PHONON MODEL DESCRIPTION OF SPHERICAL NUCLEI*

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A completely numerical quadrupole phonon model calculation enables one to obtain a satisfactory description of anharmonic energy levels, reasonable B(E2) ratios and quadrupole moment ratio for even-even spherical nuclei. The failure of the random phase approximation is clarified.

Phonon description of vibrational energy levels in spherical nuclei has been attempted with some success [1]. Starting with the Hamiltonian for the pairing plus quadrupole-quadrupole interactions, one may define a phonon operator in terms of quasiparticle pair operators,

$$d^{+}_{IM} = \sum_{\nu} \left[a_{\nu} A^{+\nu}_{IM} + (-1)^{M} b_{\nu} A^{\nu}_{I-M} \right] \qquad (1)$$

to satisfy an equation

$$[H_d, d^{+}] = \omega d^{+} \qquad (2)$$

with a condition

$$d\,|\,0\rangle_q = 0\ , \qquad (3)$$

where $|\,0\rangle_q$ is the quasiparticle vacuum state including the ground state correlations. It has been a common practice to use a random phase (RPA) or a higher random phase (HRPA) approximation to find an approximate solution for the equation (2) [1], but there has been no successful solution in these attempts.

On the other hand, Baranger and Kumar [2] have successfully described many properties of even-even nuclei in deformed regions, using the collective model Hamiltonian [3]. They have shown that their solution can be expanded in the phonon model scheme to connect the description in the deformed region to the one in the spherical region and successfully predicted level properties of nuclei in a transitional region [4].

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Recently, Kisslinger and Kumar [5] calculated energy levels of odd-even nuclei in the spherical region using an extended method of BK, and got an improved result, which is reminiscent of the calculation of Tamura and Udagawa to explain the large quadrupole moment of the first excited 2^{+} state of ^{114}Cd [6].

The main purpose of this note is to point out that a numerical expansion of low excited state wavefunctions of spherical nuclei in the quadrupole phonon scheme is quite adequate and that the numerical method can describe the level properties well. The phonon model Hamiltonian is given in terms of the phonon creation operator B^{+}_{IM} and the annihilation operator $\bar{B}_{IM} = (-1)^{I-M} B_{I-M}$,

$$H = \omega_0 (2I+1)^{\frac{1}{2}} [B^{+}\bar{B}]^{0} + w_2 \left\{ [B^{+}B^{+}]^{0} + [\bar{B}\bar{B}]^{0} \right\} +$$
$$+ w_3 \left\{ [B^{+}B^{+}B^{+}]^{0} + B^{+}B^{+}\bar{B}]^{0} + [B^{+}\bar{B}\bar{B}]^{0} + [\bar{B}\bar{B}\bar{B}]^{0} \right\} +$$
$$+ w_4 \left\{ [B^{+}B^{+}B^{+}B^{+}]^{0} + [B^{+}B^{+}B^{+}\bar{B}]^{0} + \ldots \right\} + \ldots \qquad (4)$$

and wavefunctions of low excited states are [5]

$$\begin{aligned}
|0^{+}\rangle &= a_0|0\rangle + a_1(B^{+}B^{+})^{0}|0\rangle + a_2(B^{+}B^{+}B^{+})^{0}|0\rangle + \ldots \\
|0'^{+}\rangle &= a'_0|0\rangle + a'_1(B^{+}B^{+})^{0}|0\rangle + a'_2(B^{+}B^{+}B^{+})^{0}|0\rangle + \ldots \\
|2^{+}\rangle &= b_0 B^{+}|0\rangle + b_1(B^{+}B^{+})^{2}|0\rangle + b_2(B^{+}B^{+}B^{+})^{2}|0\rangle + \ldots \\
|2'^{+}\rangle &= C_1 B^{+}|0\rangle + C_0(B^{+}B^{+})^{2}|0\rangle + C_3(B^{+}B^{+}B^{+})^{2}|0\rangle + \ldots \\
|2''^{+}\rangle &= d_2 B^{+}|0\rangle + d_3(B^{+}B^{+})^{2}|0\rangle + d_0(B^{+}B^{+}B^{+})^{2}|0\rangle + \ldots
\end{aligned} \qquad (5)$$

We calculate matrix elements of H which contain unknown parameters ω_0, w_2, w_3, w_4, and all coefficients in the eq. (5). Since these coefficients

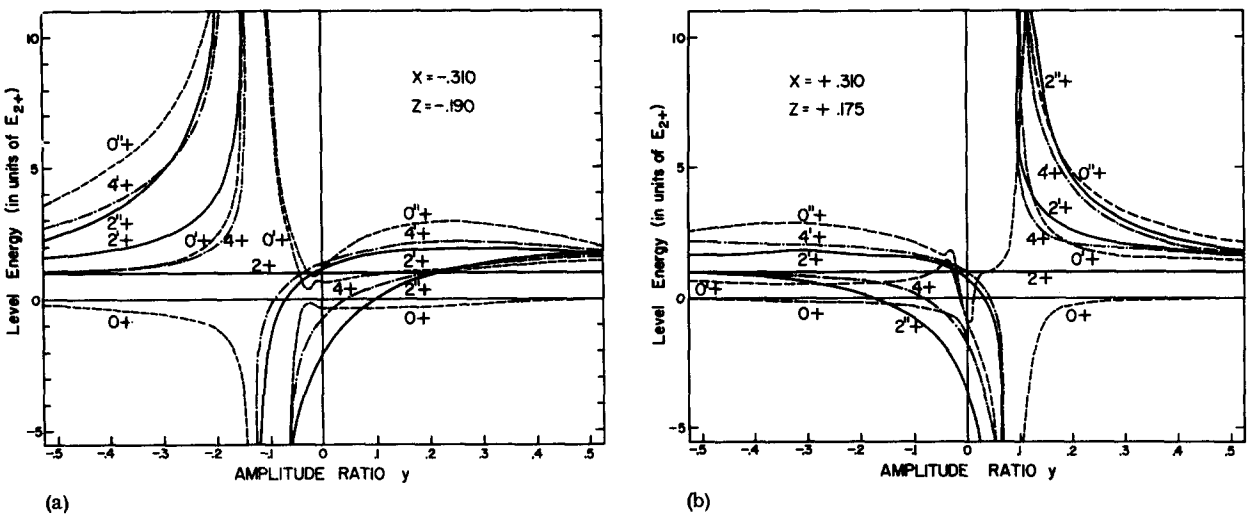

Fig. 1. Schematic diagram of 0^+, 2^+ and 4^+ state energy levels for a) Pt region and b) Cd region. For simplicity, values of y's (x's and z's) for 2^+ states are set equal in their magnitude. ($y_b = b_3/b_0$, $y_c = c_3/c_0$ and $y_d = d_3/d_0$, $z_b = b_2/b_0$, $z_c = c_2/c_0$ and $z_d = d_2/d_0$, etc.)

define matrix elements of a unitary transformation from a representation, in which the 'true' Hamiltonian is diagonal, to the phonon model representation, we can transform the 'true' Hamiltonian, whose diagonal elements are experimentally observed level energies, into the one in the phonon model scheme, and equate all matrix elements, term by term, with those which have been calculated. For a given set of parameters w_2, w_3 and w_4, one can solve the simultaneous linear equations, in principle, with all energies expressed in the unit of ω_0. In our actual calculation, however, these coefficients are first assumed and varied until we can get the best fit of the observed 2+ state energy levels. Then the coupling strengths w_2, w_3 and w_4, thus determined, are used to calculate 0^+ and 4^+ energy levels, and the whole procedure is repeated until the overall best fit is attained. Using these wavefunctions, ratios of various $B(E2)$'s and a quadrupole moment ratio [5] are calculated for ^{114}Cd. The results are shown in figs. 1-3 and table 1.

It should be noted that since it is impractical to include all phonon states in the model space, we truncated the model spacing including up to

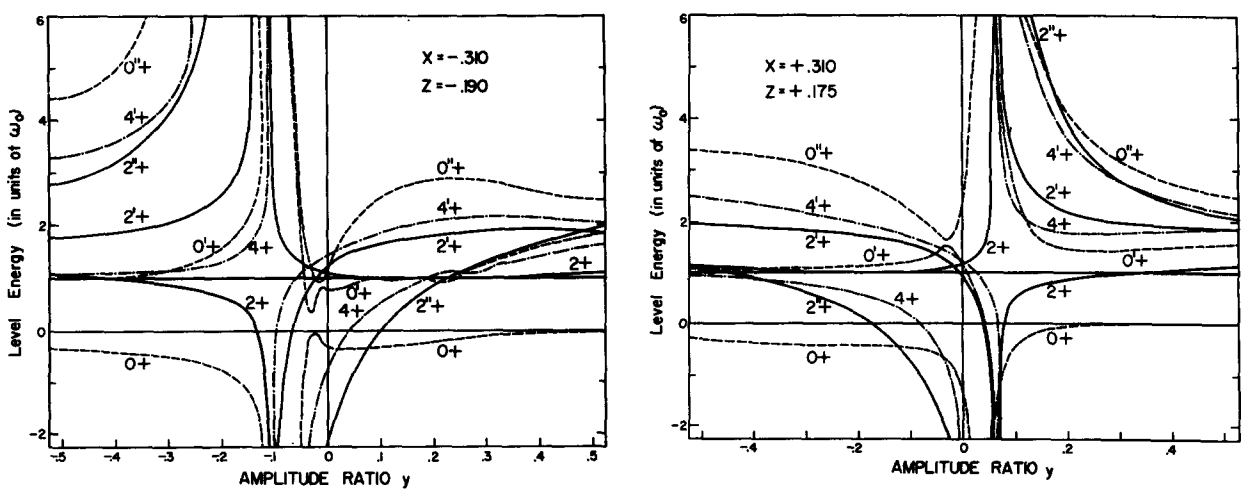

Fig. 2. Schematic diagram of energy levels in the unit of ω_0. Notations are the same as those in fig. 1.

Table 1
Comparison of experimental and theoretical values of $0^+, 2^+, 4^+$ state energies, B(E2) ratios and quadrupole moment ratio in ^{114}Cd.

	B(E2) ratios	
	exp.	this work
$B(E2, 2' \rightarrow 0)/B(E2, 2 \rightarrow 0)$	0.015 ± 0.005	0.016
$B(E2, 2' \rightarrow 2)/B(E2, 2 \rightarrow 0)$	1.21 ± 0.25	0.96
$B(E2, 0' \rightarrow 2)/B(E2, 2 \rightarrow 0)$	0.85 ± 0.17	1.13
	Quadrupole moment ratio	
	exp.	this work
Q_{22}/Q_{20}	+ 0.81 ± 0.20	+ 0.63
	Energy level	
	exp.	this work
$E(2^+)$	0.5585	0.5585
$E(2'^+)$	1.208	1.23
$E(2''^+)$	1.840	1.86
$E(0'^+)$	1.133	0.92
$E(0''^+)$	1.862	2.14
$E(4^+)$	1.282	1.09
$E(4'^+)$	1.730	1.91

either three phonon states or four phonon states, and the effect of the truncation is revealed as a lack of the normalization of the wavefunctions [5]. On the other hand, the orthogonality property within a truncated space is best attained whenever the best fit is approached. It is observed that the one phonon state energy, ω_0, is always higher than that of the $|2+\rangle$ state, and this may be understood as a reason why the RPA (or HRPA) method failed. The situation is quite easily seen from the fact that our calculation is made with a subsidiary condition, which is always satisfied automatically though, in such a way that the energy shifts, due to the anharmonicity, from the harmonic spectrum should have a set of non-trivial solutions: otherwise one should get a trivial solution which is nothing but the harmonic solution. Another reason why the RPA failed may be that the calculation is very sensitive to the choice of the parameters involved and only a little change in a single parameter can make the energy levels either unphysical or extremely high relative to that of the first $|2^+\rangle$ state. The latter case may be said 'deformation started'. This reminds us that the BK calculation for ^{196}Pt corresponds to the very edge of the nearly anharmonic spectrum [4] (see fig. 1). The former case is encountered, on the other hand, in the Cd region, where the calculated value of the quadrupole moment, Q_{22}, yields a correct sign only when the ratio $x_b = b_1/b_0$ is positive, which, in turn, makes the equation for the 2^+ state level energies to be given in terms of second order quantities with respect to the ratio x_b, and, therefore, one would get very easily the unphysical solutions.

It has been supposed that the anharmonic level structure might be very close to a case of harmonicity and hence it should be calculated by the perturbation calculation. This is, in fact, true because the interaction strengths w_2, w_3 and w_4 are very small quantities and the deviation from the harmonic spectrum must be very small. In the case of ^{114}Cd, these strengths are about 0.02 MeV, - 0.08 MeV and - 0.06 MeV, respectively, and, in general, these are about $\pm 0.04\ \omega_0$, $\pm 0.10\ \omega_0$, and $\pm 0.04\ \omega_0$, respectively, as long as there exists a reasonable solution. The failure of all previous calculations is, however, simply because of their accidentally bad choice of the phonon scheme, as an unperturbed scheme, which is always fairly away from the 'true' scheme: in other words, the choice of the ω_0 was not the right one to start with. Our calculation shows that ω_0-E(2^+) should be about 10% of ω_0, and seems not quite small at all, and that if the ω_0 is not chosen correctly, this affects the condition for the existence of the non-trivial solution substantially.

The qualitative discussions of Kisslinger and Kumar [5] all agreed with our calculation, however, we do not agree with their choice of the parameters, especially for the ratios $x_b = b_1/b_0$ and $x_a = a_1/a_0$. They assume that all these ratios are about ± 0.33. This is actually a good choice as a starting point, but our calculation ends up with these ratios x_b to be about $0.17 \sim 0.19$ and a_1/a_0 to be about ± 0.0001. Our calculation of a's assumes exact orthonormality for 0^+ state wavefunctions, and, therefore, the values of a's may not necessarily be close to those of ref. 5. In addition, we often find the ratio a_1/a_0 to be positive, whereas they assume a negative value for the ratio. Since this ratio has not come into a calculation of any physical quantity which is sensitive to the sign of this ratio, it is difficult to see which sign agrees better with an experimental value of the physical quantity.

It should be mentioned that we also used an expanded form of E2 transition operator in terms of the phonon operators [7]. This subject will be discussed elsewhere in a paper coming shortly.

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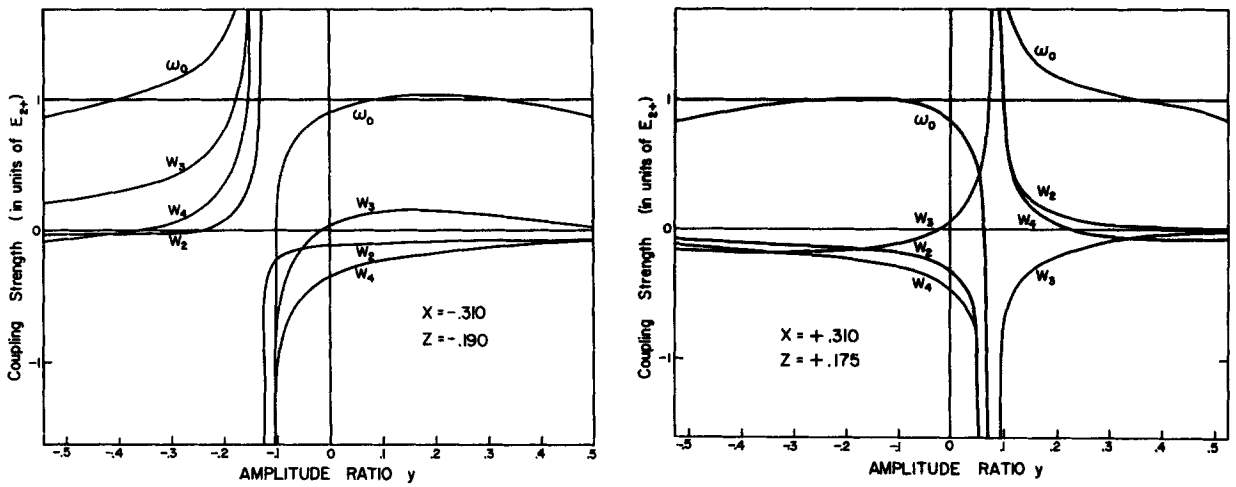

Fig. 3. Schematic diagram of coupling strengths in the unit of E_{2^+}.

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It was also pointed out by Tamura and Udgawa of ref. 1 that the additional term in the expansion would be essential.

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