2c_5d_4 - c_3 + a_{19}d_1a_5^2\big) + T_{\mathrm{L}}\big(-a_{15}b_{12}d_4d_1 + a_{22}b_9d_1 + 2c_5d_4 - c_3 + a_{19}d_1a_6^2\big),$$

$$2f_4 M^{0+}_{+-} = U_{\mathrm{L}}\big(-a_{15}b_{12}d_4d_2 + a_{22}b_9d_2 + 2c_5d_4 + c_3 - a_{19}d_2a_6^2\big) + T_{\mathrm{L}}\big(-a_{15}b_{11}d_4d_2 - a_{22}b_9d_2 + 2c_5d_4 + c_3 - a_{19}d_2a_5^2\big),$$

$$2f_4 M^{0-}_{-+} = U_{\mathrm{L}}\big(a_{15}b_{12}d_4d_2 - a_{22}b_9d_2 - 2c_5d_4 + c_3 - a_{19}d_2a_6^2\big) + T_{\mathrm{L}}\big(a_{15}b_{11}d_4d_2 + a_{22}b_9d_2 - 2c_5d_4 + c_3 - a_{19}d_2a_5^2\big),$$

$$2f_4 M^{0-}_{+-} = U_L\left(a_{15}b_{11}d_4 d_1 + a_{22}b_9 d_1 - 2c_5 d_4 - c_3 + a_{19}d_1 a_5^2\right) + T_L\left(a_{15}b_{12}d_4 d_1 - a_{22}b_9 d_1 - 2c_5 d_4 - c_3 + a_{19}d_1 a_6^2\right),$$

$$2f_5 M^{00}_{++} = m_L\left[U_L(-a_{14}b_{14}d_4 - 2c_5 d_4 + b_{13}a_{24} - c_2) - 8S_H a_{15}b_{14} + T_L(a_{14}b_{14}d_4 - 2c_5 d_4 + b_{13}a_{25} - c_2)\right],$$

$$2f_5 M^{00}_{-+} = U_L(-e_2 d_4 + b_6 b_{10}/2 + e_1 - b_8 b_{16} + b_7 b_{16} d_3) + T_L(-e_2 d_4 - b_6 b_{10}/2 + e_1 + b_8 b_{16} - b_7 b_{16} d_3),$$

$$2f_5 M^{00}_{+-} = U_L(-e_2 d_4 + b_6 b_{10}/2 - e_1 + b_8 b_{16} + b_7 b_{16} d_3) + T_L(-e_2 d_4 - b_6 b_{10}/2 - e_1 - b_8 b_{16} - b_7 b_{16} d_3).$$

A.3. $W^-W^+ \to N\bar{N}$

Use the same definitions (A.2)–(A.4), and define $\theta$ to be the angle between N and $W^-$.

21 of the 36 helicity amplitudes are given below. The remaining 15 helicity amplitudes can be obtained from these using the symmetry properties, c.f. eq. (4.10).

$$M^{++}_{++} = m_N\left[U_L(a_6 d_2 + a_{12})/2 - S_H a_{15} - S_Z a_{14}\right],$$
$$M^{++}_{--} = m_N\left[U_L(-a_5 d_1 + a_{12})/2 - S_H a_{15} - S_Z a_{14}\right],$$
$$M^{++}_{-+} = U_L a_1 a_9 d_3/2 + S_Z a_1 a_{13},$$
$$M^{++}_{+-} = U_L a_2 a_9 d_3/2 + S_Z a_2 a_{13},$$
$$M^{-+}_{++} = M^{+-}_{++} = -m_N U_L a_{12}/2, \quad M^{-+}_{-+} = -U_L a_1 a_{11} d_1/2,$$
$$M^{-+}_{+-} = U_L a_2 a_{11} d_2/2. \quad M^{+-}_{-+} = U_L a_1 a_{11} d_2/2, \quad M^{+-}_{+-} = -U_L a_2 a_{11} d_1/2, \quad /$$
$$f_1 M^{0+}_{++} = m_N\left[U_L(b_2 d_2 - a_3 a_5 d_3/2) - S_Z b_6\right],$$
$$f_1 M^{0+}_{--} = m_N\left[-U_L(b_2 d_1 + a_3 a_6 d_3/2) - S_Z b_6\right],$$
$$f_1 M^{0-}_{++} = m_N\left[U_L(b_2 d_1 + a_4 a_6 d_3/2) + S_Z b_6\right],$$
$$f_1 M^{0-}_{--} = m_N\left[U_L(-b_2 d_2 + a_4 a_5 d_3/2) + S_Z b_6\right],$$
$$f_1 M^{0+}_{-+} = U_L a_1(-a_3 a_5 d_1/2 + b_3) - S_Z a_1 b_5 d_1,$$
$$f_1 M^{0+}_{+-} = U_L a_2(a_3 a_6 d_2/2 + b_3) + S_Z a_2 b_5 d_2,$$
$$f_1 M^{0-}_{-+} = U_L a_1(-a_4 a_6 d_2/2 - b_3) - S_Z a_1 b_5 d_2,$$
$$f_1 M^{0-}_{+-} = U_L a_2(a_4 a_5 d_1/2 - b_3) + S_Z a_2 b_5 d_1,$$
$$f_2 M^{00}_{++} = -m_N\left[U_L(b_{15}a_7/2 + c_5) + S_H b_{14}a_{15} + S_Z a_{14}b_{16}\right],$$
$$f_2 M^{00}_{-+} = U_L a_1\left[-c_6 + a_{13}(b_{15} - b_1)/2\right] + S_Z a_1 a_{13} b_{16},$$
$$f_2 M^{00}_{+-} = U_L a_2\left[-c_6 + a_{13}(b_{15} + b_1)/2\right] + S_Z a_2 a_{13} b_{16}.$$

A.4. $Z^0 Z^0 \to N\bar{N}$

Use the same definitions (A.2)–(A.4), and define $\theta$ to be the angle between N and $Z^0$.

21 of the 36 helicity amplitudes are given below. The remaining 15 helicity amplitudes can be obtained from these using the symmetry properties, c.f. eq. (4.10).

$$f_3 M^{++}_{++} = m_N [U_N(a_6 d_2 + a_{12}) - 4S_H a_{15} + T_N(a_6 d_1 + a_{12})],$$

$$f_3 M^{++}_{--} = m_N [U_N(-a_5 d_1 + a_{12}) - 4S_H a_{15} + T_N(-a_5 d_2 + a_{12})],$$

$$f_3 M^{++}_{-+} = a_1 d_3 (U_N a_9 - T_N a_{10}), \quad f_3 M^{++}_{+-} = a_2 d_3 (U_N a_9 - T_N a_{10}),$$

$$f_3 M^{-+}_{++} = f_3 M^{+-}_{++} = -m_N (U_N + T_N) a_{12}, \quad f_3 M^{-+}_{-+} = -(U_N + T_N) a_1 a_{11} d_1,$$

$$f_3 M^{-+}_{+-} = (U_N + T_N) a_2 a_{11} d_2, \quad f_3 M^{+-}_{-+} = (U_N + T_N) a_1 a_{11} d_2,$$

$$f_3 M^{+-}_{+-} = -(U_N + T_N) a_2 a_{11} d_1,$$

$$f_4 M^{0+}_{++} = m_N [U_N(2b_2 d_2 - a_3 a_5 d_3) + T_N(-2b_2 d_1 + a_4 a_5 d_3)],$$

$$f_4 M^{0+}_{--} = m_N [U_N(-2b_2 d_1 - a_3 a_6 d_3) + T_N(2b_2 d_2 + a_4 a_6 d_3)],$$

$$f_4 M^{0-}_{++} = m_N [U_N(2b_2 d_1 + a_4 a_6 d_3) + T_N(-2b_2 d_2 - a_3 a_6 d_3)],$$

$$f_4 M^{0-}_{--} = M_N [U_N(-2b_2 d_2 + a_4 a_5 d_3) + T_N(2b_2 d_1 - a_3 a_5 d_3)],$$

$$f_4 M^{0+}_{-+} = U_N a_1(-a_3 a_5 d_1 + 2b_3) + T_N a_1(a_4 a_6 d_1 + 2b_3),$$

$$f_4 M^{0+}_{+-} = U_N a_2(a_3 a_6 d_2 + 2b_3) + T_N a_2(-a_4 a_5 d_2 + 2b_3),$$

$$f_4 M^{0-}_{-+} = U_N a_1(-a_4 a_6 d_2 - 2b_3) + T_N a_1(a_3 a_5 d_2 - 2b_3),$$

$$f_4 M^{0-}_{+-} = U_N a_2(a_4 a_5 d_1 - 2b_3) + T_N a_2(-a_3 a_6 d_1 - 2b_3),$$

$$f_5 M^{00}_{++} = -m_N [U_N(b_{15} a_7 + 2c_5) + 4S_H b_{14} a_{15} + T_N(b_{15} a_8 + 2c_5)],$$

$$f_5 M^{00}_{-+} = U_N a_1(-2b_2 a_{26} + a_{13} b_{15}) + T_N a_1(2b_2 a_{27} - a_{13} b_{15}),$$

$$f_5 M^{00}_{+-} = U_N a_2(2b_2 a_{27} + a_{13} b_{15}) + T_N a_2(-2b_2 a_{26} - a_{13} b_{15}).$$

A.5. $W^-Z^0 \to L^-\bar{N}$

Following the notation in subsects 4.1 and 4.2, we can write eq. (4.5) as

$$A^{-1} = \frac{1}{N} P_+(1,2) + \frac{1}{\varepsilon N} P_-(1,2), \quad \varepsilon \ll 1, \tag{A.5}$$

with

$$N = \left[2\left(E_1 E_2 + P^2 - m_L m_N\right)\right]^{1/2}, \tag{A.6}$$

$P$ and $K$ are as defined in eq. (4.1). $E_1$, $E_2$, $E_3$ and $E_4$ are $L^-$, $\bar{N}$, $W^-$ and $Z^0$ energies, respectively. $\theta$ is the angle between $L^-$ and $W^-$ in WZ c.m. frame. In addition to eqs. (A.2) and (A.3), we define the following variables

$a_1 = 2a_{29}$, $a_2 = a_{29} S_w^2 / C_w$, $a_3 = (E_3 - E_4) C_w$, $a_4 = P$, $a_5 = vP$, $a_6 = P S_\theta^2$,

$a_7 = 2E_1$, $a_8 = K C_\theta$, $a_9 = 2C_w a_8$, $a_{10} = 2v a_8$, $a_{11} = 2C_w K S_\theta$, $a_{12} = K + E_4$

$a_{13} = K - E_4$, $a_{14} = K + E_3$, $a_{15} = K - E_3$, $a_{16} = P + E_1$, $a_{17} = P - E_1$,

$a_{18} = E_1 + P C_\theta$, $a_{19} = E_1 - P C_\theta$, $a_{20} = a_{16} - E_4$, $a_{21} = a_{16} - E_3$,

$a_{22} = a_{17} + E_4$, $a_{23} = a_{17} + E_3$, $a_{24} = P S_\theta$, $a_{25} = K C_w$, $a_{26} = K$, $a_{27} = E_3$,

$a_{28} = E_4$, $a_{29} = \sqrt{S}$, $a_{30} = E_9 - E_4$, $a_{31} = E_3 + vK$, $a_{32} = E_3 - vK$,

$a_{33} = E_4 + vK$, $a_{34} = E_4 - vK$, $a_{35} = P + E_2$, $a_{36} = P - E_2$,

$b_1 = m_L a_{35} + m_N a_{17}$, $b_2 = m_L m_N - a_{16} a_{35}$, $b_3 = m_L m_N - a_{17} a_{36}$,

$b_4 = m_L a_{36} + m_N a_{16}$, $b_5 = E_1 a_{14} + P a_{31}$, $b_6 = E_1 a_{15} + P a_{32}$, $b_7 = E_1 a_{15} - P a_{32}$,

$b_8 = E_1 a_{14} - P a_{31}$, $b_9 = M_W^2$, $b_{10} = M_Z^2$, $b_{11} = PK + E_1 a_{33}$, $b_{12} = PK - E_1 a_{33}$,

$b_{13} = PK + E_1 a_{34}$, $b_{14} = PK - E_1 a_{34}$, $b_{15} = K^2$, $b_{16} = E_3 E_4$, $b_{17} = -b_9$,

$b_{18} = P E_4$, $b_{19} = v P E_4$, $b_{20} = 2 P E_3$, $b_{21} = 2 P E_3 C_\theta$, $b_{22} = 2 P E_3 S_\theta^2$,

$b_{23} = 2 P E_4 C_\theta$, $b_{24} = 2 P E_4 S_\theta^2$, $b_{25} = K^2 + E_3 E_4$,

$b_{26} = -b_{10} + 2 E_1 E_4$, $b_{27} = -b_9 - b_{10} - a_{29}^2$, $b_{28} = 2 P a_{29}$, $b_{29} = E_1 a_{30}$,

$b_{30} = v(-b_{10} - 2 E_1 E_3)$, $c_1 = 2 P E_3 E_4$, $c_2 = c_1 C_\theta$, $c_3 = c_1 S_\theta^2$,

$c_4 = E_3 b_{10} + 2 E_1 K^2$, $d_1 = 1 + C_\theta$, $d_2 = 1 - C_\theta$, $d_3 = S_\theta$, $d_4 = 1 - v$,

$f_1 = \sqrt{2}\,\mathrm{N}$, $f_2 = 2 M_W \mathrm{N}$, $f_3 = 2 M_Z \mathrm{N}$, $f_4 = \sqrt{2}\, M_W M_Z \mathrm{N}$, and

$$S_{\mathrm{W}}=\frac{1}{-S+M_{\mathrm{W}}^{2}}, \quad U_{\mathrm{L}}=\frac{1}{4\mathrm{C}_{\mathrm{W}}}\frac{1}{-U+m_{\mathrm{L}}^{2}}, \quad T_{\mathrm{N}}=\frac{1}{2\mathrm{C}_{\mathrm{W}}}\frac{1}{-T+m_{\mathrm{N}}^{2}}. \qquad (\mathrm{A}.7)$$

The 36 helicity amplitudes are shown below

$$f_1M^{++}_{++}=b_1\big[S_{\mathrm{W}}(a_2+a_9+a_3)+U_{\mathrm{L}}(-d_4d_2a_{12}+2d_2a_{16}-d_4a_6)$$
$$+T_{\mathrm{N}}(d_1(a_{14}-a_{16})+a_6)\big],$$

$$f_1M^{++}_{--}=b_4\big[S_{\mathrm{W}}(-a_2+a_9-a_3)+U_{\mathrm{L}}(d_4d_1a_{12}+2d_1a_{17}-d_4a_6)$$
$$+T_{\mathrm{N}}(-d_2(a_{14}+a_{17})+a_6)\big],$$

$$f_1M^{--}_{++}=b_1\big[S_{\mathrm{W}}(a_2+a_9+a_3)+U_{\mathrm{L}}(d_4d_1a_{13}+2d_1a_{16}-d_4a_6)$$
$$+T_{\mathrm{N}}(-d_2(a_{15}+a_{16})+a_6)\big],$$

$$f_1M^{--}_{--}=b_4\big[S_{\mathrm{W}}(-a_2+a_9-a_3)+U_{\mathrm{L}}(-d_4d_2a_{13}+2d_2a_{17}-d_4a_6)$$
$$+T_{\mathrm{N}}(d_1(a_{15}-a_{17})+a_6)\big],$$

$$f_1M^{++}_{-+}=b_2d_3\big[2S_{\mathrm{W}}a_{25}+U_{\mathrm{L}}(d_1a_4+d_4a_{12}-a_7+a_5d_2)+T_{\mathrm{N}}(a_{14}-a_{18})\big],$$

$$f_1M^{++}_{+-}=b_3d_3\big[-2S_{\mathrm{W}}a_{25}+U_{\mathrm{L}}(d_2a_4-d_4a_{12}+a_7+a_5d_1)-T_{\mathrm{N}}(a_{14}-a_{18})\big],$$

$$f_1M^{--}_{-+}=b_2d_3\big[2S_{\mathrm{W}}a_{25}+U_{\mathrm{L}}(-d_2a_4+d_4a_{13}+a_7-a_5d_1)+T_{\mathrm{N}}(a_{15}+a_{19})\big],$$

$$f_1M^{--}_{+-}=b_3d_3\big[-2S_{\mathrm{W}}a_{25}+U_{\mathrm{L}}(-d_1a_4-d_4a_{13}-a_7-a_5d_2)-T_{\mathrm{N}}(a_{15}+a_{19})\big],$$

$$f_1M^{-+}_{++}=f_1M^{+-}_{++}=b_1a_6(U_{\mathrm{L}}d_4-T_{\mathrm{N}}), \quad f_1M^{-+}_{-+}=b_2a_{24}d_1(-U_{\mathrm{L}}d_4+T_{\mathrm{N}}),$$

$$f_1M^{+-}_{-+}=b_2a_{24}d_2(U_{\mathrm{L}}d_4-T_{\mathrm{N}}), \quad f_1M^{-+}_{+-}=b_3a_{24}d_2(-U_{\mathrm{L}}d_4+T_{\mathrm{N}}),$$

$$f_1M^{+-}_{+-}=b_3a_{24}d_1(U_{\mathrm{L}}d_4-T_{\mathrm{N}}), \quad f_1M^{-+}_{--}=f_1M^{+-}_{--}=b_4a_6(U_{\mathrm{L}}d_4-T_{\mathrm{N}}),$$

$$f_2M^{0+}_{++}=b_1d_3\big[S_{\mathrm{W}}a_{25}a_1+U_{\mathrm{L}}(d_4(a_{12}a_{14}+b_{21})-2b_5)$$
$$+T_{\mathrm{N}}(b_9-a_{17}a_{26}-a_{16}a_{27}-b_{21})\big],$$

$$f_2M^{0+}_{--}=b_4d_3\big[S_{\mathrm{W}}a_{25}a_1+U_{\mathrm{L}}(d_4(a_{12}a_{14}+b_{21})-2b_8)$$
$$+T_{\mathrm{N}}(b_9+a_{16}a_{26}+a_{17}a_{27}-b_{21})\big],$$

$$f_2 M^{0-}_{--} = b_4 d_3 \big[ -S_{\rm W} a_{25} a_1 + U_{\rm L} ( d_4 ( a_{13} a_{15} - b_{21} ) + 2 b_6 ) + T_{\rm N} ( b_9 - a_{16} a_{26} + a_{17} a_{27} + b_{21} ) \big] ,$$

$$f_2 M^{0-}_{++} = b_1 d_3 \big[ -S_{\rm W} a_{25} a_1 + U_{\rm L} ( d_4 ( a_{13} a_{15} - b_{21} ) + 2 b_7 ) + T_{\rm N} ( b_9 + a_{17} a_{26} - a_{16} a_{27} + b_{21} ) \big] ,$$

$$f_2 M^{0+}_{-+} = b_2 d_1 \big[ -S_{\rm W} a_{25} a_1 - U_{\rm L} ( d_4 a_{12} a_{14} + 2 a_{14} a_{17} - d_4 b_{20} d_2 ) - T_{\rm N} ( b_9 + a_{16} a_{15} + b_{20} d_2 ) \big] ,$$

$$f_2 M^{0+}_{+-} = b_3 d_2 \big[ -S_{\rm W} a_{25} a_1 - U_{\rm L} ( d_4 a_{12} a_{14} - 2 a_{14} a_{16} + d_4 b_{20} d_1 ) - T_{\rm N} ( b_9 - a_{17} a_{15} - b_{20} d_1 ) \big] ,$$

$$f_2 M^{0-}_{-+} = b_2 d_2 \big[ -S_{\rm W} a_{25} a_1 + U_{\rm L} ( d_4 a_{13} a_{15} - 2 a_{15} a_{17} - d_4 b_{20} d_1 ) + T_{\rm N} ( b_9 - a_{16} a_{14} + b_{20} d_1 ) \big] ,$$

$$f_2 M^{0-}_{+-} = b_3 d_1 \big[ -S_{\rm W} a_{25} a_1 + U_{\rm L} ( d_4 a_{13} a_{15} + 2 a_{15} a_{16} + d_4 b_{20} d_2 ) + T_{\rm N} ( b_9 + a_{17} a_{14} - b_{20} d_2 ) \big] ,$$

$$f_3 M^{+0}_{++} = b_1 d_3 \big[ S_{\rm W} a_{25} a_1 + U_{\rm L} ( d_4 ( b_{10} + b_{23} ) - 2 ( b_{11} + b_{18} ) ) + T_{\rm N} ( a_{12} a_{14} + a_{17} a_{26} - a_{16} a_{28} - b_{23} ) \big] ,$$

$$f_3 M^{+0}_{--} = b_4 d_3 \big[ S_{\rm W} a_{25} a_1 + U_{\rm L} ( d_4 ( b_{10} + b_{23} ) + 2 ( b_{12} + b_{18} ) ) + T_{\rm N} ( a_{12} a_{14} - a_{16} a_{26} + a_{17} a_{28} - b_{23} ) \big] ,$$

$$f_3 M^{-0}_{--} = b_4 d_3 \big[ -S_{\rm W} a_{25} a_1 + U_{\rm L} ( d_4 ( b_{10} - b_{23} ) - 2 ( b_{13} - b_{18} ) ) + T_{\rm N} ( a_{13} a_{15} + a_{16} a_{26} + a_{17} a_{28} + b_{23} ) \big] ,$$

$$f_3 M^{-0}_{++} = b_1 d_3 \big[ -S_{\rm W} a_{25} a_1 + U_{\rm L} ( d_4 ( b_{10} - b_{23} ) + 2 ( b_{14} - b_{18} ) ) + T_{\rm N} ( a_{13} a_{15} - a_{17} a_{26} - a_{16} a_{28} + b_{23} ) \big] ,$$

$$f_3 M^{+0}_{-+} = b_2 d_2 \big[ S_{\rm W} a_{25} a_1 + U_{\rm L} ( d_4 ( b_{10} + 2 b_{18} d_1 ) + 2 ( b_{12} + b_{19} ) ) + T_{\rm N} ( a_{12} ( a_{14} + a_{17} ) - 2 b_{18} d_1 ) \big]$$

$$f_3 M^{+0}_{+-} = b_3 d_1 \big[ S_{\rm W} a_{25} a_1 + U_{\rm L} \big( d_4 ( b_{10} - 2 b_{18} d_2 ) - 2 ( b_{11} + b_{19} ) \big) + T_{\rm N} \big( a_{12} ( a_{14} - a_{16} ) + 2 b_{18} d_2 \big) \big] ,$$

$$f_3 M^{-0}_{-+} = b_2 d_1 \big[ S_{\rm W} a_{25} a_1 - U_{\rm L} \big( d_4 ( b_{10} + 2 b_{18} d_2 ) - 2 ( b_{13} - b_{19} ) \big) - T_{\rm N} \big( a_{13} ( a_{15} - a_{17} ) - 2 b_{18} d_2 \big) \big] ,$$

$$f_3 M^{-0}_{+-} = b_3 d_2 \big[ S_{\rm W} a_{25} a_1 - U_{\rm L} \big( d_4 ( b_{10} - 2 b_{18} d_1 ) + 2 ( b_{14} - b_{19} ) \big) - T_{\rm N} \big( a_{13} ( a_{15} + a_{16} ) + 2 b_{18} d_1 \big) \big] ,$$

$$\begin{aligned} f_4 M^{00}_{++} = b_1 \big[ & S_{\rm W} \big( b_{25} ( a_2 + a_3 ) + a_9 b_{27}/2 \big) \\ & + U_{\rm L} \big( d_4 \big( a_8 ( b_{26} + b_{20} ) + c_3 - c_4 - c_1 \big) + a_{16} ( a_{10} a_{29} + 2 b_{25} ) \big) \\ & + T_{\rm N} \big( a_8 ( b_{17} - a_{17} a_{30} ) - c_3 - b_{15} a_{22} + b_{16} a_{23} \big) \big] , \end{aligned}$$

$$\begin{aligned} f_4 M^{00}_{--} = b_4 \big[ & S_{\rm W} \big( - b_{25} ( a_2 + a_3 ) + a_9 b_{27}/2 \big) \\ & + U_{\rm L} \big( d_4 \big( a_8 ( b_{26} - b_{20} ) + c_3 + c_4 - c_1 \big) + a_{17} ( - a_{10} a_{29} + 2 b_{25} ) \big) \\ & + T_{\rm N} \big( a_8 ( b_{17} + a_{16} a_{30} ) - c_3 - b_{15} a_{20} + b_{16} a_{21} \big) \big] , \end{aligned}$$

$$f_4 M^{00}_{-+} = b_2 d_3 \big[ S_{\rm W} a_{25} b_{27} + U_{\rm L} \big( - d_4 c_2 + a_{26} ( b_{26} - b_{28} - b_{30} ) \big) + T_{\rm N} \big( c_2 + a_{26} ( b_{17} - b_{28}/2 + b_{29} ) \big) \big] ,$$

$$f_4 M^{00}_{+-} = b_3 d_3 \big[ - S_{\rm W} a_{25} b_{27} + U_{\rm L} \big( d_4 c_2 - a_{26} ( b_{26} + b_{28} - b_{30} ) \big) + T_{\rm N} \big( - c_2 - a_{26} ( b_{17} + b_{28}/2 + b_{29} ) \big) \big] .$$

## References


[1] S. Dawson, Nucl. Phys. B249 (1985) 42;
M. Chanowitz and M. Gaillard, Phys. Lett. B142 (1984) 85;
G. Kane, W. Repko and W. Rolnick, Phys. Lett. B148 (1984) 367;
J. Lindfors, Z. Phys. C28 (1985) 427

[2] R. Cahn and S. Dawson, Phys. Lett. B136 (1984) 196;
S. Dawson and J. Rosner, Phys. Lett. B148 (1984) 497;
H. Georgi and M. Machacek, Nucl. Phys. B262 (1985) 463

[3] M. Chanowitz and M. Gaillard, Nucl. Phys. B261 (1985) 379;
M. Duncan, G. Kane and W. Repko, Nucl. Phys. B272 (1986) 517

[4] S. Willenbrock and D. Dicus, Phys. Lett. B156 (1985) 429;
O. 'Eboli et al., Phys. Rev. D34 (1986) 771; O. 'Eboli et al., Phys. Lett. B178 (1986) 77;
S. Dawson, G. Kane, S. Willenbrock and C.-P. Yuan, *in* Proc. 1986 Summer study on the design and utilization of the superconducting supercollider, Snowmass, Colorado, ed. R. Donaldson and J. Marx;
W. Repko and Wu-Ki Tung, *in* Proc. 1986 Summer study on the design and utilization of the superconducting supercollider, Snowmass, Colorado, ed. R. Donaldson and J. Marx;
S. Dawson and S. Willenbrock, Nucl. Phys. B284 (1987) 449;
Z. Kunszt and D.E. Soper, Nucl. Phys. B296 (1988) 253

[5] M. Chanowitz and M. Gaillard, Nucl. Phys. B261 (1985) 379;
B. Lee, C. Quigg and H. Thacker, Phys. Rev. D16 (1977) 1519;
J. Cornwall, D. Levin and G. Tiktopoulous, Phys. Rev. D10 (1974) 1145

[6] M. Veltman, SLAC Lecture (1980);
K. Aoki, Z. Hioki, R. Kawabe, M. Konuma and T. Muta, Prog. Theor. Phys. (Suppl.) (1973)

[7] A.A. Slavnov, Theor. Math. Phys. 10 (1972) 153;
G. 't Hooft and M. Veltman, Nucl. Phys. B50 (1972) 318

[8] M. Caffo and E. Remiddi, Helvetica Physica Acta, 55 (1982) 339

[9] R. Brown and K. Mikaelian, Phys. Rev. D19 (1979) 922;
R. Philippe, Phys. Rev. D26 (1982) 1588

[10] M. Chanowitz, M. Furman and I. Hinchiliffe, Phys. Lett. B78 (1978) 285; Nucl. Phys. B153 (1979) 402;
M. Veltman, Nucl. Phys. B123 (1977) 89