Planet. Space Sci. 1965, Vol. 13, pp. 1131 to 1136. Pergamon Press Ltd. Printed in Northern Ireland

# TRANSITION PROBABILITIES IN HIGHLY IONIZED $p^2$ AND $p^4$ CONFIGURATIONS

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(*Received* 28 *June* 1965)

**Abstract**—Line strengths of magnetic dipole and electric quadrupole transitions in the configurations $2p^2$, $2p^4$, $3p^2$ and $3p^4$ are calculated using interpolated values of spin-orbit interaction parameters and unperturbed energies of the levels. Probabilities of the important ground state transitions are tabulated in the four isoelectronic sequences through the following net charges: $17(2p^2)$, $13(2p^4)$, $15(3p^2)$, $13(3p^4)$.

We present the results of calculations of probabilities of magnetic dipole and electric quadrupole transitions for high members of the carbon, oxygen, silicon and sulphur isoelectronic sequences. The computations are based on interpolations of the form

$$\theta^{1/3} = \alpha Z + \beta \qquad \sigma^{1/3} = \alpha' Z + \beta'$$

where, in Garstang's[1] notation,

$$\theta = \frac{\zeta}{DP} \qquad \sigma = \frac{\zeta}{SP}$$

TABLE 1. CARBON SEQUENCE

| Ion | Magnetic dipole transition probabilities ($sec^{-1}$) | | | | | Electric quadrupole transition probabilities ($sec^{-1}$) | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $^1S_0$–$^3P_1$ | $^1D_2$–$^3P_2$ | $^1D_2$–$^3P_1$ | $^3P_2$–$^3P_1$ | $^3P_1$–$^3P_0$ | $^1S_0$–$^1D_2$ | $^1S_0$–$^3P_2$ | $^1D_2$–$^3P_0$ | $^3P_2$–$^3P_0$ |
| C* | 0·$^2$26 | 0·$^3$23 | 0·$^4$78 | 0·$^6$27 | 0·$^7$78 | 0·50 | 0·$^4$19 | 0·$^7$55 | 0·$^{13}$20 |
| N+* | 0·$^1$34 | 0·$^3$30 | 0·$^3$103 | 0·$^5$74 | 0·$^5$21 | 1·08 | 0·$^3$16 | 0·$^6$42 | 0·$^{11}$13 |
| O+$^2$* | 0·23 | 0·$^1$21 | 0·$^2$71 | 0·$^4$97 | 0·$^4$26 | 1·6 | 0·$^3$71 | 0·$^5$19 | 0·$^{10}$35 |
| F+$^3$* | 1·1 | 0·$^1$98 | 0·$^1$34 | 0·$^3$78 | 0·$^3$20 | 2·1 | 0·$^2$23 | 0·$^5$64 | 0·$^9$50 |
| Ne+$^4$* | 4·2 | 0·38 | 0·14 | 0·$^3$45 | 0·$^3$13 | 2·6 | 0·$^2$68 | 0·$^4$19 | 0·$^8$52 |
| Na+$^5$ | 13·4 | 1·20 | 0·44 | 0·$^1$21 | 0·$^2$60 | 3·1 | 0·$^1$169 | 0·$^4$47 | 0·$^7$38 |
| Mg+$^6$ | 37 | 3·3 | 1·26 | 0·$^1$79 | 0·$^1$26 | 3·5 | 0·$^1$36 | 0·$^3$107 | 0·$^6$23 |
| Al+$^7$ | 93 | 8·4 | 3·4 | 0·26 | 0·$^1$94 | 4·0 | 0·$^1$79 | 0·$^3$22 | 0·$^5$118 |
| Si+$^8$ | 210 | 19·1 | 8·1 | 0·76 | 0·31 | 4·4 | 0·150 | 0·$^3$45 | 0·$^5$52 |
| P+$^9$ | 370 | 33 | 15·0 | 1·98 | 0·96 | 4·5 | 0·22 | 0·$^3$75 | 0·$^4$20 |
| S+$^{10}$ | 860 | 82 | 40 | 4·7 | 2·7 | 4·7 | 0·47 | 0·$^2$154 | 0·$^4$69 |
| Cl+$^{11}$ | 1610 | 157 | 82 | 10·3 | 7·1 | 4·9 | 0·80 | 0·$^2$26 | 0·$^3$22 |
| Ar+$^{12}$ | 2900 | 290 | 166 | 23 | 17·7 | 4·9 | 1·31 | 0·$^2$44 | 0·$^3$66 |
| K+$^{13}$ | 4900 | 510 | 320 | 42 | 42 | 4·9 | 2·0 | 0·$^2$72 | 0·$^2$185 |
| Ca+$^{14}$† | 7800 | 860 | 580 | 77 | 95 | 4·9 | 3·1 | 0·$^1$14 | 0·$^3$48 |
| Sc+$^{15}$ | 13,000 | 1400 | 1090 | 138 | 200 | 5·0 | 4·7 | 0·$^1$167 | 0·$^1$126 |
| Ti+$^{16}$ | 20,000 | 2200 | 1940 | 260 | 410 | 4·9 | 6·7 | 0·$^1$24 | 0·$^1$33 |

* Garstang.[1]
† Line strengths from Garstang.[8]

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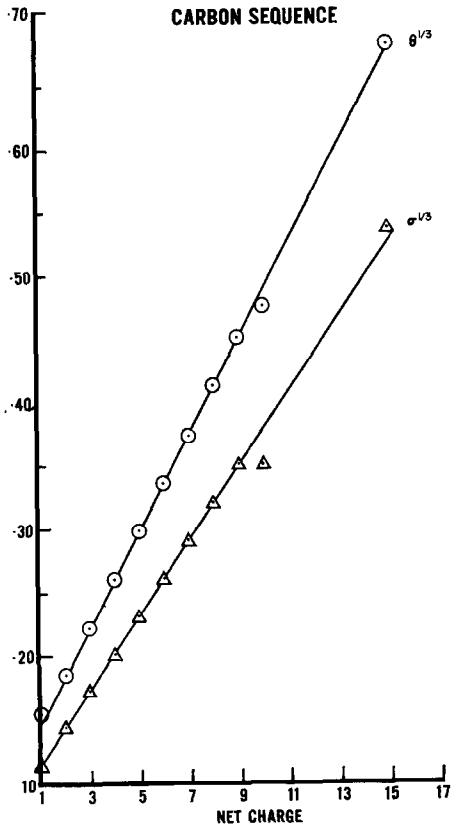


FIGS. 1–4. THE VARIATION OF $\theta^{1/3}$ AND $\sigma^{1/3}$ ALONG FOUR ISOELECTRONIC SEQUENCES.

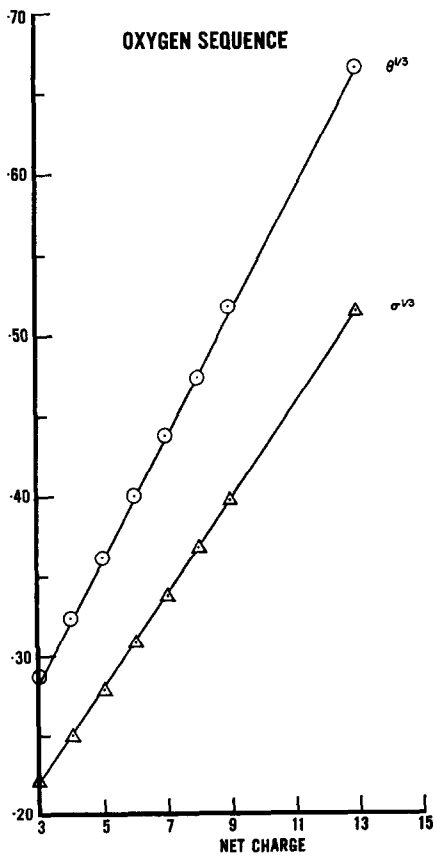


FIG. 2.

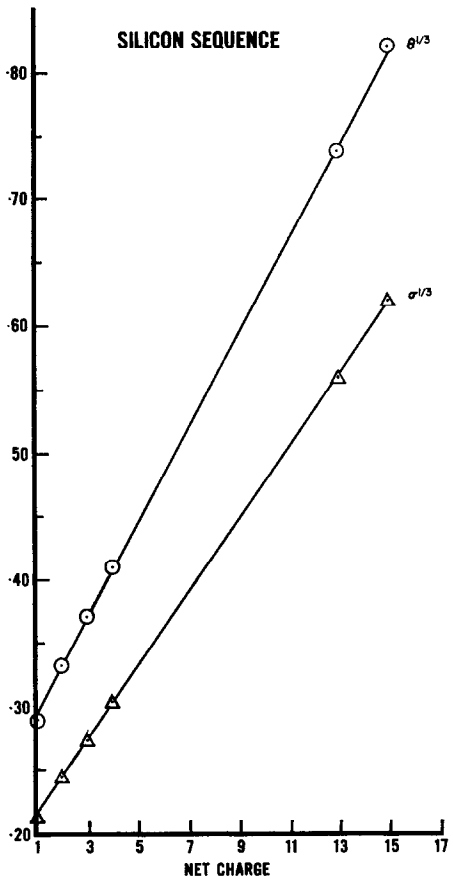
SILICON SEQUENCE
$\theta^{1/3}$
$\sigma^{1/3}$
·80
·70
·60
50
·40
·30
·20
1
3
5
7
9
11
13
15
17
NET CHARGE

FIG. 3.

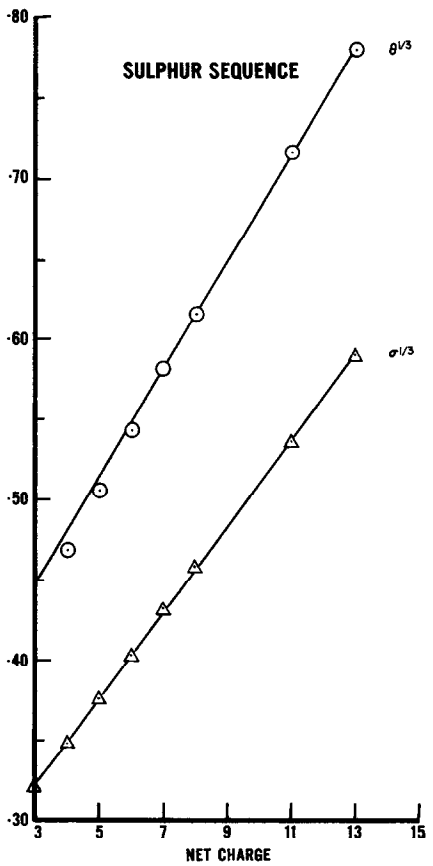
SULPHUR SEQUENCE
$\theta^{1/3}$
$\sigma^{1/3}$
·80
·70
·60
·50
·40
·30
3
5
7
9
11
13
15
NET CHARGE

FIG. 4.

TABLE 2. OXYGEN SEQUENCE

| Ion | Magnetic dipole transition probabilities ($sec^{-1}$) | | | | | Electric quadrupole transition probabilities ($sec^{-1}$) | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $^1S_0$–$^3P_1$ | $^1D_2$–$^3P_2$ | $^1D_2$–$^3P_1$ | $^3P_0$–$^3P_1$ | $^3P_1$–$^3P_2$ | $^1S_0$–$^1D_2$ | $^1S_0$–$^3P_2$ | $^1D_2$–$^3P_0$ | $^3P_0$–$^3P_2$ |
| O* | 0·178 | 0·269 | 0·222 | 0·417 | 0·489 | 1·28 | 0·837 | 0·511 | 0·9103 |
| F+* | 0·49 | 0·144 | 0·1138 | 0·818 | 0·889 | 2·1 | 0·816 | 0·541 | 0·818 |
| Ne+2* | 2·2 | 0·20 | 0·160 | 0·2113 | 0·261 | 2·8 | 0·851 | 0·412 | 0·720 |
| Na+3 | 7·6 | 0·66 | 0·197 | 0·255 | 0·130 | 3·5 | 0·1121 | 0·430 | 0·6148 |
| Mg+4 | 23 | 1·96 | 0·56 | 0·122 | 0·125 | 4·2 | 0·127 | 0·467 | 0·888 |
| Al+5 | 61 | 5·3 | 1·44 | 0·171 | 0·45 | 4·8 | 0·156 | 0·3132 | 0·543 |
| Si+6 | 148 | 12·9 | 3·3 | 0·194 | 1·45 | 5·5 | 0·105 | 0·325 | 0·4182 |
| P+7 | 340 | 29 | 6·8 | 0·47 | 4·2 | 6·2 | 0·178 | 0·346 | 0·468 |
| S+8 | 710 | 65 | 14·2 | 0·95 | 11·4 | 6·9 | 0·33 | 0·376 | 0·322 |
| Cl+9 | 1410 | 129 | 25 | 1·95 | 28 | 7·7 | 0·51 | 0·3125 | 0·368 |
| Ar+10 | 2700 | 250 | 44 | 3·0 | 67 | 8·6 | 0·78 | 0·320 | 0·3181 |
| K+11 | 4900 | 460 | 71 | 3·6 | 149 | 9·6 | 1·14 | 0·330 | 0·344 |
| Ca+12† | 8560 | 840 | 107 | 3·6 | 330 | 11·0 | 1·60 | 0·342 | 0·111 |

* Garstang.[1]
† Garstang.[9]

TABLE 3. SILICON SEQUENCE

| Ion | Magnetic dipole transition probabilities ($sec^{-1}$) | | | | | Electric quadrupole transition probabilities ($sec^{-1}$) | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $^1S_0$–$^3P_1$ | $^1D_2$–$^3P_2$ | $^1D_2$–$^3P_1$ | $^3P_2$–$^3P_1$ | $^3P_1$–$^3P_0$ | $^1S_0$–$^1D_2$ | $^1S_0$–$^3P_2$ | $^1D_2$–$^3P_0$ | $^3P_2$–$^3P_0$ |
| Si | 0·1355 | 0·2269 | 0·3964 | 0·4417 | 0·5818 | 0·798 | 0·2114 | 0·6624 | 0·9356 |
| P+* | 0·22 | 0·1168 | 0·2625 | 0·3374 | 0·4824 | 1·95 | 0·8630 | 0·532 | 0·8606 |
| S+2* | 0·85 | 0·1635 | 0·1246 | 0·2241 | 0·3467 | 2·54 | 0·1163 | 0·587 | 0·747 |
| Cl+3* | 2·61 | 0·195 | 0·1796 | 0·2819 | 0·2211 | 3·15 | 0·38 | 0·422 | 0·628 |
| Ar+4* | 6·82 | 0·512 | 0·221 | 0·1271 | 0·2796 | 3·78 | 0·1811 | 0·451 | 0·5132 |
| K+5 | 15·7 | 1·14 | 0·53 | 0·176 | 0·126 | 4·1 | 0·141 | 0·3110 | 0·554 |
| Ca+6 | 34 | 2·5 | 1·22 | 0·197 | 0·176 | 4·3 | 0·25 | 0·321 | 0·4189 |
| Sc+7 | 67 | 5·0 | 2·7 | 0·46 | 0·21 | 4·5 | 0·41 | 0·338 | 0·460 |
| Ti+8 | 125 | 9·5 | 5·6 | 0·98 | 0·53 | 4·6 | 0·66 | 0·365 | 0·8174 |
| V+9 | 220 | 17·5 | 11·3 | 1·92 | 1·28 | 4·6 | 1·02 | 0·2108 | 0·346 |
| Cr+10 | 380 | 31 | 22 | 3·5 | 3·0 | 4·5 | 1·54 | 0·2170 | 0·2116 |
| Mn+11 | 630 | 53 | 41 | 6·0 | 6·5 | 4·4 | 2·3 | 0·226 | 0·227 |
| Fe+12 | 1000 | 87 | 72 | 9·6 | 13·8 | 4·3 | 3·2 | 0·241 | 0·261 |
| Co+13 | 1570 | 145 | 132 | 14·3 | 28 | 4·1 | 4·7 | 0·251 | 0·1132 |
| Ni+14 | 2400 | 230 | 230 | 22 | 56 | 3·9 | 6·7 | 0·261 | 0·128 |

* Czyzak and Krueger.[5]

Values of the spin orbit interaction parameter, $\zeta$, and the intervals, *DP* and *SP*, between the unperturbed terms were obtained from Garstang,[1] Edlen,[2] Rohrlich.[4] Figures 1–4 show the variation of the known values of $\theta^{1/3}$ and $\sigma^{1/3}$ along the four sequences together with the interpolations used in the calculations.

Estimates of the quadrupole moment

$$s_q = \frac{2}{5}\int_0^\infty r^2 p^2\, dr,$$

required for the determination of the electric quadrupole line strengths, were obtained by extrapolating

$$s_q^{-1/2} = \alpha'' Z + \beta''$$

TABLE 4. SULPHUR SEQUENCE

| Ion | Magnetic dipole transition probabilities ($sec^{-1}$) | | | | | Electric quadrupole transition probabilities ($sec^{-1}$) | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $^1S_0$–$^3P_1$ | $^1D_2$–$^3P_2$ | $^1D_2$–$^3P_1$ | $^3P_0$–$^3P_1$ | $^3P_1$–$^3P_2$ | $^1S_0$–$^1D_2$ | $^1S_0$–$^3P_2$ | $^1D_2$–$^3P_0$ | $^3P_0$–$^3P_2$ |
| S* | 0·350 | 0·$^1$1273 | 0·$^3$800 | 0·$^2$295 | 0·$^2$139 | 1·78 | 0·$^2$731 | 0·$^5$495 | 0·$^7$709 |
| Cl+* | 1·34 | 0·10 | 0·$^1$1292 | 0·$^2$143 | 0·$^2$753 | 2·29 | 0·$^1$18 | 0·$^4$120 | 0·$^6$478 |
| Ar+2* | 4·02 | 0·32 | 0·$^1$829 | 0·$^2$512 | 0·$^1$306 | 3·10 | 0·$^1$425 | 0·$^4$290 | 0·$^5$272 |
| K+3 | 10·4 | 0·83 | 0·20 | 0·$^1$146 | 0·104 | 3·9 | 0·$^1$86 | 0·$^4$60 | 0·$^4$122 |
| Ca+4 | 24 | 1·92 | 0·43 | 0·$^1$35 | 0·31 | 4·6 | 0·156 | 0·$^3$112 | 0·$^4$45 |
| Sc+5 | 50 | 4·1 | 0·83 | 0·$^1$71 | 0·83 | 5·3 | 0·26 | 0·$^3$191 | 0·$^3$142 |
| Ti+6 | 97 | 8·4 | 1·53 | 0·133 | 2·0 | 6·1 | 0·45 | 0·$^3$29 | 0·$^3$41 |
| V+7 | 181 | 16·0 | 2·6 | 0·21 | 4·7 | 6·9 | 0·66 | 0·$^3$46 | 0·$^3$108 |
| Cr+8 | 320 | 29 | 4·0 | 0·30 | 10·3 | 7·6 | 0·92 | 0·$^2$72 | 0·$^2$26 |
| Mn+9 | 560 | 50 | 6·1 | 0·29 | 22 | 8·5 | 1·24 | 0·$^3$111 | 0·$^2$56 |
| Fe+10 | 920 | 86 | 8·8 | 0·28 | 43 | 9·3 | 1·60 | 0·$^3$168 | 0·$^1$121 |
| Co+11 | 1470 | 145 | 12·4 | 0·164 | 84 | 10·2 | 2·1 | 0·$^2$25 | 0·$^1$24 |
| Ni+12 | 2300 | 240 | 16·8 | 0·$^1$72 | 156 | 11·0 | 2·5 | 0·$^2$37 | 0·$^1$48 |
| Cu+13 | 3400 | 380 | 22 | | 280 | 11·8 | 3·0 | | |
| Zn+14 | 4900 | 610 | 29 | | 490 | 12·7 | 3·4 | | |

* Czyzak and Krueger.[5]

TABLE 5. EXTRAPOLATED QUADRUPOLE MOMENTS

| Net charge | $2p^2$ | $2p^4$ | $3p^2$ | $3p^4$ |
|---|---|---|---|---|
| 4 | | 0·234 | | 0·804 |
| 5 | | 0·176 | | 0·649 |
| 6 | 0·177 | 0·138 | 0·671 | 0·536 |
| 7 | 0·138 | 0·110 | 0·543 | 0·449 |
| 8 | 0·111 | 0·$^1$905 | 0·449 | 0·382 |
| 9 | 0·$^1$912 | 0·$^1$756 | 0·377 | 0·329 |
| 10 | 0·$^1$760 | 0·$^1$641 | 0·321 | 0·286 |
| 11 | 0·$^1$645 | 0·$^1$550 | 0·277 | 0·252 |
| 12 | 0·$^1$554 | 0·$^1$477 | 0·241 | 0·223 |
| 13 | 0·$^1$481 | 0·$^1$418 | 0·212 | 0·198 |
| 14 | 0·$^1$420 | | 0·188 | 0·178 |
| 15 | 0·$^1$372 | | 0·167 | 0·161 |
| 16 | 0·$^1$331 | | | |
| 17 | 0·$^1$296 | | | |

using values of $s_q$ determined by Garstang[1] and Czyzak and Krueger[5] from self-consistent field wave functions. The extrapolated quadrupole moments, which provide the greatest source of errors in our transition probabilities, are given in Table 5. When better values of $s_q$ become available the electric quadrupole probabilities can be accordingly revised. In the case of Si, a quadrupole moment, $s_q = 3{\cdot}40$, was calculated from the wave function of neutral silicon given by Herman and Skillman.[7]

Magnetic dipole and electric quadrupole probabilities were determined by the usual methods[1,6] using the wave numbers estimated by Rohrlich and Pecker,[3] and are given in Tables 1–4. These tables contain also the calculations of Garstang[1,8,9] and of Czyzak and Krueger[5] for some members of the sequences.

*Acknowledgment*—This work was supported in part by the Alfred P. Sloan Foundation Research Fellowship of one of the authors for which we are very grateful.

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**Резюме**—Вычисляются силовые линии магнитного диполя и электрические четырехполюсные переходы в конфигурации $2p^2$, $2p^4$, $3p^2$ и $3p^4$ используя интерполированные значения параметров спино-орбитального взаимодействия, а также невозмущенные энергии уровней. Возможность значительных переходов наземного состояния расположены в виде диаграммы в четырех изоэлектронных последовательностях посредством следующих зарядов нетто: $17(2p^2)$, $13(2p^4)$, $15(3p^2)$, $13(3p^4)$.