For the Journal of the Franklin Institute.

Work and Vis-viva. By De Volson Wood, Prof. of C. E., University of Michigan.

I have just read the article "On the Elements of Physical Work, &c." by J. W. Nystrom, which is published in the November number of the Journal. I regret to find so many errors in an article which possesses so much merit both in the original manner in which the subject is presented, and in the apt illustrations which accompany his analysis. The geometrical magnitudes used to illustrate his ideas of force, power, and work, struck me very forcibly, and I think they will ever be found serviceable in impressing upon the mind the truths which he has presented.

But I have intimated that there are errors in the article; and it is my present purpose to note them, and not to write a review of the article; and as he comes before us as a truth-seeker, I doubt not he will kindly receive any criticism which may be passed upon it.

If the terms which he uses, as *force*, *work*, *power*, *vis-viva*, &c., are definitions or the names of algebraic expressions, we have only to consider the propriety of their use; but if the terms or phrases involve principles, they must stand the test of analysis.

For instance; if "power," is *defined* to be "the product of the force by the velocity;" or "work," to be "the continued product of the force, velocity, and time;" or vis-viva, to be "the product of mass by the square of the velocity," we have only to consider the propriety of their use; to see if they agree with the terms which have long been used to express the same idea, and if not to see if his terms are better than those used; or if they are new, to see if they are good terms for expressing the ideas intended. Now suppose that we have accepted the definitions which have just been indicated, and that it be asserted that work equals the vis-viva; or $F V T = M V^2$. Now this cannot be *assumed* or *defined*, but it must be capable of proof if it be true. I will observe that the equation is true when the force is constant and the body free to move, in which case we have $F T = M V$, also from the law of falling bodies $V = g T$, which in the preceding gives $F = M g =$ weight of the body. It must be observed that I admitted the above equality merely for the sake of the argument, and, that I may not be misunderstood, I will state that the first member, $F V T$, *does not express the work which the force* F *does upon a body free to move in producing a velocity* V, *in time* T, *but when the body is free to move, as here supposed, it equals one-half the work*, as may be shown hereafter. If it be said, for the sake of the argument, that F is constant and is operating upon a body which offers a constant resistance so that V is also constant, then I say that $F V T$ cannot equal $M V^2$; for we have $F V T = M V^2$ or $F T = M V$; but in this equation all the quantities are constant except T, hence the equation is not true. If, still further, we admit that $F T = s$, and $V^2 = 2 g s$, we have by substitution and reduction, $F = 2 W$, which cannot be true. We see then that the equation, $F V T = M V^2$, reduces to an absurdity under every hypothesis except the first, and in that it simply gives a true equation without expressing the value of the work. We see then that *it never expresses the true relation between work and vis-viva.*

I have taken more space for this hypothetical case, than I intended to, and yet it will be observed that it is exactly the expression which Mr. Nystrom has given us on pages 326 and 327, vol. xlviii.

But let us notice more specifically some of the statements. The writer says, p. 326, "work appears to be the most confused function in physics," and among other things has been called "vis-viva, living force, power, momentum, &c." I can hardly conceive what class of authors the writer has had access to; for I have yet to find the author —Nystrom excepted—who considers that work means the same as any of the terms above named. It is true that work is known by many terms: for instance, Smeaton called it *mechanical power*; Carnot, moment of activity; Monge and others, *dynamic effect*; M. Navier, *quantity of action*; Poncelet, *quantity of work*; and by most modern writers, simply, *work*. On page 358, he calls F V, or as he gives it in a few lines after, M V, *moment of inertia*, while all other writers call it *momentum*; and $\Sigma\, m\, r^2$ is called moment of inertia. I fail to find where "Mosley calls work, 'moment of inertia'" (see p. 359). He says "$S = V\, T$" (see p. 326) which is true when V is constant, and yet in deducing the expression $\frac{M\, V^2}{g}$ (p. 358) he marks $T = \frac{V}{g}$, in which V is variable and increases with the time.

He says, p. 358, that the weight of the moving body at the surface of the earth where $g = 32{\cdot}166$, is equal to its mass; whereas, mechanical writers say that the weight equals the mass at the point where $g = 1$. Surely if "the difference between force, power, and work, appears not to be generally understood," this is making confusion worse confounded.

I have not made these criticisms in a captious spirit, but rather to help those who read from falling into the same errors.

If Mr. Nystrom has used any expression inadvertently, I hope I have not used it to his disadvantage.

I shall neglect the more agreeable part of my duty, if I do not show the true values and relations of work and vis-viva. I published an article in the November number of the Journal for 1862, and another in the February number for 1864, in which I endeavored to explain their meaning and relation, but as those articles may not be available to the readers of this, I shall make no further reference to them.

Work is overcoming resistance. If the resistance overcome is the inertia of a body, the work is called, *work of inertia*; but the work which has been done upon a body free to move, and which is stored in it, is rarely referred to in this way. It is generally referred to in terms of the vis-viva. The work commonly referred to is that of overcoming physical resistance; and if the resistance be constant and equal F, the work $= F\, S$; if F be variable the work is $\int F\, d\, s$. The expression $F s$, is, I think the primitive formula, not $F V T$ as given on page 327. Now the work done in the space s, is the same whether it be done in one hour, day, or week, it is independent of the time. But if we wish to compare the *efficiency* of machines we must observe the *time* which it takes the different machines to do the same amount of work ; then will their efficiencies be inversely as their times of doing the work. It is for this reason that time appears in the English unit of work which is, 33,000 lbs. raised one foot per *minute.* A force may be doing a work of resistance and of inertia at the same time, and the expression $\Sigma P\, d\, s$ shows how many pounds feet it has overcome and the expression $\frac{1}{2} M V^2$, how many pounds feet of resistance the moving body is capable of overcoming in being brought to rest. The latter statement requires proof. For this we will suppose that the resistance overcome is constant and equal P; and that it is overcome for a space S. From mechanics we know that the acceleration due to a force, is the force divided by the mass, or $\frac{P}{M}$. We also know that the square of the velocity equals twice the acceleration multiplied by the space, or $V^2 = 2\frac{P}{M}s$; from which we immediately find $\frac{1}{2} M V^2 = P S$ and as the latter is the work, the former must be also, and hence equals *one-half of the vis-viva.*

This may be proved quite as satisfactorily in a more elementary way. For, the work done may always be made equivalent to a weight W raised to a proper height h. The work of doing this will be $W h$. Now if this weight be permitted to fall freely through the height h, it will acquire a velocity V, and the amount of work stored in the mass must equal $W h$. But from the law of falling bodies we have $h = \frac{V^2}{2g}$ hence, we have $W h = W\frac{V^2}{2g} = \frac{1}{2} M V^2$, as before. I might illustrate these principles by numerous examples, and deduce the expression for vis-viva by different processes, but I trust I have given enough to establish my position. This is all I wish to do at present, but if it seems advisable I may refer to the subject at a future time.

The writer says on p. 359, "that the subject is nowhere properly explained." I confess that much obscurity has hung over the subject principally on account of the brevity of treatment by most authors, but I am pleased to refer your readers to "Morin's Mechanics," as ranslated by Joseph Bennett, and published by D. Appleton & Co. for a good exposition of the subject. For accuracy, simplicity, and thoroughness, on this subject, it is unequalled by any work within my knowledge. Whewhell in his "Mechanics and Engineering" gives a good exposition of it, and applies the principles of vis-viva to a higher class of problems, than Morin does.

While all standard writers agree upon the terms used in these articles, yet few pretend to define "power." With its present vague meaning it should not be considered a *scientific* term, but if we agree upon the definition given by Mr. Nystrom, it will be as specific as any other term used in science. Morin, in the work above referred to, says, p. 12, " we use the words *power* and *resistance* to denote those forces which favor motion, or those which oppose it;" while the general meaning is, *ability* to do something. We speak of water-power, horse-power, steam-power, power of a lens, power of the wind, mechanical powers, physical power, moral power, and many other abilities, without any idea of the measure of those abilities. It would be a decided gain to physics if it had a specific meaning, and the term force substituted for power in those indefinite expressions.

On the Incrustation of Marine Boilers. By Mr. James R. Napier.

From the London Mechanics' Magazine, November, 1864.

Read before the Institution of Engineers in Scotland, February 17, 1864.

In the Transactions of this Institution, 1859,60, will be found a paper which I wrote, chiefly for the purpose of showing that regenerators, as ordinarily constructed, were much too small for the object intended. But I have there also stated (page 46) that "when these regenerators are made with a sufficient amount of surface, so that abundance of water can be supplied to and discharged from the boilers with little loss of heat, then there will be *no incrustations*," &c. In the last paragraph of the paper I have, with more caution, said, "that this amount of discharge and surface, *it is expected*, will prevent incrustation, and save nine-tenths of the heat at present lost by the ordinary method of blowing off."

The object of the present communication is to show that the practice here recommended leads to results the very opposite of what was expected. Believing, as I then did, in the ordinary theory of blowing off from the boiler before the water became saturated with salts, that an abundant feed and blow-off would prevent the lime depositing, and therefore prevent the incrustations; and being desirous of saving the heat which would otherwise be lost by the great amount of blow-off which I believed to be necessary, I had a regenerator made for the S.S. "Lancefield" with about ten times the surface which it had been customary to give to such apparatus; but the results, as stated at a recent meeting, were so much at variance with my understanding of the ordinary theory, that I think a statement of the facts will help others to a clearer knowledge of the matter.

The vessel then, sailed from Glasgow about noon every Thursday for the Hebrides, lay in one of the lochs there from Saturday evening till Monday morning, and arrived again in Glasgow on Wednesday, to recommence on Thursday a similar voyage. The steam was up or at hand all the voyage; about fifteen stops, of two or three hours each, were made each week, during which time the boiler was supplied with feed by a Giffard's injector, but little or no blow-off.