

Mergers of producers of perfect complements competing in price

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The endogenous merger model of Kamien and Zang (QJE, 1990) is generalized to price competition with perfect complements and used to show that some socially desirable mergers will fail to occur. We also clarify the link between this merger model and the 'exogenous merger' literature.

1. Introduction

In a recent paper, Kamien and Zang (1990) endogenize the decision to merge by firms producing substitutes in a quantity-setting oligopoly. The equilibrium market structure is obtained as a subgame-perfect equilibrium of a two-stage game.¹ In the first stage of the game, each of the n firm owners announces a vector of bids for each of the other firms and an asking price for his own firm. Each firm for which some bid exceeds its asking price is then sold to the highest bidder for the value of the bid (subject to a rule for resolving ties). In the second stage, quantity competition takes place given the market structure resulting from the first stage. Kamien and Zang show that some mergers will not occur as an equilibrium of this merger game.

Market forces limit the degree to which an industry may be monopolized by acquisition. For example, consider an industry where three firms produce perfect substitutes with identical, linear total cost functions. Assume the firms face linear demand. Then in the Kamien and Zang model, neither a two-firm merger nor a merger to monopoly will occur endogenously. If two firms merged, then it follows from the results in Salant, Switzer and Reynolds (1983) that the merged entity would earn smaller profits than its components did prior to the merger. As we show below, mergers with this characteristic never occur in Kamien and Zang's model. Moreover, three firms cannot merge since no firm can afford to pay the other two what each firm conjectures it could earn as the lone

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¹ Their so-called *decentralized game* is actually a three-stage game. We restrict our attention here to their *centralized game*, which reduces to a two-stage game.

holdout. The policy implication drawn from this result is that some socially undesirable mergers need not be of concern since they will fail to occur endogenously.

This result, however, raises a quite different concern: some socially *desirable* mergers will also fail to occur endogenously. In illustrating this point, we generalize their analysis to price competition with perfect complements and clarify the link between this endogenous merger model and the 'exogenous merger' literature following Salant, Switzer and Reynolds (hereafter, SSR).

2. The model

Mergers are sometimes socially desirable. Some economize on fixed costs. Others eliminate adverse pecuniary externalities. We focus here on the latter case. Frequently, the good or service a customer wants requires dealing with several firms simultaneously. For example, the customer may want gas, electricity, or cargo shipped from point A to point B but segments of the pipeline, cable, or railroad track are owned by different firms charging separate fees. The charge to get something shipped from A to B is therefore the sum of the fees set by the owners of the different segments.

The firms in each of these examples sell perfect complements. If they set prices simultaneously, the equilibrium of the game is what Cournot (1838) first identified in his long-neglected example of copper and zinc producers selling to someone wanting brass.² For convenience, we will use the railroad example hereafter. As described in Velturo (1988), the U.S. government was for decades unable to induce railroads adjoining end-to-end to 'consolidate' despite the efficiency gains which such mergers would have created. We will show that this refusal to consolidate – even when consolidation would be socially beneficial – is precisely what the Kamien and Zang result sometimes implies when firms selling perfect complements compete in price.

Assume there are originally n identical segments of track owned by n different players. Each simultaneously sets fares on his own segment. As in Kamien and Zang, assume that the n firms in the industry have constant marginal cost of production, denoted c . There are no fixed costs.

Suppose in the second stage a player controls a subset $s \leq n$ of the firms, in the sense that he centralizes the decision concerning the behavior of those s firms. In quantity competition, Kamien and Zang assumed he chooses the aggregate output of those firms in order to maximize their joint profit. In price competition, we assume he chooses the aggregate price (the sum of the fares on each segment) in order to maximize joint profit.

Given the assumption of linear, identical cost functions, the cost to the owner of several firms of producing a given aggregate output is independent of the interfirm distribution of output between his firms. Since each firm faces the same price and this price depends only on aggregate production, the profit of a player controlling more than one firm in quantity competition depends only on the aggregate output he produces and not on which firms are used to produce it. Hence, Kamien and Zang assume for simplicity that he sets to zero the output of all but one of his firms.

Similarly, in price competition with perfect complements, since the owner of each segment of adjoining track sells the same volume and this volume depends only on the aggregate price, the profit of a player controlling more than one firm depends only on the aggregate price he charges and not on how much he collects on particular segments. To keep the parallel with quantity competition, we assume for simplicity that the player sets to zero all but one of the fares he controls.

² See Cournot's treatment of 'Mutual Relations of Producers' in his chapter 9. Elaborating on Cournot's example, Sonnenschein (1968) showed that whatever may be said about quantity competition with perfect substitutes can be applied to price competition with perfect complements by simply interchanging the role of prices and quantities.

If we let P denote the sum of all the fares paid by the consumer and let $D(P)$ denote the market demand function, then the profit of firm i in the second stage is given by

$$\pi(p_i; m) = [p_i - c]D(P),$$

where p_i is the fare charged by firm i and m is the number of players operating at least one segment after the merger decisions have been taken. Thus $P = \sum_{i=1}^m p_i$.

As the counterpart of Kamien and Zang's assumptions I through III, we assume for our analysis of price competition with perfect complements:

Assumption A. $D(P)$ is twice continuously differentiable, $D(P) > 0$ and $D'(P) < 0$ for all $P \geq 0$.

Assumption B. The industry total profit function, $[P - mc]D(p)$, has a second derivative with respect to P which is negative and bounded from below. We denote this derivative $([P - mc]D(P))''$.³

Given these assumptions, there exists a unique symmetric equilibrium in pure strategies.

Consider now the equilibrium of the first stage of the game. Merger behavior in the first stage depends on the profits which each player anticipates receiving in each second-stage subgame. Whether under quantity competition with perfect substitutes or price competition with perfect complements, each player operating at least one firm in the second stage receives an equilibrium profit which is a strictly decreasing function of the number of such players.⁴ We now clarify the relationship between the prior literature on losses from exogenous merger and the generalized endogenous merger model of Kamien and Zang.

Proposition 1. If a merger would be unprofitable, it will not occur in the Kamien and Zang model.

Assume there is a loss from a merger of s firms. Now write the second-stage equilibrium profit of an individual firm when m active firms remain at the end of the first stage as $\hat{\pi}(m) = \pi(p(m); m)$. Then

$$\hat{\pi}(n - s + 1) < s\hat{\pi}(n), \tag{1}$$

or, equivalently,

$$\hat{\pi}(n - s + 1) < (s - 1)\hat{\pi}(n) + \hat{\pi}(n). \tag{2}$$

Now since $\hat{\pi}(m)$ is a strictly decreasing function of m , $\hat{\pi}(n - s + 2) \geq \hat{\pi}(n)$ for $s \geq 2$. It follows from (2) that a loss from an exogenous merger implies

$$\hat{\pi}(n - s + 1) < (s - 1)\hat{\pi}(n - s + 2) + \hat{\pi}(n). \tag{3}$$

Suppose, for the sake of argument, that such a merger occurred endogenously. Then the acquiring player would have to pay at least what the owner of each acquired firm believes it could

³ Assumption B is the counterpart of Kamien and Zang's assumption III, which, since marginal production cost is constant, reduces under quantity competition with perfect substitutes to the industry total revenue function, $QP(Q)$, having a negative second derivative which is bounded from below.

⁴ For quantity competition with substitutes, see Corollary 1 of Kamien and Zang; for price competition with complements, see our appendix.

earn by holding out unilaterally, in this case $\hat{\pi}(n - s + 2)$.⁵ Let B_j denote the acquiring player's bid for firm j . Without loss of generality, we may assume $j = 1, \dots, s - 1$. Hence

$$\sum_{j=1}^{s-1} B_j \geq (s - 1) \hat{\pi}(n - s + 2). \quad (4)$$

Substituting into (3), we obtain

$$\hat{\pi}(n - s + 1) < \sum_{j=1}^{s-1} B_j + \hat{\pi}(n). \quad (5)$$

But if this inequality holds, the acquiring player is better off bidding zero for the $s - 1$ firms and operating solo. Hence the merger would not occur in equilibrium. Note that the only assumptions made about the second-stage is that a pure-strategy, symmetric equilibrium exists in each subgame and that the profit to each player is strictly decreasing in the number of players. Hence, this result applies both to the case of quantity competition with perfect substitutes and to the case of price competition with perfect complements. This result is useful because SSR partially characterize those mergers which would be unprofitable and hence identify a subset of those mergers which will fail to occur in the Kamien and Zang model.

The following proposition also holds:

Proposition 2. Some socially desirable mergers may fail to occur in the Kamien and Zang model.

It suffices to construct an example. Consider the case of price competition among providers of perfect complements. Assume marginal cost to be zero and $D(P) = \beta - P$. In that case, the sum of consumer and producer surplus in an m -firm industry is given by

$$W(m) = \frac{(2m + 1)\beta^2}{2(m + 1)^2}$$

and $W'(m) < 0$. Net social surplus being a strictly decreasing function of the number of active firms in the second stage, any merger is socially desirable in this context.⁶ SSR showed for the case of quantity competition and perfect substitutes that with linear demand and identical linear costs, any merger which includes less than eighty percent of the firms in the industry will be unprofitable. The same proposition holds for the case of price competition with perfect complements. Hence, a merger of two of three railroads with adjoining tracks would cause a loss and, given Proposition 1 above, will not occur even though it would be socially desirable.⁷

Indeed, complete consolidation, although privately profitable, will also fail to occur in this three-firm example. With a linear demand and zero cost, individual firm profit in an m -firm industry is $\hat{\pi}(m) = \beta^2 / (m + 1)^2$. Suppose one player attempted to acquire the firms of the other

⁵ To hold out unilaterally, the owner of a targeted firm need merely set its asking price higher than anyone bids to acquire it.

⁶ Such mergers cause output to expand toward the competitive (socially optimal) level.

⁷ The corresponding point can also be made in the context of quantity competition with perfect substitutes. SSR (1983, p. 195) show that if firms have fixed costs there exist cases where an exogenous merger causes a loss to the merging parties but would nonetheless be socially beneficial. Our Proposition 1 holds in this case as well and implies that such desirable mergers would not occur endogenously. For the inclusion of fixed costs, see also Gaudet and Salant (1992).

two players. The owner of each target firm would insist on at least the duopoly profit of $\beta^2/9$, since each of them expects to earn that much by unilaterally deviating and operating solo in a two-firm industry. But this is greater than $\beta^2/4 - \beta^2/16$, which is the gain to the acquiring firm of turning this three-firm industry into a monopoly. Hence in this example every possible merger would be preferable to the outcome of no consolidation which occurs in equilibrium.

3. Conclusion

Nearly a decade ago, SSR (1983) pointed out an unnoticed peculiarity in the way mergers and cartelizations had been analyzed: in some exogenous mergers the post-merger profits of the merged entity are smaller than the sum of the component profits prior to the merger. SSR took this as strong evidence of the need to endogenize the merger decision. Kamien and Zang (1990) have provided one of the first models where mergers arise endogenously.

As we showed in Proposition 1, unprofitable mergers never arise endogenously in Kamien and Zang's model. Moreover, as the monopolization example at the outset illustrates, *some* profitable mergers will not arise in their bidding model because of the holdout problem.

We have generalized Kamien and Zang's model and have used it to show that some socially desirable mergers will not occur endogenously. Indeed what occurs in equilibrium may be socially inferior to every other potential market structure. Consolidation failures of this sort are, or ought to be, an issue of policy concern.

Appendix

The purpose of this appendix is to verify that $\hat{\pi}(m)$ is a strictly decreasing function of m .

At the unique and symmetric equilibrium, it follows from Assumption A that each player operating at least one segment in the second stage charges a price which is greater than marginal cost. Let $p = p(m) > c \geq 0$ denote this common equilibrium price. From the first-order conditions, it must satisfy

$$[p - c]D'(mp) + D(mp) = 0. \quad (\text{A.1})$$

If we multiply through by m , it follows that $P(m) = mp(m)$ must satisfy

$$[P - mc]D'(P) + D(P) = 0. \quad (\text{A.2})$$

Differentiating (A.2) totally we find that

$$P'(m) = - \frac{D(P(m)) - cD'(P(m))}{(m - 1)D'(P(m)) + ([P - mc]D'(P(m)))''} > 0 \quad \text{for } m \geq 1, \text{ from Assumption B.} \quad (\text{A.3})$$

This is the counterpart of Kamien and Zang's Proposition 1.

Now denote the second-stage aggregate equilibrium profit when m active firms remain at the end of the first stage by $\hat{\Pi}(m) = m\hat{\pi}(m) = m\pi(p(m); m)$. We easily verify that

$$\begin{aligned}\hat{\Pi}'(m) &= (m-1)[p(m) - c]D'(P(m))P'(m) \\ &< 0 \quad \text{by Assumption A and inequality (A.3)}.\end{aligned}\tag{A.4}$$

It follows immediately that

$$\hat{\pi}'(m) < 0.\tag{A.5}$$

Inequality (A.4) is the counterpart of Kamien and Zang's Proposition 2, while (A.5) is the counterpart of their Corollary 1.

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