$$\langle\sigma_E\rangle/\sigma_s = E_{c.m.}^{-1} \exp[-3.27\,(E_{c.m.} - 1.88)]\,, \tag{1}$$

where $\langle\sigma_E\rangle$ and σ_s do not include diffraction elastic scattering. We will take $\sigma_s = 3 \times 10^{-26}$ cm^2 independent of energy.

In fig. 1 we plot the quantity $d\langle\sigma_E\rangle/d\Omega$ where

$$d\langle\sigma_E\rangle/d\Omega = (3 \times 10^{-26}/2\pi)(\langle\sigma_E\rangle/\sigma_s)\ .$$

We also plot the experimental data from ref. [1] for comparison. It should be recalled that the statistical model does not predict or even address the question of the angular distribution of products. As may be seen from the figure, the agreement with the statistical prediction is surprisingly good, particularly if we consider the large angle points where $\theta_{c.m.} \gtrsim 50^{\circ}$. Probably a more sophisticated evaluation of the sticking probability and inclusion of angular momentum information in the statistical model are necessary in order to make a more significant comparison. Perhaps the strongest statement that can be made is that the statistical prediction is as satisfactory an explanation as any other alternative at this time.

Note added in proof: G. Cocconi has independently reached similar conclusions (private communication).

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$\pi^{\pm}$-p AND p - p ELASTIC SCATTERING AT 8.5, 12.4 AND 18.4 GeV/c

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An experiment on elastic scattering, using the spark chamber technique, has been carried out at the CERN proton synchrotron. In this paper results are given for π^--p and p-p elastic scattering at incident momenta of 8.5, 12.4 and 18.4 GeV/c, and for π^+-p at 8.5 and 12.4 GeV/c. For these three incident momenta, the measurements cover a range of values for the square of the four-momentum transfer $|t|$ between 0.12 - 1.15, 0.12 - 2.36 and 0.18 - 4.86 (GeV/c)^2. The limits on $|t|$ were determined by the geometry, which covered all angles between 1.25° and 7.45°, and by the limited range of the recoil proton in the hydrogen target. Only measurements for $|t|$ values smaller than 1.6 (GeV/c)^2 are reported in this paper.

Besides the intrinsic importance of obtaining data on the fundamental π-p and p-p processes, the interest in these phenomena was enhanced by the theoretical predictions of Regge pole theory [1]. In the limit of high energies and small momentum transfers, the Pomeranchuk trajectory (vacuum trajectory) was expected to dominate. The elastic scattering of any two elementary particles was

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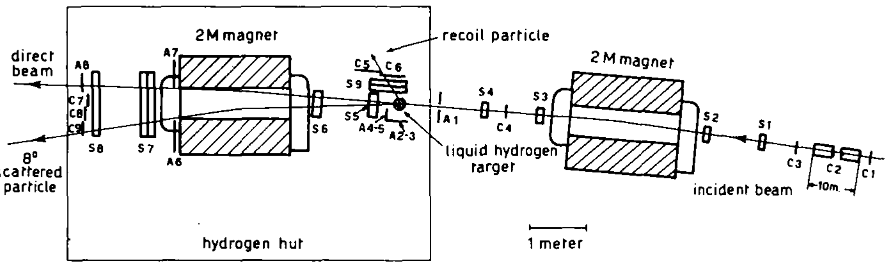

Fig. 1. Experimental layout. Spark chambers S_1 - S_4 detect the incident particle, S_5 - S_8 the scattered particle and S_9 the recoil proton. C_2 is a 10 m long threshold Čerenkov counter. The counters C_1 - C_4 define the incident beam, C_5 and C_6 detect the recoil proton and C_7 - C_9 the scattered particle. A_1 - A_8 are anticoincidence counters.

then predicted to show a logarithmic shrinking of the diffraction pattern with increasing energy. This behaviour was found in high-energy p-p scattering by Diddens et al. [2], and confirmed by Foley et al. [3]. Semi-classically this is interpreted as an increase of the radius of the interaction. The behaviour of the relevant cross sections measured so far nevertheless indicate that the present Regge pole theory is over-simplified.

Our results, based on about one third of our data, confirm previously reported observations from other experiments on the non-shrinking of the $\pi^{\pm}$-p diffraction peaks [3-5]. We find no significant difference in the behaviour of π^+-p scattering compared with π^--p scattering. Our results confirm the shrinking of the p-p diffraction peak between 8.5 and 18.4 GeV/c if the data at all t values are included. Within the accuracy of our data the diffraction pattern for $|t| < 0.5$ (GeV/c)^2 does not shrink significantly in this energy range, but it is definitely narrower than for incident momenta below 7 GeV/c.

The experimental layout is shown in fig. 1. The beam, from an integral target, had a momentum spread of $\pm$ 2.5%. To ensure the correct particle assignment, a 10 m long hydrogen gas threshold Čerenkov counter, C_2, was incorporated in the beam layout, and used in coincidence for π-p scattering, in anticoincidence for p-p scattering. The pion beams had a muon contamination of (6 $\pm$ 1)% at 8.5 GeV/c, (5 $\pm$ 1)% at 12.4 and 18.4 GeV/c incident momentum. The beam was focused onto a cylindrical liquid hydrogen target of 20 cm diameter with 0.25 mm thick mylar walls.

A scintillation counter system preselected the possible elastic scattering events. It consisted essentially of a triple coincidence of the incident, scattered, and recoil particles, in addition to an anticoincidence from a counter which limited the incident beam to 5 cm horizontal and 2.5 cm vertical cross section. The azimuthal angular interval accepted was between 14° and 20°. The momenta of incident and scattered particles are determined by magnetic deflection to an accuracy of $\pm$ 1.5%. The 18 views of the chambers (one horizontal and one vertical view for each chamber) were brought together in a single picture by a system of 38 plane mirrors. The camera imaged an object plane at 17 m distance with a linear demagnification of 55 onto 35 mm film. Spherical field lenses were used over the larger spark chambers S_5 - S_9 to reduce parallax.

Events not obviously inelastic were measured by means of digitized scanning tables. The position of a track is located to a precision of about 0.5 mm, its direction to a precision of about 0.7 mrad. We measured the momenta of the incident and scattered particle, together with the angles of the scattered particle and recoil proton. Thus, the two-body kinematics is twice over-determined. The kinematics are calculated using only the measured incident particle momentum and the scattered angle of the pion. The selection criteria for elastic events are the measured deviations from the calculated scattered particle momentum and proton angle. Limits are placed on the distributions of these two quantities corresponding to approximately three times their standard deviations. The inelastic contamination within these limits changes from 1.5% at $|t| = 0.13$ to 11% at $|t| = 1.0$ (GeV/c)^2 for all energies. The scanning efficiency is checked to be better than 98%. Pictures with two or more incident particles are not measured and a correction is applied for this.

Due to the shape of the target and the incident beam distribution, the recoil proton needs a minimum initial energy of 50 MeV in order to always reach the counter C_5 or C_6. This is the case for laboratory scattering angles larger than 2.2° at 8.5 GeV/c, 1.5° at 12.4 GeV/c and 1° at 18.4 GeV/c. A correction factor has been calculated to take into account the probability that the proton makes a nuclear interaction between its point of origin and its arrival at counter C_5 or C_6. This correction is largest at 8.5 GeV/c where it is 3.3% for the smallest angles. An additional correction for particle loss between counter C_4 and counters C_7 - C_9

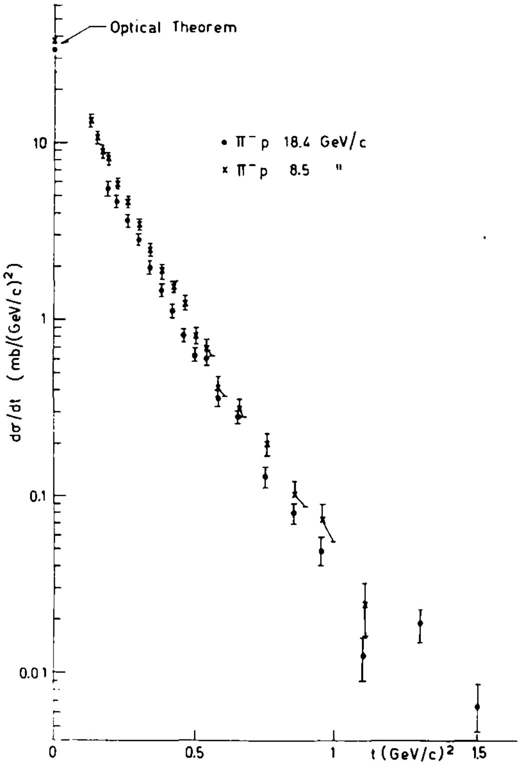

Fig. 2. π^--p elastic scattering differential cross sections at 8.5 and 18.4 GeV/c.

amounts to 3.7% of the incident beam for $\pi^{\pm}$-p scattering and to 5.2% for p-p scattering.

The results of this experiment are at present based on approximately 3000 events for each momentum and each type of incident particle. As an example, the differential cross sections for π^--p scattering at 8.5 and 18.4 GeV/c are shown in fig. 2 as $d\sigma/dt$ versus $|t|$. The $|t|$ scale is uncertain by about 2%. It is estimated that the absolute cross sections have an uncertainty of ± 7%.

The data have been fitted by the least mean squares method with the functions

$$d\sigma/dt = e^{a+bt} \text{ mb}/(\text{GeV}/c)^2 \tag{1}$$

and

$$d\sigma/dt = e^{A+Bt+Ct^2} \text{ mb}/(\text{GeV}/c)^2 \tag{2}$$

The results of this analysis applied to our data are given in tables 1 and 2.

For a limited $|t|$ range, e.g. $0.13 < |t| < 0.5$ (GeV/c)^2, the fit of our data to eq. (1) gives quite a reasonable χ^2. For the complete $|t|$ range, eq. (2) is required, e.g., C is significantly different from zero.

The deviation of the diffraction patterm from a pure exponential can be represented by the dimensionless quantity C/B^2, which is the second coefficient in the expansion

$$\log\left[\frac{d\sigma/dt}{(d\sigma/dt)_{t=0}}\right] = Bt\left[1 + \frac{C}{B^2}(Bt) + \ldots\right]. \tag{3}$$

The values for C/B^2 of table 2 show that the deviation from a pure exponential is small. The errors are calculated under the assumption that the errors in B and C are uncorrelated and are therefore probably overestimated.

To describe the behaviour of the cross sections in the region of small t values, we choose the radius of interaction, which for an exponential diffraction pattern is usually defined as

$$r = 2\sqrt{-b}\ . \tag{4}$$

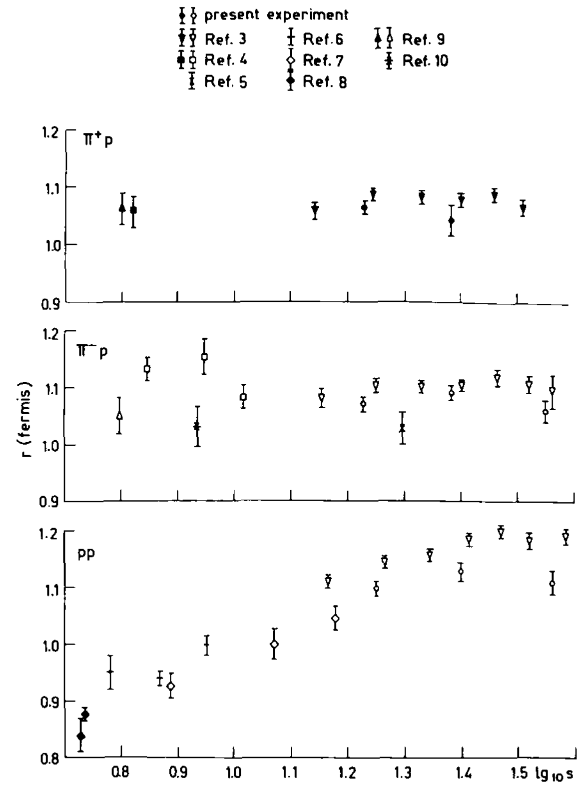

Fig. 3. Radii of interaction for π^--p, π^+-p and p-p elastic scattering against $\log_{10} s$; the radius is defined as $r = 2\sqrt{-b}$ where b is obtained with a purely exponential fit of the data over the region $0.13 < |t| < 0.5$.

In fig. 3 the results for r from our experiment are shown as a function of $\log_{10} s$ and compared with those from other experiments. The data taken from the literature have been reanalysed so that they apply to approximately the same t interval as our data ($0.13 < |t| < 0.5$). While the radii for π^--p and π^+-p interaction remain constant, the p-p interaction radius increases with s.

The description of the slope of the diffraction pattern in terms of r is based only on data at low t values. A test for the shrinking or non-shrinking,

Table 1
Results of the best fit analysis according to formula (1).
Errors represent standard deviations independent of the goodness of fit (internal errors).

	Laboratory pion momentum (GeV/c)	s (GeV)^2	$\lvert t\rvert$ range (GeV/c)^2	$-b$ (GeV/c)$^{-2}$	$r = 2\sqrt{-b}$ (fm)
π^-	8.5	16.8	0.13 - 0.50	7.33 ± 0.18	1.069 ± 0.013
	12.4	24.2	0.13 - 0.50	7.64 ± 0.18	1.091 ± 0.013
	18.4	35.4	0.19 - 0.50	7.21 ± 0.28	1.060 ± 0.021
π^+	8.5	16.9	0.13 - 0.50	7.23 ± 0.17	1.062 ± 0.012
	12.4	24.2	0.13 - 0.50	6.93 ± 0.38	1.039 ± 0.028
p	8.5	17.8	0.13 - 0.50	7.74 ± 0.19	1.098 ± 0.013
	12.4	25.1	0.13 - 0.50	8.15 ± 0.24	1.127 ± 0.016
	18.4	36.3	0.19 - 0.50	7.86 ± 0.30	1.107 ± 0.021

Table 2
Results of the best fit analysis according to formula (2).

	Laboratory pion momentum (GeV/c)	$\lvert t\rvert$ range (GeV/c)^2	A	$-B$ (GeV/c)$^{-2}$	C (GeV/c)$^{-4}$	$\frac{C}{B^2}$
π^-	8.5	0.13 - 1.10	3.608 ± 0.068	8.42 ± 0.37	1.78 ± 0.41	0.025 ± 0.006
	12.4	0.13 - 1.50	3.504 ± 0.059	8.62 ± 0.30	1.84 ± 0.30	0.025 ± 0.004
	18.4	0.19 - 1.50	3.361 ± 0.100	8.55 ± 0.32	1.97 ± 0.25	0.027 ± 0.004
π^+	8.5	0.13 - 1.10	3.509 ± 0.061	7.94 ± 0.35	1.47 ± 0.40	0.023 ± 0.007
	12.4	0.13 - 1.30	3.166 ± 0.138	7.42 ± 0.70	1.13 ± 0.74	0.021 ± 0.014
p	8.5	0.13 - 1.10	4.264 ± 0.067	8.34 ± 0.40	1.01 ± 0.47	0.014 ± 0.007
	12.4	0.13 - 1.50	4.250 ± 0.074	9.59 ± 0.38	2.24 ± 0.38	0.024 ± 0.005
	18.4	0.19 - 1.50	4.095 ± 0.134	9.14 ± 0.55	1.16 ± 0.51	0.014 ± 0.006

based on the data at all t can be obtained from the behaviour of $d\sigma/dt/[\sigma^2(s)/16\pi]$ at fixed t as a function of $\log_{10} s$ [2,3].

For large values of s the dependence on s of the shape of the diffraction peak is usually expressed by the function $\alpha(t)$ in the parametrisation

$$\log\,[d\sigma/dt/\frac{\sigma^2(s)}{16\pi}] = K + [2\alpha(t)-2]\,\log s\ , \qquad (5)$$

which is suggested by the Regge pole theory.

From our results we find

$$\begin{aligned} \text{p-p} \quad & \alpha(t) = (0.89\pm0.10) - (0.45\pm0.22)\,t \\ \pi^-\text{-p} \quad & \alpha(t) = (0.89\pm0.06) + (0.06\pm0.14)\,t \\ \pi^+\text{-p} \quad & \alpha(t) = (0.71\pm0.19) + (0.24\pm0.50)\,t\,. \end{aligned}$$

Thus $\alpha(t)$ has a significant t dependence for p-p scattering, i.e., the p-p diffraction peak shrinks. For π^--p and π^+-p scattering, however, $\alpha(t)$ is, within the errors, independent of t and therefore the diffraction peak does not change its shape as a function of s.

The extrapolations of our differential cross sections to $t = 0$ are not particularly significant because of the large t range over which the extrapolation has to be made. In general, however, the extrapolated values agree with the optical theorem point within the errors.

Fig. 4 shows all total elastic cross sections at present available, as a function of incident momentum. Our values were obtained by integration of the differential cross sections, extrapolated to the optical theorem point at $t = 0$. The values from other authors were taken directly from their respective papers. The errors shown include statistical as well as systematic errors.

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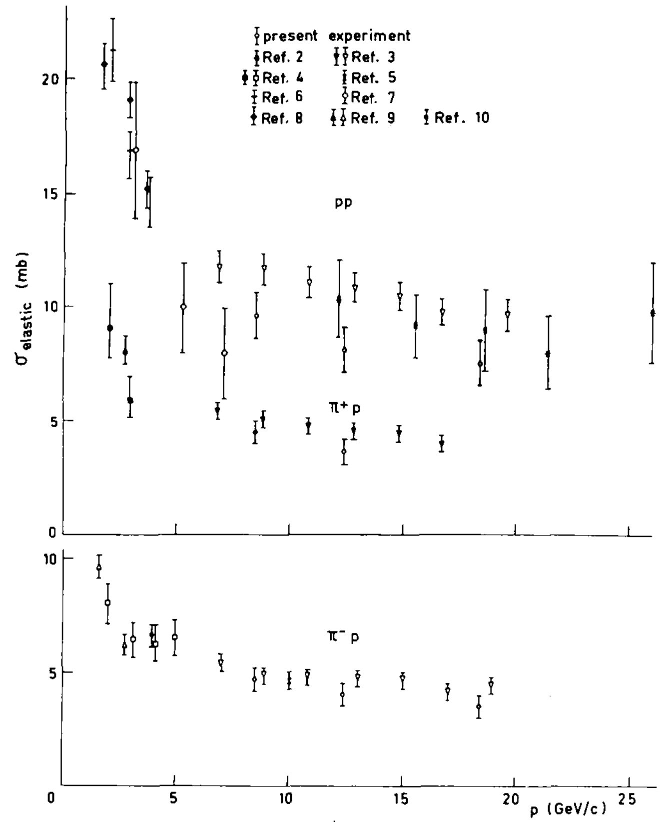

Fig. 4. Total elastic cross sections for π^--p, π^+-p and p-p data. Errors include statistical as well as systematic errors.

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