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A Simulation Model of a Decentralized Metropolitan Area with Two-Worker, "Traditional," and Female-Headed Households*

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An urban simulation model is developed in which there are multiple household types: two-worker households, traditional households, and female-headed households, and multiple employment locations: a CBD and two subcenters. Female workers are assumed to have lower wages. Labor force participation and choice of job and residential locations are endogenous. The model shows that a fairly realistic city can be represented, particularly when a random component to wages is introduced. The distribution of household types within the city is shown to be quite sensitive to relative wages of male versus female workers, to commuting costs, and to the level of CBD versus suburban wages—for example, when CBD wages fall, the proportion of female-headed households declines sharply.

1. INTRODUCTION

While urban models have traditionally assumed that each urban household has one—presumably male—worker and that all urban jobs are at the CBD, in reality the majority of urban households have either multiple workers or a single female worker and most urban jobs are located outside the CBD. Married women's labor force participation has increased dramatically in the United States, thus raising the proportion of urban

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households that have two workers.[1] Divorce rates have also risen rapidly, as has the number of children born to unwed mothers.[2] Both trends have led to an increase in the number of urban households that are female headed. In addition, women workers are paid less than men workers and commute less than men workers because of their household responsibilities.[3] These changes have many unexplored urban spatial implications. In particular, rising numbers of two-workers households increase the dispersion of income in cities, with two-worker households having the highest incomes and female-headed households the lowest. Their low incomes may make it difficult for female-headed households to find affordable housing anywhere in the city. Also, increasing numbers of female workers may cause increased employment suburbanization, since women workers favor jobs close to home. Or, alternately, if convenient jobs are not available, women may choose to work only part time or not at all.

In this paper we develop an urban simulation model in which these issues can be addressed. The model is intended to capture the characteristics of equilibrium in a Mills–Muth urban model,[4] but to extend the theoretical model by allowing for multiple household types and decentralized employment. In particular, we assume that male workers earn high wages and female workers low wages and that households can have a single male worker, a single female worker, or two workers—one male and one female. These three household types are referred to as "traditional" households, female-headed households, and two-worker households. There are also three job locations: the CBD and two suburban subcenters.[5] The model determines whether potential workers choose to work and how much, which types of households live in the metropolitan area, where each type of household lives within the metropolitan area, the distribution of jobs across employment locations, and the pattern of commuting behavior. The model also breaks new ground by investigating how random variation in wage levels affects location behavior.[6]

[1]Between 1960 and 1989, women's labor force participation rose from 32 to 57% in the U.S. *Statistical Abstract of the United States 1991*, 111th ed., Table 641.

[2]The divorce rate per thousand population rose from 2.2 in 1960 to 4.8 in 1987. The proportion of children born to unwed mothers rose from 0.11 in 1970 to 0.25 in 1988. See *Statistical Abstract of the United States 1991*, 111th ed., Tables 128 and 92, respectively.

[3]Juster and Stafford [3], Table 3, indicates that women workers in the United States in 1981 spent only 57% as much time commuting as men workers did. See Madden [5] for discussion of various explanations and a model.

[4]See Mills [6] and Muth [7].

[5]See Wieand [9], White [11], and Yinger [14] for recent research on metropolitan areas with decentralized employment.

[6]See White [10] for an earlier version of a simulation model which explores the effects of congestion and growth in a decentralized city.

2. THE MODEL

Suppose that metropolitan area is placed on a graph, with the CBD at the origin (0, 0). Draw two numbers, each from the same uniform distribution whose range is from $-k$ to $+k$. The resulting pair, denoted (x_i, y_i), becomes the coordinates of a potential residential location i on the map of the metropolitan area. Residential locations are evaluated according to a grid search procedure, described below.

In the model, the set of job locations and the wages at each job location are exogenously determined. The three job locations are the CBD, the north subcenter at (6, 2), and the south subcenter at (−5, 0).[7] At each job location, there may be high wage and/or low wage jobs. As a shorthand representation of sex differences in wage patterns, we assume that high wage jobs are held by male workers and low wage jobs by female workers. Hourly wages at the CBD are assumed to be w_m^C and w_f^C for male and female workers, respectively, where $w_m^C > w_f^C$. Hourly wages at the north subcenter for male and female workers are w_m^N and w_f^N, respectively, where $w_m^N > w_f^N$. Wages are assumed to be lower at the north subcenter than at the CBD for both types of job, so that $w_m^C > w_m^N$ and $w_f^C > w_f^N$. Both male and female workers' wages are assumed to be the same at the south subcenter as at the north subcenter, so that $w_m^N = w_m^S$ and $w_f^N = w_f^S$. The spatial pattern of residential locations and the distribution of jobs of each type across employment locations are determined endogenously by the model.

Turn now to the utility function of households living in the metropolitan area. For two worker households, household utility is defined over land consumption L, other goods consumption (including consumption of housing capital) g, husband's leisure l_m, and wife's leisure l_f. All of these depend on residential location i. Suppose that the household utility function is Cobb–Douglas. Then the utility level of a two-worker household living at residential location i is

$$U_i = L(i)^{\alpha} g(i)^{\beta} l_f(i)^{\gamma} l_m(i)^{\delta}, \tag{1}$$

where $\alpha + \beta + \gamma + \delta = 1$. In contrast to the normal monocentric urban model, here consumption of land, leisure, and other goods all may vary

[7]The two subcenters are assumed to be widely separated, based on White's [11] model in which there is an incentive for employers moving to the suburbs to locate in different directions away from the CBD, since doing so enables them to hire more workers at lower wages. On the other hand, there are likely to be agglomeration economies for some types of suburban jobs which cause them to cluster in subcenters. See Ogawa and Fujita [8] for a general model in which agglomeration economies and congestion trade off to determine the amount of employment suburbanization in a metropolitan area.

depending on both radial distance from the CBD and direction away from the CBD.

Commuting speed and out-of-pocket commuting costs are assumed to be constant per mile all over the metropolitan area, i.e., there is no congestion. Commuting speed is s miles per hour and out-of-pocket commuting costs per mile are c. Commuting distance is denoted d. Commuting distance to the CBD from residential location i is $d^{\mathrm{C}}(i) = \sqrt{x_i^2 + y_i^2}$. Commuting distances to the north and south subcenters from residential location i are $d^{\mathrm{N}}(i) = \sqrt{(x_i - 2)^2 + (y_i - 6)^2}$ and $d^{\mathrm{S}}(i) = \sqrt{x_i^2 + (y_i + 5)^2}$, respectively. Thus if the husband in a two-worker household works at the CBD and the wife works at the north subcenter, the household's commuting costs are $2c(d^{\mathrm{C}}(i) + d^{\mathrm{N}}(i))$ in out-of-pocket expenditures, $2d^{\mathrm{C}}(i)/s$ in time spent by the husband, and $2d^{\mathrm{N}}(i)/s$ in time spent by the wife.

Each worker has a time budget constraint. Work hours of male workers are denoted h_{m} and work hours of female workers are h_{f}. Work hours plus commuting hours plus leisure time are assumed to sum to 16, so that the time budget for a male worker who works at the CBD is

$$16 = h_{\mathrm{m}}^{\mathrm{C}}(i) + l_{\mathrm{m}}^{\mathrm{C}}(i) + 2d^{\mathrm{C}}(i)/s \tag{2}$$

and the time budget for a female worker who works at the north subcenter is

$$16 = h_{\mathrm{f}}^{\mathrm{N}}(i) + l_{\mathrm{f}}^{\mathrm{N}}(i) + 2d^{\mathrm{N}}(i)/s. \tag{3}$$

Each household also has a money budget constraint. Willingness-to-pay rent for land per unit at location i is denoted $r(i)$ and it varies depending on what type of household occupies site i. For example, willingness-to-pay rent for site i is $r^{\mathrm{C/N}}(i)$ when the site is occupied by a two-worker household whose male worker works at the CBD and whose female worker works at the north subcenter. Since the price of other goods per unit is assumed to be one, the same household's money budget constraint becomes

$$h_{\mathrm{m}}^{\mathrm{C}}(i)w_{\mathrm{m}}^{\mathrm{C}} + h_{\mathrm{f}}^{\mathrm{N}}(i)w_{\mathrm{f}}^{\mathrm{N}}(i) - 2c(d^{\mathrm{C}}(i) + d^{\mathrm{N}}(i)) = g(i) + L(i)r^{\mathrm{C/N}}(i). \tag{4}$$

The time and money budget constraints of households whose workers work at other job locations are similarly defined.

Households are assumed to maximize their utility function (1) subject to their time and money budget constraints (2)–(4). The indirect utility function for a household whose male worker works at the CBD and whose female worker works at the north subcenter is

$$V = \frac{\alpha^{\alpha}\beta^{\beta}\gamma^{\gamma}\delta^{\delta}\left[w_{\mathrm{m}}^{\mathrm{C}}(16 - 2d^{\mathrm{C}}(i)/s) + w_{\mathrm{f}}^{\mathrm{N}}(16 - 2d^{\mathrm{N}}(i)/s) - 2c(d^{\mathrm{C}}(i) + d^{\mathrm{N}}(i))\right]}{(r^{\mathrm{C/N}}(i))^{\alpha}(w_{\mathrm{f}}^{\mathrm{N}})^{\gamma}(w_{\mathrm{m}}^{\mathrm{C}})^{\delta}}. \tag{5}$$

The term in square brackets in the numerator is the household's "full income," or the amount it could earn if both workers worked 16 hours per day minus commuting time, i.e., neither consumed any leisure. The indirect utility functions for two-worker households having other job locations can be derived by the same procedure. Since there are three possible job locations for both husbands and wives, there are nine possible types of two-worker households that might occupy residential location i.

Alternately, households may have only one worker, who may be either male or female. The utility function of "traditional" households having one male worker is

$$U = L(i)^{\alpha'}g(i)^{\beta'}l_{\mathrm{m}}(i)^{\delta'}, \tag{6}$$

where $\alpha' + \beta' + \delta' = 1$. The utility function of female-headed households having one female worker is

$$U = L(i)^{\alpha''}g(i)^{\beta''}l_{\mathrm{f}}(i)^{\gamma''}, \tag{7}$$

where $\alpha'' + \beta'' + \gamma'' = 1$. The (single) time budget constraint for a traditional household is Eq. (2) and for a female-headed household is Eq. (3). The money budget constraint for a "traditional" household is the same as (4), except that the terms involving the wife's earnings and the wife's out-of-pocket commuting costs drop out. The money budget constraint for a female-headed household is similarly defined. Thus, for example, the indirect utility function for a traditional household in which the husband works at the CBD becomes

$$V = \frac{(\alpha')^{\alpha'}(\beta')^{\beta'}(\delta')^{\delta'}\left[w_{\mathrm{m}}^{\mathrm{C}}(16 - 2d^{\mathrm{C}}(i)/s) - 2c(d^{\mathrm{C}}(i))\right]}{(r^{\mathrm{C/-}}(i))^{\alpha'}(w_{\mathrm{m}}^{\mathrm{C}})^{\delta'}}, \tag{8}$$

where $r^{\mathrm{C/-}}(i)$ represents willingness-to-pay rent for site i by a traditional household whose male worker works at the CBD. The indirect utility function for a female-headed household in which the female head works at the north subcenter becomes

$$V=\frac{(\alpha'')^{\alpha''}(\beta'')^{\beta''}(\gamma'')^{\gamma''}\left[w_f^{N}(16-2d^{N}(i)/s)-2c(a^{N}(i))\right]}{\left(r^{-/N}(i)\right)^{\alpha''}\left(w_f^{C}\right)^{\gamma''}},\tag{9}$$

where $r^{-/N}(i)$ in (9) denotes willingness-to-pay rent for site i by a female-headed household whose worker works at the north subcenter.

Since both male and female workers may work at any of the 3 job locations, there are 3 types of traditional households and 3 types of female-headed households. This means that there are 15 types of households in total in the model.

Turn now to the determination of which type of household occupies each residential location i. The urban model is assumed to be an "open city," meaning that household utility levels are determined exogenously by the level of utility that households would achieve by locating in an alternative city. The utility level of two-worker households is denoted $\bar{V}_2$ and the utility levels of traditional and female-headed households are denoted $\bar{V}_m$ and $\bar{V}_f$, respectively. Since utility is assumed to be increasing in household income, $\bar{V}_2 > \bar{V}_m > \bar{V}_f$.

Suppose that the exogenous utility level for a particular type of household is substituted into its indirect utility function. Then the willingness-to-pay rent term is the only remaining unknown. For example, the indirect utility function (5) for the two-worker household located at i whose male worker works at the CBD and whose female worker works at the north subcenter can be solved to determine $r^{C/N}(i)$. In this case, $r^{C/N}(i)$ is the maximum amount that the household can pay in rent for location i consistent with achieving utility level $\bar{V}_2$. Similarly, we can solve for the maximum willingness-to-pay rent for site i by each of the other eight types of two-worker households and by each of the six types of one-worker households.

Each residential location i is assumed to be offered for rent by a landlord who implicitly conducts an auction. At each site i, each of the 15 types of households bids its maximum willingness-to-pay rent and, in addition, there is a fixed bid of r_A for each site which represents its value when used for non-urban purposes. The landlord of each site i thus has 16 bids. The highest bid for each site is selected. If the highest bid is r_A, then the site is assumed to be outside the metropolitan area and is used for agricultural purposes. If the highest bid is by some type of household, then site i is in the metropolitan area and the landlord rents it to the household type whose bid is highest. The maximum rent offered for the site becomes its market rent, denoted $r_M(i)$. The auction thus determines what type of household will occupy each site and where its worker(s) will work.

Once the rent level at site i has been determined, the residential density level at site i can also be determined. From the relevant utility function and budget constraints, we can solve for a land demand function for each type of household. For example, suppose a two-worker household whose male worker works at the CBD and whose female worker works at the north subcenter wins the auction for site i. Then land demand at i becomes

$$L(i) = \frac{\alpha\left[w_{\mathrm{m}}^{\mathrm{C}}(16 - 2d^{\mathrm{C}}(i)/s) + w_{\mathrm{f}}^{\mathrm{N}}(16 - 2d^{\mathrm{N}}(i)/s) - 2c\left(d^{\mathrm{C}}(i) + d^{\mathrm{N}}(i)\right)\right]}{r_{\mathrm{M}}(i)}. \tag{10}$$

The inverse of land demand at i, $1/L(i)$, is the number of households per unit of land at i, or residential density $N(i)$.

Turn now to the grid search procedure used to evaluate residential locations. The area evaluated is a square of dimension $2k$ on each side, centered on the CBD. k is assumed to equal 15, corresponding to a maximum urban distance from the CBD to the urban–rural boundary of 15 miles in the four compass directions. We divide the large square into smaller squares of size 0.2 by 0.2. In total, there are $(15*2*5)^2 = 22{,}500$ smaller squares, each of which must be evaluated for each run of the model. We evaluated positions by starting at the upper left corner at $(k, -k)$, working across the top row to the upper right, and then repeating the procedure for each of the lower rows. However, the positions can be evaluated in any order.

The grid search procedure implies that the distribution of locations i is uniform over the city. Therefore in order to compute average statistics describing the metropolitan area, such as number of households of each type in the metropolitan area or average commuting distance, we need to weight each location i by the density level $N(i)$ of households living there.

3. PARAMETER VALUES

Turn now to the parameter values used in the benchmark simulation. Our general approach was to choose realistic parameter values based on existing empirical research whenever possible. The only parameter values which were adjusted to obtain particular results were the parameter values of the household utility function: the α's through δ's and the $\bar{V}$'s for each household type. These were adjusted to result in male and female workers choosing a realistic number of hours of work and the urban area containing a fairly realistic distribution of households by type in the benchmark case. We allowed the model itself to determine the spatial distribution of household types, i.e., we did not adjust any parameter values to change the patterns of residential or employment location. This approach—which does not force the model to reflect existing urban spatial patterns—allows us to learn from the results what important phenomena have been satisfactorily explained by the model versus what extensions are needed to make the model more realistic.

In the Cobb–Douglas utility function, the utility parameters represent how expenditure of "full income" is divided among land, other goods, and leisure. Since the value of time spent at the margin in any activity must equal the wage rate, households would "spend" about half their full income on leisure if their workers devoted 8 hours per day to work and 8 hours per day to leisure, neglecting time spent commuting. This suggests that total expenditure on leisure, which is $\gamma + \delta$ for two-worker households, δ' for traditional households, and γ'' for female-headed households, must be fairly close in value to 0.5. We therefore set both δ' and γ'' equal to 0.5. However, since married women workers tend to work less than male workers, we set $\gamma = 0.30$ and $\delta = 0.25$ for two-worker households. The heavier weighting of female workers' leisure in the two-worker household utility function has the effect of making wives in two-worker households work less than their husbands.[8] Expenditure on housing versus other goods is assumed to be divided in the proportion 1 : 4. Therefore for two-worker households, $\alpha = 0.09$ and $\beta = 0.36$, and for traditional and female-headed households, $\alpha' = \alpha'' = 0.10$ and $\beta' = \beta'' = 0.40$.

In the benchmark case, wages at the CBD for males are assumed to be $w_m^C = \$15$ per hour and for females $w_f^c = \$12.50$. For full time workers working 2000 hours per year, these figures correspond to yearly earnings of \$30,000 and \$25,000, respectively. However, actual salary differentials for full time male versus female workers are greater than these figures imply. We therefore also run a simulation in which wages for male workers remain the same, but wages for female workers drop by \$1 per hour to \$11.50 at the CBD and \$10.50 at the subcenters.[9] Wages at both subcenters are assumed to be \$1 per hour lower for both types of workers, or \$14 for males and \$11.50 for females.[10]

[8] Average weekly hours of work for U.S. workers are 44 for males and 24 for females, according to Juster and Stafford [3], Table 3.

[9] Full-time weekly wages for male and female workers over 25 years old were \$500 and \$350 per week in 1989, according to *Statistical Abstract of the United States 1991*, 111th ed., Table 678, p. 415. Dividing these figures by 40, we get hourly wages of \$14 for male workers and \$9 for female workers. However, when we tried using a wage differential as high as \$5 in the simulations, we found that no women chose to work! See the discussion of results below.

[10] The \$1 per hour wage differential for CBD versus suburban workers is based on wage gradients estimated by Ihlanfeldt [2]. His results imply wage differentials between jobs located at the CBD versus 5 miles from the CBD of \$1.15 per hour for professional/managerial workers, \$0.87 for sales workers, and \$0.71 for administrative support workers.

Out-of-pocket commuting costs c are assumed to be \$0.40 per mile in each direction, based on the actual combined costs of owning and operating a car.[11] Commuting speed s is assumed to be 20 miles per hour everywhere in the metropolitan area. Therefore for male workers having CBD jobs, the round trip cost of a five-mile commute is \$4 in out-of-pocket expenses plus \$7.50 in time costs, or a total of \$11.50. Urban economic models imply that there is a relationship between wage differentials for identical jobs at the CBD versus the subcenters and the savings in workers' commuting costs when they work at a subcenter versus at the CBD. In particular, workers living on the ray connecting the CBD and a subcenter, but further out than the subcenter, will choose to work at the subcenter if the wage differential is less than the extra cost of commuting from the subcenter to the CBD and will choose to work at the CBD if the wage differential exceeds the extra cost of commuting. Since the wage differential of \$1 per hour between jobs at the CBD versus the subcenters implies a wage differential of \$8 per day for an 8-hour day, the wage differential is smaller than the cost of commuting across the subcenter to the CBD. As a result, workers whose residential locations are more suburbanized than the subcenters will prefer suburban jobs.[12]

The agricultural land rent r_A is \$75 per acre per year, corresponding to a value of \$3000 per acre if the real interest rate is 0.025.

The benchmark levels of utility are assumed to be $\bar{V}_2 = 17.586$ for two-worker households, $\bar{V}_m = 14.103$ for traditional households, and $\bar{V}_f = 12.8$ for female-headed households, or about 1.4 : 1.1 : 1. These figures were calibrated to result in a realistic overall distribution of household types in the metropolitan area in the benchmark case. The target distribution of household types, based on current data, is one-half two-worker households, one-third traditional households, and one-sixth female-headed households.[13] Note that the values of $\bar{V}$ are not proportional to the full incomes of the various types of households, which are in the proportions 2.2 : 1.2 : 1 if commuting costs are ignored. This is because, while utility in the Cobb–Douglas utility function is proportional to full income holding the taste parameters and price terms constant, the taste parameters in fact differ for single-worker versus two-worker households. If we evaluate the

[11] As of 1987, the fixed cost of operating a car was \$0.29 per mile and the variable cost was \$0.09. See White [10] for data and discussion.

[12] This ignores circumferential commuting. Note that it will not necessarily hold in the random wage simulations.

[13] The distribution of household types is based loosely on data from the *Statistical Abstract of the United States 1991*, 111th ed., Tables 644 and 733, which indicate that as of 1989 there were 30 million married couple households in which the wife works, 22 million married couple households in which the wife does not work, and 11 million female-headed households. Other types of households, such as households in which no one works, are ignored.

taste parameter term $\alpha^\alpha \beta^\beta \gamma^\gamma \delta^\delta$ for two-worker households using the assumed values, we get 0.275. However, if we evaluate the taste parameter term $(\alpha')^{\alpha'}(\beta')^{\beta'}(\delta')^{\delta'}$ for traditional households using the relevant assumed values, we get 0.389. (The result is the same for female-headed households). Thus the utility constants are in effect adjusting for tastes not being held constant across households with different numbers of workers.

4. SIMULATION RESULTS—NON-RANDOM WAGES

In this section, we discuss the results of simulating the model. The first simulation is the benchmark case. In the second, out-of-pocket commuting costs are raised from $0.40 to $0.55 per mile. In the third, CBD wages are lowered by $0.20 per hour but suburban wages remain the same, and in the fourth, female workers' wages are lowered by one dollar per hour at all job locations. In each simulation, all parameter values except the one specified remain equal to the values in the benchmark case. The results of simulating the model with random variation in wage levels are discussed in the next section.

The Benchmark Case

The benchmark simulation results are given in Fig. 1 and Table 1. In all tables and figures, the designation CBD/CBD refers to two-worker households and refers to the location of the male worker's job and of the female worker's job, respectively; the designation N/− refers to traditional households whose male workers work at the north subcenter and who have no female workers; −/CBD refers to female-headed households who have no male workers and whose female workers work at the CBD, etc. Table 1 gives average values by household group for all groups that exist in the metropolitan area. Numbers in parentheses in the table indicate standard deviations. Figure 1 shows that the metropolitan area bears a striking resemblance to a single cell in the process of dividing. Two-worker households having both jobs at the CBD occupy the center of the city. This group is large, consisting of 28% of all households in the metropolitan area. They are surrounded on the east and west sides by a ring of female-headed households with jobs at the CBD, who are in turn surrounded by a ring of traditional households whose workers have jobs at the CBD. To the north and south of the CBD, the regions around the two subcenters are occupied by two-worker households in which both workers have jobs at the nearby subcenter. Between the N/N and the CBD/CBD commuting regions, there is a narrow band occupied by two-worker households in which the husband works at the CBD and the wife at the north subcenter, while beyond the N/N commuting region, there is a band of traditional households in which the husband works at the north subcenter. The pattern around the south subcenter is analogous. In general,

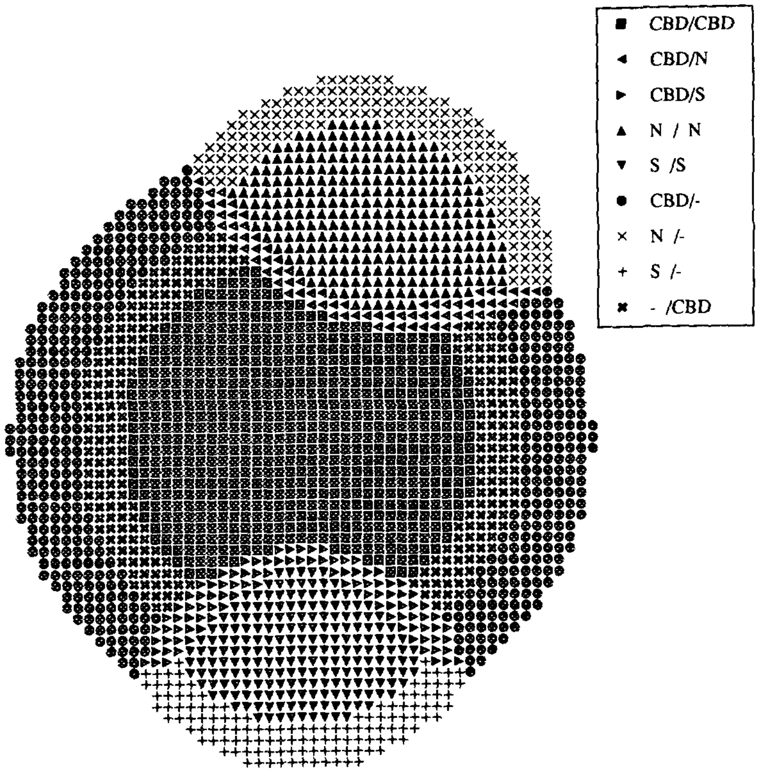

FIG. 1. The benchmark case.

two-worker households outbid single-worker households for the most accessible locations around all three employment centers, leaving single-worker households to occupy more distant locations involving longer commutes. This is due to two-worker households' higher incomes.

Several possible household types do not exist in the city. There are no two-worker households in which the husband works at either of the subcenters and the wife at the CBD (N/CBD or S/CBD households). Examining Table 1, we see that in the average CBD/N households, husbands commute 6.7 miles and wives 3.4 miles. Thus if the two spouses reversed their jobs, wives would commute more and husbands less. The CBD/N pattern dominates the N/CBD pattern because the parameters of the two-worker household utility function place a higher weight on wives' leisure than on husbands' leisure. The same explanation applies to the S/CBD job pattern. The N/S and S/N job combinations do not appear because they involve both low suburban wages and long commutes.

TABLE 1
Results for the Benchmark Case

	Proportion	Commute distance Males	Commute distance Females	Average density	Work hours Males	Work hours Females
CBD/CBD	0.28	3.8 (1.5)	3.8 (1.5)	4.1 (0.6)	8.5 (0.04)	5.5 (0.01)
CBD/N	0.025	6.7 (1.4)	3.4 (1.4)	3.1 (0.5)	8.6 (0.04)	5.2 (0.04)
CBD/S	0.033	6.6 (1.6)	3.3 (1.5)	3.2 (0.5)	8.6 (0.05)	5.2 (0.05)
N/N	0.099	3.0 (1.4)	3.0 (1.4)	3.2 (0.5)	8.6 (0.04)	5.4 (0.02)
S/S	0.070	3.1 (1.4)	3.1 (1.4)	3.2 (0.5)	8.6 (0.04)	5.4 (0.02)
CBD/ –	0.21	9.2 (0.7)	—	3.9 (0.3)	7.8 (0.02)	—
N/ –	0.070	6.1 (0.5)	—	3.9 (0.2)	7.9 (0.01)	—
S/ –	0.053	6.1 (0.5)	—	3.9 (0.2)	7.9 (0.01)	—
– /CBD	0.16	—	7.2 (0.5)	5.7 (0.3)	—	7.9 (0.02)

Note. Proportion of households with some CBD job = 0.71; proportion of jobs in CBD = 0.66; average commute distance = 4.88 miles.

There are also no female-headed households having suburban jobs (– /N and – /S households). This is because the low incomes of female-headed households make them prefer the relatively inaccessible but cheap land in the east and west directions and they are willing to make long commutes to get higher CBD wages.

As indicated, parameter values were set so as to result in a relatively realistic pattern of work hours. Married women workers work between 5.2 and 5.5 hours per day and male workers work between 7.8 and 8.6 hours per day, close to the actual levels. Note that variation in hours of work is quite low; for example, 95% of all male workers in CBD/ – households work between 7.76 and 7.84 hours per day. Married women workers commute shorter distances and therefore spend less time commuting than male workers, which is realistic. If the utility function parameters for husbands' and wives' leisure were made more equal, then their work hours and commuting hours would tend to move closer together. Since the pattern shown is realistic, it suggests that two-worker households' location behavior supports a model in which they pick a residential location taking only the husband's job into account and then find a job near the home for the wife, rather than a model in which they pick a residential location halfway between their two jobs.[14] Parameter values were also set to result in a realistic distribution of household types across the metropolitan area —about half of the households in the metropolitan area have two workers, one-third are "traditional," and one-sixth are female-headed.

[14]See White [12] for an exploration of alternate models of two-worker location choice within a metropolitan area.

But several aspects of the simulation results are quite unrealistic. One is that a high fraction of jobs are located in the CBD, 66%, and another is that high-income households occupy the center of the city. Density patterns are also different than those in most urban models, since the region of highest average density is not around any of the employment centers. Instead, female-headed households, who live fairly far from the CBD, have the highest average density levels. (However, density levels nonetheless fall monotonically with distance from the CBD or the subcenters within any particular commuting region.) Low incomes tend to make them prefer high-density housing, while low commuting costs make them prefer relatively distant, inaccessible locations. Thus the metropolitan area resembles a European rather than an American city, since high-income households prefer to locate near the center and lower income households occupy the suburbs. These results are not unrealistic, but they are uncharacteristic of U.S. cities, where high-income households generally prefer to locate in the suburbs.

In an attempt to explore why high-income households occupy the center rather than the suburbs, we reran the model with a higher demand for housing to see if this might cause the location pattern to reverse. In particular, we raised the $\alpha/(\alpha + \beta)$ ratio from 0.2 to 0.3 for all types of households. But this change had little effect on the results and the location pattern remained the same. This suggests that high-income households in the United States may prefer suburban locations because of the importance of disamenities associated with living near the CBD, such as higher crime rates, congestion, and crowded, dirty streets. These are not built into the model and suggest an avenue for future research.[15] Another implication of the model is that if disamenities at the CBD cause households to prefer residential locations in the suburbs, then they will also cause jobs to move from the CBD to the suburbs.

Increased Commuting Costs

Figure 2 and Table 2 give the results of a simulation in which out-of-pocket commuting costs rise from \$0.40 to \$0.55 per mile. Since only about one-quarter of the out-of-pocket cost of operating a car is attributable to the cost of gasoline (the rest consists of insurance, repair, and depreciation costs), this change can be interpreted either as the effect of imposing a hefty excise tax on gasoline (equivalent to a tax of (\$0.15)(20) = \$3.00 per gallon if cars get 20 miles per gallon of gas) or as the effect of substantial increases in the level of congestion in the city. Note that since the benchmark utility levels are assumed to remain constant, the implicit assumption is that commuting costs in other cities remain unchanged.

[15]See LeRoy and Sonstelie [4] for discussion of the conditions under which low versus high-income households locate closer to the CBD.

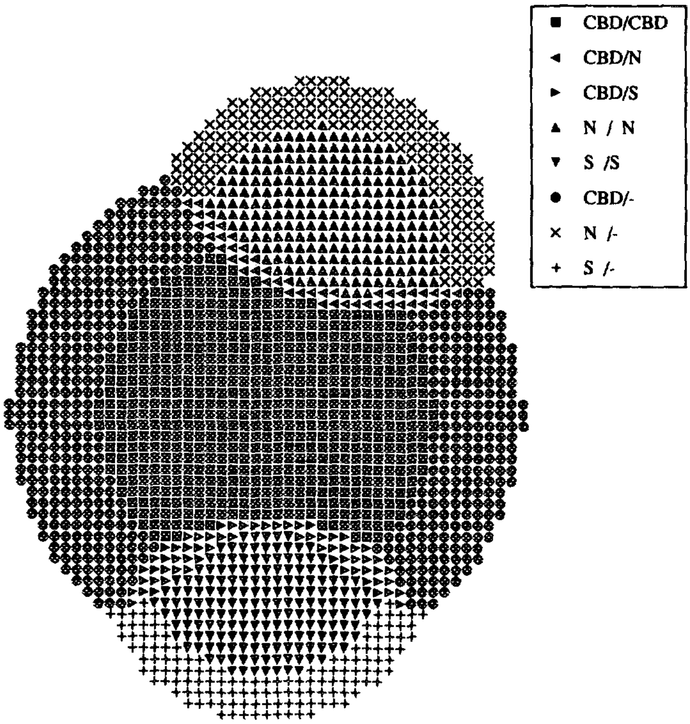

FIG. 2. City with higher commuting costs.

TABLE 2
Results with Higher Commuting Cost

	Proportion	Commute distance Males	Commute distance Females	Average density	Work hours Males	Work hours Females
CBD/CBD	0.31	3.5 (1.5)	3.5 (1.5)	3.9 (0.7)	8.6 (0.03)	5.5 (0.05)
CBD/N	0.020	5.9 (1.1)	3.0 (1.0)	3.1 (0.4)	8.7 (0.03)	5.2 (0.06)
CBD/S	0.027	5.7 (1.2)	2.8 (1.2)	3.2 (0.5)	8.7 (0.03)	5.2 (0.07)
N/N	0.10	2.6 (1.1)	2.6 (1.1)	3.3 (0.5)	8.7 (0.02)	5.4 (0.05)
S/S	0.078	2.5 (1.2)	2.5 (1.2)	3.3 (0.5)	8.7 (0.02)	5.4 (0.05)
CBD/–	0.30	7.6 (0.91)	—	4.1 (0.4)	7.9 (0.01)	—
N/–	0.090	5.3 (0.49)	—	3.9 (0.2)	7.9 (0.01)	—
S/–	0.071	5.3 (0.49)	—	3.9 (0.2)	7.9 (0.01)	—

Note. Proportion of households with some CBD job = 0.66. Proportion of jobs in CBD = 0.63. Average commute distance = 4.07 miles. Total population relative to benchmark case = 0.59.

Comparing Figs. 1 and 2, we see that the metropolitan area gets smaller in size when commuting costs rise and the population, measured in number of households, falls by 41%. More surprisingly, the group of female-headed households disappears completely. Female-headed households are squeezed out by traditional households with CBD jobs, who react to higher commuting costs by moving closer to their jobs and outbidding female-headed households for land. What happens to these households? One interpretation is that they could choose to locate in the city if they accepted a lower utility level, but that they choose instead to relocate in other cities where commuting costs have not risen and they can maintain their benchmark utility level. Another interpretation is that increased competition for housing causes female-headed households to become homeless.[16]

Other household groups increase in size as a result of the disappearance of female-headed households. The group of traditional households having CBD jobs increases from 21 to 30% of all households in the metropolitan area and the group of CBD/CBD households increases from 28 to 31% of all households. These groups expand to fill the commuting regions previously occupied by female-headed CBD households. Despite the increase in commuting costs, the proportion of households which have two earners rises slightly, from 50.7% in the base case to 53.5%. Thus households do not economize on commuting costs by having fewer workers.

Other changes are less surprising. Comparing Tables 1 and 2, we find that commuting distances fall for all types of workers, reflecting the fact that they cluster more closely around their workplaces. Also, jobs move from the CBD to the subcenters. The proportion of all jobs located at the CBD falls from 0.66 to 0.63 and the proportion of households having at least one CBD worker falls from 0.71 to 0.66. This reflects the fact that some workers can reduce commuting by taking suburban rather than CBD jobs. In general, higher commuting costs reduce the attractiveness of CBD jobs relative to suburban jobs and also reduce the size and population of the city. They also drive lower income households out of the city completely. However, if commuting costs rose in all cities by about the same amount, then the changes that occur in the particular city would be much less pronounced.[17]

[16]A third interpretation is that commuting costs rise in all cities and female-headed households therefore must accept lower utility levels to live in any city. However, this is ruled out by the open city assumption, which takes utility levels as fixed.

[17]Note that higher commuting costs may make it worthwhile to add additional employment subcenters further from the CBD—a possibility not considered in the model. This would tend to offset the reduction in size and population of the city. See White [10] for an exploration of this effect in a model that allows the number of subcenters to vary.

Reduced CBD Wages

Figure 3 and Table 3 examine the effect of lowering CBD wages by \$0.20 per hour for both male and female workers, while holding suburban wages constant. Again since benchmark utility levels remain constant, the implicit assumption is that wages in other cities are unaffected. This change causes the incomes of all households having only CBD jobs, i.e., CBD/CBD, CBD/ –, and – /CBD households, to fall by an equal proportion. However, the results of the income decline differ strikingly for the three groups. Female-headed households can no longer outbid other types of households for land anywhere in the city and they disappear. In contrast, both CBD/CBD and CBD/ – households increase relative to the base case. CBD/CBD households increase from 0.28 to 0.31 as a proportion of all households and CBD/ – households increase from 0.21 to 0.24. Both types of households benefit from reduced competition in the land market and their commuting areas expand to fill the regions previously occupied by female-headed CBD households. Thus the same pro-

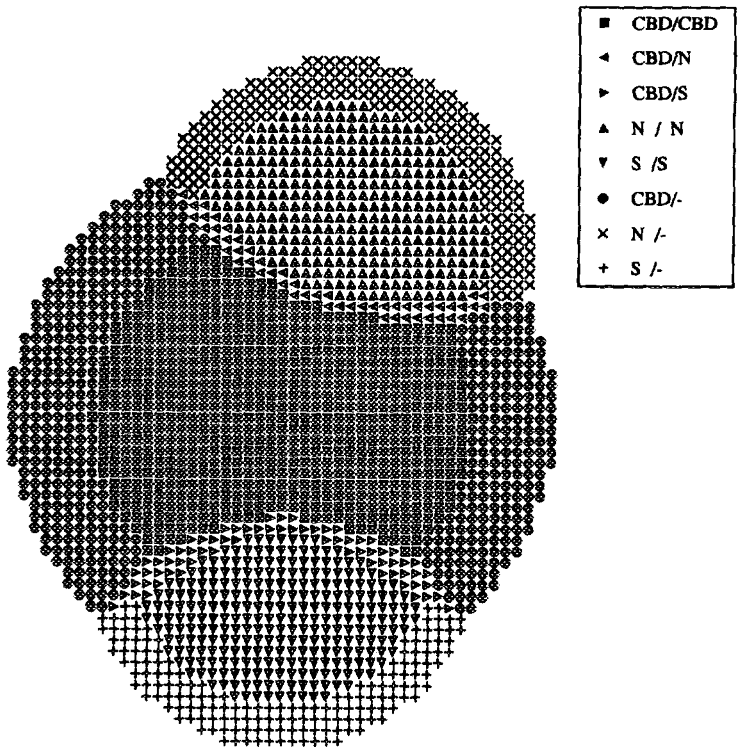

FIG. 3. City with lower CBD wages.

TABLE 3
Results with Lower CBD Wages

	Proportion	Commute distance		Average density	Work hours	
		Males	Females		Males	Females
CBD/CBD	0.31	3.9 (1.7)	3.9 (1.7)	3.8 (0.7)	8.6 (0.05)	5.4 (0.02)
CBD/N	0.019	6.1 (1.3)	3.5 (1.2)	3.1 (0.4)	8.6 (0.04)	5.2 (0.04)
CBD/S	0.026	5.9 (1.4)	3.3 (1.4)	3.2 (0.5)	8.6 (0.04)	5.2 (0.04)
N/N	0.13	3.1 (1.4)	3.1 (1.4)	3.2 (0.5)	8.6 (0.04)	5.4 (0.02)
S/S	0.10	3.1 (1.4)	3.1 (1.4)	3.2 (0.5)	8.6 (0.04)	5.4 (0.02)
CBD/ –	0.24	8.3 (0.8)	—	4.0 (0.3)	7.8 (0.02)	—
N/ –	0.091	6.1 (0.5)	—	3.9 (0.2)	7.9 (0.1)	—
S/ –	0.075	6.1 (0.5)	—	3.9 (0.2)	7.9 (0.01)	—

Note. Proportion of households with some CBD job = 0.60. Proportion of jobs in CBD = 0.58. Average commute distance = 4.57 miles. Total population relative to benchmark case = 0.84.

portionate change in income has widely different effects on different household groups. Since the household group having the highest density disappears, overall household population falls by 16%.

Not surprisingly, the reduction in CBD wages reduces jobs at the CBD. Relative to the base case, the proportion of jobs at the CBD falls from 0.66 to 0.58 and the proportion of households with at least one CBD worker falls from 0.71 to 0.60. The implied elasticity of the number of CBD jobs with respect to CBD wages is extremely high—between 7.6 and 9.1.

Lower Wages for Female Workers

In this simulation, wages for female workers are lowered by $1.00 at all work locations. The motivation for this simulation is that actual average wages for full-time female workers relative to full-time male workers in the United States are lower than the figure used in the benchmark case.

The results of this simulation, given in Fig. 4 and Table 4, are easy to describe: no women work. The lower wages paid to female workers reduce the incomes of two-worker and female-headed households relative to traditional households and therefore reduce their willingness-to-pay for land relative to that of traditional households. The result is that traditional households outbid other types of households for sites everywhere in the metropolitan area. The city consists of CBD/ – households living around the CBD and N/ – and S/ – households living around their respective subcenters. The suggestion of the model that at realistic wage levels, no households containing women workers can bid successfully for urban land is rather startling!

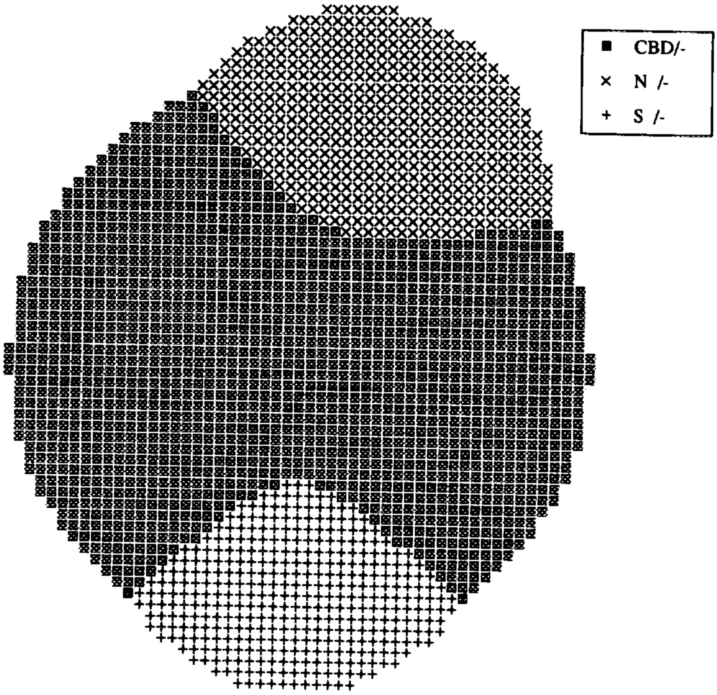

FIG. 4. City with reduced wages for female workers.

TABLE 4
Results with Reduced Wages for Female Workers

	Proportion	Commute distance Males	Commute distance Females	Average density	Work hours Males	Work hours Females
CBD/−	0.67	5.8 (2.6)	—	5.2 (1.2)	7.9 (0.06)	—
N/−	0.19	4.0 (1.8)	—	4.7 (0.8)	7.9 (0.04)	—
S/−	0.14	4.3 (1.8)	—	4.6 (0.8)	7.9 (0.04)	—

Note. Proportion of households with some CBD job = 0.67. Proportion of jobs in CBD = 0.67. Average commute distance = 4.62 miles. Total population relative to benchmark case = 1.23.

5. SIMULATION RESULTS—RANDOM WAGES

Now turn to the simulations using random wages. In these simulations, wages for each worker are multiplied by a random number drawn from a uniform distribution ranging from 0.95 to 1.05. Separate and independent random terms are drawn for each worker. Thus single-worker households have one random draw, while two-worker households have two. We first repeat the benchmark case but with random wages, and then repeat the higher commuting cost and lower female wage simulations with random wages.

The Benchmark Case with Random Wages

The results of repeating the benchmark case with random wages are shown in Table 5 and Fig. 5. Examining Fig. 5, we see that the metropolitan area changes greatly. Instead of precisely delineated commuting regions, household types overlap greatly, with no particular type dominating any particular region of the city. The urban–rural boundary also becomes very fuzzy and the city expands relative to the base case. At the urban–rural boundary, there is some "leapfrogging," i.e., some sites developed for urban use surround land that remains rural and vice versa. Another striking result is that the number of household types represented within the city increases. Four types of households, N/CBD, S/CBD, – /N, and – /S, appear in this simulation, while only two types of households, N/S and S/N, still do not appear. Thus slight randomness in wages causes a tremendous increase in the diversity of the household population and in the realism of the model's results.

TABLE 5

Results for the Benchmark Case with Random Wages

		Commute distance		Average	Work hours	
	Proportion	Males	Females	density	Males	Females
CBD/CBD	0.11	5.4 (1.7)	5.4 (1.7)	3.6 (0.6)	8.6 (0.08)	5.4 (0.20)
CBD/N	0.022	7.7 (2.1)	4.5 (2.2)	3.1 (0.4)	8.6 (0.17)	5.1 (0.36)
CBD/S	0.025	7.4 (1.4)	4.4 (1.9)	3.2 (0.4)	8.7 (0.11)	5.1 (0.29)
N/CBD	0.005	4.2 (2.1)	7.8 (2.1)	3.0 (0.3)	8.5 (0.18)	5.6 (0.35)
N/N	0.036	4.1 (0.9)	4.1 (0.9)	3.1 (0.3)	8.7 (0.05)	5.3 (0.14)
S/CBD	0.008	4.1 (1.9)	7.4 (1.3)	3.1 (0.4)	8.5 (0.13)	5.6 (0.24)
S/S	0.022	4.2 (0.8)	4.2 (0.8)	3.1 (0.2)	8.6 (0.03)	5.3 (0.09)
CBD/ –	0.23	7.2 (2.2)	—	5.1 (1.1)	7.8 (0.05)	—
N/ –	0.082	5.2 (1.0)	—	4.7 (0.4)	7.8 (0.05)	—
S/ –	0.056	5.3 (0.9)	—	4.8 (0.4)	7.9 (0.02)	—
– /CBD	0.32	—	6.5 (2.1)	6.7 (1.4)	—	7.9 (0.10)
– /N	0.054	—	4.5 (0.9)	6.1 (0.6)	—	8.1 (0.22)
– /S	0.036	—	4.8 (0.8)	5.8 (0.5)	—	7.9 (0.03)

Note. Proportion of households with some CBD job = 0.72. Proportion of jobs in CBD = 0.67. Average commute distance = 5.42 miles. Total population relative to benchmark case = 1.56.

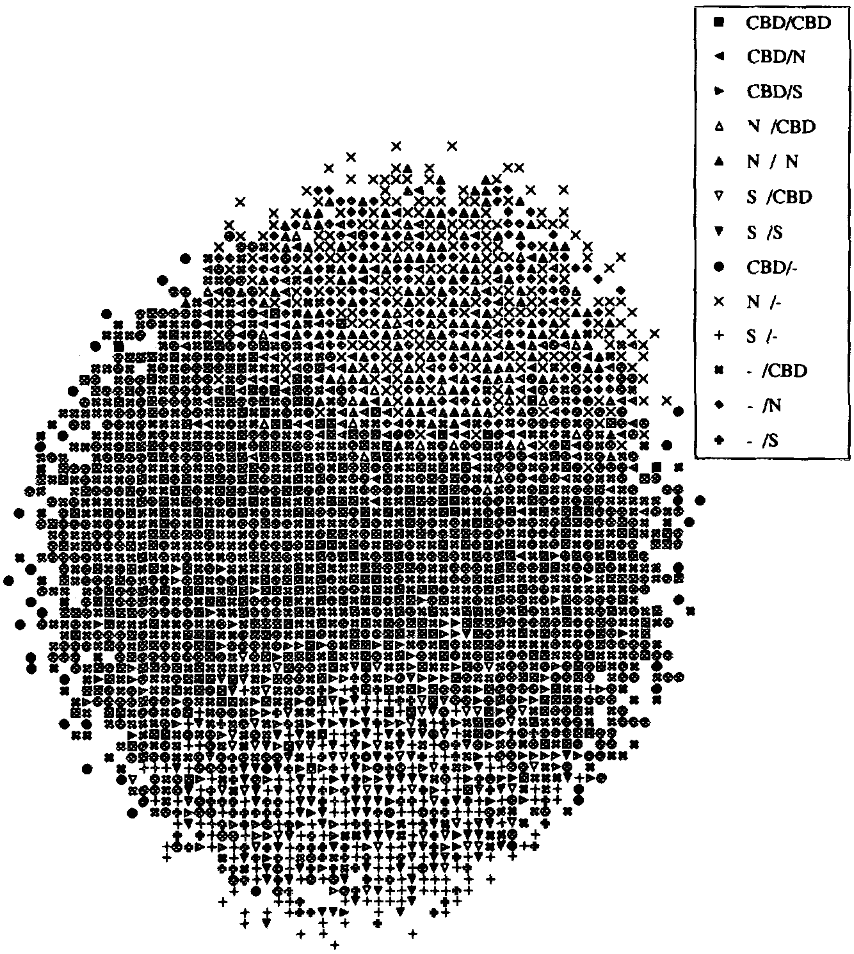

FIG. 5. The benchmark case with random wages.

The mixing of household types results from the fact that sites are likely to go to the household type that happens to benefit most from random variation in incomes. For example, a female-headed CBD household might locate at a site near the CBD since it happens to have higher than usual income and willingness-to-pay for land when the two-worker CBD/CBD household that would normally have won the auction for the site happens to have lower than usual income and willingness-to-pay. The random variation in incomes thus breaks up the strict homogeneity of commuting regions seen in previous runs of the model. It also allows household types not seen previously to appear in the city since they occasionally win auctions for land if they happen to have higher than usual income when other household types do not. Note that there is much greater variation in average commuting distances and, especially, in work hours for each group, as measured by the standard deviations. For example, male workers in CBD/ – households previously worked an average of 7.8 hours with a standard deviation of ± 0.02. Now they still work the same average number of hours, but the standard deviation is ± 0.05. This is because each group of households is more spread out. Relative to the base case, the size of the city also expands since in any auction for a site, some household group is likely to have higher than its average income and willingness-to-pay for land. Therefore the maximum bid for each site is higher than that in the base case and more sites are added to the city. (The value of land for non-urban use remains fixed and has no random component.) Total household population expands by 56% relative to the base case.

The distribution of household types also changes dramatically. The proportion of two-worker households falls from 0.51 to 0.23, the proportion of female-headed households rises from 0.16 to 0.41, and the proportion of traditional households rises slightly from 0.33 to 0.37. Relative to the base case, two worker households tend to lose more auctions for land since their range of income variation in dollar terms is greater than that of single-worker households. Therefore their probability of having a small positive increase in income due to randomness in wages is smaller than that of single-worker households. As a result, they are likely to lose some auctions that they would have previously won and their population falls. The other types of households expand because they win more auctions against two-worker households. In addition, female-headed households' incomes also vary over a smaller dollar range than traditional households, so that they are more likely than traditional households to have a small positive income increase. This causes them to win more auctions and their population to expand relative to that of traditional households.

Commuting distances also rise in the random wage model, from 4.88 miles in the benchmark case to 5.42 miles. This is both because the city gets larger and—because commuting regions are no longer sharply defined—"wasteful" commuting occurs, i.e., commuting that could be eliminated by households switching residential or job locations.[18] Randomness in wages also causes the density pattern to change. For example, the average density of CBD/CBD households falls by 12%, because they are now dispersed across much of the metropolitan area instead of being concentrated near the high-density CBD. The opposite change occurs for

[18]See Hamilton [1] and White [13].

CBD/− households. They previously occupied the most suburban locations, but now they are more scattered, so their average density rises by 31%.

An interesting implication of the random wage model is that high- and low-density housing exist side by side. For example, east and west of the CBD, many CBD/CBD households locate adjacent to −/CBD households. The CBD/CBD households occupy low-density housing—particularly in the suburbs—while the much poorer −/CBD households occupy high-density housing. Such heterogeneity in the housing stock is not usually observed in actual cities, since exclusionary zoning prevents mixing of apartments with single-family housing in the same neighborhood.

Increased Commuting Costs with Random Wages

Table 6 and Fig. 6 repeat the raised commuting cost simulation with random wages. Recall that without random wages, raising out-of-pocket commuting cost from $0.40 to $0.55 per mile caused female-headed households to disappear completely from the model. (See Tables 1 and 2.) This does not occur in the random wage simulations. Instead, the proportion of households that are female-headed declines only slightly when commuting costs rise, from 0.41 and 0.37 (see Tables 5 and 6). Thus the large swings in household composition in the metropolitan area that occur

TABLE 6
Results with Higher Commuting Cost and Random Wages

		Commute distance		Average	Work hours	
	Proportion	Males	Females	density	Males	Females
CBD/CBD	0.109	4.6 (1.4)	4.6 (1.4)	3.6 (0.6)	8.61 (0.06)	5.5 (0.20)
CBD/N	0.017	7.0 (2.1)	4.0 (2.3)	3.1 (0.5)	8.73 (0.17)	5.1 (0.38)
CBD/S	0.022	6.8 (1.1)	4.0 (1.3)	3.2 (0.3)	8.73 (0.09)	5.1 (0.21)
N/CBD	0.005	3.7 (2.2)	6.5 (1.9)	3.1 (0.4)	8.53 (0.19)	5.7 (0.38)
N/N	0.044	3.6 (0.5)	3.6 (0.5)	3.1 (0.2)	8.69 (0.03)	5.4 (0.08)
S/CBD	0.007	3.4 (1.5)	6.7 (1.1)	3.1 (0.3)	8.55 (0.13)	5.7 (0.24)
S/S	0.031	3.5 (0.4)	3.5 (0.4)	3.2 (0.1)	8.70 (0.01)	5.4 (0.04)
CBD/−	0.238	6.4 (2.0)	—	5.2 (1.1)	7.91 (0.03)	—
N/−	0.086	4.5 (0.8)	—	4.8 (0.4)	7.95 (0.01)	—
S/−	0.068	4.6 (0.6)	—	4.8 (0.3)	7.95 (0.01)	—
−/CBD	0.283	—	5.5 (1.9)	6.8 (1.5)	—	8.0 (0.04)
−/N	0.055	—	3.8 (0.7)	6.2 (0.5)	—	8.2 (0.20)
−/S	0.035	—	4.0 (0.5)	5.2 (0.3)	—	8.0 (0.00)

Note. Proportion of households with some CBD job = 0.68. Proportion of jobs in CBD = 0.64. Average commute distance = 4.71 miles. Total population relative to benchmark case = 1.23.

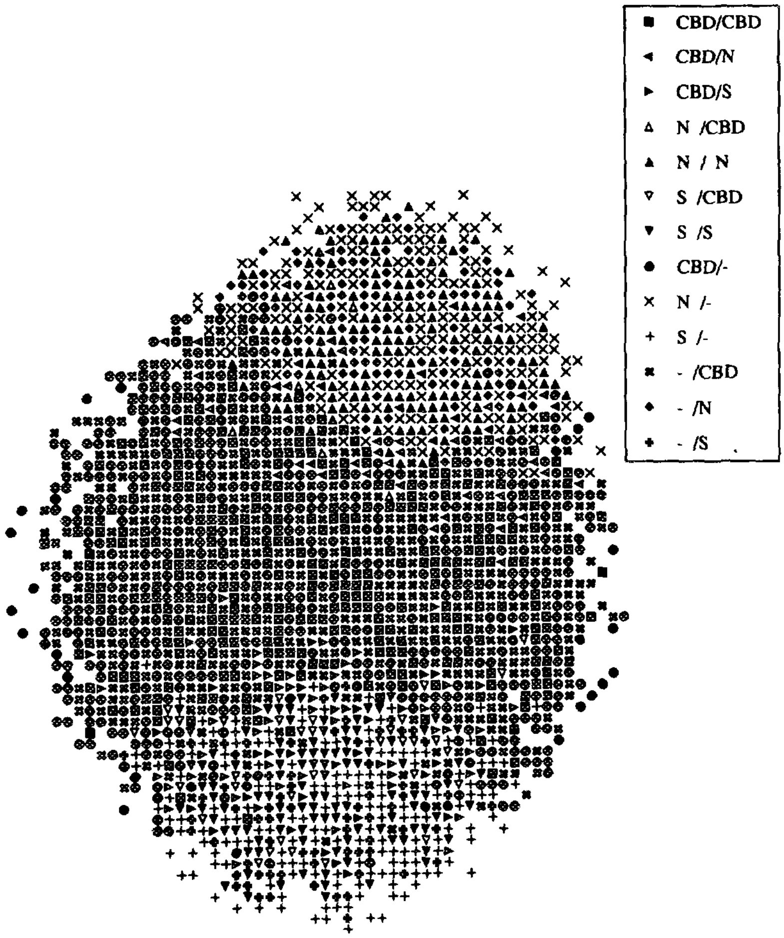

FIG. 6. City with higher commuting cost and random wages.

in the non-random wage simulations when parameter values change are muted in the random wage model. Instead, the distribution of households in the metropolitan area is much more stable.

Other changes that occur when commuting costs rise are similar under both the random and the non-random wage assumptions. In the random wage case, average commuting distance falls by 13%, the metropolitan area becomes smaller, and population drops by 21%. There are slightly fewer households with jobs at the CBD. These results are similar to the results in the non-random wage case.

Lower Female Wages with Random Wages

Finally, suppose that wages of women workers again fall by $1 per hour, but that the random factor affecting wages is introduced. The results are in Table 7 and Fig. 7. In the non-random wage simulations, lowering female wages caused women workers to disappear completely from the model, i.e., the proportion of households having two workers dropped from 0.51 to 0 and the proportion of households having a female head dropped from 0.16 to zero (see Tables 1 and 4). In contrast in the random wage simulations, the same reduction in female wages causes the proportion of two-worker households to fall from 0.23 to 0.165 and the proportion of female-headed households to fall from 0.41 to 0.001 (see Tables 5 and 7). (In Fig. 7, which is more aggregated than the model itself, there are no female-headed households.) Since the reduction in female wages causes the total income of female-headed households to fall by more than that of two-worker households, the fact that the proportion of female-headed households still drops steeply but the proportion of two-worker households drops only gradually in the random wage simulation seems reasonable. The results of the simulation with random wages appear to be much more realistic than the results of the non-random simulation.

A surprising result is that the reduction in female wages causes quite different changes in the size of the city in the two cases. In the random wage case, total urban population falls by 19% (see Tables 5 and 7), while in the non-random wage case, total urban population rises by 23% (see

TABLE 7

Results with Reduced Wages for Female Workers and Random Wages

		Commute distance		Average	Work hours	
	Proportion	Males	Females	density	Males	Females
CBD/CBD	0.092	4.8 (1.3)	4.8 (1.3)	3.5 (0.5)	8.8 (0.07)	4.9 (0.19)
CBD/N	0.015	6.9 (2.0)	4.1 (2.5)	3.1 (0.4)	9.0 (0.17)	4.5 (0.39)
CBD/S	0.017	6.4 (0.9)	4.0 (1.4)	3.2 (0.3)	9.0 (0.07)	4.4 (0.19)
N/CBD	0.003	3.5 (2.0)	7.1 (2.1)	2.9 (0.2)	8.8 (0.20)	5.1 (0.40)
N/N	0.022	3.4 (0.6)	3.4 (0.6)	3.1 (0.2)	9.0 (0.03)	4.7 (0.08)
S/CBD	0.004	3.2 (1.3)	6.6 (0.6)	3.1 (0.2)	8.8 (0.08)	5.2 (0.15)
S/S	0.012	3.5 (0.4)	3.5 (0.4)	3.1 (0.1)	9.0 (0.02)	4.7 (0.04)
CBD/–	0.543	6.7 (2.0)	—	5.2 (1.0)	7.8 (0.05)	—
N/–	0.165	4.8 (1.0)	—	4.8 (0.4)	7.9 (0.02)	—
S/–	0.125	4.9 (0.8)	—	4.8 (0.3)	7.9 (0.02)	—
–/CBD	0.001	—	4.3 (1.0)	6.3 (0.9)	—	7.9 (0.42)

Note. Proportion of households with some CBD job = 0.675. Proportion of jobs in CBD = 0.66. Average commute distance = 4.76 miles. Total population relative to benchmark case = 1.27.

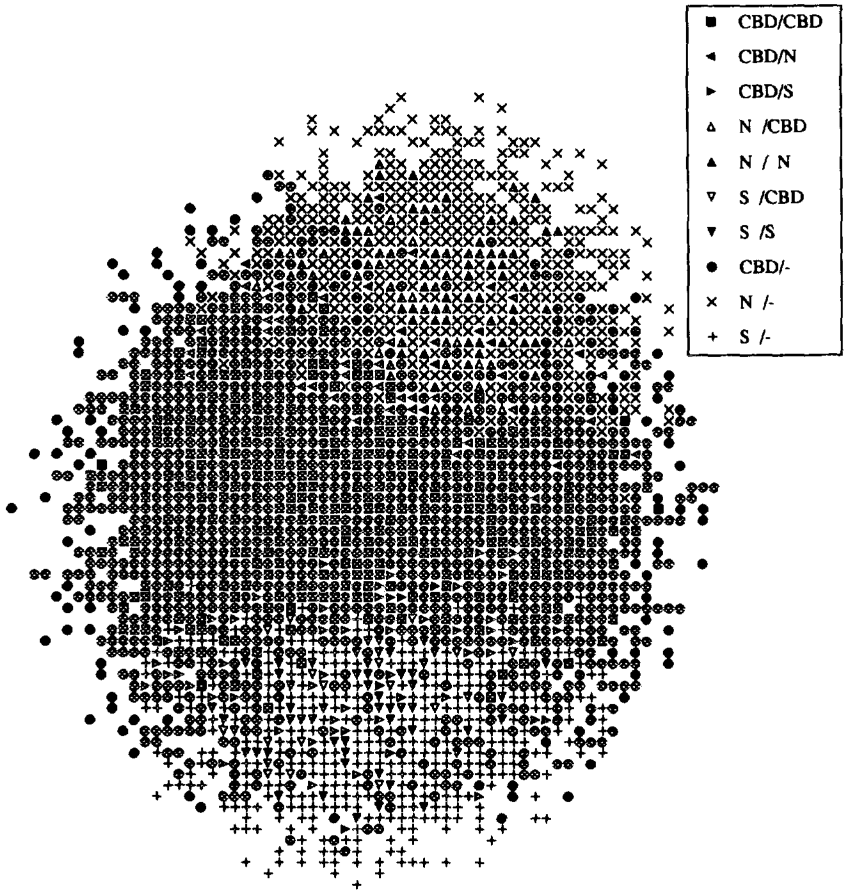

FIG. 7. City with reduced wages for female workers and random wages.

Tables 1 and 4). In both simulations, the reduction in female wages causes traditional households to replace two-worker and female-headed households. The average density level of traditional households is lower than that of female-headed households, but higher than that of two-worker households. In the non-random wage case, the group of households which is replaced by traditional households contains a relatively high proportion of two-worker relative to female-headed households, so that overall average density and total population of the city rise. In the random wage case, the group of households which are replaced by traditional households contains relatively more female-headed households, so that overall average density and total population fall.

To summarize, the results of the random wage model suggest that urban models can be made much more realistic by introducing even a small amount of randomness—whether it be in wages, tastes, or willingness-to-pay for land. The largest effect of random wages is on the distribution of household types within the city, which becomes both more diverse and more stable.

6. CONCLUSION

The paper has two major, and somewhat contradictory, results. The main result of the non-random wage simulations is that seemingly unrelated exogenous changes have important effects on the distribution of household types within the city and particularly on the availability of housing for groups with low income. Both higher commuting costs and lower CBD wages were shown to cause female-headed households to disappear completely from the metropolitan area because they could no longer compete against other household groups for housing. Slightly lower wages were shown to cause married women workers or all women workers to stop working. This suggests that policy makers need to be aware that almost any change—whether due to policy initiatives or not—may have important consequences for vulnerable groups. On the other hand, the random wage simulations suggest a somewhat different conclusion: that the household composition and the composition of the workforce in the metropolitan area is fairly stable and responds much less to parameter value changes.

In general, introduction of even a fairly minor random component in wages causes the results of the simulation model to change substantially. The number of different household types increases, neighborhoods shift from being perfectly homogeneous to being quite heterogeneous, borders become fuzzy, and a substantial amount of wasteful commuting appears. All of these changes, of course, make the model much more realistic.

Despite the incorporation of 15 different types of households and three employment locations, the model remains simplified in many important dimensions. An important excluded factor is the disamenities associated with living in high-density neighborhoods, such as crime, congestion, littering, crowding, and the presence of undesirable neighbors, which presumably account for high-income households tending to choose suburban locations in U.S. cities. Incorporating these disamenities would be a desirable direction for future research. Another approach to increasing the realism of the model would be to design it specifically to represent a particular city. The locations of suburban subcenters could be chosen to correspond to actual subcenter locations in the particular city (such as Roslyn, Bethesda, and Rockville in Washington, DC) and more subcenters could be incorporated as relevant. Congestion could also be incorporated simultaneously, using the technique developed in White [10]. Randomness in income levels could be introduced using a more realistic income distribution than the uniform distribution employed here. The historic development of a city could be modeled by running the simulation for parameter values corresponding to a particular historic time period and assuming that housing built in that period remains fixed in density in later periods. Then the model could be rerun for parameter values corresponding to later periods to determine how the city changes and where new housing is built. Such a procedure could be followed over several periods to model how a metropolitan area grows and changes over time.

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