

The possibility of factor price equalization, revisited

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This paper derives a condition for factor price equalization (FPE) in a Heckscher–Ohlin model with many goods, factors, and countries. Using Dixit and Norman's integrated world economy (IWE), two sets, called lenses, are constructed: one spanned by the factor vectors used to produce goods in the IWE; the other spanned by the countries' factor endowments. If the factor–endowment lens ever passes outside the factor–use lens, then FPE is impossible. In this sense, therefore, FPE requires that factor endowments vary less across countries than factor intensities vary across industries.

1. Introduction

Much depends, in modern international trade theory, on whether prices of factors are equalized internationally. With factor price equalization (FPE), industries in different countries with identical, constant-returns-to-scale technologies use identical techniques of production, and the analysis of trade and production is greatly simplified.¹ Even when there are elements of increasing returns to scale, Helpman and Krugman (1985) have gotten great mileage out of the assumption of FPE in simplifying otherwise intractable problems. In contrast, without FPE, trade patterns are perhaps more starkly de-

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¹For example, the factor content version of the Heckscher–Ohlin Theorem, introduced to the theoretical literature by Vanek (1968) and to the empirical literature by Leamer (1980), depends in its simplest form on the assumption of FPE.

limited,² but the different techniques of production used in different countries make comparisons across them much more difficult.³ As a contribution, therefore, to understanding the presence or absence of FPE, this paper presents a necessary condition for FPE, stated in terms of the allocation of factor endowments across any number of countries relative to the demands for, and the factor intensities of, any number of goods.⁴

Our knowledge of the conditions for FPE has evolved over the years. Samuelson (1949) identified the crucial condition for FPE in a two-factor model – that the two countries should produce at least two goods in common – and he recognized that this in turn requires that countries' factor endowments differ by less than the factor intensities of the industries. McKenzie (1955) generalized this idea to arbitrary numbers of factors (and at least as many goods) by focusing on the sets spanned by input requirement vectors and factor endowments. In current terminology, he showed that FPE required that factor endowments lie in the diversification cone. These conditions and others have been used over the years to try to ascertain the 'likelihood' of FPE and whether that likelihood rises or falls with the numbers of goods and factors.⁵

Perhaps the most useful and enlightening approach to FPE, though, and the one that I will build upon here, was presented by Dixit and Norman (1980, pp. 110–122). They spoke in terms of an 'integrated world economy' (IWE), in which both factors and goods are perfectly mobile across countries.⁶ In an IWE the world as a whole attains an equilibrium with a single set of prices of goods, prices of factors, and techniques of production, and with certain equilibrium quantities of all goods demanded on world markets. Then in the same world but with immobile factors, if it is possible to allocate factors within countries to industries, using the techniques of production of the IWE, in such a way as to duplicate the world outputs of the IWE, then FPE is possible and this allocation is one that could arise with FPE. If it is not possible to allocate the given factor endowments in this way, then FPE is not possible.

²See Jones (1974) and Deardorff (1979) who show how, with free trade but without FPE, countries specialize in production of a small number of traded goods and import all others.

³In Deardorff (1982), for example, I was able to prove generalizations of both the factor-content and commodity versions of the Heckscher–Ohlin Theorem even in the absence of FPE. Without FPE, however, the measurement of factor intensity for these purposes becomes cumbersome.

⁴Wu (1987) has provided a necessary condition for *the absence* of FPE in a model with two factors, two countries, and many goods. His condition, which therefore implies an analogous sufficient condition for FPE, is quite different from the necessary condition derived here.

⁵Samuelson (1949, p. 192) started this discussion himself with the claim that adding more than two goods 'increases the likelihood of complete factor-price equalization'. This was followed by McKenzie (1955), Land (1959), Johnson (1967), Vanek and Bertrand (1971), Wu (1987) and Deardorff and Courant (1990).

⁶Samuelson (1949) had also used this device, though without the name or the accompanying diagram, to motivate and explore the conditions for FPE. Helpman and Krugman (1985) have also used it for a variety of problems.

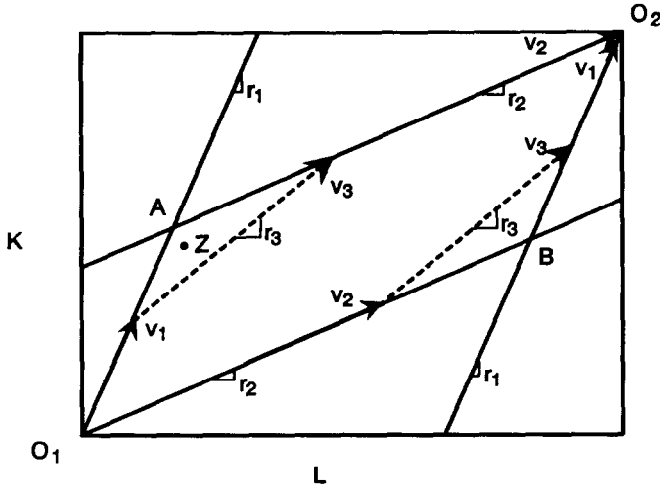


Fig. 1. FPE with two and three goods and two countries.

2. The model

With two factors, two goods, and two countries, this formulation leads to the simple and familiar visual representation of FPE factor allocations shown by the solid lines in fig. 1.⁷ It shows a box diagram, the dimensions of which are L and K , the world factor endowments of labor and capital. Letting the lower-left corner at O_1 be the origin for measuring factor endowments of country 1, and the upper-right corner at O_2 be the origin for measuring factor endowments of country 2, any point in the box describes an allocation of the world's factor endowments between the two countries. Inserting into this diagram rays from both O_1 and O_2 with slopes equal to the ratios of capital to labor employed in the two industries, r_1 and r_2 , Dixit and Norman (1980) have shown that FPE occurs if and only if factor endowments lie within the parallelogram formed by these rays, O_1AO_2B .

With a third good (here assumed to be of intermediate capital intensity, between goods 1 and 2), factor intensities alone are not enough to determine where FPE is possible; one needs also to know the outputs of each good in the IWE, and the amounts of the factors needed to produce them. Let X_i , $i=1, \dots, 3$, be the quantities of the goods demanded on the world market in the IWE, and let v_i be the vectors of factors needed to produce them. Since these vectors of factors must exhaust the world endowments, if placed end to end in the box diagram they must extend from one origin to the other. In fig.

⁷This diagram is often attributed to Lancaster (1957), who used (p. 31) something similar to show that two countries trading with FPE are equivalent to a single closed economy.

1, these vectors are drawn both in decreasing order of capital intensity, v_1, v_3, v_2 , and in increasing order of capital intensity, v_2, v_3, v_1 .

Together these vectors outline the hexagonal area $O_1v_1v_3O_2v_3v_2$, which is the only portion of the box in which FPE can occur. To see this, consider a point such as Z , inside the two diversification cones but above the path $v_1v_3v_2$. At such a point both countries are capable of keeping both factors fully employed using the three available techniques, but only by producing too much of two of the goods. Country 1, for example, must use more than the vector v_1 in industry 1, or else it would have too much capital per worker left over to be fully employed in the more labor-intensive industries 2 and 3.⁸

Thus it is not enough that countries have their endowments inside the same diversification cone. In addition they must be far enough inside of it to be able to produce the quantities of goods demanded on the world market. In the case shown in fig. 1, the danger is that if both countries are too near the opposite sides of the diversification cone, then both will need to produce larger quantities of goods of extreme factor intensity than are demanded on the world market. Correspondingly, the output of the good of intermediate factor intensity will be too small.

This technique can easily be extended to additional goods. Let there be n goods, $j=1, \dots, n$. Define X_j for each good to be the quantities demanded on the world market in the IWE, and define $v_j=(l_j, k_j)$ as the vectors of labor and capital needed to produce these quantities using the techniques of the IWE. Again these vectors could be laid end-to-end in a box like fig. 1, and they would reach exactly from O_1 to O_2 . By arranging them first in decreasing order of capital intensity (as measured by the ratio $r_j=k_j/l_j$), and then a second time in increasing order of capital intensity, two paths can be constructed between which lie all possible allocations of two countries' factor endowments that are consistent with FPE. The space between these two paths, which are shown as the solid lines in fig. 2, has roughly the shape of a lens. All of this has been shown, or at least suggested, by Dixit and Norman (1980, p. 117).

In order to extend this technique to more than two countries and more than two factors, as well as more than two goods, it is useful to reinterpret slightly what has just been said. The area in the box between the two strings of vectors – what I will call a polyhedral lens – is the set of all points that can be reached by the vectors, or by parts of them. One could also form an analogous shape using the endowment vectors of the countries. With only two of them, this would be a parallelogram; with more it too would be a polyhedral lens as shown by the broken lines in fig. 2. With two countries, it

⁸This can be verified geometrically by drawing lines parallel to v_2 and v_3 , down and to the left from point Z . See Dixit and Norman (1980, pp. 116–117).

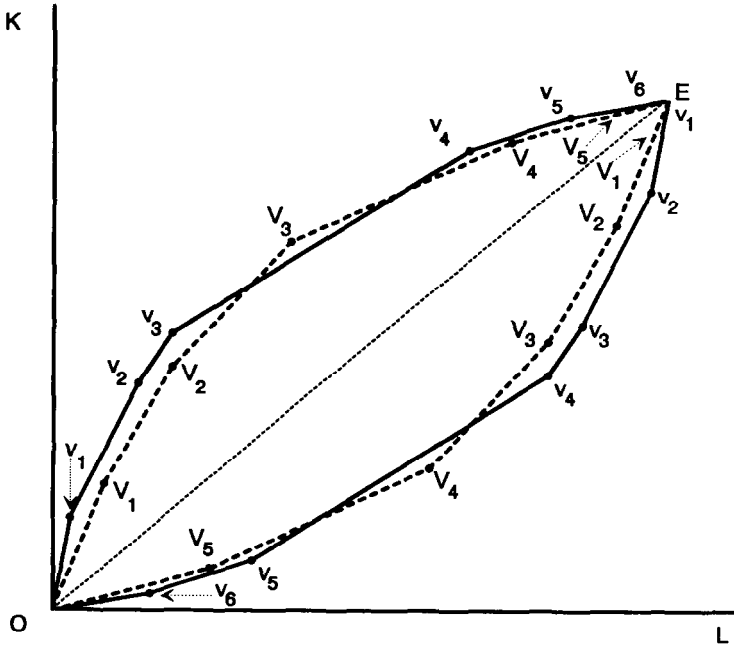


Fig. 2. Factor use (solid) and endowment (broken) lenses with six goods and five countries.

is clear that if the endowment point lies inside the factor intensity lens, then this endowment parallelogram will lie inside it as well. My claim is that even with more countries and more factors, a necessary condition for FPE is that the lens formed by the factor–endowment vectors must be a subset of the lens formed by the factor–use vectors.

To make the idea of these lenses precise, consider any collection of m n -dimensional row vectors, $a_i = (a_{i1}, \dots, a_{in})$, arranged as rows of an $m \times n$ matrix $a = (a_{ij})$. Then the (polyhedral) lens spanned by these vectors is defined by

Definition (lens):

$$\mathcal{L}(a) = \{x \in \mathbb{R}^n \mid x = ba \text{ for some } m\text{-vector } b \geq 0 \leq b_i \leq 1, \text{ where } a \text{ is } m \times n\}. \tag{1}$$

Now suppose that there are m countries indexed i , n goods indexed j , and f factors indexed h . Define the $n \times f$ matrix $v = (v_{jh})$, where v_{jh} is the amount of factor h required for production of the world demand for good j in the IWE. Similarly, define the $m \times f$ matrix $V = (V_{ih})$, where V_{ih} is country i 's endowment of factor h . Factor price equalization is possible if and only if the

rows of v can be allocated across the m countries so as to exhaust the factor endowments in V . That is, letting u_m and u_n represent vectors of lengths m and n consisting entirely of ones,

Definition (factor price equalization). The factor-use vectors in v and the factor-endowment vectors in V permit FPE only if \exists an $m \times n$ matrix $\lambda = (\lambda_{ij})$, with $0 \leq \lambda_{ij} \leq 1$, such that

$$\lambda v = V, \quad (2)$$

$$u_m \lambda = u_n. \quad (3)$$

With these definitions, it can then be shown that FPE implies that the factor-endowment lens lies wholly inside the factor-use lens:

Theorem.

$$\text{FPE} \Rightarrow \mathcal{L}(V) \subseteq \mathcal{L}(v). \quad (4)$$

Proof. Consider any vector of factors $x \in \mathbb{R}^f$ such that $x \in \mathcal{L}(V)$. From (1) \exists an m -vector b , with $0 \leq b_i \leq 1$, such that

$$x = bV. \quad (5)$$

From FPE $\exists \lambda$, with $0 \leq \lambda_{ij} \leq 1$, satisfying (2) and (3). Let

$$c = b\lambda. \quad (6)$$

Then from (5) and (2),

$$x = bV = b\lambda v = cv. \quad (7)$$

From (3), elements of c are weighted averages of elements of b . Since the latter are between zero and one, the same is true of c , and it follows from (1) that $x \in \mathcal{L}(v)$. Q.E.D.

This is a general result. With only two factors, but arbitrarily many goods and countries, the factor-use and factor-endowment lenses are easily depicted as in fig. 2. In each case, the vectors are laid end to end, in decreasing order of their slopes to get the top of the lens and in increasing order of their slopes to get the bottom of the lens. The case shown is one in

which FPE does not hold, because the endowment vectors of three of the five countries, 1, 2, and 3, extend outside of the factor–use lens.⁹

With more than two factors, the lenses are of higher dimension and cannot easily be graphed. However, for one lens to be inside another it must also be true that the projections of the first lens onto any plane must also be inside the projection of the second onto the same plane. Therefore, FPE also implies that these projections for any pair of factors will display the factor–endowment lens lying inside the factor–use lens. Thus, if a picture like fig. 2 arises for any pair of factors, with those two factor endowments for a subset of countries extending outside the limits that can be reached by factor–use vectors for those same factors, then FPE is not possible.

The theorem provides only a necessary condition for FPE, not a sufficient condition. It seems plausible to me that the same condition may in fact be sufficient for FPE, but I have as yet been unable to show that. All I know at this point is that the condition is sufficient for the case of two countries (and arbitrarily many goods and factors) as shown in the appendix, and that I have been unable to construct a counter-example in the easily graphed case of two factors (and arbitrarily many goods and countries). I would note only that the necessary condition mentioned in the preceding paragraph, involving not the full lenses but only their projections onto the various two-factor planes, is surely not sufficient for FPE since higher dimensional lenses could easily intersect without that ever showing in these projections.

3. Discussion

This result gives analytical content to the idea from the $2 \times 2 \times 2$ Heckscher–Ohlin model that FPE requires factor endowments to differ across countries by less than factor intensities differ across industries. With only two goods and two countries, this merely means that the endowments must lie in the same diversification cone. That is, the difference in endowment ratios of the two countries must be literally smaller than the difference in factor intensity ratios of the two industries. With additional goods this simple result no longer holds, as was seen in fig. 1, and with additional countries and factors the requirements of FPE become even more stringent, as was shown here. It is now necessary to compare not only the extremes of factor endowments with the extremes of factor intensities, but also to account for differences in the distribution of both endowments and intensities for goods and countries that are not extreme.

⁹In Deardorff (1991) I examined the two-factor case in more detail and showed somewhat more explicitly how the placement of point V_3 above the upper edge of the factor–use lens was inconsistent with FPE.

The comparison of the two lenses in the theorem does this. For example, if all vectors contributing to a lens pointed in the same direction, then the lens would be a straight line with no thickness. Thus if factor intensities do not differ across industries, the factor–use lens will be such a line, and any variation at all in relative factor endowments will preclude FPE. Likewise, if relative factor endowments were all the same and factor intensities were not, then the factor–endowment lens would be such a line and would necessarily lie within the factor–use lens. In either case the extent to which the lens is bowed out away from its diagonal depends on how different are the vectors that contribute to it. Thus, for the factor–endowment lens to lie wholly within the factor–use lens as required for FPE, the variation across countries in relative factor endowments must be less, in some sense, than the variation across industries of factor intensities.¹⁰

4. Conclusion

I have deliberately not attempted here to draw conclusions about the ‘likelihood’ of FPE. Such conclusions are often difficult to interpret in any case, and in this case they would require a prior sense of the likelihood, in the real world, of various distributions of factor intensities, factor endowments, and demands for goods. What is really needed is to measure these distributions and to match them against the requirement of the theorem. However, since the theorem deals with factor intensities and demands as they would appear in the IWE, and since we certainly do not observe that kind of equilibrium in the world, that empirical exercise cannot be done.

Appendix: Sufficiency with two countries

To see that $\mathcal{L}(V) \subseteq \mathcal{L}(v) \Rightarrow$ FPE in the case of two countries, n goods and f factors, let V_1 and V_2 be the endowment vectors of the two countries. $V_1 \in \mathcal{L}(V)$ since $V_1 = (10)V$, and $\dots V_1 \in \mathcal{L}(v)$. From (1) \exists n -vector b , with $0 \leq b_j \leq 1$, such that $V_1 = bv$. Define

$$\lambda = \begin{pmatrix} b \\ u_n - b \end{pmatrix}.$$

¹⁰These variations must also be weighted by levels of output in the IWE and by country size. Wu (1987) argued that FPE is made less likely if demands are greatest for goods of intermediate rather than extreme factor intensity. That is clear here, as well, where high demand for such goods in the IWE will make the factor–use lens thin. Similarly, if the largest countries tend to have extreme factor endowments, then this will thicken the factor–endowment lens and also make the condition for FPE more stringent.

It is then easily checked that the conditions for FPE are satisfied: $0 \leq \lambda_{ij} \leq 1$ by construction; $u_m \lambda = b + (u_n - b) = u_n$; and

$$\lambda v = \begin{pmatrix} bv \\ (u_n - b)v \end{pmatrix} = \begin{pmatrix} V_1 \\ V - V_1 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = V.$$

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