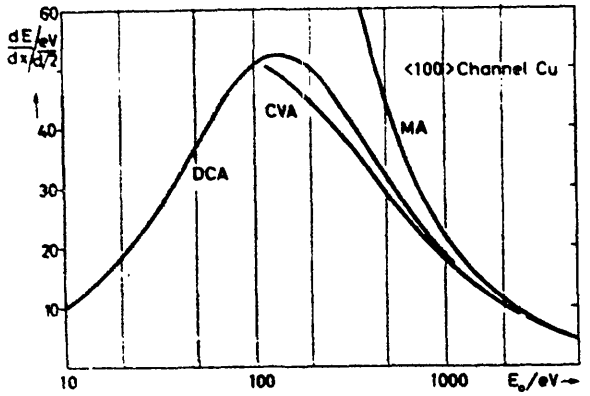

Fig. 1. d*E*/d*x* for a Cu atom in the center of the ⟨100⟩ channel in Cu. *d* = lattice constant. Potential [4]: Gibson 2.

by making use of Robinson's tables [7]. Fig. 1 shows the result for a Cu atom in the ⟨100⟩ channel in Cu. Together with these results (DCA) we plot ΔE in the constant velocity approximation (CVA) suggested by Weijsenfeld [5], where the deflections of ring atoms by a *uniformly* moving projectile are calculated in an exact way. This approach requires ΔE to be small, but fig. 1 shows that the agreement between our results and the constant velocity approximation is still good, when this assumption is no more valid at all.

Fig. 1 indicates that the momentum approximation is somewhat better than in the pure two-particle collision, but nevertheless breaks down below 1 keV. It is furthermore seen that ΔE_{max} is slightly greater than 50 eV, so that some ring atoms might receive approximately the displacement energy $E_d \approx 25$ eV.

Calculations were also made on the energy loss of ⟨111⟩ collision sequences in Cu. According to Nelson and Thompson [1] these should be focusing at energies below about 300 eV. The momentum approximation used in this calculation turns out to overestimate scattering considerably in this energy range. Applications to other focusing rows and channels are being done and will be published shortly.

It is a pleasure to thank Professor J. Linhard for many enlightning remarks and for his great interest in this work. Drs. C. H. Weijsenfeld has been so kind to permit reference to unpublished material. We express our gratitude to lektor E. Bork for providing excellent working conditions at Strandkrogen, Fakse Ladeplads.

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ATTAINMENT OF HIGH RESOLUTIONS IN HOLOGRAPHY BY MULTI-DIRECTIONAL ILLUMINATION AND MOVING SCATTERERS

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The possibility of attaining high resolutions in X-ray holography was pointed out in our previous paper [1]. Here was describe in particular how the phase in high-resolution holograms [1] can be made recordable in interferometric arrangements by illuminating a stationary object from a number of directions, for instance by means of a *moving* mirror (or scatterer), provided that the coherent background is stationary with respect to the object. Such illumination permits one to solve the "phase problem" in high-resolution holography by directing the ordinarily unrecordable high-order diffracted waves (representing high-resolution spatial frequencies in the object) into the direction of the zero order, where both the amplitude and the phase in the scattered electromagnetic field can be losslessly recorded.

Three methods for attaining high resolutions, in excess of those associated with conventional holography [2 - 5] * have now been investigated by us theoretically, and confirmed experimentally, in view of X-ray extensions. In (i) a focussing element leads to the high-resolution Fraunhofer holograms of eq. (4) and the Fourier-transform reconstruction of our previous paper [1]. In (ii) a scheme such as heterodyning of a spherical reference beam against the diffracted field also leads to Fraunhofer holograms. Method (iii) discussed here solves the phase problem in high-resolution holography by multi-directional illumination.

The theory of multi-directional illumination in holography is given by means of the following one-dimensional model. The theory applies to moving and stationary mirrors, or scatterers, except for intensity scale factors. Only a (temporal)steady-state coherence between the object and the coherent background is required [6]. Let $T(\xi)$ be an X-ray diffracting structure whose complex amplitude transmittance is given by the Fourier series in eq. (1) in our ref. [1]. With an "ideal" scatterer placed on the source side next to the object, the transmittance through the object becomes

$$T'(\xi) = \mathrm{rect}(\xi/l) \sum_m \exp(-\mathrm{i}\theta_m - \mathrm{i}m\beta\xi) \times \\ \times \sum_n A_n \exp(\mathrm{i}\varphi_n + \mathrm{i}n\beta\xi) \quad (1)$$

where θ_m are the random phases describing the scatterer, $2l(2\pi/\beta)$ is the length(period) of the structure, and $A_n\exp(\mathrm{i}\varphi_n)$ are its Fourier coefficients. The diffracting structure will act as a diffraction grating for each of the plane-wave components in the field originating from the scatterer and produce a diffraction order for each term in the series of eq. (1). With a coherent plane-wave reference beam (wavenumber k) superposed at an angle $\theta = \alpha/k$ on the diffracted field in the conventional way, the amplitude of the field indicent on the photographic plate will be proportional to (see our ref. [1])

$$A_0\exp(-\mathrm{i}\alpha x) + \sum_{n,m} \mathrm{rect}[x/l - f(n-m)\beta/kl] \times \\ \times A_n \exp\{\mathrm{i}[\varphi_n - \theta_m + (n-m)\beta\xi - f(n-m)^2\beta^2/2k)]\}, \quad (2)$$

where f is the distance between the diffracting plane ξ and the hologram plane x. In X-ray holography each term in the double sum of eq. (2), *except* those with $(n-m) = 0$, will generally be averaged out by the photographic plate [1]. In this case, the portion of the incident field intensity which actually exposes the photographic plate will be proportional to

$$A_0^2 - 2A_0 \sum_n \mathrm{rect}(x/l)\, A_n \cos(\alpha x + \varphi_n - \theta_n) + \\ + |A_S(x)|^2, \quad (3)$$

where $|A_S(x)|^2$, the square of the magnitude of the second term in eq. (2), is generally negligible. The remarkable feature of the result in eq. (3) is that the phases φ_n of the diffracting structure have now been made recordable. In high-resolution X-ray holography, recording may in addition involve some generalization of Buerger's two-wavelength microscopy principles [1], or the use of method (i) or (ii). The fact that the random phases θ_n are also present in eq. (3) means only that in the reconstruction process the diffracting structure will appear in the way it would have appeared if it had been observable directly.

Fig. 1 shows a magnified (3 times) image of a two-dimensional grating reconstructed from a hologram illuminated through a *moving* scatterer (see fig. 2; recording and reconstruction in 6328 Å laser light). Point-reference interferometric backgrounds were apparently first introduced for microscopy by Nomarski [8, 9], and independently by Dyson [10]. The quality of the reconstructed image and the degree to which the "grain" in the rather coarse scatterer used has been averaged out by the motion during the exposure are quite apparent. The three-dimensional character of the image shows the use of the scatterer at some small distance behind the object. Of course, not any scatterer or diffuser, stationary or moving, placed at arbitrary distance between the source and the object will lead to high-resolution holograms, but indeed only arrangements which permit one to bring the high-order diffracted spectra down towards the zero order may be used.

* In projection holography the linear resolution ϵ attainable with a source-pinhole of radius r, and a photographic emulsion having a resolution capability of N lines/unit length, can be shown to be limited by the inequality $\epsilon^{-1} < (r^{-1} + N)$ (J. Winthrop, private communication).

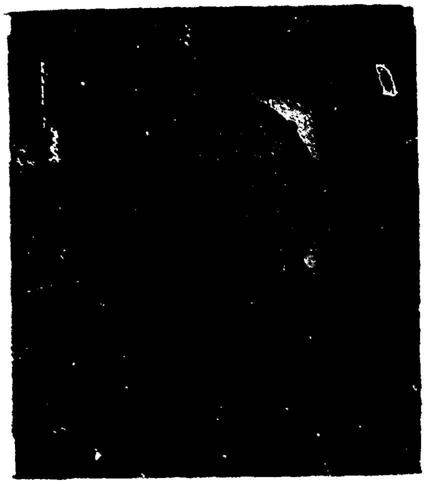

Fig. 1.

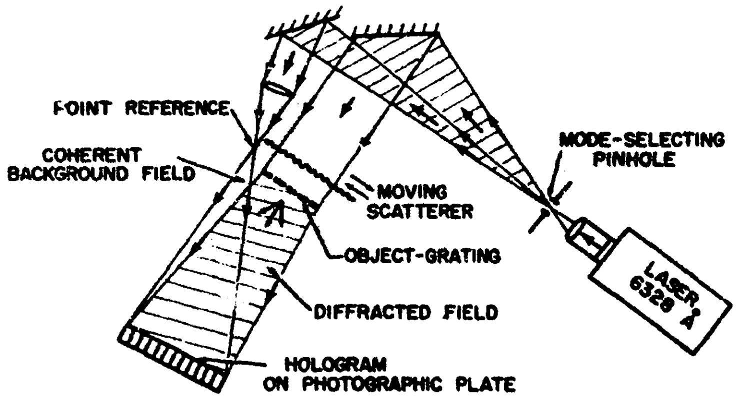

Fig. 2.

In a subsequent paper the use of the extremely coherent Mössbauer sources [11] and of luminosity-increasing matched reticles [12] as source-apertures in X-ray holography will be discussed. Possibilities of astronomical applications will be dealt with in separate papers. We wish to acknowledge the kind assistance of D. Brumm and A. Funkhouser with the experimental work in our laboratory.

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DRIFT MOBILITY OF LARGE POLARONS IN THE INTERMEDIATE COUPLING REGION

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An intermediate coupling calculation of the drift mobility of large polarons has been carried out [1], using Kubo's formula for the frequency dependent electrical conductivity together with a modified L. L. P. polaron model. The modification of the L. L. P. polaron model consists in applying the unitary transformation, given by the unitary operator:

$$U = \exp[-\Sigma_{kq}\, a^+_q\, a_{q+q}\, (F^*_{-q}\, b_{-q} - F_q\, b^+_q)]\ , \qquad (1)$$