THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

A STABILITY STUDY OF AN ATOMIC POWER PLANT

Robert W. Albrecht

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I. INTRODUCTION

A. Description of Problem

This paper presents a relatively simple, straight forward analysis of the stability of a nuclear power plant. The analysis is undertaken for only one operating condition; namely full power with a constant flow rate.

This is the type of study which should be carried out early in the design or feasibility stage of power plant development because it yields some approximate information concerning stability, and is valuable for making early decisions as to necessary parameter adjustments before the design has proceeded beyond the point of no return. This type of study would also serve as a rough guide to control engineers who must decide upon the degree of complexity which will be required in the control system. The analysis undertaken here would also serve as a guide in making a more comprehensive computer study as the design proceeds.

As an example, the method has been used to study the Enrico Fermi Power Plant.

B. General Characteristics of Enrico Fermi Reactor Plant

The Enrico Fermi Atomic Power Plant is a fast breeder power reactor which is designed for a power output of 300 megawatts, with 268 megawatts being released in the reactor core and 32 megawatts in the blanket. The coolant is liquid sodium with a total flow rate of 13,200,000 pounds per hour, resulting in a coolant temperature rise of about 250°F across the core.

Heat is removed from the reactor core and blanket by the primary coolant, transferred to the secondary coolant in the intermediate heat exchangers, and then transferred to water and steam in once-through steam generators. The overall system is composed of three primary coolant loops having a common point in the reactor vessel and three independent secondary coolant loops having no common hydraulic point.

The primary system sodium flows by gravity from the free surface pool of the reactor upper chamber to the shell side of the intermediate heat exchanger and then to the pump tank. The sodium is pumped from the pump tank back to the reactor.

The secondary coolant system is the intermediate link that transfers the heat from the primary coolant system to the steam generator and consists of the tube side of the intermediate heat exchanger, the steam generator, and the centrifugal sump-type pump.

The steam generators are counter-flow, shell and tube, once-through type of units with water and steam in the tubes and sodium on the shell side of the tubes. Electricity is then generated in a conventional steam-turbine generator.

II. APPROXIMATIONS AND LIMITATIONS

This stability analysis uses the concepts of linear servomechanism theory, and is therefore subject to the usual limitations of this technique. Some of the limitations are: 1) Because of the linearity assumption, this analysis is only valid for small perturbations in δK about a nominal operating power level, 2) loading between transfer functions is not considered, 3) since the study does not use a computer, it is necessary to represent distributed components of the system by point transfer functions.

Further approximations have been made in this study in order to facilitate the use of a mathematical model of the system which can be handled expediently by analytical methods.

Some of the more serious of these approximations are listed below:

1) Reactor core and blanket are represented by point functions.

2) Intermediate heat exchanger and secondary piping represented by point functions.

3) Frequency dependent portion of core upper structure and core lower structure is neglected.

4) Fuel element and coolant reactivity coefficients are both taken as a function of the average temperature of the coolant (for both core and blanket).

5) Core and blanket are represented as independent point functions.

6) Only one equivalent delayed neutron group is considered.

7) Coolant transport is represented by the transport term e^{-st} and the mixing term $\frac{1}{1 + st.}$.

III. ILLUSTRATION OF TYPICAL POINT TRANSFER FUNCTION

In order to illustrate the derivation of a typical point transfer function, the reactor core was chosen as a suitable example. In the derivation below, the heat transfer equations are written, Laplace transform notation is employed, some manipulations are carried out, and then the transfer function of the core is displayed as a block diagram.

A. Symbols to be used:

M_f = mass of fuel

C_f = specific heat of fuel

$\overline{T_f}$ = average temperature of fuel element

Q_f = heat generated in core

A = heat transfer area

U = heat conductivity

$\overline{T}_{cc}$ = average temperature of core coolant

M_{cc} = mass of core coolant

C_{cc} = specific heat of core coolant

W = weight flow rate through core

T_{cc} in = inlet temperature of core coolant

T_{cc} out = outlet temperature of core coolant

B. Derivation of transfer function: (ref. 3)

1. Heat transfer equations.

a) Rate of heat increase in core = heat content of core - heat transfer from core

$$M_f C_f \frac{d\,\overline{T}_f}{dt} = Q_f - AU(\overline{T}_f - \overline{T}_{cc})$$

b) Rate of heat increase of coolant = heat transfer from core + heat increase due to incoming sodium - heat decrease due to outgoing sodium.

$$M_{cc} C_{cc} \frac{d\,\overline{T}_{cc}}{dt} = AU(\overline{T}_f - \overline{T}_{cc}) + W\,C_{cc}\,T_{cc\,in} - W\,C_{cc}\,T_{cc\,out}$$

c) Definition of $\overline{T}_{cc}$

$$\frac{T_{cc\,in} + T_{cc\,out}}{2}$$

rewriting the heat transfer equations;

$$a)\quad \frac{d\overline{T}_f}{dt} = \frac{Q_f}{M_f C_f} - \frac{AU}{M_f C_f}\overline{T}_f + \frac{AU}{M_f C_f}\overline{T}_{cc}$$

$$b)\quad \frac{d\overline{T}_{cc}}{dt} = \frac{AU}{M_{cc}C_{cc}}\overline{T}_f - \frac{AU}{M_{cc}C_{cc}}\overline{T}_{cc} + \frac{2W}{M_{cc}}T_{cc\ in} - \frac{2W}{M_{cc}}\overline{T}_{cc}$$

$$c)\quad \overline{T}_{cc} = \frac{T_{cc\ in} + T_{cc\ out}}{2}$$

now, defining the time constants;

$$\frac{AU}{M_f C_f} = \frac{1}{t_1};\quad \frac{AU}{M_{cc}C_{cc}} = \frac{1}{t_2};\quad \frac{2W}{M_{cc}} = \frac{2}{t_o}$$

then, substituting these into the heat transfer equations and changing to Laplace transform notation;

$$a)\quad \overline{T}_f + t_1 S\,\overline{T}_f = \overline{T}_{cc} + t_1 \frac{Q_f}{M_f C_f}$$

$$b)\quad \overline{T}_{cc}\left(1 + 2\frac{t_2}{t_o}\right) + t_2 S\,\overline{T}_{cc} = \overline{T}_f + \frac{2t_2}{t_o}T_{cc\ in}$$

$$c)\quad \frac{T_{cc\ in} + T_{cc\ out}}{2} = \overline{T}_{cc}$$

now, eliminating $\overline{T}_f$ between equations a) and b), we get;

$$\overline{T}_{cc}\left(1 + \frac{2t_2}{t_o}\right) + t_2 S\,\overline{T}_{cc} = \frac{\overline{T}_{cc} + t_1 \frac{Q_f}{M_f C_f}}{1 + S\,t_1} + \frac{2t_2}{t_o}\,T_{cc\ in}$$

now, substituting c) into the above and separating $T_{cc\ in}$ and $T_{cc\ out}$ and arranging;

$$T_{cc\ out}\left\{\frac{t_1 t_o}{2}S^2 + \left[\frac{t_o}{2}\left(\frac{t_1}{t_2} + 1\right) + t_1\right]S + 1\right\} = T_{cc\ in}\left\{-\frac{t_1 t_o}{2}S^2 - \left[\frac{t_o}{2}\left(\frac{t_1}{t_2} + 1\right) - t_1\right]S + 1\right\} + \frac{t_o t_1}{t_2}\frac{Q_f}{M_f C_f}$$

so, solving for $T_{cc\ out}$;

$$T_{cc\ out} = \frac{\left\{-\frac{t_1 t_o}{2} S^2 - \left[\frac{t_o}{2}\left(\frac{t_1}{t_2}+1\right) - t_1\right] S + 1\right\} T_{cc\ in} + \frac{t_o t_1}{t_2} \frac{Q_f}{M_f C_f}}{\frac{t_1 t_o}{2} S^2 + \left[\frac{t_o}{2}\left(\frac{t_1}{t_2}+1\right) + t_1\right] S + 1}$$

factoring the polynomials appearing above, the equation becomes;

$$T_{cc\ out} = \frac{1}{\left(1+\frac{S}{\alpha_h}\right)\left(1+\frac{S}{\alpha_h'}\right)} \left[\left(1-\frac{S}{\alpha_c}\right)\left(1+\frac{S}{\alpha_c'}\right) T_{cc\ in} + \frac{t_o t_1}{2} \frac{Q_f}{M_f C_f}\right]$$

so, the core transfer function can be represented by the block diagram;

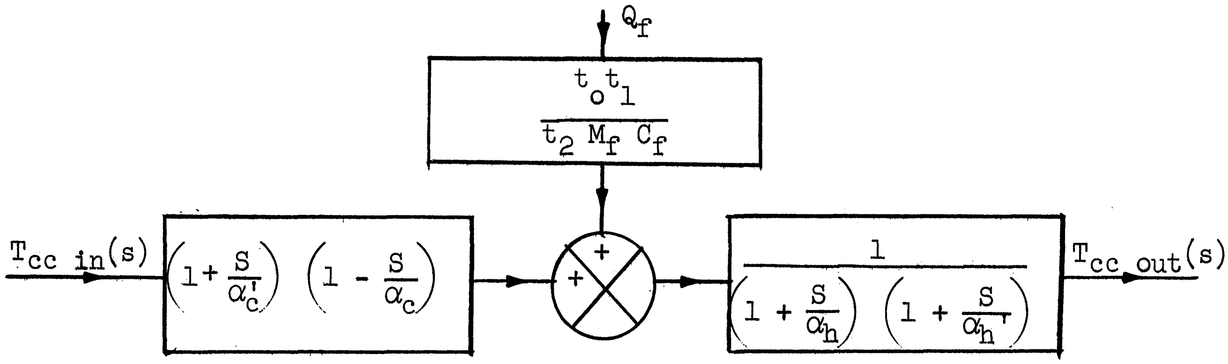


Figure 1

Note that in the above representation, the heat generated in the core is artificially fed into the center of the core between the transfer functions which deal with the transport of coolant through the core. The α's and t's above are functions only of the parameters of the system and the operating condition being considered. $T_{cc\ in}(s)$ is a function of the coolant flow throughout the power plant and the value of Q_f is a function of the neutron kinetics which in turn is a function of the temperatures in the system and the reactivity coefficients.

IV. THE REPRESENTATION OF THE SYSTEM

The transfer functions of all parts of the system must be derived from basic heat transfer equations and assembled into a block diagram which represents the entire power plant system. Time limitations prevent the discussion of all of these transfer functions, so the system block diagram will merely be presented. Figure 2 shows the system block diagram and Figure 3 shows an enlargement of the block entitled "Plant Thermal System."

The block diagram of Figure 2 shows δK affecting both the core and blanket neutron kinetics. Thus, the outlet temperatures of the reactor ($T_{BC\ out}$ & $T_{cc\ out}$) are functions of both the inlet temperature and the heat generated in fuel and blanket elements. The reactor coolant is then fed into the plant thermal system and thus back into the reactor inlet.

It can be seen that four reactivity coefficients are considered. The core element and coolant reactivity coefficient is taken as a function of the average temperature of the core coolant, the blanket element and coolant reactivity coefficient is taken as a function of the blanket coolant average temperature, and the upper and lower structure reactivity coefficients are taken as a function of the coolant temperature of the upper and lower structure respectively.

The block diagram of Figure 3 shows the primary piping transfer functions, the intermediate heat exchanger, the secondary sodium piping and the steam generator. For this study the transfer function of the steam generator was taken as a constant attenuation and a transport delay.

In order to investigate stability by use of the Nyquist criterion, the open loop system must be considered. In order to do this, the loop is broken at the reactivity feedback summer, and the open loop system is drawn as an open-loop signal flow diagram (ref. 1). This signal flow diagram is shown in Figure 4.

V. SOLVING FOR THE OPEN-LOOP FUNCTION

By investigating the forward gain paths and the closed loops of the signal flow diagram the open-loop system function can be solved for by use of a standard form (ref. 4). The closed loops are shown in Figure 5, and the forward gain paths are shown in Figure 6. Using standard techniques, (ref. 4) the open loop function is found by finding the function.

$$KG = \frac{F_1(1 - E_2) + F_2 + F_3(1 - E_1) + F_4(1 - E_1) + F_5 + F_6}{1 - E_1 - E_2}$$

where the F's and the E's are shown in Figures 5 and 6.

The above open loop function is solved for by drawing a Bode plot for each forward gain path transfer function and each closed loop transfer function and then manipulating these plots to arrive at a Bode plot of the open-loop system function, KG. This plot is then transformed into a Nyquist plot of the open-loop function. The Nyquist plot for this power plant is shown in Figure 7.

VI. CONCLUSIONS OF THIS STUDY

The Nyquist plot of the open-loop function indicates stability for the operating condition considered. The plot also indicates that at a low value of gain (power level) and the same flow rate, an instability could occur.

The purpose of this analysis is just to get a feeling for system behavior and to serve as a guide for future computer studies; so in this case a study of this nature would indicate to a designer that the possibility of instability should be investigated more thoroughly at low power levels.

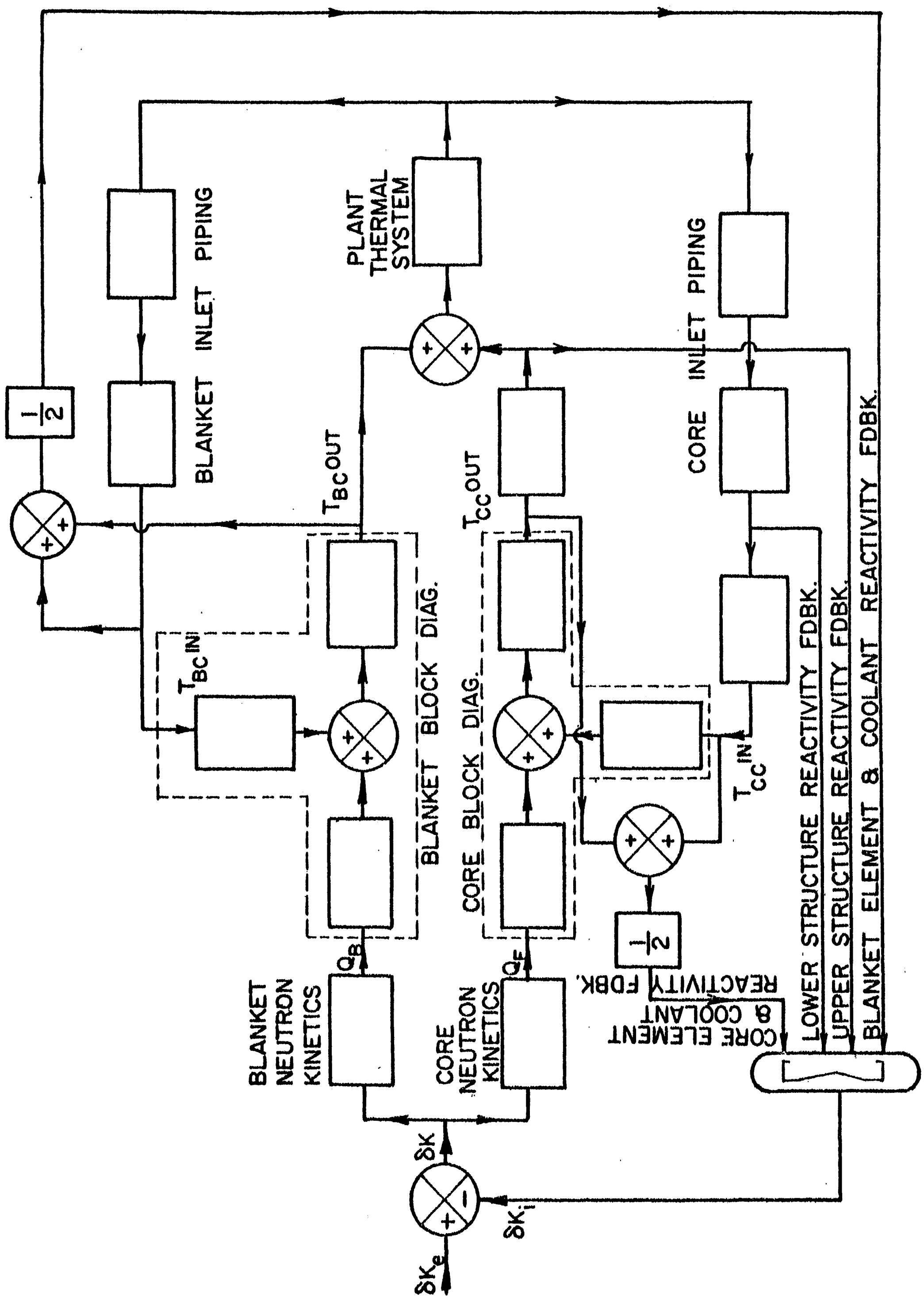
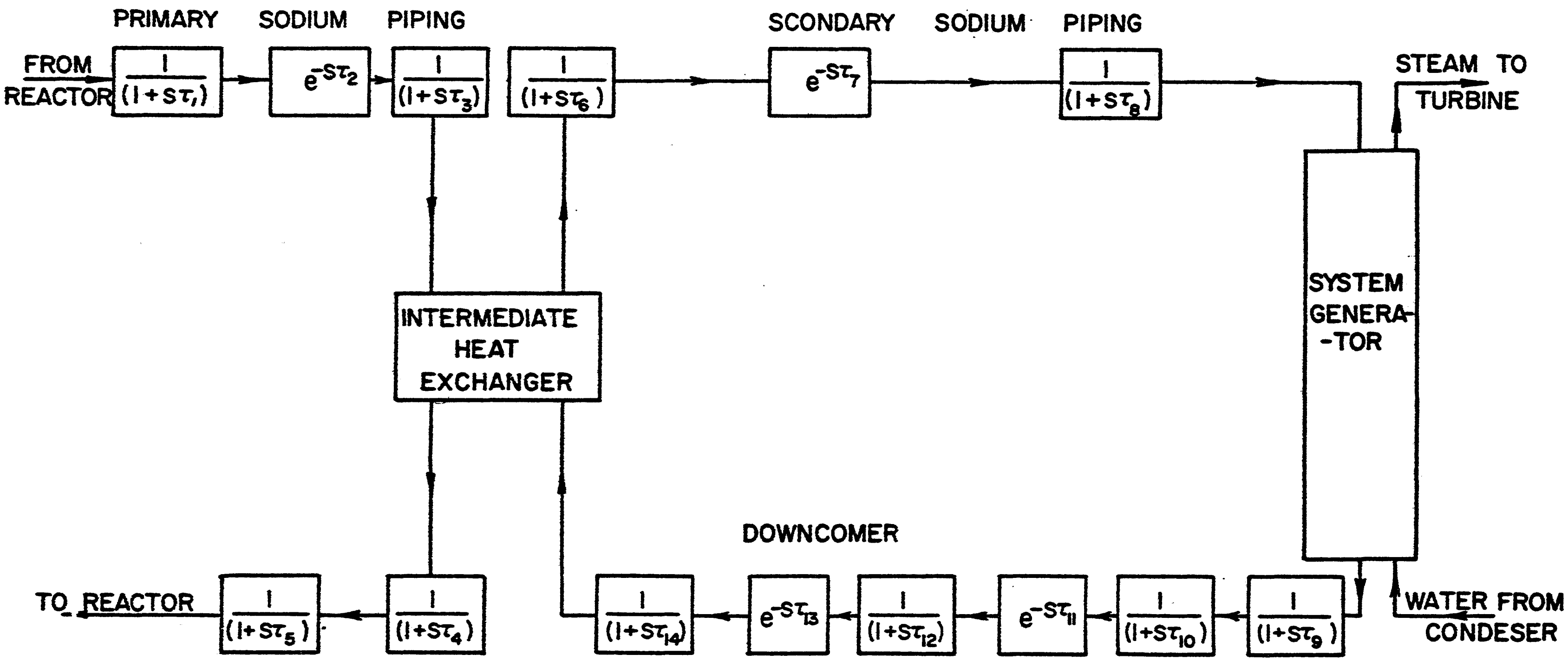


Figure 2. Servo Block-Diagram of Enrico Fermi Plant



τ_1 = 16.56 SEC.		τ_8 = 0.535 SEC.	
τ_2 = 14.08 SEC.		τ_9 = 2.25 SEC.	
τ_3 = 2.04 SEC.		τ_{10} = 5.8 SEC.	
τ_4 = 2.90 SEC.		τ_{11} = 5.87 SEC.	
τ_5 = 6.63 SEC.		τ_{12} = 1.83 SEC.	
τ_6 = 1.19 SEC.		τ_{13} = 1.16 SEC.	
τ_7 = 9.49 SEC.		τ_{14} = 0.67 SEC.	

Figure 3. Plant Thermal System Block Diagram.

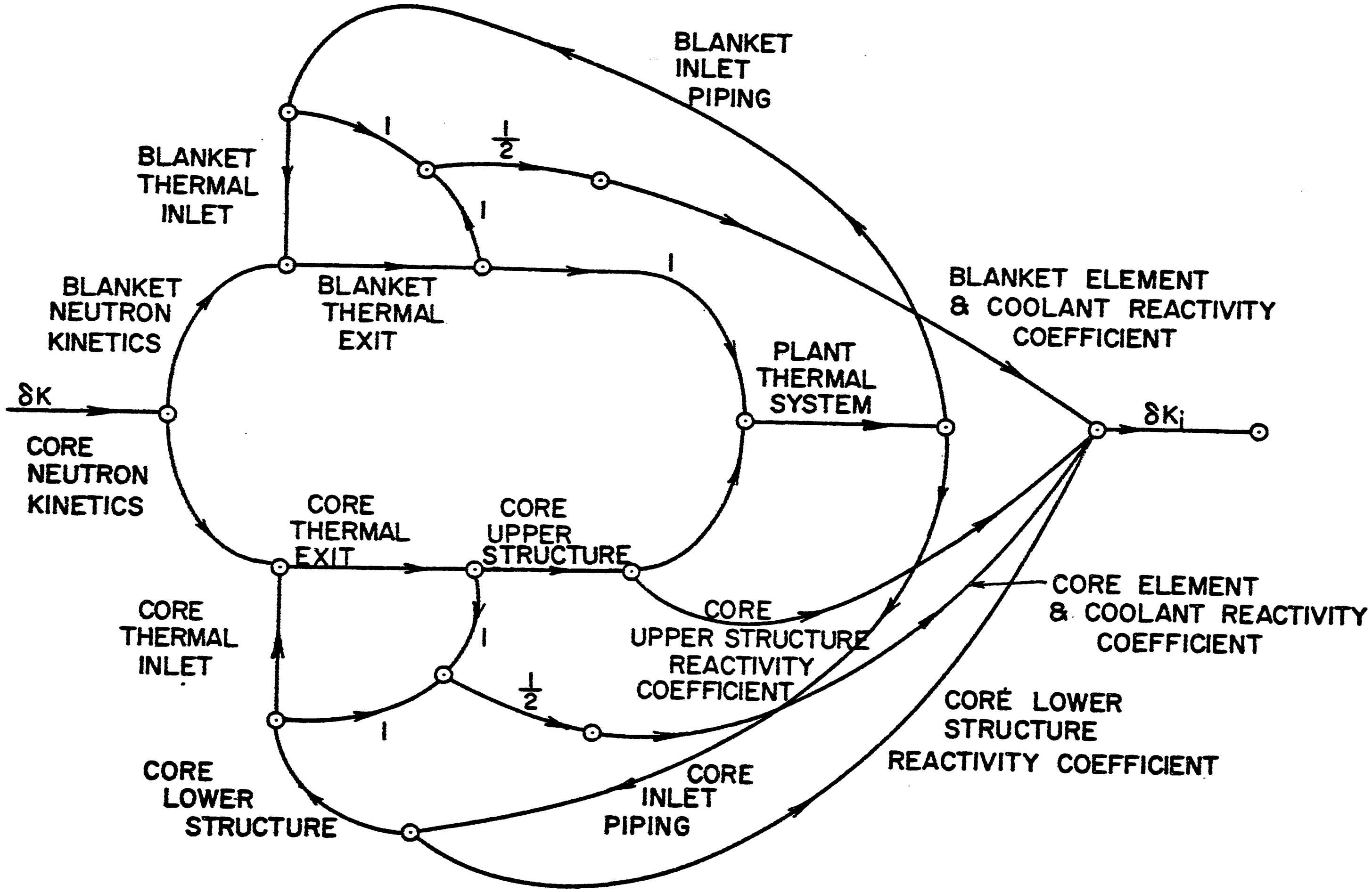


Figure 4. Open-Loop Signal Flow Chart.

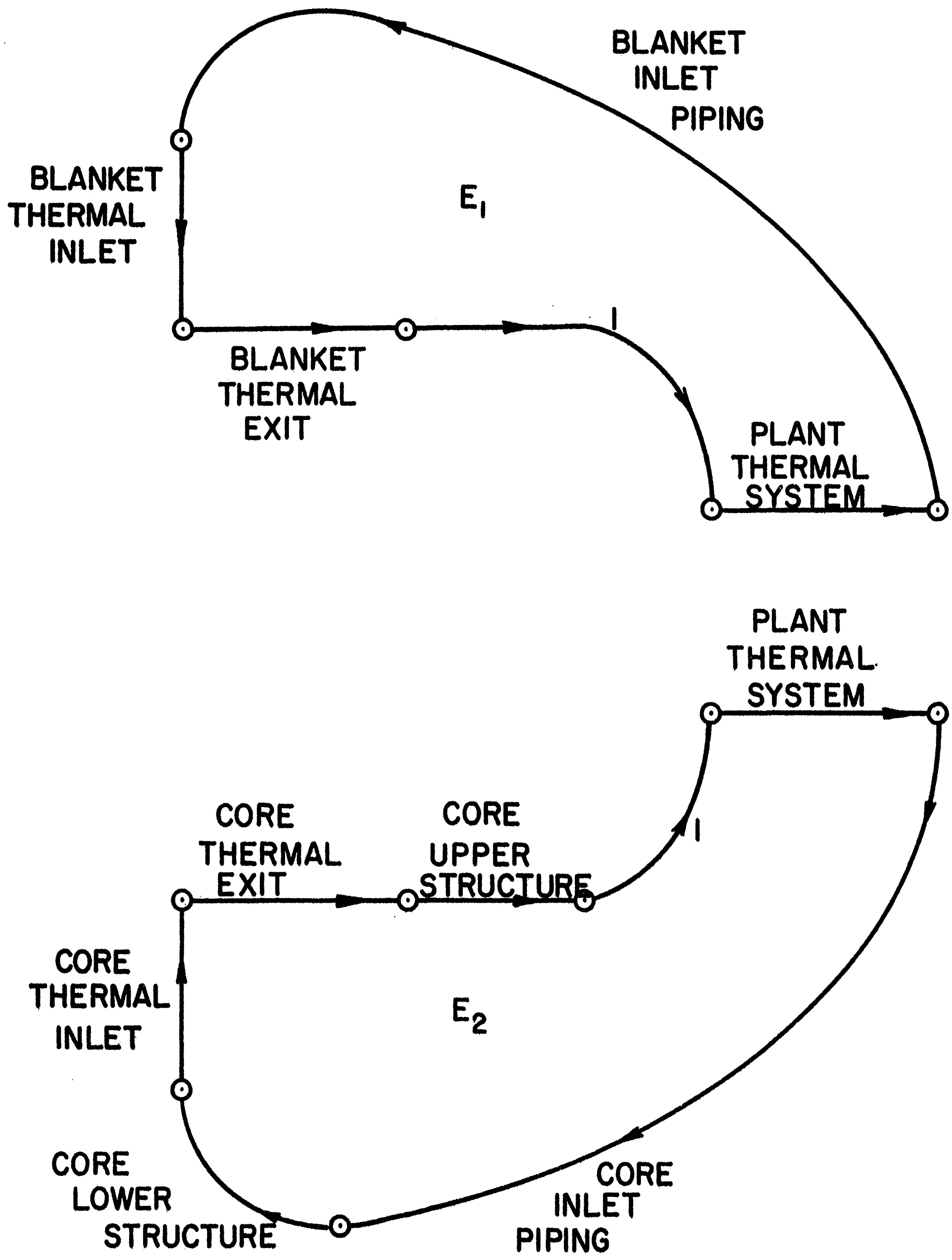


Figure 5. Untouching Loops from Signal Flow Chart.

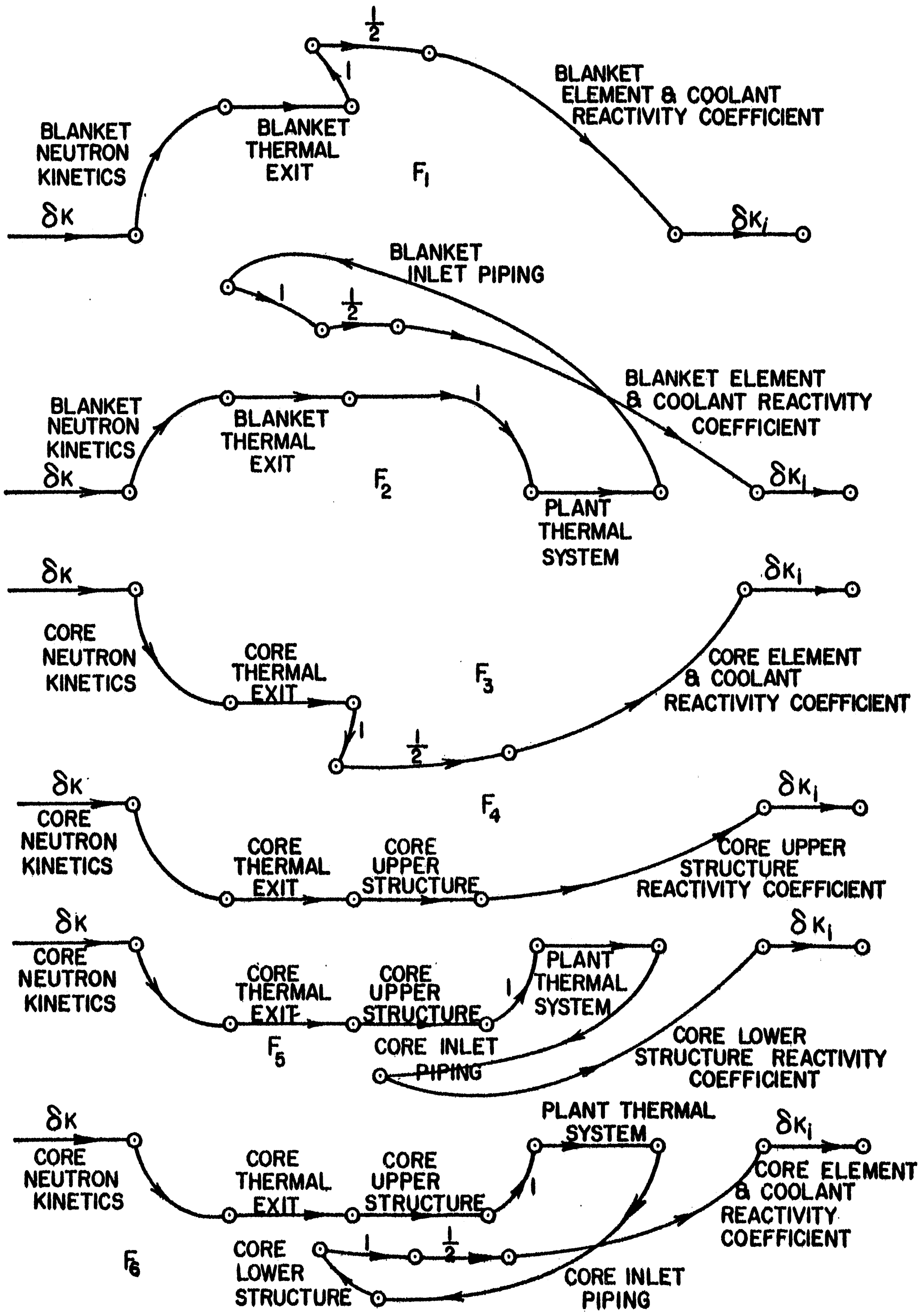


Figure 6. Forward Gain Paths from Signal Flow Chart.

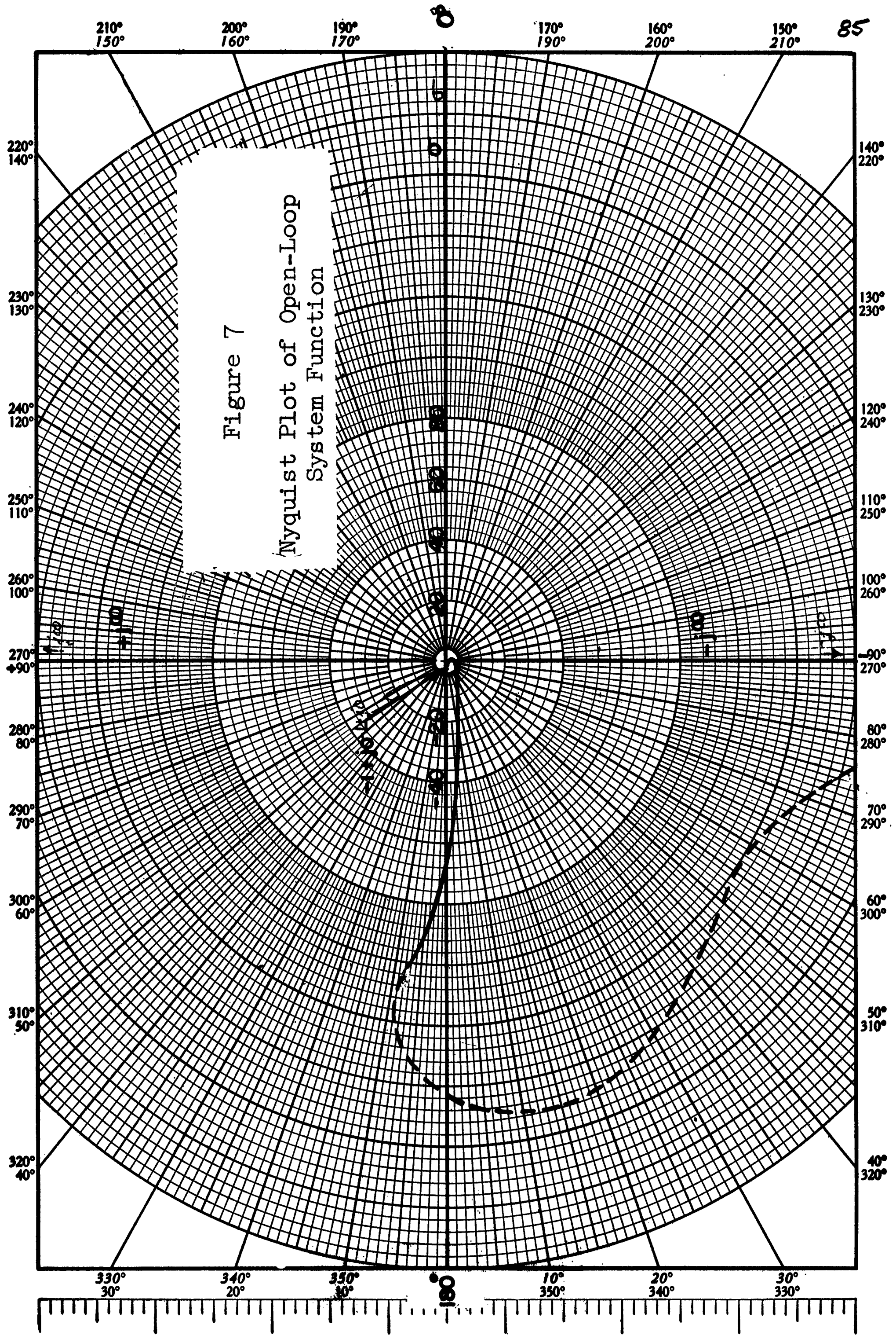


Figure 7
Nyquist Plot of Open-Loop System Function

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