

A quasiparticle version of the Ginocchio S,D fermion pair algebra^{*}

K.T. Hecht

Physics Department, University of Michigan, Ann Arbor, MI 48109, USA

Received 30 November 1992

Abstract: The automorphisms of SO(8) which exchange vector and spinor representations do not lead to new physically meaningful subgroup chains for the nuclear S,D fermion pair algebra; but a simpler transformation leads to a physically meaningful quasiparticle version of this algebra.

1. Introduction

Recently the automorphisms of the group SO(8) have been used to advantage in several branches of physics. The very special nature of SO(8) arises from the unique three-fold symmetry of its Dynkin diagram. This has been highlighted by Georgi in a chapter entitled "Automorphisms: Fun with SO(8)" in his book on Lie algebras¹⁾ and leads to the embedding of three distinct SO(7) subgroups within SO(8). These have been used in particle physics in superstring and supergravity theories²⁾ and in the study of octonionic superstring solitons³⁾. Very recently they have been used to great advantage by Judd and Lister⁴⁻⁶⁾ to explain many unexpected simple proportionalities and unexplained zeros in atomic f-shell matrix elements. These successes have led to the question: Are there three distinct physically relevant SO(7) subgroups in the most important nuclear physics application of SO(8), the Ginocchio S,D fermion pair algebra⁷⁾ and its outgrowth, the fermion dynamic symmetry model⁸⁾?

The automorphisms of SO(8) interchange the three fundamental 8-dimensional irreducible representations of this group, the 8-vector defining representation (1000), and the two real spinor representations ($\frac{1}{2}\frac{1111}{2222}$) and ($\frac{1}{2}\frac{1111}{2222} - \frac{1}{2}$), and lead to the transformation of the general representation ($\Omega_1\Omega_2\Omega_3\Omega_4$) into ($\frac{1}{2}(\Omega_1 + \Omega_2 + \Omega_3 - \Omega_4)$, $\frac{1}{2}(\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4)$, $\frac{1}{2}(\Omega_1 - \Omega_2 + \Omega_3 + \Omega_4)$, $\pm\frac{1}{2}(-\Omega_1 + \Omega_2 + \Omega_3 + \Omega_4)$). These three SO(8)

Correspondence to: Dr. K.T. Hecht, Department of Physics, University of Michigan, 500 East University, MI 48109-1120 Ann Arbor, USA.

* It is a pleasure to dedicate this short contribution to Hans Weidenmüller on the occasion of his 60th birthday.

irreducible representations have the same dimension and the same values for the quadratic Casimir invariants, but differ markedly in the embedding of the three types of SO(7) irreducible representations⁹⁾, which are given by the usual “betweenness” conditions for the quantum numbers. E.g., the generalized seniority-zero irrep. (3000) of the Ginocchio pair algebra leads to the D pair SO(7) irreps. ($\omega 00$) with $\omega = 3, 2, 1, 0$. The associated SO(8) irreps. ($\frac{333}{222} \pm \frac{3}{2}$) on the other hand would lead to a single SO(7) irrep. ($\frac{333}{222}$). Is this a physically relevant irrep. for the Ginocchio S,D pair algebra? The mappings of the SO(8) generators into their counterparts under the automorphisms is straightforward¹⁾. To be physically relevant, however, for the nuclear physics applications these mappings would have to preserve the subgroup characteristics of the angular momentum SO(3) generators. In the atomic applications⁴⁻⁶⁾ the 28 generators of SO(8) are constructed from spherical tensors of rank $r = 1$ and 5 and two different tensors of rank 3. The three distinct SO(7) subgroups are generated by the 21 operators which include the $r = 1$ and 5 spherical tensors and three distinct linear combinations of the two independent $r = 3$ spherical tensors. Since the SO(8) automorphism does not mix spherical tensors of different rank, the angular momentum subgroup generators remain intact. The nuclear S,D fermion pair algebra, on the other hand, is generated by 28 SO(8) operators which decompose into single spherical tensors of ranks $r = 1$ and 3 and three distinct spherical tensors of ranks $r = 0$ and 2 each. To keep the angular momentum subgroup generators intact, the SO(7) subgroups can only be generated by 21 operators which include the $r = 1$ and 3 spherical tensors and linear combinations of the $r = 2$ tensors and the $r = 0$ tensors. The familiar D pair SO(7) subalgebra^{7,10)} of the S,D pair SO(8) algebra is generated by the 5 D pair creation operators, their companion D pair annihilation operators, the nucleon number operator which is an $r = 0$ operator, and the $r = 1$ and 3 spherical tensors. It is the purpose of this note to show that there are other realizations of the SO(8) \supset SO(7) algebra corresponding to different linear combinations of the $r = 2$ and 0 tensors. These can be interpreted as quasi-particle realizations of the SO(8) algebra. Their SO(7) subalgebras correspond to quasi-particle realizations of the D pair algebra and they are not generated via the transformations of the SO(8) automorphisms which exchange vector and spinor representations. In the nuclear application these automorphisms would mix spherical tensors of different rank. Despite the failure of the SO(8) automorphism to lead to new physics in the case of the nuclear S,D pair algebra, the new quasi-particle realizations of the Ginocchio SO(8) algebra may be of special interest in their own right. This SO(8) algebra applies to fermion pairs created from identical nucleons in the normal parity orbits of the shell model; e.g., the $s_{1/2}$, $d_{3/2}$, $d_{5/2}$ and $g_{7/2}$ orbits for $50 \leq N \leq 82$. Since the nucleon occupation probabilities for these orbits must compete with the wrong-parity $h_{11/2}$ intruder, a normal-parity pair model built from quasi-particles rather than particles may lead to the physically more relevant symmetry.

2. The S,D pair algebra revisited

In the Ginocchio S,D fermion pair model the single-nucleon creation operators with definite total angular momentum quantum numbers j, m are expressed in terms of single particle pseudo orbital (k) and pseudo-spin (i) angular momenta, such that $k + i = j$,

$$a_{jm}^\dagger = \sum_{m_k, m_i} \langle km_k im_i | jm \rangle a_{km_k, im_i}^\dagger. \quad (1)$$

With $i = \frac{3}{2}$ (the SO(8) algebra) the single particle quantum numbers j can take on the values,

$$\begin{aligned} \text{with } k = 1: \quad & j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \quad (\Omega = 6), \\ \text{with } k = 2: \quad & j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2} \quad (\Omega = 10), \end{aligned}$$

e.g. the normal-parity orbits of the nuclear p,f shells ($28 \leq N \leq 50$) or s,d,g shells ($50 \leq N \leq 82$), with $\Omega = \sum_j (j + \frac{1}{2}) = 2(2k + 1)$. The SO(8) algebra is generated by the fermion pair operators coupled to angular momentum zero in the pseudo orbital k -space

$$A_{JM}^\dagger = \sum_{m_k} \sum_{m_1, m_2} (-1)^{k-m_k} \langle \frac{3}{2} m_1 \frac{3}{2} m_2 | JM \rangle a_{km_k, \frac{3}{2} m_1}^\dagger a_{k-m_k, \frac{3}{2} m_2}^\dagger \equiv [a^\dagger \times a^\dagger]_{JM}, \quad (2)$$

$$A_{JM} = (A_{JM}^\dagger)^\dagger, \quad (3)$$

$$\begin{aligned} C_\mu^r &= \left\{ \sum_{m_k} \sum_{m_1, m_2} \langle \frac{3}{2} m_1 \frac{3}{2} -m_2 | r\mu \rangle a_{km_k, \frac{3}{2} m_1}^\dagger a_{km_k, \frac{3}{2} m_2} (-1)^{\frac{3}{2}-m_2} \right\} - \frac{1}{2} \Omega \delta_{r0} \\ &\equiv [a^\dagger \times \tilde{a}]_\mu^r - \frac{1}{2} \Omega \delta_{r0}, \end{aligned} \quad (4)$$

where the pair creation and annihilation operators, A_{JM}^\dagger, A_{JM} are restricted to $J = 0, 2$ only (S,D pairs), while the one-body operators C_μ^r range through $r = 0, 1, 2, 3$ leading to the spherical tensor classification of the SO(8) generators enumerated in the introduction. Note also that $C_0^0 = \frac{1}{2} N_{\text{op}} - \frac{1}{2} \Omega$, where N_{op} is the nucleon number operator. The 28 group generators $A_{JM}^\dagger, A_{JM}, C_\mu^r$ are put into 1:1 correspondence with standard SO(8) generators $J_{ab} = -i(x_a \partial_b - x_b \partial_a)$ with $a, b = 1, \dots, 8$ through table 1 of ref. ¹⁰⁾. This SO(8) group has two physically relevant SO(n) subgroup chains,

$$\left. \begin{aligned} \text{SO}(8) \supset \text{SO}(7) \\ \text{SO}(8) \supset \text{SO}(6) \times \text{SO}(2) \end{aligned} \right\} \supset \text{SO}(5) \times \text{SO}(2) \supset \text{SO}(3) \times \text{SO}(2), \quad (5)$$

The SO(7) subgroup is generated by the D pair creation and annihilation operators A_{2M}^\dagger, A_{2M} and the one body-operators, C_μ^r , with $r = 0, 1$ and 3 where these can be put into 1:1 correspondence ¹⁰⁾ with J_{ab} , with $a, b = 1, \dots, 7$ while the subgroup SO(6) is generated by the C_μ^r with $r = 1, 2, 3$ corresponding to J_{ab} , with $a, b = 1, \dots, 5$ and 8. Note that these commute with C_0^0 corresponding to the SO(2) generator J_{67} , leading to the SO(6) \times SO(2) subgroup which can be used in the full classification of the SO(8) irreducible representations $(\Omega_1 \Omega_2 \Omega_3 \Omega_4) = (\frac{1}{2} \Omega - \frac{1}{2} u, \rho_1 \rho_2 \rho_3)$. Here Ω_1

is given by the highest possible weight of C_0^0 corresponding to maximum nucleon number $2\Omega - u$, where u , the generalized S,D pair seniority number, is the number of nucleons entirely free of the favored S and D pairs of eq. (2). The remaining SO(8) quantum numbers ρ_1, ρ_2, ρ_3 give the SO(6) symmetry of the u nucleons free of favored S and D pairs. The full set of states of the irrep., $(\frac{1}{2}\Omega - \frac{1}{2}u, \rho_1\rho_2\rho_3)$, with nucleon numbers $n = u, u+2, \dots, 2\Omega - u - 2, 2\Omega - u$ can then be generated by coupling $p = \frac{1}{2}(n - u)$ favored nucleon S,D pairs with SO(6) labels $(p00), (p - 2, 00), \dots, (100)$ or (000) , resulting from p symmetrically coupled SO(6) (100) pairs, with the SO(6) representations $(\rho_1\rho_2\rho_3)$ of the u nucleons free of favored S,D pairs to resultant SO(6) representations $(\sigma_1\sigma_2\sigma_3)$. Table 1 shows a very simple example, the SO(8) irrep. (3000) corresponding to the states of generalized seniority $u = 0$ in the $\Omega = 6$ shell. Finally, the subgroup SO(5) common to both group chains of eq. (5) is generated by the C'_μ with $r = 1$ and 3 corresponding to J_{ab} with $a, b = 1, \dots, 5$, whereas the final SO(3) subgroup is generated by the C''_μ with $r = 1$. Its irreps. give the I-spin and in this subspace of states with a zero pseudo-orbital angular momentum, the total angular momentum, J , of the states. This is shown as a right subscript on the SO(5) irreps $(\tau_1\tau_2)$ of table 1. The SO(7) irreps. $(\omega_1\omega_2\omega_3) = (\frac{1}{2}\Omega - \frac{1}{2}w, \theta_1\theta_2)$ embedded within $(\Omega_1\Omega_2\Omega_3\Omega_4)$ are given by the "betweenness" conditions: $\Omega_1 \geq \omega_1 \geq \Omega_2, \Omega_2 \geq \omega_2 \geq \Omega_3, \Omega_3 \geq \omega_3 \geq |\Omega_4|$. Now, w is the D pair seniority number which counts the number of nucleons entirely free of favored D pairs (note that $w \geq u$). In the example of table 1 the 2-particle state generated by the action of A_{00}^\dagger is an SO(7) state, (200), with $w = 2$; while the 4-particle state generated by the action of two pair-creation operators A_{00}^\dagger is in the SO(7) irrep. (100) with $w = 4$, etc. States with larger particle numbers in these SO(7) irreps. are then generated by the action of $p = \frac{1}{2}(n - w)$ D pair-creation operators with SO(5) symmetries $(p0), (p - 2, 0), \dots$, as shown specifically in table 2. The SO(5) labels (τ_1, τ_2) are shown as right superscripts whereas the angular momentum labels are given by right subscripts in this table. States with $n > 6$ (more than half-full shell) are given by the hole equivalents, involving pair annihilation operators acting on the full shell state.

TABLE 1
States of the SO(8) irrep. (3000) in the
SO(8) \supset SO(6) \times SO(2) \supset SO(5) \times SO(2) \supset SO(3) \times SO(2) chain

SO(2) n	SO(6) $(\sigma_1\sigma_2\sigma_3)$	SO(5) \supset SO(3) $(\tau_1\tau_2)_J$
12	(000)	(00) _{$J=0$}
10	(100)	(10) _{$J=2$} (00) _{$J=0$}
8	(200)(000)	(20) _{$J=2,4$} (10) _{$J=2$} (00) _{$J=0$} ²
6	(300)(100)	(30) _{$J=0,3,4,6$} (20) _{$J=2,4$} (10) _{$J=2$} (00) _{$J=0$} ²
4	(200)(000)	(20) _{$J=2,4$} (10) _{$J=2$} (00) _{$J=0$} ²
2	(100)	(10) _{$J=2$} (00) _{$J=0$}
0	(000)	(00) _{$J=0$}

TABLE 2
State vectors of SO(7) irreps. ($\omega 00$) embedded within the SO(8) irrep. (3000)

(300)	(200)	(100)	(000)
Full)			
$A_{2M}^{(10)} Full)$	$A_{00}^{(00)} Full)$		
$[A_2 \times A_2]_{J=0,2,4}^{(00),(20)} Full)$	$[A_2 \times A_0]_{2M}^{(10)} Full)$	$[A_0 \times A_0]_{00}^{(00)} Full)$	
$[A_2^\dagger \times A_2^\dagger \times A_2^\dagger]_{J=2,0,3,4,6}^{(10),(30)} 0)$	$[A_2^\dagger \times A_2^\dagger \times A_0^\dagger]_{J=0,2,4}^{(00),(20)} 0)$	$[A_2^\dagger \times A_0^\dagger \times A_0^\dagger]_{2M}^{(10)} 0)$	$[A_0^\dagger \times A_0^\dagger \times A_0^\dagger]_{00}^{(00)} 0)$
$[A_2^\dagger \times A_2^\dagger]_{J=0,2,4}^{(00),(20)} 0)$	$[A_2^\dagger \times A_0^\dagger]_{2M}^{(10)} 0)$	$[A_0^\dagger \times A_0^\dagger]_{00}^{(00)} 0)$	
$A_{2M}^{(10)} 0)$	$A_{00}^{(00)} 0)$		
0)			

The question to be examined is now the following: Are there different physically relevant subgroups SO(7) or SO(6) in the S,D fermion-pair algebra, i.e. are there different subgroups generated by linear combinations of the SO(8) generators which preserve the spherical tensor character of these operators and contain the SO(3) subgroup generators C_μ^r with $r=1$. Clearly this will have to be limited to linear combinations of the $r=2$ and 0 tensors. The following is a solution. The S,D pair-creation operators can be replaced by the two combinations

$$U^2 A_{JM}^\dagger - V^2 A_{J-M} (-1)^{J-M} \mp 2UV C_M^J \quad (6)$$

with $U^2 + V^2 = 1$.

If the D pair-creation operators of the standard SO(7) subgroup of SO(8) are replaced by the $J=2$ operators of eq. (6) we are led to the two new SO(7) subgroups, SO(7)' and SO(7)". However, these are not related to the automorphisms of SO(8) which exchange vector and spinor irreps. Instead they are merely quasiparticle realizations of the standard $SO(8) \supset SO(7)$ group. For simplicity only the case for the upper sign of eq. (6) will be discussed. This arises from the quasiparticle transformation

$$b_{km_k, \frac{3}{2}m_i}^\dagger = U a_{km_k, \frac{3}{2}m_i}^\dagger - V a_{k-m_k, \frac{3}{2}-m_i} (-1)^{k+m_k+\frac{3}{2}+m_i}. \quad (7)$$

Using the shorthand notation introduced in eqs. (2) and (4) the operators of eq. (6) are simply $[b^\dagger \times b^\dagger]_{JM}$, again with $J=2, 0$. Together with the conjugate $[\tilde{b} \times \tilde{b}]_{JM}$ and the operators $([b^\dagger \times \tilde{b}]_\mu^r - \frac{1}{2}\Omega\delta_{r0})$ they generate a new copy of SO(8). The $[b^\dagger \times \tilde{b}]_\mu^r$ operators are simply:

For $r = \text{odd}$ ($r=1, 3$):

$$[b^\dagger \times \tilde{b}]_\mu^r = [a^\dagger \times \tilde{a}]_\mu^r. \quad (8)$$

For $r = \text{even}$ ($r=0, 2$):

$$[b^\dagger \times \tilde{b}]_\mu^r - \frac{1}{2}\Omega\delta_{r0} = UV(A_{r\mu}^\dagger + (-1)^{r-\mu}A_{r-\mu}) + (U^2 - V^2)C_\mu^r. \quad (9)$$

The SO(8) irreps. generated by the 28 operators $[b^\dagger \times b^\dagger]_{JM}, [\tilde{b} \times \tilde{b}]_{JM}, ([b^\dagger \times \tilde{b}]'_\mu - \frac{1}{2}\Omega\delta_{r0})$ can still be labelled by $(\frac{1}{2}\Omega - u', \rho'_1\rho'_2\rho'_3)$, where u' is now the number of quasiparticles entirely free of favored S and D pair quantum numbers. With $u' = 0$ and $\Omega = 6$ the complete set of states can still be given by table 2 if the a^\dagger are replaced by b^\dagger and the true vacuum, $|0\rangle$, is replaced by the quasiparticle vacuum for the U 's and V 's of eq. (7). The full SO(8) irrep. (3000) still breaks apart into four SO(7)' irreps (300), (200), (100) and (000). The SO(7)' generators do not mix states in the quasiparticle analogs of different columns of table 2. Note that this is very different from the groups SO(7)' and SO(7)'' generated by the automorphisms of SO(8) which exchange vector and spinor representations. For both of these the generalized $u = 0$ state for $\Omega = 6$ would be spanned by a single 112-dimensional irrep. $(\frac{333}{222})$ of SO(7)' or SO(7)''. The action of SO(7)' generators would thus have to mix all states of table 2.

Finally, it may be instructive to exhibit explicitly the particle character of the SO(7)' states. For this purpose we choose the four states with $J = 4$ in the irrep. (3000). The 3 states of the SO(7)' irrep (300) are given by:

$$\begin{aligned} |\tilde{n} = 8\rangle &= -V^2\mathcal{N}[A_2^\dagger \times A_2^\dagger]_{J=4,M}|0\rangle - \sqrt{2}UV\mathcal{N}'[A_2^\dagger \times A_2^\dagger]_{4M} \times A_{00}^\dagger|0\rangle \\ &\quad + U^2\mathcal{N}[A_2 \times A_2]_{4,-M}|\text{Full}\rangle, \\ |\tilde{n} = 6\rangle &= \mathcal{N}''[A_2^\dagger \times A_2^\dagger \times A_2^\dagger]_{J=4,M}|0\rangle, \\ |\tilde{n} = 4\rangle &= U^2\mathcal{N}[A_2^\dagger \times A_2^\dagger]_{J=4,M}|0\rangle - \sqrt{2}UV\mathcal{N}'[A_2^\dagger \times A_2^\dagger]_{4M} \times A_{00}^\dagger|0\rangle \\ &\quad - V^2\mathcal{N}[A_2 \times A_2]_{4,-M}|\text{Full}\rangle. \end{aligned} \quad (10)$$

For the SO(7)' irrep. (200):

$$\begin{aligned} |\tilde{n} = 6\rangle &= \sqrt{2}UV\mathcal{N}[A_2^\dagger \times A_2^\dagger]_{J=4,M}|0\rangle + (U^2 - V^2)\mathcal{N}'[A_2^\dagger \times A_2^\dagger]_{4M} \times A_{00}^\dagger|0\rangle \\ &\quad + \sqrt{2}UV\mathcal{N}[A_2 \times A_2]_{4,-M}|\text{Full}\rangle, \end{aligned} \quad (11)$$

where \tilde{n} denotes the eigenvalue of $[b^\dagger \times \tilde{b}]_0^0$ and the $\mathcal{N}, \mathcal{N}', \mathcal{N}''$ are trivial normalization constants.

3. Discussion

The transformations associated with the automorphisms of SO(8) which transform the 8-dimensional vector representation (1000) into the spinor representations $(\frac{1111}{2222})$ or $(\frac{111}{222} - \frac{1}{2})$ unfortunately do not lead to physically relevant distinct subgroups SO(7), SO(7)', SO(7)'' for the Ginocchio S,D fermion pair algebra. These transformations place pieces of the angular momentum algebra outside the subgroups SO(7)' or SO(7)'' so that states of irreps. of these groups no longer carry good angular momentum, a basic requirement for a physically relevant subgroup. Despite the failure of these SO(8) automorphisms to lead to new physics in the case of the nuclear S,D pair algebra, there are now simpler transformations which mix the

spherical tensors of rank 2 and 0 of this algebra to lead to new subgroups $SO(7)'$ and $SO(7)''$ which are, however, simple copies of the original subgroup chains, with similar irreducible representation embeddings. The mixing coefficients U and V can be given a simple quasiparticle interpretation. The irreducible representation labels for the quasiparticle realization are to be interpreted as generalized quasiparticle seniority quantum numbers in place of the usual real particle seniority numbers. The $SO(8)$ S,D pair algebra applies to fermion pairs created from identical nucleons in the normal parity orbits of the shell model; with $k=2$, $i=\frac{3}{2}$, e.g., the $s_{1/2}$, $d_{3/2}$, $d_{5/2}$ and $g_{7/2}$ orbits for $50 \leq N \leq 82$. Since the nucleon occupation probabilities for these orbits must compete with the abnormal parity $h_{11/2}$ intruder orbit, a normal parity pair model built from quasiparticles rather than particles may lead to the physically more meaningful symmetry.

The Ginocchio S,D pair model was originally introduced as a toy model to study the fermionic foundation of the interacting boson model of Arima and Iachello. More recently it has been proposed as a fermion dynamical symmetry model meriting serious consideration for nuclear spectra throughout the periodic chart⁸⁾. Since the Ginocchio S,D pair algebra is based on fermion pairs built from normal parity shell model orbits only, it fails to properly take into account the contributions of the abnormal parity high- j intruder orbits. By making a transformation to quasiparticle S and D pairs with U^2 and V^2 which give the occupation probabilities of the essentially degenerate normal-parity orbits of a major shell, the quasiparticle version of this fermionic pair model permits a distribution of occupation probabilities among the abnormal parity orbits as well as the orbits above and below the valence shells. The quasiparticle version of the $SO(8)$ algebra may thus alleviate one of the shortcomings of the fermion dynamic symmetry model. Since the new version of the algebra does not change the angular momentum characteristics of the favored D pair it will, however, not cure one of the serious problems of this model. Since the favored D pair of the $SO(8)$ algebra has a j_1-j_2 structure quite different from that predicted for realistic low seniority 2^+ states in real nuclei^{11,12)}, the successes of the fermion dynamic symmetry model⁸⁾ may involve a mapping from real space onto a model space which is quite as complicated as the mapping from fermion space onto a boson space which is the basis for the interacting boson model of Arima and Iachello.

References

- 1) H. Georgi, Lie algebras in particle physics (Addison-Wesley, Reading, MA, 1982) ch. XXV
- 2) R. Dündarar, F. Gursev and C.H. Tze, Nucl. Phys. **B266** (1986) 440
- 3) J.A. Harvey and A. Strominger, Phys. Rev. Lett. **66** (1991) 549
- 4) B.R. Judd and G.M.S. Lister, Phys. Rev. Lett. **67** (1991) 1720
- 5) B.R. Judd and G.M.S. Lister, J. de Phys. II **2** (1992) 853
- 6) B.R. Judd and G.M.S. Lister, in Group theory and special symmetries in nuclear physics, eds. J. Jänecke and J.P. Draayer (World Scientific, Singapore, 1992) pp. 225, 258
- 7) J.N. Ginocchio, Ann. of Phys. **126** (1980) 234

- 8) J.Q. Chen, D.H. Feng and C.L. Wu, Phys. Rev. **C34** (1986) 2269;
C.L. Wu, D.H. Feng, X.G. Chen, J.Q. Chen and M.W. Guidry, Phys. Rev. **C36** (1987) 1157
- 9) B.G. Wybourne, J. of Phys. B **25** (1992) 1683
- 10) K.T. Hecht, Nucl. Phys. **A475** (1987) 276
- 11) P. Halse, Phys. Rev. **C36** (1987) 372
- 12) Sudha Swaminathan, University of Michigan dissertation (1993) unpublished