

Clearly, a solution of equations (30)–(32) is given by

$$g_0(\bar{\eta}) = 1/\beta. \quad (39)$$

By rigorous, but rather involved, phase-plane arguments it is possible to show [5] that there does exist another solution of equations (30)–(32). However, this other solution would imply a temperature at  $\eta = 1$  which is greater than  $1/\beta$ ; therefore this solution is rejected upon the grounds that such a temperature profile is impossible for the solution of the steady state conduction equations, (1)–(5).

Now we shall find  $g_1(\bar{\eta})$ . Substituting equation (39) into equations (33) and (34) and solving for  $g_1(\bar{\eta})$  yields

$$g_1(\bar{\eta}) = \frac{\gamma C_0 (\beta - \alpha)^{\frac{1}{2}}}{\beta(1 + \alpha C_0 \ln \eta_0)} \exp[\beta \bar{\eta}/(\beta - \alpha)^{\frac{1}{2}}]. \quad (40)$$

Therefore, in the boundary layer starting at  $\eta = 1$  and penetrating to a distance of order  $\epsilon^{\frac{1}{2}}$  the asymptotic expansion of the solution as  $\epsilon \rightarrow 0$  is given by

$$\theta(\eta) = \frac{1}{\beta} - \epsilon^{\frac{1}{2}} \frac{\gamma C_0 (\beta - \alpha)^{\frac{1}{2}}}{\beta(1 + \alpha C_0 \ln \eta_0)} \exp[\beta(\eta - 1)/(\beta - \alpha)^{\frac{1}{2}} \epsilon^{\frac{1}{2}} + O(\epsilon)] \quad (41)$$

Upon comparing equation (41) with equation (39) it can be seen that equation (41) is uniformly valid on the whole interval  $0 \leq \eta \leq 1$ .

It is now possible to determine the value of  $C_0$  from equations (7) and (41); the result is

$$C_0 = \frac{-1}{\beta \ln \eta_0} \left[ 1 + \epsilon^{\frac{1}{2}} \frac{\gamma}{(\beta - \alpha)^{\frac{1}{2}}} + O(\epsilon) \right]. \quad (42)$$

Thus, the asymptotic expressions of the temperature profiles for the conductor and for the insulator are respectively

$$\theta(\eta) = \frac{1}{\beta} - \frac{1}{J^{\frac{1}{2}} \beta (\beta - \alpha)^{\frac{1}{2}} \ln \eta_0} \exp \left\{ \left[ \frac{\beta J^{\frac{1}{2}}}{(\beta - \alpha)^{\frac{1}{2}}} \right] (\eta - 1) \right\} + O\left(\frac{1}{J}\right), \quad 0 \leq \eta \leq 1, \quad (43)$$

and

$$\theta(\eta) = \frac{-1}{\beta \ln \eta_0} \left[ 1 + \frac{1}{J^{\frac{1}{2}} (\beta - \alpha)^{\frac{1}{2}} \ln \eta_0} \right] \ln \left( \frac{\eta}{\eta_0} \right), \quad 1 \leq \eta \leq \eta_0. \quad (44)$$

As expected, equations (23) and (43) both yield  $\theta(\eta) = 1/\beta$  when  $J$  is large and  $\gamma$  is small.

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## FUEL DROPLETS EVAPORATION IN A MOVING GASEOUS MEDIUM

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(Received 2 January 1970 and in revised form 6 February 1970)

### NOMENCLATURE

$A$ ,	area;
$C_D$ ,	drag coefficient;
$c_p$ ,	specific heat;
$D$ ,	diffusivity;
$h$ ,	heat-transfer coefficient;
$k$ ,	thermal conductivity;
$k_m$ ,	mass-transfer coefficient;
$M$ ,	mass of droplet;

$N_{ur}$ ,	Nusselt number;
$P$ ,	pressure;
$Pr$ ,	Prandtl number;
$P_s$ ,	duct-static pressure;
$Q_r$ ,	total heat transfer to vapor film;
$Q_D$ ,	heat transfer to liquid droplet;
$Q_{sh}$ ,	heat carried with diffusing vapor in form of super-heat;
$Q_\lambda$ ,	heat to vaporize diffusing vapor;
$Q_D$ ,	equals $Q_l + Q_\lambda$ ;
$R$ ,	gas constant;

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$r$ ,	radial distance from center of droplet;
$Re$ ,	Reynolds number;
$Sc$ ,	Schmidt number, $\mu/D\rho$ ;
$Sh$ ,	Sherwood number, $2r_D RT k_m/D$ ;
$T$ ,	temperature;
$t$ ,	time, $t_0$ = time lag;
$U$ ,	velocity difference between the gas and the droplet;
$V$ ,	gas velocity;
$V_D$ ,	droplet velocity;
$w$ ,	vaporization rate;
$Z$ ,	correction factor for heat transfer;
$z$ ,	defined as $wc_{p,f}\delta/4\pi r_D(r_D + \delta)k_{f,o}$ ;
$\alpha$ ,	correction factor for mass transfer;
$\lambda$ ,	latent heat of vaporization;
$\mu$ ,	viscosity, droplet size in microns;
$\rho$ ,	density;
$\delta$ ,	vapor film thickness;
$\omega$ ,	equals $2\pi/\tau$ ;
$\tau$ ,	period, corresponding to 2000 rpm engine, taken as = 0.0225 s;
$b$ ,	bulk or surrounding medium;
$D$ ,	pertaining to droplet; droplet surface;
$f$ ,	vaporizing fuel;
$f, 0$ ,	fuel gas mixture;
$f, D$ ,	fuel at surface of droplet;
$l$ ,	liquid;
$m$ ,	mass;
$0$ ,	oxygen gas;
$sh$ ,	superheat;
$s$ ,	gas static condition;
$cs$ ,	cross sectional area.

## INTRODUCTION

IN THE present study the effects of a varying gas velocity and a varying static pressure on the evaporation rate, temperature, and velocity of moving fuel droplets are examined. The analysis follows the model presented by Priem and Heidman [1] for the transport of heat and mass from moving fuel droplets in a combustion chamber. Attention is given in the present analysis to individual fuel droplets that are drawn at virtually zero velocity into the flowing gaseous medium. The gas and the droplets temperatures are taken different with droplet temperature being different than the wet bulb temperature.

The history of a group of fuel droplets of varying sizes is examined when each of the following conditions is imposed on the surrounding gaseous medium. (a) A constant gas velocity and a constant gas static pressure, (b) A constant gas velocity and a linearly varying static pressure, (c) A gas velocity and a static pressure that vary with time with droplets and gas velocities starting together at zero values, and (d) A gas velocity and a static pressure that vary with time with fuel droplets introduced into the duct when the gas velocity is finite.

## ANALYSIS

In this analysis we assume spherical symmetry of the droplets, one dimensional quasi-steady transport of heat and mass with no shattering of the droplets, non-reacting system, viscous dissipation and radiant energy transport are ignored along with thermal diffusion.

### Heat transfer

The liquid fuel droplet is assumed to be surrounded by a vapor film of thickness  $\delta$ . The rate of heat transfer to this film goes, then, into heating the liquid, to vaporizing a quantity of it, and to superheating the diffusing vapor.

Therefore, we can write

$$Q_i = Q_l + Q_z + Q_{sh} \quad (1)$$

The quantity of heat that reaches the surface of the droplet is equal to the sum of  $Q_l$  and  $Q_z$  and is defined as  $Q_D$ . An energy balance across a spherical layer in the film of thickness  $dr$  leads to

$$Q_D = 4\pi r^2 k \frac{dT}{dr} - wc_{p,f}(T - T_l) \quad (2)$$

For constant fluid properties, equation (2) can be integrated between  $r = r_D$  where  $T = T_l$  and  $r = r + \delta$  where  $T = T_b$  to yield

$$Q_D = wc_{p,f} \frac{T_b - T_l}{\exp \left[ \frac{wc_{p,f} \delta}{4\pi k_{f,o} r_D (r_D + \delta)} \right] - 1} \quad (3)$$

Because the film thickness,  $\delta$  is unknown we follow a similar analysis to that of [1, 2] and represent the rate of heat transfer to the droplet by the following relation\*

$$Q_D = hA_D(T_b - T_l)Z \quad (4)$$

where

$$Z = z/(e^z - 1)$$

and

$$z = \frac{wc_{p,f} \delta}{4\pi r_D (r_D + \delta) k_{f,o}}$$

The average heat transfer coefficient,  $h$ , is expressed by [3, 4]

$$N_u = \frac{2hr_D}{k} = 2 + 0.6(Pr)^{1/4}(Re)^{1/2} \quad (5)$$

An energy balance on the droplet yields the following expression for the change in the droplet temperature

$$\frac{dT_l}{dt} = \frac{1}{Mc_{p,l}} [Q_D - w\lambda] \quad (6)$$

From equation (6) we conclude that if  $w\lambda$  (the heat transfer carried by the vapor) is greater than  $Q_D$ , the droplet temperature decreases with time while if  $Q_D$  is greater than  $w\lambda$ , the

\*  $Z$  in equation (4), represents the ratio of the heat that would be conducted to the surface with pure convection heat transfer and no mass transfer, to the heat transfer with convection and mass transfer.

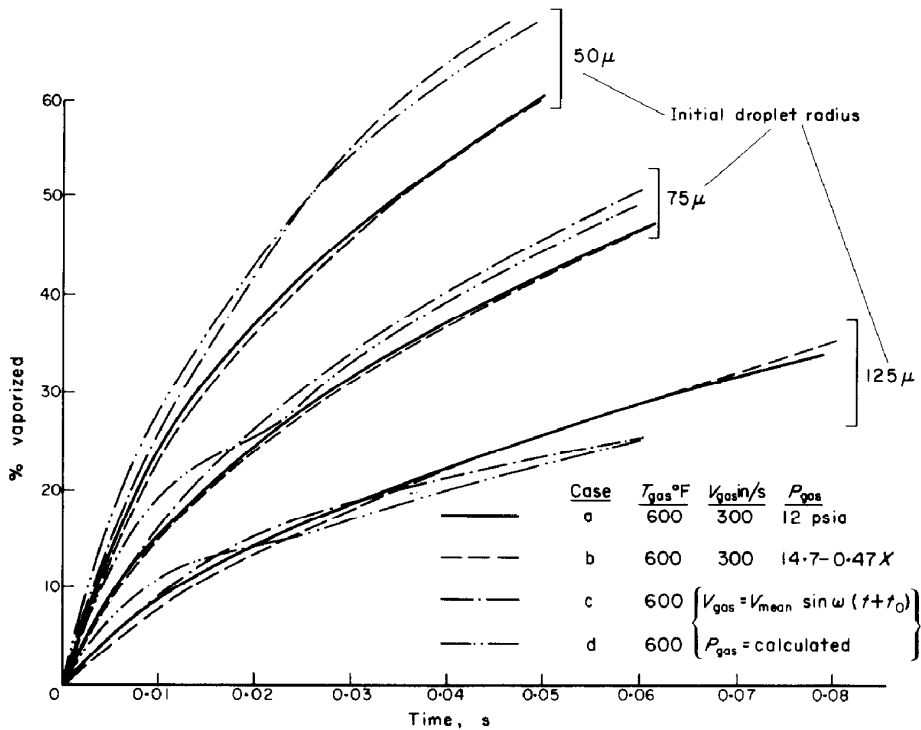


FIG. 1. The per cent of mass vaporized from droplets.

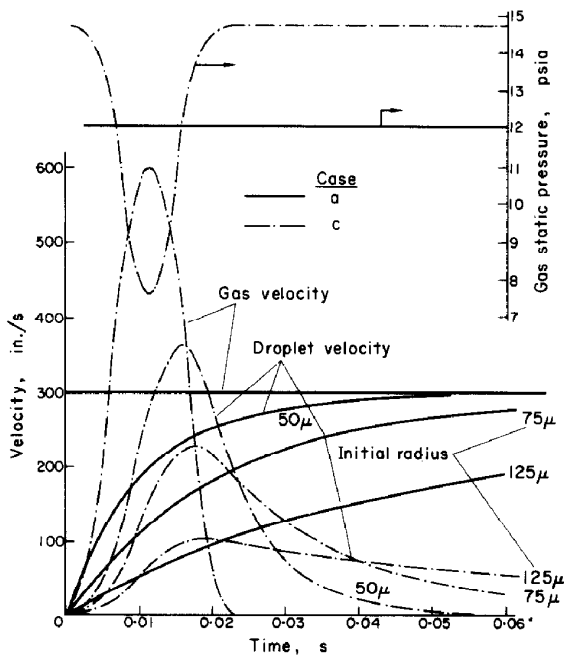


FIG. 2. Droplet velocity in oxygen.

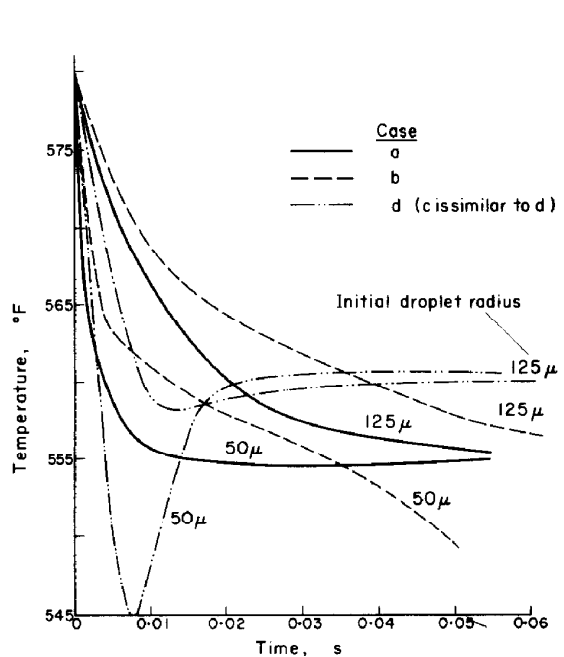


FIG. 3. Droplet temperature.

temperature increases. When  $Q_D$  equals  $w\lambda$ , the droplet temperature remains unchanged as it moves in the medium. This equilibrium temperature represents the wet bulb temperature.

#### Mass transfer

A mass balance on a spherical shell surrounding the droplet yields for the mass-transfer rate the following expression\*

$$w = \frac{A_D D_{f,0} P_s}{RT} \left( \frac{1}{\delta} + \frac{1}{r_D} \right) \ln \frac{P_s}{P_s - P_{f,D}} \quad (7)$$

In equation (7) the following conditions have been used

$$P_f(r_D + \delta) = 0, \quad P_f + P_0 = P_s$$

In view of the unknown value of  $\delta$ , a more convenient form for the mass-transfer rate will be used and is given by [1, 2, 4]

$$w = A_D k_m P_{f,D} \alpha \quad (8)$$

where

$$\alpha = \frac{P_s}{P_{f,D}} \ln \frac{P_s}{P_s - P_{f,D}}$$

The mass-transfer coefficient,  $k_m$ , is obtained from the empirical relation† [3-5]

$$Sh = \frac{2r_D RT k_m}{D_{f,0}} = 2 + 0.6(Sc)^{\dagger} (Re)^{\dagger} \quad (9)$$

which results in the following expression for  $w$

$$w = \frac{2\pi D_{f,0} r_D}{RT} \alpha Sh P_{f,D} \quad (10)$$

#### Momentum transfer

The aerodynamic drag force on the droplet will either speed it up or slow it down so that the velocity of the droplet approaches that of the moving gas. For a spherical droplet we write [6]

$$\begin{aligned} \text{Drag force} &= -M \frac{dV_D}{dt} = C_D A_{cs} \rho_{f,0} \frac{U^2}{2} \\ \frac{dV_D}{dt} &= -\frac{3}{8} C_D \frac{\rho_{f,0}}{\rho_l} U^2 / r_D \end{aligned} \quad (11)$$

where  $C_D$  is the drag coefficient and taken for evaporating droplets as [7]

$$C_D = 27(Re)^{-0.84} \quad (12)$$

#### Discussion of results

The calculation of the droplet history is broken into small increments of time. At every increment the droplet velocity,

\* In equation (7), thermal diffusion and diffusion from oxygen into the liquid are ignored.

† In reference 5, the constant multiplying  $Sc$  and  $Re$  in equation (9) is taken as 0.55 instead of 0.6.

size, vaporization rate and droplet temperature are calculated using the relations presented earlier. Liquid heptane\* and gaseous oxygen are considered for the fuel and the gas respectively. In the four cases considered, Fig. 1 shows that for the same period of time, the finer droplets lose a greater percentage of their mass as compared to the larger ones. This is so in spite of the fact that the velocity of the finer droplets approaches that of the gas at an earlier time as shown (for cases a and c) in Fig. 2. In cases c and d the gas velocity† changes as  $V = V_{\text{mean}} \sin \omega(t + t_0)$  and the static pressure follows a similar change calculated assuming a quasi-steady process. The droplet temperature is shown in Fig. 3 to be decreasing (and then increasing for cases c and d) with time until a steady situation is reached in which the rate of heat transfer to the droplets is equal to the energy transported to the diffusing vapor.

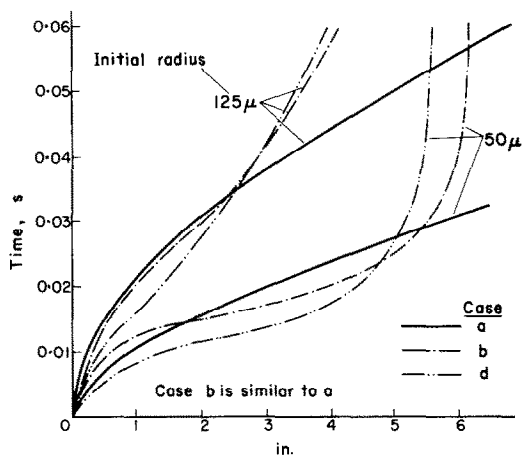


FIG. 4. Distance-time diagram.

It is further concluded that a low fraction of the total mass of fuel drawn into the inlet duct is vaporized before reaching the combustion chamber. Some droplets that are introduced into the duct at certain times do not reach the combustion chamber if the duration of the intake process is within the range of time shown on the figures. This conclusion is reflected in Fig. 4 which shows the distance covered by the various droplets as a function of time.

\* The properties of heptane and oxygen are taken from [1].

† In cases c and d,  $V_{\text{mean}}$  was taken as 300 in./s and  $\omega$  is taken as  $278 \text{ s}^{-1}$ .  $t_0$  is the time lag between the time the droplet is introduced into the gaseous medium and the time the gas starts accelerating. For case c,  $t_0 = 0$ , and for case d,  $t_0 = 0.0056 \text{ j}$ .

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## THE EFFECT OF FREE STREAM TURBULENCE ON THE HEAT TRANSFER FROM THE STAGNATION POINT OF A SPHERE

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(Received 13 November 1969 and in revised form 19 February 1970)

AN EXPERIMENTAL investigation was conducted to define quantitatively the effect of free stream turbulence on the heat transfer to the stagnation point of a sphere. The present data expands the existing heat-transfer results for turbulence intensities up to 36 per cent. It is shown that the heat transfer remains generally constant for the low turbulence intensities (1-10 per cent), following existing data, but at higher intensities a significant increase in heat transfer is noted.

### INTRODUCTION

Heat transfer from solid bodies immersed in a stream of fluid is of importance in many engineering problems. For most common cases, heat-transfer rates can be adequately predicted from available engineering analyses or correlations. One serious shortcoming of the available techniques is that there is no systematic way of predicting the effects of the stream turbulence on heat transfer; yet experiments by Giedt [1] have shown that even moderate turbulence levels may increase the heat transfer by as much as 70 per cent over the theoretical value.

Many analytical models have been attempted to describe turbulence effects, including vorticity stretching models [2] and eddy diffusion models [3]. Of the many models proposed, only these two have been successful in predicting the dramatic effect of turbulence on the heat transfer from the stagnation line of a cylinder.

Neither theory has been applied, however, to other geometries; therefore an analytical prediction of the heat

transfer from the stagnation point of a sphere for high turbulence intensities cannot be made.

The current investigation was prompted by experimental results showing the stagnation line heat-transfer rate on cylinders to increase by 40 per cent for free stream turbulence levels increasing from approximately 0.05 to 2 per cent. Prior to this study, no such trend was evident for heat transfer from a spherical stagnation point, since the existing data for the spherical stagnation point falls on both sides of theory.

### EXPERIMENTAL EQUIPMENT

The test model was constructed from a 1.5 in. dia. aluminum-bronze bearing spherical to  $10^{-6}$  in. The heat transfer was measured by an equilibrium method similar to that used in guarded-hot-plate thermal-conductivity measurements. A portion at the stagnation region of the sphere, called the center calorimeter, was insulated thermally from the rest of the electrically heated sphere by an air gap of 0.005 in., as shown in Fig. 1. The calorimeter was provided with a heater and thermocouple.

The sphere was heated by a series of four electrical resistance heaters; four thermocouples were used to measure the temperature within the sphere. The sphere heaters and thermocouples were molded to four cylindrical capsules that fit machined shafts in the sphere.

The turbulence was generated by a series of 50 per cent porosity perforated drilled plates with holes ranging from  $\frac{1}{4}$  to  $\frac{1}{2}$  in. The range in turbulence intensity produced by