Nuclear Physics B409 (1993) 69–86
North-Holland

NUCLEAR
PHYSICS B

μ–e conversion in nuclei and Z′ physics

J. Bernabéu [a], E. Nardi [b] and D. Tommasini [a]

[a] *Instituto de Física Corpuscular - CSIC and Departament de Física Teòrica, Universitat de València, E-46100 Burjassot, València, Spain*

[b] *Randall Laboratory of Physics, University of Michigan, Ann Arbor, MI 48109-1120, USA*

Received 14 June 1993
Accepted for publication 1 July 1993

Together with the existence of new neutral gauge bosons, models based on extended gauge groups (rank > 4) often predict also new charged fermions. A mixing of the known fermions with new states with *exotic* weak-isospin assignments (left-handed singlets and right-handed doublets) will induce tree level flavour changing neutral interactions mediated by Z exchange, while if the mixing is only with new states with *ordinary* weak-isospin assignments, the flavour changing neutral currents are mainly due to the exchange of the lightest new neutral gauge boson Z′. We show that the present experimental limits on μ–e conversion in nuclei give a nuclear-model-independent bound on the Z–e–μ vertex which is twice as strong as that obtained from $\mu \to$ eee. In the case of E_6 models these limits provide quite stringent constraints on the Z′ mass and on the Z–Z′ mixing angle. We point out that the proposed experiments to search for μ–e conversion in nuclei have good chances to find evidence of lepton flavour violation, either in the case that new exotic fermions are present at the electroweak scale, or if a new neutral gauge boson Z′ of E_6 origin lighter than a few TeV exists.

1. Introduction

The search for the conversion of muons into electrons in nuclei provides a very stringent test of muon number conservation. The present experimental bound on the branching for μ–e conversion in titanium, $R \lesssim 4 \times 10^{-12}$ at TRIUMF [1] and PSI [2], gives a very powerful constraint on possible flavour changing neutral currents (FCNCs) violating the muon and electron number conservation. Due to the enhancement by the coherent contribution of all the nucleons in the nucleus, the limits on lepton flavour violation resulting from this process are already more stringent than the ones obtained from the purely leptonic decays $\mu \to$ eee, $\mu \to$ eγ, etc.. Furthermore, new experiments searching for μ–e conversion in nuclei are planned, aiming to test branching ratios up to $R \lesssim 4 \times 10^{-14}$ [3], or possibly even up to $R \lesssim 10^{-16}$ [4]. In the next few years, they could either provide the first accelerator evidences for lepton flavour violation, or give particularly strong constraints on several possible extensions of the Standard Model (SM).

In the SM, lepton flavour violating (LFV) currents are strictly forbidden. This is not true in most of its extensions. For instance, if right-handed neutrinos are present, LFV currents are generated radiatively, proportional to very small GIM-like factors involving neutrino masses. Other extensions of the SM which include new neutral fermions and/or new Higgses, have been discussed in ref. [5]. In model building, it is generally required that some natural mechanism exists to suppress LFV currents at a level compatible with the present experimental constraints.

Recently it has been stressed [6] that extended gauge models, characterized by additional U(1) factors and by the presence of new charged fermions, predict FCNCs mediated by the additional neutral gauge boson Z′. Since the flavour changing Z′ vertices are expected to be naturally large, these FCNCs must be suppressed by a large Z′ mass. In order to be consistent with the limit on $\mu \to$ eee and for natural assumptions on the fermion mass matrix the additional gauge boson should not be much lighter than $\sim$ O(TeV) [6].

In this paper we will consider the constraints implied by the present limit on μ–e conversion in nuclei for the LFV currents mediated either by the standard Z boson, or by a new Z′. By now these data have not been used to constrain Z′ physics, and we show that in most cases they give the strongest bounds on the FC Z′ effects. We also discuss the implications of the planned future experiments [3,4] on μ–e conversion in Ti. If the underlying physics is described by an extended gauge model like E_6, these experiments are expected to reveal evidence for lepton flavour violation. If no signal for LFV processes is detected, this will result in very powerful constraints on the structure of these models, implying vanishingly small values for the parameters describing fermion mixing, and/or very large masses for the additional gauge bosons ($M_{Z'} \gtrsim 5$ TeV). In sect. 2, we derive the effective LFV interaction between the charged leptons and the nucleons, in terms of the fundamental lepton and quark neutral current couplings. The μ–e conversion rate for the coherent nuclear process is then obtained in a nuclear-model-independent way. Following ref. [6], in sect. 3 we show how possibly large FCNC could naturally arise in extended gauge theories. The case of E_6 models will be considered explicitly. In sect. 4, we relate the E_6 parameters of sect. 3 to the effective couplings relevant for the nuclear μ–e conversion process. From the present experimental bound for μ–e conversion in Ti we derive new stringent constraints on the Z–e–μ vertex and on the Z′ parameters, and we also discuss how these constraints will be improved thanks to the proposed future experiments. Finally in sect. 5 we present our conclusions.

2. Coherent μ–e conversion

We will concentrate on the case in which the LFV interactions are mediated only by the exchange of massive gauge bosons, and not by photon or scalar exchange. In this case, the general lepton–quark effective lagrangian can be written in terms of a sum of contact interactions between the leptonic and quark currents of the form

$$\mathscr{L}_{\text{eff}} = \sqrt{2}\ G\bar{e}\gamma^{\lambda}(k_{\text{V}} - k_{\text{A}}\gamma_5)\mu \sum_{q=\text{u,d,s},\dots} \bar{q}\gamma_{\lambda}(v_q - a_q\gamma_5)q, \tag{2.1}$$

where $q =$ u, d, s,... are the relevant quark flavours. k_{V}, k_{A} are the LFV lepton couplings, and v_q, a_q the quark flavour diagonal couplings to the physical massive gauge boson (Z or Z$'$) exchanged, which depend on the particular model considered. For the contribution corresponding to Z exchange, $G = G_{\text{F}}$, the Fermi constant. The Z$'$-exchange term has an overall strength $G = G_{\text{F}}M_{\text{Z}}^2/M_{\text{Z}'}^2$, and whenever we will need to single out this case explicitly we will also prime the couplings in eq. (2.1), $k_{\text{V,A}} \to k'_{\text{V,A}}$, v_q, $a_q \to v'_q$, a'_q.

Since the maximum momentum transfer q^2 involved in the μ–e conversion process is much smaller than the scale associated with the structure of the nucleon, we can neglect the q^2 dependence in the nucleon form factors. Then, in the limit $q^2 \approx 0$, the matrix elements of the quark current for the nucleon $N =$ p, n can be written as

$$\begin{aligned} \langle N|\bar{q}\gamma_{\lambda}q|N\rangle &= G_{\text{V}}^{(q,N)}\bar{N}\gamma_{\lambda}N, \\ \langle N|\bar{q}\gamma_{\lambda}\gamma_5 q|N\rangle &= G_{\text{A}}^{(q,N)}\bar{N}\gamma_{\lambda}\gamma_5 N. \end{aligned} \tag{2.2}$$

In the limit in which strong isospin is a good symmetry, that is up to terms proportional to the up and down mass difference, the neutron and proton form factors are related as follows

$$\begin{aligned} G^{(\text{u,n})} &= G^{(\text{d,p})} \equiv G^{(\text{d})}, \\ G^{(\text{d,n})} &= G^{(\text{u,p})} \equiv G^{(\text{u})}, \\ G^{(\text{s,n})} &= G^{(\text{s,p})} \equiv G^{(\text{s})}. \end{aligned}$$

The conserved vector current and its coherent character, with the vector charge equal to the quark number, determine the couplings

$$G_{\text{V}}^{(\text{u})} = 2, \quad G_{\text{V}}^{(\text{d})} = 1, \quad G_{\text{V}}^{(\text{s})} = 0. \tag{2.3}$$

This argument cannot be applied to the axial-vector current. In terms of definite U(3)-flavour transformation properties, one can introduce the following combination of couplings:

$$\begin{aligned} G_{\text{A}}^{(3)} &= G_{\text{A}}^{(\text{u})} - G_{\text{A}}^{(\text{d})}, \\ G_{\text{A}}^{(8)} &= G_{\text{A}}^{(\text{u})} + G_{\text{A}}^{(\text{d})} - 2G_{\text{A}}^{(\text{s})}, \\ G_{\text{A}}^{(0)} &= G_{\text{A}}^{(\text{u})} + G_{\text{A}}^{(\text{d})} + G_{\text{A}}^{(\text{s})}. \end{aligned} \tag{2.4}$$

The weak currents transform as an octet under flavour SU(3). The two axial form factors $G_A^{(3)}$ and $G_A^{(8)}$ can be expressed in terms of the reduced amplitudes F and D extracted from the semi-leptonic decays of baryons,

$$G_A^{(3)} = F + D = 1.254 \pm 0.006,$$
$$G_A^{(8)} = 3F - D = 0.68 \pm 0.04. \qquad (2.5)$$

The EMC [7] measurement of the polarization-dependent structure function of the proton determines an additional independent combination of $G_A^{(3)}$, $G_A^{(8)}$ and of the singlet $G_A^{(0)}$. One then obtains

$$G_A^0 = 0.12 \pm 0.17. \qquad (2.6)$$

As a result all the axial form factors are determined.

At the nucleon level, the LFV lagrangian (2.1) can then be written as

$$\mathscr{L}_{\text{eff}} = \sqrt{2}\ G\bar{e}\gamma^{\lambda}(k_V - k_A\gamma_5)\mu \sum_{N=p,n} \bar{N}\gamma_{\lambda}(C_{1N} - C_{2N}\gamma_5)N, \qquad (2.7)$$

where the nucleon couplings are [8] vector:

$$\begin{cases} C_{1p} = 2v_u + v_d, \\ C_{1n} = v_u + 2v_d, \end{cases} \qquad (2.8)$$

and axial:

$$\begin{cases} C_{2p} = G_A^{(u)}a_u + G_A^{(d)}a_d + G_A^{(s)}a_s, \\ C_{2n} = G_A^{(d)}a_u + G_A^{(u)}a_d + G_A^{(s)}a_s. \end{cases} \qquad (2.9)$$

We will now discuss the four nucleon couplings (2.8) and (2.9) in the isospin formalism for the nucleon, as appropriate for nuclear physics studies. Introducing the nucleon spinor $\psi_N = \binom{p}{n}$, and the isospin Pauli matrix τ_3, (2.7) reads

$$\mathscr{L}_{\text{eff}} = \sqrt{2}\ G\bar{e}\gamma^{\lambda}(k_V - k_A\gamma_5)\mu$$
$$\times \bar{\psi}_N\gamma_{\lambda}\left[(C_{1S} + C_{1V}\tau_3) - (C_{2S} + C_{2V}\tau_3)\gamma_5\right]\psi_N, \qquad (2.10)$$

with the following couplings:
vector isoscalar:

$$C_{1S} \equiv \tfrac{1}{2}(C_{1p} + C_{1n}) = \tfrac{3}{2}(v_u + v_d), \qquad (2.11a)$$

vector isovector:

$$C_{1V} \equiv \tfrac{1}{2}(C_{1p} - C_{1n}) = \tfrac{1}{2}(v_u - v_d), \qquad (2.11b)$$

axial isoscalar:

$$C_{2S} \equiv \tfrac{1}{2}(C_{2p} + C_{2n}) = \tfrac{1}{2}(G_A^{(u)} + G_A^{(d)})(a_u + a_d) + G_A^{(s)} a_s, \qquad (2.11c)$$

axial isovector:

$$C_{2V} \equiv \tfrac{1}{2}(C_{2p} - C_{2n}) = \tfrac{1}{2}(G_A^{(u)} - G_A^{(d)})(a_u - a_d). \qquad (2.11d)$$

At the low values of the squared momentum transfer relevant for the kinematics of the μ–e conversion process ($q^2 \simeq -m_\mu^2$), the matrix element of $\mathscr{L}_{\text{eff}}$ for a nuclear transition is dominated by the coherent nuclear charge associated with the vector current of the nucleon

$$Q_W = (2Z + N) v_u + (Z + 2N) v_d, \qquad (2.12)$$

which gives an enhanced contribution to the coherent nuclear transition. In practice only the appropriate nuclear form factor for the coherent contribution is needed. The axial quark couplings $G_A^{(u,d,s)}$ do not contribute to the coherent nuclear charge, and will only give rise to nuclear-spin-dependent effects which are negligible as long as the nucleon number ($A = Z + N$) is large enough. For the nucleon numbers relevant for μ–e conversion experiments, the rate for the coherent process, proportional to Q_W^2, will indeed dominate over the incoherent excitations of the nuclear system, which are sensitive to all the vector and axial couplings given in eqs. (2.11). This expectation is supported by explicit calculations based on nuclear models [9], that show that the ratio between the coherent rate and the total μ–e conversion rate for nuclei as ^{48}Ti can be as large as 90%.

In the non-relativistic limit for the motion of the muon in the muonic atom, one can factorize the "large" component of the muon wave function. The corresponding coherent conversion rate is then given by

$$\Gamma = \frac{G^2}{\pi} p_e E_e (k_V^2 + k_A^2) Q_W^2 |M(q)|^2, \qquad (2.13)$$

where p_e (E_e) is the electron momentum (energy), $E_e \simeq p_e \simeq m_\mu$ for this process, and $M(q)$ is the nuclear matrix element of the vector charge density,

$$M(q) = \int d^3x\, \rho(\boldsymbol{x})\, e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \Phi_\mu(\boldsymbol{x}). \qquad (2.14)$$

In eq. (2.14), $\Phi_\mu(\boldsymbol{x})$ is the normalized atomic wave function of the muon and $\rho(\boldsymbol{x})$ is the nuclear density (normalized to unity) taken to be equal for proton and neutron distributions.

The form (2.13) is particularly convenient for discussing the fundamental physics involved in the μ–e conversion process, because it factorizes the model-dependent combination of couplings $(k_V^2 + k_A^2)Q_W^2$ from the nuclear matrix element squared. As said before, if both Z and Z′ exchanges mediate this FCNC process, then one has to reinterpret the product $(k_V^2 + k_A^2)Q_W^2$, but not the nuclear ingredient factorization.

For nuclei with $A \lesssim 100$ one can take, as customary in μ-capture analyses, an average value for the muon wave function inside the nucleus in eq. (2.14) in such a way that

$$|M(q)|^2 = \frac{\alpha^3 m_\mu^3}{\pi} \frac{Z_{\rm eff}^4}{Z} |F(q)|^2, \tag{2.15}$$

where $Z_{\rm eff}$ has been determined in the literature [10] and $F(q)$ is the nuclear form factor, as measured for example from electron scattering [11]. One expects in $^{48}_{22}$Ti this approximation to work within a few percent, with $F(q^2 \simeq -m_\mu^2) \simeq 0.54$ and $Z_{\rm eff} \simeq 17.6$.

The branching ratio R for μ–e conversion in nuclei normalized to the total nuclear muon capture rate $\Gamma_{\rm capture}$, which is experimentally measured with a good precision, can then be computed in any specific extension of the SM, and the informations related to the factors associated with new physics can be extracted in a nuclear-model-independent way. In the case of FCNCs mediated by both Z and Z′ exchange, we obtain

$$R \simeq \frac{G_F^2 \alpha^3}{\pi^2} m_\mu^3 p_e E_e \frac{Z_{\rm eff}^4}{Z} |F(q)|^2 \frac{1}{\Gamma_{\rm capture}} \times \left[(k_V^2 + k_A^2) Q_W^2 + 2 \frac{M_Z^2}{M_{Z'}^2} (k_V k'_V + k_A k'_A) Q_W Q'_W + \left(\frac{M_Z^2}{M_{Z'}^2} \right)^2 (k'^2_V + k'^2_A) Q'^2_W \right], \tag{2.16}$$

where

$$Q'_W = (2Z + N) v'_u + (Z + 2N) v'_d \tag{2.17}$$

and we have explicitly primed the lepton and quark couplings to the Z′ boson. For $\Gamma_{\rm capture}$ in Ti we will use the experimental determination $\Gamma_{\rm capture} \simeq (2.590 \pm 0.012) \times 10^6\ {\rm s}^{-1}$ [12].

3. FCNC in extended models

Following ref. [13] we will now assume the effective low energy gauge group is of the form $\mathscr{G} = ({\rm SU(2)_L} \times {\rm U(1)_Y} \times {\rm SU(3)_C}) \times {\rm U_I(1)}$, and that it originates from the breaking of a *simple* unification group, like E_6. The SM neutral gauge boson Z_0 can then mix with the $U_1(1)$ gauge boson Z_1, resulting in the two mass eigenstates Z and Z′. The NC lagrangian in the physical Z and Z′ basis can be written as follows [13]:

$$-\mathscr{L}_{NC} = eJ^{\lambda}_{em}A_{\lambda} + g_0\left(J^{\lambda}Z_{\lambda} + J'^{\lambda}Z'_{\lambda}\right), \tag{3.1}$$

where $g_0 = (4\sqrt{2}\ G_F M^2_{Z_0})^{1/2}$ is the SM gauge coupling of the Z_0 and J, J' are the fermionic currents coupled to the Z and Z′ bosons. They are related to the gauge currents J_0 and J_1, coupled to Z_0 and Z_1 respectively, by the rotation

$$\begin{pmatrix} J^{\lambda}_{Z} \\ J^{\lambda}_{Z'} \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} J^{\lambda}_0 \\ \sin\theta_w J^{\lambda}_1 \end{pmatrix}, \tag{3.2}$$

where ϕ is the Z–Z′ mixing angle and θ_w is the weak mixing angle *.

Besides predicting extra Z′ bosons, extended gauge models like E_6 predict also the existence of "new" fermions $\psi^0_{\mathscr{N}}$. The new fermions will in general mix with the standard "known" fermions $\psi^0_{\mathscr{K}}$ having the same electric and colour charges. Then for any specific value of the electric and colour charges, the component of chirality $\alpha = L, R$ of the light mass eigenstates ψ_l will correspond to a general superposition of gauge eigenstates that can be written as [13]

$$\psi_{l\alpha} = A^{\dagger}_{\alpha}\psi^0_{\mathscr{K}\alpha} + F^{\dagger}_{\alpha}\psi^0_{\mathscr{N}\alpha}. \tag{3.3}$$

The mixing matrices A_α and F_α describe respectively the mixing of the light states with the known and the new fermions, and satisfy the unitarity relation $A^{\dagger}_{\alpha}A_{\alpha} + F^{\dagger}_{\alpha}F_{\alpha} = I$. The presence of these mixings will affect the couplings of the gauge bosons to the light fermions ψ_l [6,13,14]. In particular, given a general current $J^{\lambda}_{\mathscr{G}}$, corresponding to a broken generator $\mathscr{G}$, its projection on the light fermions $\psi_{l\alpha}$ will read

$$J^{\lambda}_{l\mathscr{G}} = \sum_{\alpha=L,R} \overline{\Psi}_{l\alpha}\gamma^{\lambda}\left[q^{\mathscr{K}}_{\alpha}I + \left(q^{\mathscr{N}}_{\alpha} - q^{\mathscr{K}}_{\alpha}\right)F^{\dagger}_{\alpha}F_{\alpha}\right]\Psi_{l\alpha}, \tag{3.4}$$

where $q^{\mathscr{K}}_{\alpha}$ ($q^{\mathscr{N}}_{\alpha}$) is the $\mathscr{G}$ eigenvalue of the known (new) fermions $\psi^0_{\mathscr{K}\alpha}$ ($\psi^0_{\mathscr{N}\alpha}$), and for simplicity we have assumed that all the new states have the same $\mathscr{G}$ charge. We refer to refs. [6,13] for a more general discussion.

If the known fermions are mixed with new states having different assignments of weak-isospin ("exotic" fermions), then the coefficient $q^{\mathscr{N}}_{\alpha} - q^{\mathscr{K}}_{\alpha} = t_3(\psi^0_{\mathscr{N}\alpha}) - t_3(\psi^0_{\mathscr{K}\alpha})$ multiplying the mixing matrix $F^{\dagger}_{\alpha}F_{\alpha}$ in (3.4) is non-vanishing, and the current J^{λ}_{l0}

* We assume that the running of the $U_1(1)$ gauge coupling constant g_1 from the unification scale down to low energy is similar to the running of the hypercharge coupling constant. Normalizing the $U_1(1)$ charge as the hypercharge generator $\frac{1}{2}Y$ then yields $g_1/g_0 \simeq \sin\theta_w$.

coupled to the Z_0 boson is affected. In this case extremely stringent bounds on the off-diagonal terms can be obtained from the limits on FC processes. For example $(F^\dagger F)_{e\mu} \lesssim 2 \times 10^{-6}$ was obtained in ref. [6] from the non-observation of the $\mu \to$ eee decay, however we will see in sect. 4 that the limit from μ–e conversion in nuclei is stronger by a factor two. The diagonal elements of the matrix $F^\dagger F$ are also constrained mainly from LEP, NC and charged current precision data [13,14], and the corresponding limits are in general $\lesssim 10^{-2}$. On the other hand, the mixings between the ordinary fermions and the new exotic ones are theoretically expected to be very small, since they arise in general from seesaw-like formulas [6,15], so that the corresponding limits are not very effective in constraining the models under examination.

If instead the mixing is with new states having the same SU(2) assignments as the SM fermions ("ordinary" fermions), the coefficient of the mixing term in the J_0^λ current is vanishing, and the couplings to the Z_0 boson are not affected. In this case no phenomenological bounds can be set on the elements of $F^\dagger F$, with the exception of the ordinary mixings of the left-handed quarks, that are constrained by the unitarity tests of the CKM matrix [14]. However, ordinary–ordinary fermion mixing *does* affect the J_1 current, since in general $q_{1\alpha}^{\mathscr{N}} \neq q_{1\alpha}^{\mathscr{K}}$. Clearly at low energy the possible effects of the ordinary–ordinary mixings is suppressed with respect to the effects of the ordinary–exotic mixings as the ratio of the gauge boson mass squared. However, this suppression could be largely compensated by the fact that in general these mixings do not originate from seesaw-like mass matrices, and then *all* the entries in the mixing matrix $F^\dagger F$ can be large [6].

For definiteness we will now consider the case of E_6 models, in which new gauge bosons as well as new ordinary and new exotic fermions are present. Since E_6 is rank six, as many as two additional neutral gauge bosons could appear in the low energy effective gauge group. It is useful to consider the embedding of the SM gauge group $\mathscr{G}_{SM}$ in E_6 through the following pattern of subgroups: $E_6 \to U(1)_\psi \times SO(10) \to U(1)_\chi \times SU(5) \to \mathscr{G}_{SM}$. Then the lightest additional gauge boson will in general correspond to an effective extra $U_1(1)$ resulting as a combination of the $U(1)_\psi$ and $U(1)_\chi$ factors. We will parametrize this combination in terms of an angle β. This will define an entire class of Z′ models in which each fermion f is coupled to the new boson through the effective charge

$$q_1(f) = q_\psi(f)\sin\beta + q_\chi(f)\cos\beta. \tag{3.5}$$

Particular cases that are commonly studied in the literature [13,16,17] correspond to $\sin\beta = -\sqrt{\frac{5}{8}}$, 0, 1 and are respectively denoted as Z_η, Z_χ and Z_ψ models. Z_ψ occurs in $E_6 \to SO(10)$, while Z_η occurs in superstring models when E_6 directly breaks down to rank five. As we will see, this model plays a peculiar role in the present analysis, since it evades completely the kind of constraints that we are investigating. Finally, a Z_χ boson occurs in $SO(10) \to SU(5)$ and couples to the known fermions in the same way as the Z′ present in SO(10) GUTs. However, since SO(10) does not contain additional charged fermions, the kind of FC effects that we are studying here is absent. In contrast, new charged quarks and leptons are present in E_6. The fundamental **27** representation contains, beyond the standard 15 fermion degrees of freedom, 12 additional states for each generation, among which we have a vector doublet of new leptons $H=(N\ E^-)_L^T$, $H^c=(E^+\ N^c)_L^T$.

The chiral couplings of the leptons to the Z_1 as well as the coefficient of the LFV term $F^\dagger F$ are determined by the q_ψ and q_χ charges of the new and known states, which are

$$q_\psi(E_L) = -q_\psi(E_R) = -\tfrac{1}{3}\sqrt{\tfrac{5}{2}}, \quad q_\chi(E_L) = q_\chi(E_R) = -\tfrac{1}{3}\sqrt{\tfrac{3}{2}},$$

$$q_\psi(e_L) = -q_\psi(e_R) = \tfrac{1}{6}\sqrt{\tfrac{5}{2}}, \qquad q_\chi(e_L) = 3q_\chi(e_R) = \tfrac{1}{2}\sqrt{\tfrac{3}{2}}. \tag{3.6}$$

With respect to the $SU(2)_L$ transformation properties, the E_L^+ new leptons are exotic and then the mixings of their *CP* conjugate states E_R^- with the standard right-handed leptons e_R violates weak-isospin by $\frac{1}{2}$. As is discussed for example in ref. [6] this kind of mixings are generally suppressed as the ratio of the light to heavy masses, and then for the e and μ leptons they are expected to be particularly small. In contrast, the E_L^- leptons are ordinary and their mixings with the light leptons are not expected to be suppressed by any small mass ratio since they do not violate weak-isospin. These mixings are generated by entries in the mass matrix corresponding to VEVs of singlet Higgs fields $\langle\phi_S\rangle_0$ which, since they also contribute to the masses of the new (heavy) gauge bosons, are expected to be larger than the doublet VEVs. We note that in E_6 the ordinary–ordinary lepton mixings occur between SU(2) doublets. Then it is clear that for each entry in the charged lepton mass matrix of the form $E_R e_L\langle\phi_S\rangle_0$ there must be a corresponding entry $N^c\nu\langle\phi_S\rangle_0$ in the mass matrix for the neutral states, that would generate a large Dirac mass for the light neutrinos. Even if in the **27** of E_6 several new neutral states (including two SU(2) singlets) are present, in the minimal E_6 models it is not possible to generate naturally any small value eigenvalue for the mass matrix if these Dirac mass entries are present, since the Higgs representation that could generate large Majorana masses and lead to a seesaw mechanism is absent. Then, in the frames of these models, the limits on the neutrino masses automatically guarantee that any possible ordinary–ordinary mixing in the charged lepton sector should be unobservably small. However, as was discussed by Nandi and Sarkar [18], large Majorana masses for the singlet neutral fermions can be generated due to gravitational effects, leading to a rather complicated mass matrix for the neutral states for which a seesaw mechanism is effective, and produce naturally small masses for the light doublet neutrinos. In this scenario, in order not to conflict with the limits on the neutrino masses, there is no need to tune the Dirac mass entries to any unnaturally small value. Then the weak-isospin conserving mixings of the charged leptons are no more constrained, and in the limit in which the singlet VEVs are much larger than the doublet VEVs are theoretically expected to be O(1) [6].

The LFV lagrangian in E_6 models can be obtained from eqs. (3.1), (3.4). For the charged leptons of the first two generations it reads

$$-\mathscr{L}^{e\mu}_{LFV} = g_0\big[k_0(\cos\phi\, Z_\lambda - \sin\phi\, Z'_\lambda)\bar{e}_R\gamma^\lambda\mu_R + k_1(\sin\phi\, Z_\lambda + \cos\phi\, Z'_\lambda)\bar{e}_L\gamma^\lambda\mu_L\big], \tag{3.7}$$

where

$$k_0 = -\tfrac{1}{2}(F_R^\dagger F_R)_{e\mu} \tag{3.8}$$

is induced by the mixing with the exotic charged leptons E_R^-, while

$$k_1 = \sin\theta_w\,[q_1(E_L) - q_1(e_L)](F_L^\dagger F_L)_{e\mu} \tag{3.9}$$

results from the mixing with the new ordinary leptons E_L.

From the second term in eq. (3.7), we see that ordinary–ordinary fermion mixing can still induce a LFV vertex for the physical Z boson. However, this vertex is suppressed by the Z_0–Z_1 mixing, which is severely constrained by present data to $|\phi| \lesssim 0.02$ [13,16], and then we can expect that in the presence of a "light" Z' the FCNC processes would be mainly induced by direct Z' exchange.

4. Constraints from μ–e conversion in nuclei

The LFV parameters can now be constrained by comparing the theoretical expression for the branching ratio R for the μ–e conversion process in eq. (2.16) to the experimental bound B. Presently $B = 4\times10^{-12}$ [1,2] at 90% CL, however we will also discuss the limits on the LFV parameters achievable with the planned future experiments.

First, the limits on Z-mediated FCNC can be obtained in the limit in which the Z' is decoupled from low energy physics ($M_{Z'}\to\infty$ and $\phi\to0$). In this case, the quark vector couplings v_f ($f=$ u, d) entering eq. (2.12) are given by the standard expression $v_f = t_3(f_L) - 2q_{em}(f)\sin^2\theta_w$. We obtain

$$k_V^2 + k_A^2 < 5.2\times10^{-13}\left(\frac{B}{4\times10^{-12}}\right), \tag{4.1}$$

Independent limits on the LFV mixings of the right-handed or left-handed leptons can be given in terms of the chirality couplings $k_{L,R} = \frac{1}{2}(k_V \pm k_A)$. Then (4.1) implies $|k_L^{e\mu}|$, $|k_R^{e\mu}| < 0.51 \times 10^{-6}$. These limits are twice as strong as the corresponding ones from the non-observation of the decay $\mu \to eee$ obtained in ref. [6]. In the case of E_6 models the LFV couplings of the charged leptons to the Z boson originate only in the R-sector ($k_R = k_0$, $k_L = 0$). From (4.1) we obtain

$$(F_R^\dagger F_R)_{e\mu} < 1.0 \times 10^{-6} \left(\frac{B}{4 \times 10^{-12}} \right)^{1/2},$$

that is tighter than the limit $(F_R^\dagger F_R)_{e\mu} < 2.4 \times 10^{-6}$ from $\mu \to eee$ [6].

As we see, the limits from μ–e conversion in nuclei on the LFV ordinary–exotic mixing of the first two families are indeed quite strong. We stress that due to the coherent enhancement of the rate, this process gives the strongest constraint on the Z–e–μ vertex, twice more stringent than that from $\mu \to eee$.

However, as we have already discussed, these vertices are expected to be suppressed as the ratio of the light and heavy masses, that is by a factor of the order $m_\mu^2/M_E^2 \lesssim 10^{-6}$ for $M_E \gtrsim 100$ GeV. As a conclusion, at present these limits are still not strong enough to effectively constrain the models under examination, since the possible FCNCs induced by such naturally small ordinary–exotic mixings are still compatible with the present experimental data.

However, the planned experiments [3,4], aiming to test branching ratios down to $B \sim 4 \times 10^{-14}$–10^{-16}, do have good chances to reveal signals of violation of the lepton flavour number induced by this kind of new physics. If no signals are detected, the present limits will be improved to $|k_L^{e\mu}|$, $|k_R^{e\mu}| < 0.51 \times 10^{-7}$–$0.25 \times 10^{-8}$ corresponding to a LFV ordinary–exotic mixing $(F_R^\dagger F_R)_{e\mu} < (10\text{–}0.5) \times 10^{-8}$. This bound will indeed represent a serious constraint on E_6 models, if the exotic states are assumed to be not much heavier than the electroweak scale.

Let us now consider the effect of the mixing of the left-handed charged leptons with the new ordinary states E_L^- present in E_6. In order to do this we will henceforth set the ordinary–exotic mixing term $(F_R^\dagger F_R)_{e\mu}$ to zero, and we will concentrate on the consequences of having a non-vanishing ordinary–ordinary mixing parameter $\mathscr{F}_{e\mu} \equiv (F_L^\dagger F_L)_{e\mu}$. This is a safe procedure, since in the limit in which we neglect the electron mass, there are no interference terms relating the left-handed and right-handed lepton sectors, and the experimental limit on the conversion of muons into electrons represents a fortiori a limit on the production of electrons in the left-handed helicity state.

The LFV parameters k_V and k_A entering eqs. (2.1)–(2.16), can be read from eq. (3.7),

$$k_V = k_A = k_1 \sin\phi,$$

$$k'_V = k'_A = k_1 \cos\phi,$$

while the quark couplings v_f, v'_f, $f = $ u, d, entering in eqs. (2.12)–(2.17), are given by [13]

$$v_f = \cos\phi\,\left[t_3(f_L) - 2q_{em}(f)\sin^2\theta_w\right] + \sin\phi\,\sin\theta_w\,\left[q_1(f_L) + q_1(f_R)\right],$$
$$v'_f = -\sin\phi\,\left[t_3(f_L) - 2q_{em}(f)\sin^2\theta_w\right] + \cos\phi\,\sin\theta_w\,\left[q_1(f_L) + q_1(f_R)\right], \tag{4.2}$$

where the $U_1(1)$ charge $q_1(f)$ that was defined in (3.5) is given in terms of the q_ψ and q_χ charges for the quarks,

$$q_\psi(u_L) = -q_\psi(u_R) = q_\psi(d_L) = -q_\psi(d_R) = \tfrac{1}{6}\sqrt{\tfrac{5}{2}}\,,$$
$$q_\chi(u_L) = -q_\chi(u_R) = q_\chi(d_L) = \tfrac{1}{3}q_\chi(d_R) = -\tfrac{1}{6}\sqrt{\tfrac{3}{2}}\,. \tag{4.3}$$

Due to the approximation made, for each value of the parameter β in (3.5) the branching ratio (2.16) depends only on the values of M'_Z, ϕ and $\mathscr{F}_{e\mu} \equiv (F_L^\dagger F_L)_{e\mu}$. However, it is easy to see that since the gauge boson mixing effects in the diagonal electron couplings are in any case very small ($|\phi| \lesssim 0.02$ [13–16]), the relevant variables are actually only two, namely $\mathscr{F}_{e\mu}(M_Z^2/M_{Z'}^2)$ and $\mathscr{F}_{e\mu}\phi$. Moreover, once the Higgs sector of the model is specified, $M_{Z'}$ and ϕ are no more independent quantities. For example, an approximate relation that holds for small mixings and when $M_{Z'}$ ($\gg M_Z$) originates from a large Higgs singlet VEV [19] reads

$$\phi \simeq -\frac{M_Z^2}{M_{Z'}^2}\sin\theta_w\frac{\sum_i t_3^i q_1^i |\langle\phi^i\rangle|^2}{\sum_i t_3^{i2}|\langle\phi^i\rangle|^2}\,, \tag{4.4}$$

and in this case the branching ratio (2.16) is in practice only a function of $\mathscr{F}_{e\mu}(M_Z^2/M_{Z'}^2)$.

The limits on the Z′ LFV parameter $M_{Z'}\mathscr{F}_{e\mu}^{-1/2}$, obtained by comparing eq. (2.16) to the present 90% CL experimental bound $B = 4\times10^{-12}$ [1], are plotted in fig. 1. The thick solid line depicts the limits obtained by setting the gauge boson mixing angle ϕ to zero, so that the μ–e conversion is mediated only by Z′ exchange in this case. The resulting constraints are about twice as strong as the ones from $\mu \to$ eee found in ref. [6]. For most of the values of sin β, we find $M_{Z'}$ $(\mathscr{F}_{e\mu}/10^{-2})^{-1/2} \gtrsim 5$ TeV $\times (B/4\times10^{-12})^{-1/4}$. Clearly it is not possible to translate the limits on the μ–e conversion process directly into bounds on $M_{Z'}$, since the value of the mixing parameter $\mathscr{F}_{e\mu}$ is not known. However, as we have discussed, from the theoretical point of view the entries in the mixing matrix $\mathscr{F}$

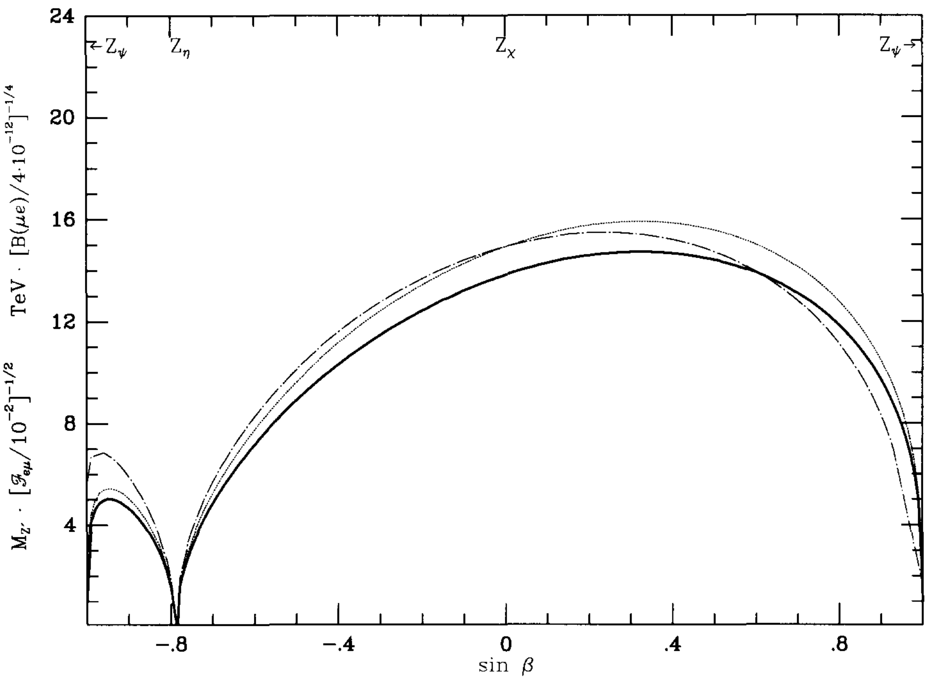

Fig. 1. Limits on the Z′ LFV parameter $M_{Z'}\mathscr{F}_{e\mu}^{-1/2}$ from the experimental limits on the μ–e conversion process, for a general E_6 neutral gauge boson, as a function of sin β. The mixing term $\mathscr{F}_{e\mu}$ is given in units of 10^{-2}. The vertical units are TeV when the current limit on the branching for μ–e conversion $B = 4\times10^{-12}$ is taken. The limits on the Z′ mass for different values of the experimental branching ratio and/or of $\mathscr{F}_{e\mu}$ can be easily read off the figure by properly rescaling the vertical units. The thick solid line is obtained by setting the Z_0–Z_1 mixing angle ϕ to zero. The bounds obtained by allowing for a non-vanishing Z_0–Z_1 mixing, consistent with the values of M_Z' when a minimal Higgs sector is assumed, are also shown. The dotted lines correspond to equal VEVs of the two Higgs doublets present in the model, i.e. $\sigma \equiv \bar{v}/v = 1$ while the dot-dashed lines correspond to $\sigma = \infty$.

are not expected to be suppressed by any particularly small factor, and they are completely unconstrained experimentally. Then it is reasonable to assume $10^{-4} \lesssim \mathscr{F}_{e\mu} \lesssim 10^{-1}$ as a natural range for the ordinary–ordinary mixing parameter. In this case, using the lower extreme $\mathscr{F}_{e\mu} = 10^{-4}$, we get a "conservative naturalness" bound $M_{Z'} \gtrsim 500$ GeV, for most of the values of sin β. These bounds are indeed quite strong, but since they are model dependent obviously they cannot replace the direct [20] or indirect [13,16] limits on the Z′ parameters, which do not depend on any assumption on the fermion mixings.

The planned experiment [4], aiming to test the branching ratios for the μ–e conversion process down to $B \sim 10^{-16}$, would allow to improve the bounds up to $M_{Z'} \gtrsim (5$–$100)$ TeV for the same range of "natural" values for $\mathscr{F}_{e\mu}$. This would be a serious constraint on E_6 models, and it is amusing to note that this kind of relatively unexpensive experiments can in principle be sensitive to the presence of a Z′ boson out of the reach of the supercolliders.

From fig. 1 it is apparent that two important exceptions are represented by the ψ and the η models, corresponding respectively to $\sin\beta = -1$ and $\sin\beta = -\sqrt{\frac{5}{8}}$, since in both these models the constraints on the Z' mass are evaded.

The absence of limits in the ψ model is due to the fact that all the standard fermions and their conjugate states belong to the same representation of the SO(10) subgroup of E_6, namely the **16**, and thus have the same q_ψ abelian charge. As a consequence the q_ψ charges of the left- and right-handed states are equal and opposite in sign, implying that the vector coupling to the Z_ψ boson is vanishing, and only the axial coupling is present. Then for this particular value of β it is not possible to obtain strong bounds from the μ–e conversion in nuclei. In particular, for $\phi = 0$ no bounds at all are obtained on the parameter $M_{Z'}\mathscr{F}_{e\mu}^{-1/2}$ due to the fact that in the present analysis we have neglected the incoherent contributions. In this case, however, a strong limit $M_{Z'}(\mathscr{F}_{e\mu}/10^{-2})^{-1/2} \gtrsim 3.7$ TeV can still be obtained from the non-observation of the decay $\mu \to$ eee [6].

The absence of limits in the η model has quite a different origin. Besides having $t_3^{\mathscr{K}} = t_3^{\mathscr{N}}$, the known and new ordinary fermions also have $q_\eta^{\mathscr{K}} = q_\eta^{\mathscr{N}}$ for the particular value $\sin\beta = -\sqrt{\frac{5}{8}}$. This implies that the coefficient of the $F_L^\dagger F_L$ term is vanishing not only in the SM J_0 current, but in the J_1 current as well. As a consequence any effect related to the ordinary–ordinary mixing is completely absent in the η model, independently of the kind of process considered. We refer to ref. [6] for a more complete discussion on this point.

To study the possible effects on these results of a non-vanishing mixing angle ϕ, i.e. when both the Z' and Z bosons contribute to the decay, we have used (4.5) assuming, consistently with the conventional E_6 models, two doublets of Higgs fields with VEVs $\bar{v}$ and v. Since $\bar{v}$ and v give mass respectively to the t and b quarks, $\sigma \equiv \bar{v}^2/v^2 > 1$ is theoretically preferred. The bounds on $M_{Z'}$ obtained by allowing for a Z_0–Z_1 mixing consistent with this minimal Higgs sector are shown in fig. 1 by the dotted and dot-dashed lines, which correspond to $\sigma = 1$ and ∞ respectively. It is apparent that by allowing for a non-vanishing value of ϕ, the limits on the Z' mass are qualitatively unchanged.

Fig. 2 depicts the constraints on the Z' LFV parameter $\phi\mathscr{F}_{e\mu}$. The solid line shows the bounds obtained by taking the limit $M_{Z'} \to \infty$. In this case the μ–e conversion process is mediated only by the Z boson, and is due to the mixing between the Z_0 and the Z_1. It is apparent that the Z_0–Z_1 mixing angle is constrained to be at most $\sim$ few $\times 10^{-4}/(\mathscr{F}_{e\mu}/10^{-2})$ almost all over the β axis. For the smallest value of the mixing in the natural range $10^{-4} \lesssim \mathscr{F}_{e\mu} \lesssim 10^{-1}$, this is comparable to the limit $|\phi| \lesssim 10^{-2}$ resulting from the fit to the available NC, charged current and LEP data [13,16]. The dotted ($\sigma = 1$) and dot-dashed lines ($\sigma = \infty$) enclose the regions of the limits obtained assuming a minimal Higgs sector. In this case the value of $M_{Z'}$ is finite and consistent, according to (4.5), with the values of ϕ at the bound. We see that with this additional condition in practice the Z and Z' bosons are constrained to be unmixed, except in a very small region in

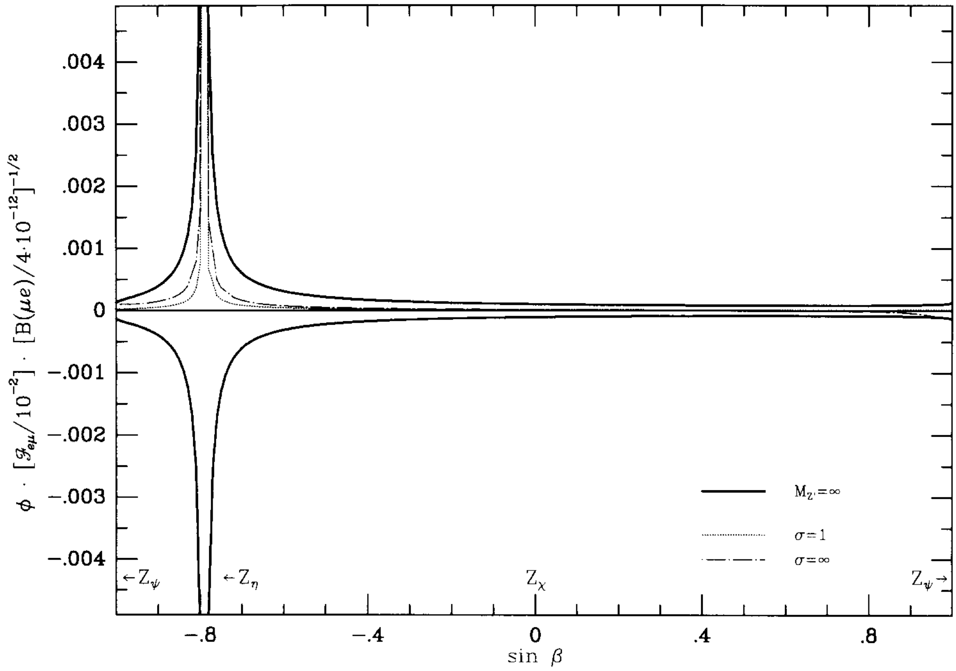

Fig. 2. Limits on the Z′ LFV parameter $\phi\mathscr{F}_{e\mu}$ from the experimental limits on the μ–e conversion process, for a general E_6 neutral gauge boson, as a function of sin β. The current limit on the branching for μ–e conversion $B = 4\times10^{-12}$ is assumed and the mixing term $\mathscr{F}_{e\mu}$ is given in units of 10^{-2}. The limits on the Z_0–Z_1 mixing angle φ for different values of the experimental branching ratio and/or of $\mathscr{F}_{e\mu}$ can be easily read off the figure by properly rescaling the vertical units. The thick solid lines are obtained in the limit $M_{Z'}\to\infty$. The dotted ($\sigma=1$) and dot-dashed ($\sigma=\infty$) lines show the limits obtained for a finite Z′ mass and assuming a minimal Higgs sector.

the vicinity of the η model. The fact that in the case in which the Higgs sector is specified the limits on φ are significantly tighter than in the case in which φ and $M_{Z'}$ are assumed independent (and the limit $M_{Z'}\to\infty$ is taken) means that the μ–e conversion in nuclei is in first place sensitive to the Z′ exchange, and thus constrains the Z′ mass, while the contribution to the LFV transition of Z_0–Z_1 mixing alone is less relevant and leads to loser constraints. It is worth noting that this behaviour is opposite to what is encountered in deriving limits on the Z′ parameters from precise electroweak data [13,16], where in fact the best bounds on the Z′ mass are obtained from the tight limits on φ implied by the LEP measurements.

5. Conclusions

We have introduced the general charged lepton–quark contact lagrangian describing LFV neutral currents, we have derived the corresponding effective lepton–nucleon interaction and we have applied it to the case of μ–e conversion in muonic atoms. The relevant nucleon vector couplings result from the coherent character of the conserved vector current. The axial couplings are determined from $SU(3)_f$ symmetry considerations and experiments, and their actual values should be used to study the incoherent contribution to the processes. However, in the case of μ–e conversion in nuclei with $A \gg 1$, the axial current contribution can be neglected with respect to the vector coherent contribution. We have determined the rate of the coherent μ–e conversion process in terms of the couplings appearing in the general lepton–quark effective lagrangian, by means of the following additional approximations:

(1) we have treated the muon as non-relativistic, which is correct up to $O(\alpha Z)$;

(2) we have taken an average for the μ wave function inside the nucleus, which is a good approximation for $A \lesssim 100$;

(3) we have used equal form factors for the proton and the neutron, which is valid for light enough nuclei.

All these approximations work up to a few percent for ${}^{48}_{22}Ti$. We have then normalized the rate for μ–e conversion in nuclei with the experimental value of the μ-capture rate, rather than with the theoretical expression which has been previously used in the literature [21].

Following ref. [6], we have discussed how extended gauge models, predicting new neutral gauge bosons Z′ as well as new charged fermions, imply flavour changing couplings between the Z and Z′ gauge bosons and the known fermions, and we have pointed out that in particular the Z′ flavour changing vertices are expected to be unsuppressed. As an example for illustrating this mechanism, we have considered the case of E_6 models.

We have then studied the constraints on LFV couplings from the limit on μ–e conversion in nuclei, obtaining the following results.

First, we have derived stringent bounds on the LFV interactions mediated by the standard Z boson, which in extensions of the SM can be induced by the mixing of the charged leptons with new exotic particles, and in particular in E_6 models could appear in the right-handed leptonic sector. The limits obtained are twice as strong as the ones from $\mu \to$ eee. We have also discussed the sensitivity that will be attained by the proposed future experiments searching for μ–e conversion in nuclei, and we have shown that signals of LFV transitions induced by ordinary–exotic lepton mixing are expected to be detected with these experiments if the exotic leptons have masses not much larger than the electroweak scale.

Second, we have considered the LFV interactions induced in E_6 models by the mixing of the known charged leptons with new ordinary states. In this case the μ–e conversion proceeds through both Z and Z′ exchange. We have derived constraints on the relevant combinations of Z′ mass and mixing angle with the Z′ LFV couplings. We have briefly discussed the reasons why the Z′ LFV couplings are theoretically expected to be large, and we have concluded that in order to account for the non-observation of μ–e conversion in nuclei, the Z′ should be sufficiently heavy (in most cases at least at the TeV scale) to suppress the transition rate, and almost unmixed with the standard Z.

We have suggested that the simultaneous presence of new charged fermions and new gauge bosons with mass up to a few TeV should give rise to LFV transitions that should be observed in future experiments looking for μ–e conversion in nuclei with improved sensitivity. On the other hand, if no effect were found, the resulting limits on these kind of FCNCs will be extremely severe, implying in most cases $M_{Z'} \gtrsim 5$ TeV unless the LFV couplings are tuned to be smaller than $\simeq 10^{-4}$.

As we have discussed in some detail, the constraints on the Z′ mass presented here do not apply to two particular E_6 models. In the ψ model the quark vector couplings to the Z′ vanish, so that there is no coherent contribution to μ–e conversion in nuclei, and then leptonic processes like $\mu \to$ eee should be used to constrain the possibly large Z′-mediated FCNCs. On the other hand, as was already stressed in ref. [6], in the superstring-inspired η model the large Z′-mediated LFV are completely absent, implying that the kind of constraints discussed here are not effective to derive limits on the Z_η parameters independently of the particular experimental process considered.

We would like to thank Professor V.M. Lobashev for stimulating discussions on this topic. One of us (D.T.) acknowledges a post-doc fellowship from the Spanish Ministry of Education and Science. This work has been partially supported by CICYT (Spain), under grant AEN/90-0040.

References

[1] D.A. Bryman et al., Phys. Rev. Lett. 55 (1985) 55;
S. Ahmad et al., Phys. Rev. Lett. 59 (1987) 970;
S. Ahmad et al., Phys. Rev. D38 (1988) 2102
[2] A. Badertscher et al., J. Phys. G17 (1991) S47;
A. Van der Schaaf, Nucl. Phys. A546 (1992) 421C
[3] SINDRUM II Collaboration, A. Badertscher et al., preprint PSI-PR-90-41 (1990)
[4] MELC Collaboration, V.S. Abadjev et al., preprint MOSCOW (1992)
[5] W.J. Marciano and A.I. Sanda, Phys. Rev. Lett. 38 (1977) 1512;
J.D. Vergados, Phys. Rep. 133 (1986) 1;
J. Bernabéu, An. Fís. 5 (1989) 36;
T.S. Kosmas and J.D. Vergados, Nucl. Phys. A510 (1990) 641
[6] E. Nardi, preprint UM-TH 92-19 (1992), Phys. Rev. D, to appear
[7] J. Ashman et al., EMC Experiment, Phys. Lett. B206 (1988) 364; Nucl. Phys. B328 (1989) 1
[8] J. Bernabéu, Z. Phys. C56 (1992) S24
[9] T.S. Kosmas and J.D. Vergados, Nucl. Phys. A510 (1990) 641;
H.C. Chiang, E. Oset, T.S. Kosmas, A. Faessler and J.D. Vergados, Nucl. Phys. A, submitted

[10] J.C. Sens, Phys. Rev. 113 (1959) 679;
K.W. Ford and J.G. Wills, Nucl. Phys. 35 (1962) 295;
R. Pla and J. Bernabéu, An. Fís. 67 (1971) 455;
H.C. Chiang, E. Oset, T.S. Kosmas, A. Faessler and J.D. Vergados, Nucl. Phys. A, submitted
[11] I. Sick and J.S. McCarthy, Nucl. Phys. A150 (1970) 631;
W. Bertozzi, J. Friar, J. Heisenberg and J.W. Negele, Phys. Lett. B41 (1972) 408;
W. Bertozzi et al., Phys. Rev. Lett. 28 (1972) 1711;
B. Dreher et al., Nucl. Phys. A235 (1974) 219;
W. Donnely and J.D. Walecka, Annu. Rev. Nucl. Sci. 25 (1975) 329;
B. Frois and C.N. Papanicolas, Annu. Rev. Nucl. Sci. 37 (1987) 133
[12] T. Suzuki, D.F. Measday and J.P. Roalsvig, Phys. Rev. C35 (1987) 2212
[13] E. Nardi, E. Roulet and D. Tommasini, Phys. Rev. D46 (1992) 3040
[14] P. Langacker and D. London, Phys. Rev. D38 (1988) 886;
E. Nardi, E. Roulet and D. Tommasini, Nucl. Phys. B386 (1992) 239; *in* Electroweak physics beyond the Standard Model, eds. J.W.F. Valle and J. Velasco (World Scientific, Singapore, 1992)
[15] P. Langacker and D. London, Phys. Rev. D38 (1988) 886
[16] J. Layssac, F.M. Renard and C. Verzegnassi, Z. Phys. C53 (1992) 97;
M.C. Gonzalez García and J.W.F. Valle; Phys. Lett. B259 (1991) 365;
G. Altarelli et al., Phys. Lett. B263 (1991) 459;
F. del Aguila, J.M. Moreno and M. Quirós, Nucl. Phys. B361 (1991) 45;
F. del Aguila, W. Hollik, J.M. Moreno and M. Quirós, Nucl. Phys. B372 (1992) 3;
P. Langacker and M. Luo, Phys. Rev. D45 (1992) 278
[17] J.L. Hewett and T.G. Rizzo, Phys. Rep. 183 (1989) 195, and references therein
[18] S. Nandi and U. Sarkar, Phys. Rev. Lett. 56 (1986) 564;
M. Cvetič and P. Langacker, Phys. Rev. D46 (1992) R2759
[19] P. Langacker and M. Luo, Phys. Rev. D45 (1992) 278
[20] CDF Collaboration, F. Abe et al., Phys. Rev. Lett. 67 (1991) 2609; 68 (1992) 1463
[21] See e.g. J.D. Vergados, Phys. Rep. 133 (1986) 1