ON THE DEFINITION OF THE WEAK MIXING ANGLE

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A discussion of the need for a definition of $\sin^2\theta$ is presented. The use of a minimal subtraction scheme is advocated.

This note is intended to clarify some issues concerning the definition of $\sin\theta$ (the electroweak mixing angle) as it appears that not only there is some confusion, but worse, that this confusion leads to a wrong interpretation of the data.

Consider some theory, supposedly described by a lagrangian depending on certain parameters. Suppose for simplicity there is only one parameter called x: $\mathscr{L}=\mathscr{L}(x)$. In the tree approximation there is no ambiguity and theoretical predictions from this lagrangian can be compared with the experiment. One data point is needed to fix x, after that any other comparison is a test of the theory. Of course, ideally one would like to combine all data and express the result as a probability, that all the data are consistent with one free parameter, but we will not dwell on that.

Now suppose that one wants to go beyond the tree approximation. Then radiative corrections must be calculated. The relation between the parameter x and the experimental data becomes much more complicated. Nonetheless it remains precisely true: one measurement is needed to fix the free parameter x, the rest is a test. Of course the value of x as determined using only the tree approximation will be different from the value determined taking into account radiative corrections. As it happens this difference is usually infinitely large because the radiative corrections contain infinities. Such infinities must be well defined and understood, but nowadays everybody uses the same regularization scheme, i.e. dimensional regularization, and there is as yet no real problem there. In still higher order there is the problem of how to define γ^5, and indeed, strictly speaking renormalizability of the standard model is not yet proven. Again, that is not the issue here, but it must be mentioned because it is a potential source for scheme diversification.

Because of the awkward situation that the *corrected* x and the *tree* x are so different one introduces the notion of a counter term. Thus in the lagrangian one writes $x(1+\delta x)$ instead of x, and δx is chosen in some well defined manner such that now x remains in the neighbourhood of the tree x. It is, however, purely a matter of convenience; the only thing that ever emerges in the confrontation with the data is $x(1+\delta x)$. In order to have meaningful communication it is necessary, when talking about x, to specify what δx is used. Stating one's conventions

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on this matter is what is usually termed the *subtraction scheme* or *renormalization scheme*. Two essentially different approaches may be distinguished:

(a.1) prescribe precisely what x is,

(a.2) prescribe precisely what δx is.

Again, only the combination $x(1+\delta x)$ appears in the confrontation with the data, and we are discussing here a matter of convention. As a matter of terminology we will call quantities such as *δx counter terms*.

In the older days of QED method (a.1) was the preferred. The convention was to prescribe x, and to use for that some very well defined experimental quantity. The quantity δx is then obtained from the data including radiative corrections. A case in point is the electron mass. The quantity $m(1+\delta m)$ was called the *bare mass* and m itself the *experimental mass*. This method also reflected some vague intuition about the physical meaning of the bare mass: if the interactions could be switched off that is what one would see. Also, the mass of the electron is very well known, and the scheme is well understood. Convention (a.1) has the advantage of not being dependent on the choice of regularization scheme but it offers a problem when there is no clear precisely known experimental quantity that can play the role of defining x. Such is the case of QCD with respect to the coupling constant g of that theory. That g, at least as seen experimentally, is a function of the scale, and, moreover, not easy to measure due to confinement. Consequently theorists, after considerable wrangling and all kinds of confusion, have more or less settled on method (a.2). The quantity δx is prescribed and x is determined from some experiment depending on x. There are two schemes, MS and $\overline{\text{MS}}$. The δx defined by these schemes differ from each other by some finite amount. The $\overline{\text{MS}}$ scheme appears at this time to be the winner.

In the case of the electroweak theory the approach (a.1) has been advocated by Sirlin [1] and the full implementation can be found in ref. [3]. Other authors previously [4] and also more recently [5], have used method (a.2). The *Sirlin scheme* or *on shell scheme* has received considerable acceptance, although not everyone is talking about the same thing. Let us consider that in some detail.

The main problem in the use of method (a.1) for the electroweak theory is really the same as in the case of QCD. What to take for x? It is in fact not even clear for the vector boson masses, because the vector bosons are unstable and their mass is not well defined. Of course, one can define the mass of an unstable particle to be the value at the peak, or the location of the zero of the real part of the propagator (which is not the same) or whatever. In other words, one can foresee the advent of the *Sirlin–XXX scheme*, where XXX is one of a class of scientists defining what the mass of an unstable particle is. The difficulty concerning the coupling constant in QCD surfaces here in the form of a difficulty concerning $\sin\theta$. Following the concepts of method (a.1) Sirlin prescribes the value of $\sin\theta$ as some function of the experimental W and Z^0 masses. If everyone would agree on some number here all would be fine. A problem arises if one insists that this number is strictly related to the experimentally measured vector boson masses.

The existing confusion in the present literature arises from the fact that physicists have come to express their data in terms of a prediction for $\sin\theta$. Thus, a typical (but not all!) experimental paper would report a measurement of $\sin\theta$ following the *Sirlin definition*. How must that be understood? What really happens is that some experiment reports a result; that result, including radiative corrections, can be expressed in terms of a prediction for the vector boson masses. That ratio is then reported and called $\cos\theta$ according to Sirlin.

Of course, there is nothing wrong in reporting the data in this form. It is, however, a different thing. Defining the subtraction scheme is not the same as making a prediction. In fact, this is an artificially convoluted situation. The input for the calculation of the result is the result itself! To be more specific the tree level $\sin\theta$ of the minimal standard model is expressible through at least two different ratios:

– the ratio between the SU(2) and the U(1) couplings in the covariant derivative of the Higgs doublet,

– the mass ratio M_W/M_Z.

In the definition of the on shell scheme one assumes that there is a θ_W with the relation $\sin^2\theta_W = 1 - M_W^2/M_Z^2$ valid to all orders while $M_W^2 \sin^2\theta_W = \pi\alpha/\sqrt{2}\,G_\mu$ is modified by radiative corrections. The $\sin^2\theta$ which appears in the previous relation should actually be identified with the ratio e^2/g^2, which is not in a one-to-one correspondence with $\sin^2\theta_W$ when varying the top quark mass or the Higgs boson mass. Predictions for the W mass and $\sin^2\theta_W$ are therefore obtained assuming, in a way, $\sin^2\theta_W$ as input; however, in principle the vector boson masses and the mixing angle are unrelated quantities.

There is another problem here. In doing this work, i.e. predicting the vector boson masses from the experiment, a certain amount of theoretical input is needed. Consider for example, some low energy neutrino–electron scattering experiment. In order to predict the vector boson masses the radiative corrections to these masses must be calculated. This involves the calculation of vector boson self-energy diagrams, containing quark loops. First there is an important and as yet unknown contribution from the top quark; as a consequence one must assume a value for the top mass before the experimental result can be quoted. Another problem relates to the contributions of the light quarks. There are large higher order QCD effects, amounting to the fact that one cannot use the perturbative approach with some values for the quark masses. Consider typically the one loop corrected photon propagator which receives a correction factor $(\alpha/4\pi)\Pi_F(p^2)$. For a fermion of mass m_f and charge Q_f, if $|p^2| \gg m_f^2$, we have

$$\Pi_F^f(p^2) = \tfrac{4}{3}Q_f^2 N_{cf}\left[\ln\left(\frac{-p^2}{m_f^2}\right) - \tfrac{5}{3} - i\pi\right],$$

where $N_c = 1$ for leptons and 3 for quarks. First there are large QCD corrections to this free field theory expression, secondly the quark masses are not unambiguously defined and we encounter a large logarithm leading to a sizeable theoretical error. Dispersion methods can be used instead, but these in general need as input the total decay rate of off-mass-shell vector bosons into hadrons. Insofar as the vector current part of these decay rates is concerned and insofar as one knows all about isospin (to infer the value of the charged currents from the neutral ones) one may use the existing measured total decay rates of a photon into hadrons off mass-shell (i.e. the total rate of e^+e^- annihilation into hadrons) [6]. However, very little is known about the axial current part [12]. Some gallant guessing is the usual practice.

Now it is of course unavoidable that combination of low and high energy data involves the above described theoretical work. The bad part about the above practise is that the experimental result as reported involves these theoretical speculations, often without really specifying them. As a consequence reported data may become useless if some day the theoretical ideas change. Data should be also reported in a form as much as possible independent of theoretical speculations or prejudice. Thus low energy data should be expressed in terms of some convenient parameter not sensitive to these extrapolations to high energy.

It is, at least to these authors, abundantly clear that one should not use a scheme of type (a.1), but rather one to type (a.2). How can one sensibly use a convoluted scheme where the method depends on the result? If one insists on using scheme (a.1) then at least one should agree on the value of $\sin\theta$ to be used in the scheme as opposed to a value of $\sin\theta$ as defined by the ratio of the vector boson masses. Indeed the main issue in comparing theoretical predictions with experimental measurements is not to define $\sin\theta$ as $\sin\theta_W$ or a variant of it, but rather to obtain the building blocks which render the calculation feasible. Therefore we must prescribe counter terms for the parameter of the lagrangian, including $\delta\sin\theta$. This can be done and the rest is pure convention, namely once the value of $\sin\theta$ to be used in the scheme, say $\sin\theta_{\overline{MS}}$, is given one can introduce a plethora of different combinations of physical quantities to be termed $\sin^2\theta_{XY}$ and decide to express the data in terms of one of them. In the following some of these experimental quantities will be considered and interpreted in terms of effective mixing angles.

How would things be if method (a.2) is used, like in QCD? There would be a $\sin\theta_{\overline{MS}}$, whose value as deduced from low energy experiments would not involve sensitivity to the top quark mass or the quark contributions to the vector boson propagators. There would still be such contributions, but much reduced. For example there would be no dependence proportional to the square of m_t. Moreover, the ρ-parameter [well defined only in scheme (a.2)] is depending on m_t^2, but is sensitive to the strong interactions of the light quarks only insofar as they violate isospin. This is because the tree relation $\rho = 1$ is essentially a consequence of isospin invariance also in the Higgs sector after symmetry breaking. Only isospin breaking effects such as $m_t \neq m_b$ will affect the ρ-

parameter. Thus theoretical uncertainties concerning quark induced self-energy effects are considerably reduced. To see in practice how this works we first consider a quantity of interest in discussing fermionic contributions to vector boson self-energies, namely a particular combination of two-point functions [7]

$$B_f(p^2, m, m) = 2B_{21}(p^2, m, m) - B_0(p^2, m, m) .$$

This combination will appear in most self-energy diagrams. Let $S_{ij} = \Pi_{ij} p^2 + \Sigma_{ij}$ be the i–j transition without explicit overall factors containing coupling constants and $\sin\theta$. For what we need in this note it is enough to consider

$$S^{\rm f}_{Z\gamma}(p^2) = S^{\rm f}_{3\gamma}(p^2) - \sin^2\theta\, p^2 \Pi^{\rm f}_{\gamma\gamma}(p^2) = 2N_{\rm cf} Q_{\rm f}(I_{3\rm f} - 2Q_{\rm f}\sin^2\theta) B_{\rm f}(p^2, m_{\rm f}, m_{\rm f}) p^2 ,$$

where $I_{3\rm f}$ denotes the third component of isospin. In particular, using $\Delta = -2/(n-4) + \gamma - \ln\pi$, we obtain

$$|p^2| \ll m^2 , \quad B_{\rm f}(p^2, m, m) = -\tfrac{1}{3}(\Delta - \ln m^2) + \frac{1}{15}\frac{p^2}{m^2} ,$$

$$|p^2| \gg m^2 , \quad B_{\rm f}(p^2, m, m) \sim -\tfrac{1}{3}[\Delta - \ln(-p^2 - {\rm i}\epsilon)] - \tfrac{5}{9} - \tfrac{1}{3}{\rm i}\pi \sin(-p^2) ,$$

$$s \gg m^2 , \quad {\rm Re}\, B_{\rm f}(-s, m, m)|_{\overline{\rm MS}} \sim \tfrac{1}{3}\ln\frac{s}{\mu^2} - \tfrac{5}{9} .$$

Let us consider the amplitude for $\nu_\mu e^- \to \nu_\mu e^-$. In real life many different contributions should be considered but to illustrate the previous points it is enough to limit the calculation by considering the quark contribution to the Z^0–Z^0 and Z^0–γ transitions. In this case we obtain

$$A(\nu_\mu e^- \to \nu_\mu e^-) = (2\pi)^4 {\rm i} \left(\frac{{\rm i}g}{4c_\theta}\right)^2 \gamma^\mu (1+\gamma^5) \otimes \gamma^\mu (a + b\gamma^5) ,$$

where $\gamma^\alpha \otimes \gamma^\alpha = \bar{\nu}_\mu \gamma^\alpha \nu_\mu \bar{e} \gamma^\alpha e$, etc ... and

$$a = \left(4s_\theta^2 - 1 - \frac{g^2 s_\theta^2}{4\pi^2}\frac{S_{Z\gamma}(p^2)}{p^2}\right)\Delta_Z(p^2) , \quad b = -\Delta_Z(p^2) , \quad \Delta_Z^{-1}(p^2) = p^2 + \frac{M^2}{c_\theta^2} - \frac{g^2}{16\pi^2 c_\theta^2} S_{ZZ}(p^2) ,$$

with $s_\theta^2 = \sin^2\theta$, M is the W bare mass and $S_{Z\gamma}$, S_{ZZ} are the corresponding transitions. Then the total cross section $\sigma_{\nu e}$ can be computed and the data point $R = \sigma_{\bar{\nu}e}/\sigma_{\nu e}$ used,

$$R = \frac{\xi_{\nu e}^2 - \xi_{\nu e} + 1}{\xi_{\nu e}^2 + \xi_{\nu e} + 1} , \quad \xi_{\nu e} = \frac{a}{b} ,$$

where we assume the approximation of zero momentum transfer. Subtracting the terms involving Δ and introducing a mass scale μ we define the counter term for $\sin\theta$ and fix our $\sin\theta_{\overline{\rm MS}}$ to first order in α. At this point we mention that Dyson resummation [3] will be not considered here, but its inclusion is straightforward also in a scheme of type (a.2). the whole renormalization procedure amounts to throwing infinities away. If one subtracts the terms involving Δ the $\overline{\rm MS}$ redefinition of the parameters is obtained (including $\sin\theta$), but we could as well assign any finite value to Δ and check for a Δ independence of the physical quantities. From the t–b doublet we get

$$\sin^2\theta_{\overline{\rm MS}} = \bar{s}_\theta^2 + \frac{\alpha}{12\pi}\left[2(1 - \tfrac{8}{3}\bar{s}_\theta^2)\ln\left(\frac{m_{\rm t}^2}{\mu^2}\right) + (1 - \tfrac{4}{3}\bar{s}_\theta^2)\ln\left(\frac{m_{\rm b}^2}{\mu^2}\right)\right] , \quad \bar{s}_\theta^2 = \tfrac{1}{4}(1 - \xi_{\nu e}) .$$

A recent measurement [8] gives $\bar{s}_\theta^2 = 0.233 \pm 0.012 \pm 0.008$. As expected there are no terms quadratic in the quark masses. At this point we are ready to make predictions. By comparing different energy scales one can even introduce an effective p^2 dependent weak mixing angle, deduced from low energy data

$$\sin^2\theta_{\nu e}(p^2)=\bar{s}_\theta^2-\frac{\alpha}{4\pi}\left(\mathrm{Re}\frac{S_{Z\gamma}(p^2)}{p^2}-\frac{S_{Z\gamma}(p^2)}{p^2}\bigg|_{p^2=0}\right)\bigg|_{s_\theta=\bar{s}_\theta}$$

for fermionic contributions we have $S_{Z\gamma}=p^2\Pi_{Z\gamma}$. For $p^2>0$ and $m_t^2\gg p^2\gg m_f^2$, f=u, d, s, c, b, the term in the bracket behaves like

$$\tfrac{2}{3}\sum_{f\neq t}N_{cf}Q_f(I_{3f}-2Q_f\bar{s}_\theta^2)\left(\ln\frac{p^2}{m_f^2}-\tfrac{5}{3}\right)+\tfrac{2}{15}(1-\tfrac{8}{3}\bar{s}_\theta^2)\frac{p^2}{m_t^2}$$

and again no quadratic term in the masses shows up. For this reason it would seem most appropriate if the results of low energy experiments were to be expressed, within the $\overline{\mathrm{MS}}$ scheme, in terms of $\sin\theta_{\overline{\mathrm{MS}}}$ and the ρ-parameter. Actually some do, notably low energy neutrino–electron scattering experiments [8]. Values for the vector boson masses deduced from those data will involve different uncertainties and can be deduced separately.

What about high energy experiments, such as the measurements of the Z^0 mass, and the asymmetries in lepton or quark decay of the Z^0? In actual fact a $\sin\theta_{\overline{\mathrm{MS}}}$ deduced from an asymmetry experiment would not involve theoretical extrapolation as mentioned above. For example, there would again be no contribution proportional to the top quark mass squared. To a very high degree one can directly confront the high and low energy data here without undue theoretical uncertainty. Since we limit our considerations to fermionic contributions in the vector boson self-energies, the amplitude for $e^+e^-\to\bar{f}f$ ($f\neq e$) becomes on-resonance and up to terms which drop in the asymmetries

$$A(\bar{f}f)\propto(a\gamma^\mu\otimes\gamma^\mu+b\gamma^\mu\gamma^5\otimes\gamma^\mu-c\gamma^\mu\otimes\gamma^\mu\gamma^5+d\gamma^\mu\gamma^5\otimes\gamma^\mu\gamma^5)$$

with

$$a=(1-4s_\theta^2)(I_{3f}-2Q_fs_\theta^2)+(4Q_fs_\theta^2-I_{3f}-\tfrac{1}{2}Q_f)X\,,\quad b=I_{3f}-2Q_fs_\theta^2-\tfrac{1}{2}Q_fX\,,\quad c=I_{3f}(1-4s_\theta^2-X)\,,\quad d=I_{3f}\,,$$

$$X=\frac{\alpha}{\pi}\frac{S_{Z\gamma}(-s)}{s}\,.$$

The corresponding forward–backward and left-right asymmetries are

$$A_{FB}=\frac{3}{2}\frac{ad+bc}{a^2+b^2+c^2+d^2}\,,\quad A_{LR}=2\frac{ac+bd}{a^2+b^2+c^2+d^2}\,.$$

Photon exchange and photon–Z^0 interference are always suppressed; they enter the on-resonance asymmetries with the imaginary part of the Z^0 self-energy, which is obviously independent from unknown heavy particles. Of course imaginary parts are always present in X but for our purposes we will drop them because their contributions are of higher order. First we take f=μ for simplicity and introduce $\xi_{FB}(s)=\tfrac{2}{3}A_{FB}(s)$. Instead of using directly $\sin\theta$ we work with $v(\theta)=4s_\theta^2-1$ and define $\bar{v}$ as a solution of

$$\xi_{MB}(M_Z^2)(1+\bar{v}^2)^2=2\bar{v}^2\,,$$

then it is easy to obtain $v(\theta_{\overline{\mathrm{MS}}})$ related to the forward-backward asymmetry at the Z^0 peak

$$v(\theta_{\overline{\mathrm{MS}}})=\bar{v}-\frac{\alpha}{\pi}\mathrm{Re}\frac{S_{Z\gamma}(-M_Z^2)}{M_Z^2}\bigg|_{\overline{\mathrm{MS}}}\,,\quad \bar{s}_\theta^2=\tfrac{1}{4}(\bar{v}+1)\,,$$

$$\frac{S_{Z\gamma}(-M_Z^2)}{M_Z^2}=-2\sum_f N_{cf}Q_f(I_{3f}-2Q_f\bar{s}_\theta^2)B_f(-M_Z^2,m_f,m_f)\,.$$

The asymptotic behavior of B_f given above proves the assertion, namely no m_t^2 dependence. Similarly we put $\xi_{LR}(s)=\tfrac{1}{2}A_{LR}(s)$ and require $\bar{v}$ to be a solution of

$$\xi_{LR}(M_Z^2)(1+\bar{v}^2)=-\bar{v}\,.$$

It follows that $v(\theta_{\overline{MS}})$, related to the left-right asymmetry at the Z^0 peak, is once more given by the same formula, with a different $\bar{v}$. Again there is no contribution proportional to m_t^2. Notice that $\sin^2\theta_{\overline{MS}}$, being related to physical quantities, is real by construction. Fixed $\sin^2\theta_{\overline{MS}}$ at one data point we predict, for instance, ξ_{LR} in the neighbourhood of M_Z^2, where, however, the photon channel should also be included:

$$\xi_{LR}(s)=\xi_{LR}(M_Z^2)+\frac{\alpha}{\pi}\frac{\bar{v}-1}{(\bar{v}^2+1)^2}\,\mathrm{Re}\left(\frac{S_{Z\gamma}(-s)}{s}-\frac{S_{Z\gamma}(-M_Z^2)}{M_Z^2}\right)\bigg|_{s\theta=s\theta}\,.$$

The bracket behaves like $\ln(M_Z^2/s)$ for $s, M_Z^2\gg m_t^2$ and goes to zero in the limit $s, M_Z^2\ll m_t^2$. Now it becomes a pure convention to define a $\sin^2\theta_{LR}(s)$

$$\xi_{LR}(s)=\frac{1-4\sin^2\theta_{LR}(s)}{1+[1-4\sin^2\theta_{LR}(s)]^2}$$

and report the data as a measure of $\sin^2\theta_{LR}$.

The Z^0 mass and the W mass are another story. Confronting low energy data and the asymmetry measurements with a vector boson mass value will always involve the contributions mentioned above. For illustration we consider a situation where α, G_F and M_Z, as the location of the zero of the propagator, are given. Let $S_{ij}=\Pi_{ij}p^2+\Sigma_{ij}$ be the i–j transition without explicit overall factors containing coupling constants or $\sin\theta$. A solution for $\sin^2\theta_{\overline{MS}}$ is given in this case, up to first order corrections, by

$$\sin^2\theta_{\overline{MS}}=\bar{s}_\theta^2\left\{1+\frac{\alpha}{4\pi}\,\mathrm{Re}\left[\frac{1}{\bar{c}_\theta^2-\bar{s}_\theta^2}\left(\bar{c}_\theta^2\Pi_F+\frac{\Sigma_F}{M_Z^2\bar{s}_\theta^2}\right)-\frac{S_{Z\gamma}(-M_Z^2)}{M_Z^2\bar{s}_\theta^2}\bigg|_{\overline{MS},s\theta=\bar{s}\theta}\right]\right\},$$

where Π_F and Σ_F are ultraviolet finite combinations

$$\Pi_F=\Pi_{\gamma\gamma}(-M_Z^2)-\Pi_{\gamma\gamma}(0)\,,\quad \Sigma_F=\Sigma_{WW}(0)-S_{33}(-M_Z^2)+S_{3\gamma}(-M_Z^2)\,,$$

where $S_{3\gamma}=S_{Z\gamma}+\bar{s}_\theta^2p^2\Pi_{\gamma\gamma}$, $S_{33}=S_{ZZ}+2\bar{s}_\theta^2S_{Z\gamma}+\bar{s}_\theta^4p^2\Pi_{\gamma\gamma}$ and the lowest order $\sin\theta$ is

$$\bar{s}_\theta^2=\frac{1}{2}\left(1-\sqrt{1-\frac{2\pi\alpha}{G_FM_Z^2}}\right).$$

Besides large logarithms to be eventually resummed we have a quadratic dependence from the unknown m_t, since from the explicit expression we derive $\Sigma_F\sim-\frac{3}{4}m_t^2$. Here there are no left over contributions simply because everything is expressible in terms of self-energy diagrams. Having defined counter terms we can predict measurable quantities, as for instance the W mass. If required $\sin^2\theta_{\overline{MS}}$ can be computed to higher orders in α by including multi-loop reducible diagrams. Similarly, as pointed out above, we may choose α, G_F and $\xi_{\nu e}$ as data points and predict the Z^0 mass. By considering the one-loop corrected Z^0 propagator and using the $\overline{MS}$ definition of g, M and $\sin\theta$ we find M_Z^2 as a solution of

$$\frac{2G_F}{\pi\alpha}\bar{s}_\theta^2\bar{c}_\theta^2M_Z^2=1+\frac{G_F}{2\pi^2}\,\mathrm{Re}\{\Sigma_F(-M_Z^2)+\bar{s}_\theta^4M_Z^2\Pi_F(-M_Z^2)+(\bar{c}_\theta^2-\bar{s}_\theta^2)M_Z^2[\Pi_{3\gamma}(-M_Z^2)-\Pi_{3\gamma}(0)]\}$$

and $\xi_{\nu e}=1-4\bar{s}_\theta^2$. In this formula we used the fact that for fermion insertions $S_{3\gamma}=p^2\Pi_{3\gamma}$. Similar formulas obtain when we use ξ_{FB} or ξ_{LR} at the peak. Terms quadratic in m_t^2 will always be there. From the on-resonance left–right asymmetry we obtain

$$-\frac{2G_F}{\pi\alpha}\bar{s}_\theta^2\bar{c}_\theta^2M_Z^2+1+\frac{G_F}{2\pi^2}\,\mathrm{Re}[\Sigma_F(-M_Z^2)+\bar{s}_\theta^2\bar{c}_\theta^2M_Z^2\Pi_F(-M_Z^2)]=0\,,$$

$$\bar{s}_{\theta}^{2}=\frac{2\xi_{LR}-1+\sqrt{1-4\xi_{LR}^{2}}}{8\xi_{LR}}\,, \quad \xi_{LR}=\tfrac{1}{2}A_{LR}(M_{Z}^{2})\,.$$

From an experimental point of view there is really no difficulty in reporting the data around the resonance; one just produces a picture of the Z^0 line-shape with experimental error bars and that is it. Then one can leave it to the theorists to have a field day defining what the mass really is. Interesting quantities, such as the top quark mass can then be worked out including the ambiguities involved. And of course, one can still talk about sin θ–Sirlin, but now understood as deduced from the experimentally measured vector boson masses defined in some way. For hadron collider experiments, where the precision of the measurements is not likely to get to the level of theoretical ambiguity, it would seem to be a practical way of reporting data.

Assuming then the use of method (a.2) one must still specify which particular scheme to use, i.e. MS or $\overline{\mathrm{MS}}$ or whatever. It would seem natural to follow the consensus with respect to QCD, i.e. to use the $\overline{\mathrm{MS}}$ scheme. Someday there will be an interplay between these two segments of the standard model, and it seems advantageous to use similar schemes.

There is, however, an additional point. In case the Higgs mass is very large there could still be a large difference between the tree and the one loop corrected quantities, because the radiative corrections involve not only unobservable infinities, but also equally unobservable terms proportional to the Higgs mass squared. It would seem appropriate to include also these terms in the definition of the counter terms [9]. A case can be made [10] for the inclusion of certain terms proportional to the logarithm of the Higgs mass, but for the time being that seems not necessary and can lead to further strife, disagreement, confusion and ambiguity.

In the above the question of the physical meaning of the various choices has not been raised. After all, it is mainly a question of convention. Even so, let us consider the situation for a moment. The mixing between Z^0 and γ is what one naturally thinks of when speaking about the weak mixing angle. That angle, understood in that sense defines the distribution of the vector current between γ and Z^0. It surfaces in the fact that the EM charge is proportional to sin θ, and that the vector part of the neutral current of fermions as coupling to the Z^0 contains sin θ. Without mixing the Z^0 couplings would be pure V – A. In principle the values of the vector boson masses are totally unrelated to this mixing. Using an appropriate Higgs system one can, for a given mixing angle, produce any desired value for these masses. It is an accident of Schwinger's σ model as used by Weinberg to construct the simplest possible Higgs system that the vector boson masses are not both free, and that in fact the ratio of the masses equals cos θ. That accident is nothing else but isospin invariance. It really seems to be a first task of the experiment to establish the truth of that relation. It is the first real information on the Higgs system. Why then define sin θ from this mass ratio? It is quite precisely like defining the fine structure constant α as $\frac{3}{2}$ times the ratio of the electron and muon mass.

As a final note an answer to a point raised by Sirlin [1,2]. The statement is this: how can one talk about the ρ-parameter if one has not defined first sin θ? The answer is this: there is no definition needed for sin θ if δ sin θ is specified. The ρ-parameter, being finite, is not sensitive to the details of the scheme used. One could even work, as stated before, without counter terms altogether. For completeness, the ρ-parameter is defined as the ratio of W and Z^0 masses squared divided by $\cos^2\theta$. Both the *bare* and the *experimental* ρ-parameter are finite and numerically very close in any scheme of type (a.2). The main correction is presumably due to the top quark.

Concerning this ρ-parameter the comparison with the data is as follows. Suppose the results of a low energy experiment are interpreted using the tree approximation. That will then result in certain values of the vector boson masses as well as sin θ, and a ρ_{tree} can be established. Next, doing all necessary one loop calculations, eventually including resummation of potentially large terms, one may use the data to deduce the values of the quantities sin θ, etc. in the lagrangian within, say, the $\overline{\mathrm{MS}}$ scheme. The ρ-parameter made of these bare quantities should be one ($\rho_{\mathrm{bare}}=1$) if indeed the simplest Higgs system applies. The difference between one and ρ_{tree} is the radiative correction to the ρ-parameter. Thus $\tau_{\mathrm{tree}}=1+\Delta\rho$.

Note that the radiative corrections to ρ requires specification of the processes considered. One may for example use low energy neutrino–electron scattering [4,9], or the measured values of the W and Z^0 masses [5,11].

The variations in these corrections for different experiments (in particular experiments at low energy) are minimal. The most interesting part is the correction proportional to the top mass squared. It is presumably the largest as well, and it is the same for all experiments. It should be mentioned that a two loop renormalization, including both reducible and irreducible diagrams, could in principle improve this correction.

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