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FLUCTUATING HEAT TRANSFER IN A SINGLE FLUID HEAT EXCHANGER

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ABSTRACT

An unsteady heat transfer analysis is presented for single fluid heat exchangers having internal heat sources. The transient effect is introduced through fluctuations of the fluid velocity about a mean value. It is found that the phase angle and amplitude for the fluid and wall temperatures, and the interface heat flux have oscillatory characteristics. These results apply to certain types of nuclear reactors.

1. INTRODUCTION

Although much work has been published on unsteady heat transfer, it appears to have been confined to problems of the boundary layer type. The customary method of solving these problems is to impose a boundary condition along the fluid-wall interface, and then solve the energy equation of the fluid. The interesting and important problem of how the wall responds to fluctuations of an external fluid flow about a steady mean has apparently remained untreated.

In this paper a start is made to investigate this subject. The fluid energy equation is simplified by neglecting the viscosity of the fluid. Then the fluid and wall energy equations are coupled along the fluid-wall interface by means of the natural boundary conditions. A specific example, taken from the field of Nuclear Engineering, is used to illustrate the analytical procedure. This example pertains to certain types of nuclear reactors which can be idealized as consisting of three elements. These are a coolant (fluid) which flows coaxially between a moderator and a rod (wall) in which heat is generated (Figure 1). Because the coolant thickness is much less than the radius of the rod, the radial variations in the coolant velocity and temperature may be neglected. The energy equation for the coolant may then be approximated by a simpler equation. In those instances where the moderator possesses a low thermal conductivity as compared to the fluid and rod, the heat transfer from the coolant to moderator may be neglected as a further simplication.

The purpose of this paper, under the foregoing assumptions, is to present solutions for the temperature fields of the rod and coolant when the coolant velocity fluctuates about a mean value.

2. FORMULATION

The energy equations for the rod and coolant may be written as:

$$\rho \frac{\partial (C_p \theta^*)}{\partial t^*} = \text{div } (k \text{ grad } \theta^*) + q''' \quad , \tag{1}$$

$$\rho' \frac{D(C_p' T^*)}{Dt^*} = \text{div } (k' \text{ grad } T^*) \quad , \tag{2}$$

where (ρ,ρ') are the densities, (θ^*,T^*) the temperatures, (C_p,C_p') the specific heats at constant pressure, and (k,k') the thermal conductivities of the rod and coolant respectively. The time is t^*, and the heat generation per unit volume in the rod is q'''.

Neglecting radial variations of the velocity and temperature of the coolant, axial conduction in both the rod and coolant, heat transfer from the coolant to the moderator, and assuming all physical properties are constant, (1) and (2) can be replaced by the following equations:

$$\rho C_p \frac{\partial \theta^*}{\partial t^*} = k\left(\frac{\partial^2 \theta^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial \theta^*}{\partial r^*}\right) + q''' \quad , \tag{3}$$

$$(\rho' C_p')A\left(\frac{\partial T^*}{\partial t^*} + U\frac{\partial T^*}{\partial x^*}\right) = - Pk\left(\frac{\partial \theta^*}{\partial r^*}\right)_{r^*=R} \quad , \tag{4}$$

where A is the cross-sectional area of the coolant, U the velocity of the collant, P the periphery of the rod, R the radius of the rod, and (r^*,x^*) are the cylindrical coordinates.

This same physical system has been considered recently for a starting type of transient phenomena by introducing a step change in the rate of heat generation within the rod.(1) It will now be assumed that the heat generation is constant with respect to time and uniform throughout the rod, but the coolant is fluctuating in a simple harmonic motion about a mean value U_o with a small amplitude ϵ and an angular frequency ω^*, so that

$$U = U_o[1 + \epsilon \exp(i\omega t)], \qquad (5)$$

where $\epsilon \ll 1$. By introducing the non-dimensional variables

$$\begin{aligned} x &= (x^*/R)/Pe' , & r &= (r^*/R) , \\ t &= \alpha t^*/R^2 , & Pe' &= \alpha U_o/R^2 , \\ \theta &= \theta^*(q''' R^2/k), & T &= T^*/(\lambda q''' R^2/k) , \\ \lambda &= (PR/A)(\rho C_p/\rho' C_p'), & \omega &= \omega^* R^2/\alpha \end{aligned} \qquad (6)$$

in which α is the thermal diffusivity of the rod, and Pé, the Péclet number, then (3) and (4) become

$$\frac{\partial\theta}{\partial t} = \left(\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r}\right) + 1 ,$$

$$\frac{\partial T}{\partial t} + [1 + \epsilon \exp(i\omega t)]\frac{\partial T}{\partial x} = -\left(\frac{\partial\theta}{\partial r}\right)_{r=1} . \qquad (8)$$

3. METHOD OF STUDY

Considering the steady periodic (or asymptotic) solutions of the rod and coolant temperatures, which is the purpose of the present study, it is appropriate to write:

$$\theta = \theta_o + \epsilon\theta_1 \exp(i\omega t) , \qquad (9)$$

$$T = T_o + \epsilon T_1 \exp(i\omega t) , \qquad (10)$$

where θ_o, T_o and θ_1, T_1 satisfy:

$$0 = \left(\frac{\partial^2\theta_o}{\partial r^2} + \frac{1}{r}\frac{\partial\theta_o}{\partial r}\right) + 1 , \qquad (11)$$

$$\frac{dT_o}{dx} = -\left(\frac{\partial\theta_o}{\partial r}\right)_{r=1} , \qquad (12)$$

and

$$i\omega\Theta_1 = \left(\frac{\partial^2\Theta_1}{\partial r^2} + \frac{1}{r}\frac{\partial\Theta_1}{\partial r}\right) , \tag{13}$$

$$i\omega T_1 + \frac{dT_1}{dx} + \frac{dT_0}{dx} = - \left(\frac{\partial\Theta_1}{\partial r}\right)_{r=1} , \tag{14}$$

with the boundary conditions:

$$T_0 = 0,\ T_1 = 0, \text{ at the inlet of the coolant,} \tag{15}$$

$$\frac{\partial\Theta_0}{\partial r} = 0,\ \frac{\partial\Theta_1}{\partial r} = 0, \text{ at the center of the rod,} \tag{16}$$

$$T_0 = \Theta_0,\ T_1 = \Theta_1, \text{ along the coolant-rod interface.} \tag{17}$$

T_0 and Θ_0 may be obtained readily by solving (11) and (12) subject to the boundary conditions (15), (16), and (17). The results are:

$$T_0(x) = x/2 , \tag{18}$$

$$\Theta_0(x,r) = x/2 + (1-r^2)/4 . \tag{19}$$

To determine T_1 and Θ_1, the method is outlined as follows: the appropriate solution of (13) satisfying (16) is

$$\Theta_1(r,x) = A(x)J_0(i^{3/2}\omega^{1/2}r), \tag{20}$$

where $A(x)$ is an integration parameter to be determined, and J_0 is the Bessel function of first kind of order zero in the usual notation.

Using the Equations (17), (18), and (20), the Equation (14) may be reduced to the following first order, ordinary and non-homogeneous differential equation:

$$\frac{dA}{dx} + [i\omega + (\omega/i)^{1/2}J_1(i^{3/2}\omega^{1/2})/J_0(i^{3/2}\omega^{1/2})]A = - 1/2\ J_0(i^{3/2}\omega^{1/2}). \tag{21}$$

where J_1 is the Bessel function of first kind of order first. The solution of this equation is:

$$A(x) = B \exp\left\{-[i\omega + (\omega/i)^{1/2} J_1(i^{3/2}\omega^{1/2})/J_o(i^{3/2}\omega^{1/2})]\right\}$$
$$- 1/2 \quad [i\omega J_o(i^{3/2}\omega^{1/2}) + (\omega/i)^{1/2} J_1(i^{3/2}\omega^{1/2})] , \qquad (22)$$

where B is an integration constant to be determined. Combining Equations (20) and (22), and applying boundary conditions (15) and (17) results in the following:

$$T_1(x) = -\frac{1}{2} \; \frac{1 - \exp\left\{-[i\omega + (\omega/i)^{1/2} J_1(i^{3/2}\omega^{1/2})/J_o(i^{3/2}\omega^{1/2})]x\right\}}{i\omega + (\omega/i)^{1/2} J_1(i^{3/2}\omega^{1/2})/J_o(i^{3/2}\omega^{1/2})} , \qquad (23)$$

and

$$\Theta_1(r,x) = -\frac{1}{2} \; \frac{1 - \exp\left\{-[i\omega + (\omega/i)^{1/2} J_1(i^{3/2}\omega^{1/2})/J_o(i^{3/2}\omega^{1/2})]x\right\}}{i\omega + (\omega/i)^{1/2} J_1(i^{3/2}\omega^{1/2})/J_o(i^{3/2}\omega^{1/2})}$$

$$\frac{J_o(i^{3/2}\omega^{1/2} r)}{J_1(i^{3/2}\omega^{1/2})} \qquad (24)$$

For numerical convenience only, the coolant and rod temperatures may be rearranged as:

$$T(x,t) = \frac{1}{2} x - \frac{\epsilon}{2} \left[\frac{\exp(-2ax) - 2\exp(-ax)\cos bx + 1}{a^2 + b^2}\right]^{1/2} \exp[i(\omega t - \phi_T)] , \qquad (25)$$

and

$$\Theta(r,x,t) = \frac{1}{2} x + \frac{1}{4}(1 - r^2)$$

$$- \frac{\epsilon}{2}(c^2 + d^2)^{1/2} \left[\frac{\exp(-2ax) - 2\exp(-ax)\cos bx + 1}{a^2 + b^2}\right]^{1/2} \exp[i(\omega t - \phi_\Theta)] , \qquad (26)$$

where

$$a = (\omega/2)^{1/2}[(U_oV_1-V_oU_1) - (U_oU_1+V_oV_1)]/(U_o^2+V_o^2), \tag{27}$$

$$b = \omega + (\omega/2)^{1/2}[(U_oV_1-V_oU_1) + (U_oU_1+V_oV_1)]/(U_o^2+V_o^2), \tag{28}$$

$$c = [U_oU_o(r) + V_oV_o(r)]/(U_o^2 + V_o^2) , \tag{29}$$

$$d = [V_oU_o(r) - U_oV_o(r)]/(U_o^2 + V_o^2) , \tag{30}$$

$$\phi_T = \tan^{-1} \frac{b - \exp(-ax)(b\cos bx + a\sin bx)}{a - \exp(-ax)(a\cos bx - b\sin bx)} , \tag{31}$$

$$\phi_\Theta = \phi_T - \tan^{-1}(d/c), \tag{32}$$

and (U_o, V_o) are the real and imaginary parts of the Bessel function of first kind of order zero with complex argument, (U_1, V_1) the real and imaginary parts of the Bessel function of first kind of order first with complex argument.(2) It may be noted that U_o, V_o, U_1 and V_1 should be calculated for the modulus $(\omega^{1/2})$ and the argument $(\pi/4)$, but $U_o(r)$ and $V_o(r)$ for the modulus $(\omega^{1/2}r)$ and the argument $(\pi/4)$. The foregoing procedure completes the required solutions for the temperature distributions.

It remains to determine the fluctuating heat transfer from the rod to the coolant. The total heat flux per unit area of the rod is:

$$q^* = -k\left(\frac{\partial\theta^*}{\partial r^*}\right)_{r^*=R} . \tag{33}$$

In terms of non-dimensional variables Equation (33) becomes

$$q = -\left(\frac{\partial\theta}{\partial r}\right)_{r=1} , \tag{34}$$

where

$$q = q^*/q''' R . \tag{35}$$

It follows that

$$q = q_o + \epsilon q_1 \exp(i\omega t) \;, \tag{36}$$

where

$$q_o = -\left(\frac{\partial\theta_o}{\partial r}\right)_{r=1} = \frac{1}{2} \;, \tag{37}$$

$$q_1 = -\left(\frac{\partial\theta_1}{\partial r}\right)_{r=1} = \frac{1}{2}\,\frac{1 - \exp\{-[i\omega + (\omega/i)^{1/2} J_1(i^{3/2}\omega^{1/2})/J_o(i^{3/2}\omega^{1/2})]x\}}{1 + (i^{3/2}\omega^{1/2}) J_o(i^{3/2}\omega^{1/2})/J_1(i^{3/2}\omega^{1/2})} \,. \tag{38}$$

Again, for numerical convenience only, the total heat flux per unit area may be written as:

$$q(x,t) = \frac{1}{2} + \frac{\epsilon}{2}\left[\frac{\exp(-2ax) - 2\exp(-ax)\cos bx + 1}{e^2 + f^2}\right]^{1/2} \exp[i(\omega t - \phi_q)] \;, \tag{39}$$

where

$$e = 1 + (\omega/2)^{1/2}[(U_oV_1 - V_oU_1) + (U_oU_1 + V_oV_1)]/(U_1^2 + V_1^2), \tag{40}$$

$$f = (\omega/2)^{1/2}[(U_oV_1 - V_oU_1) - (U_oU_1 + V_oV_1)]/(U_1^2 + V_1^2), \tag{41}$$

$$\phi_q = \tan^{-1}\left[\frac{f - \exp(-ax)(f\cos bx + e\sin bx)}{e - \exp(-ax)(e\cos bx - f\sin bx)}\right] \,. \tag{42}$$

It will now be convenient to introduce the quasi-steady value of the amplitude of the collant and rod temperatures, and the interface heat flux. Accordingly, in the Equations (23), (24), and (38) replacing the Bessel functions and the exponential terms by their series expansions for small values of the argument results in

$$T_1(x) \rightarrow \theta_1(r,x) \rightarrow -\frac{1}{2}x \;, \tag{43}$$

and

$$q_1(x) \rightarrow \frac{1}{2}\left(\frac{\omega x}{2}\right)\exp(i\pi/2). \tag{44}$$

The ratio of the amplitude of the fluctuating term to its quasi-steady value for the collant and rod temperatures, and the interface heat flux may then be written as follows:

$$AR_T = \frac{1}{x}\left[\frac{\exp(-2ax)-2\exp(-ax)\cos bx + 1}{a^2 + b^2}\right]^{1/2}, \quad (45)$$

$$AR_\Theta = (c^2+d^2)^{1/2}\,\frac{1}{x}\left[\frac{\exp(-2ax)-2\exp(-ax)\cos bx + 1}{a^2 + b^2}\right]^{1/2}, \quad (46)$$

and

$$AR_q = \frac{2}{\omega x}\left[\frac{\exp(-2ax)-2\exp(-ax)\cos bx + 1}{e^2 + f^2}\right]^{1/2}, \quad (47)$$

where AR stands for the "Amplitude ratio".

Values of ϕ_T, ϕ_Θ at the center of the rod and ϕ_q, and AR_T, AR_Θ at the center of the rod and AR_q, for three different locations and as functions of the angular frequency, are given in Figures 2, 3, 4, 5, 6, and 7.

4. CONCLUSIONS

Single fluid heat exchangers having internal heat sources serve as the basis for analysis in this paper. The problem is one of unsteady heat transfer wherein the transient phenomena is introduced through fluctuations in the coolant velocity.

It is found that the phase lag in the coolant temperature approaches zero for small values of ω and $\pi/2$ for large values of ω, being similar to boundary layer type problems.[3] The behavior of this lag, however, shows a fundamental difference when compared to boundary layer problems. Phase lags in the latter are monotonic in nature but show an oscillatory character for the situation discussed in this study.

The phase lag in the rod temperature also approaches zero for small values of ω, but, unlike the coolant temperature, approaches infinity for large values of ω.

In regard to the interface heat flux it is found that small values of ω cause the phase angle to lead the coolant velocity by $\pi/2$ whereas large values of ω produce a lag of $\pi/4$. This apparently strange result can be understood if the same angle is evaluated with respect to its quasi-steady value. If this is done, a phase lag approaching zero for small values of ω and $3\pi/4$ for large values of ω is obtained.

Amplitude ratios for the coolant and wall temperatures and the interface heat flux behave in a similar manner, approaching unity for small values of ω, and zero for large values of ω. Again, in contrast to the monotonic variations in the amplitude ratios of the boundary layer problems, here one finds damped fluctuations in the variation of the amplitude ratios.

It would seem pertinent now to mention two possible extentions of this present study. The first is related to the power generation in the rod. As is well known, heat generation in a nuclear rod varies cylindrically in the radial direction and hyperbolically in the axial direction rather than being uniform throughout.(4) This condition of space dependent power generation causes additional complexities, but could be handled in a manner that is essentially similar to the procedure presented in this paper.

A second extension of this study regards the introduction of fluctuations in the power generation instead of coolant velocity as was done in the present analysis. The first two approximations of coolant and rod temperatures would yield the exact solutions, thus, the assumption $\epsilon \ll 1$ would be unnecessary.

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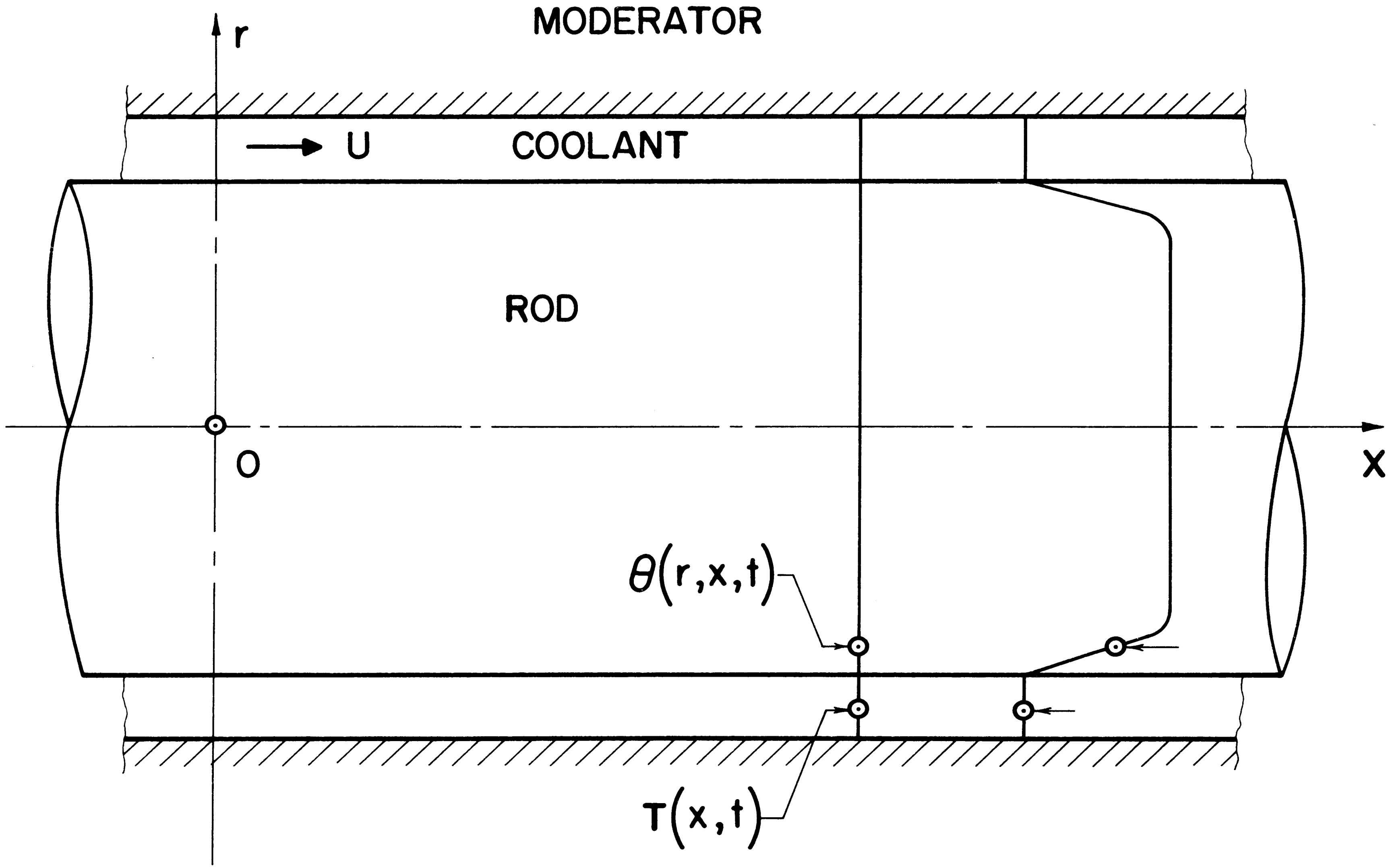

Figure 1. Physical Model of the Analysis

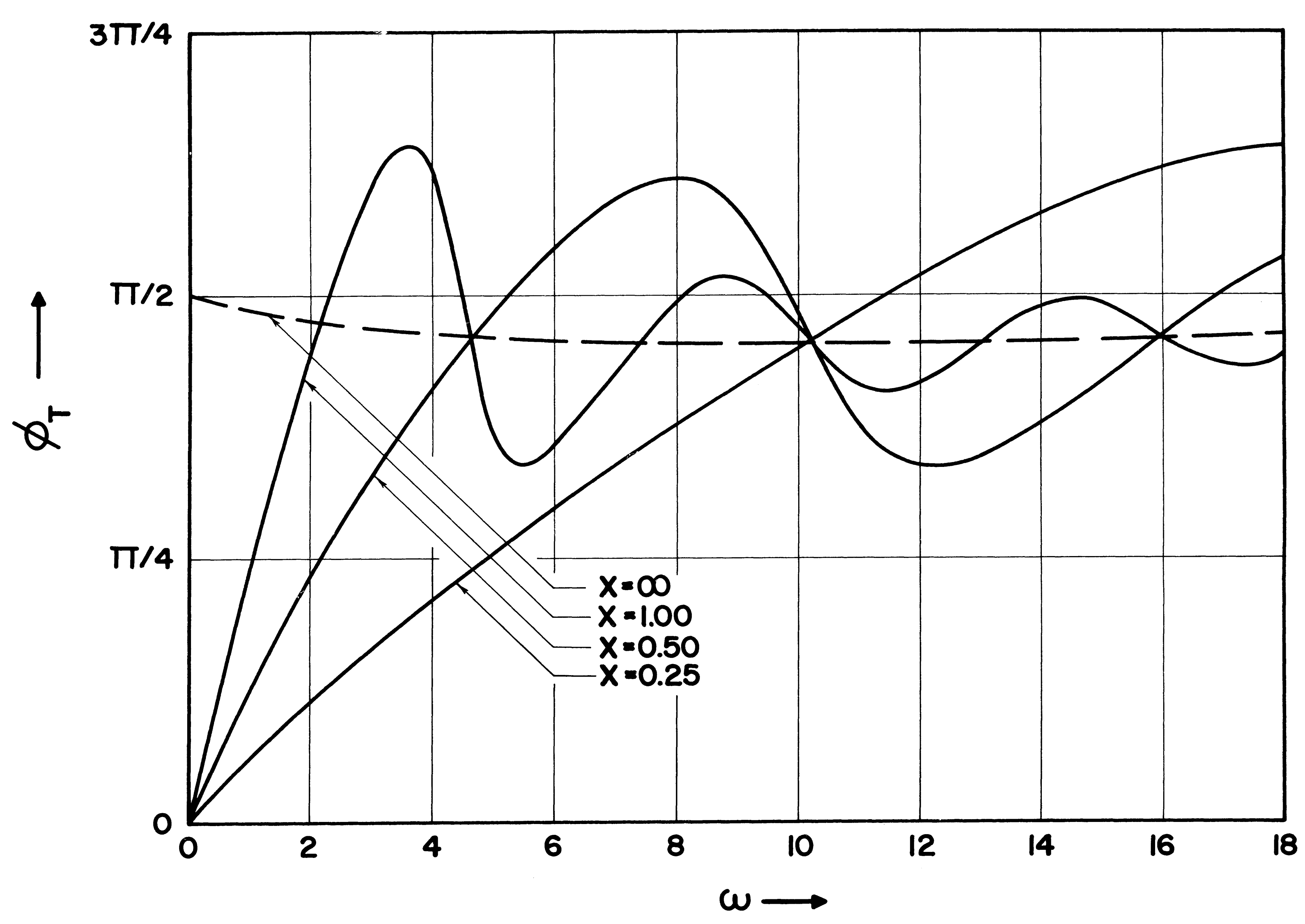

Figure 2. Phase in Coolant Temperature versus Angular Velocity

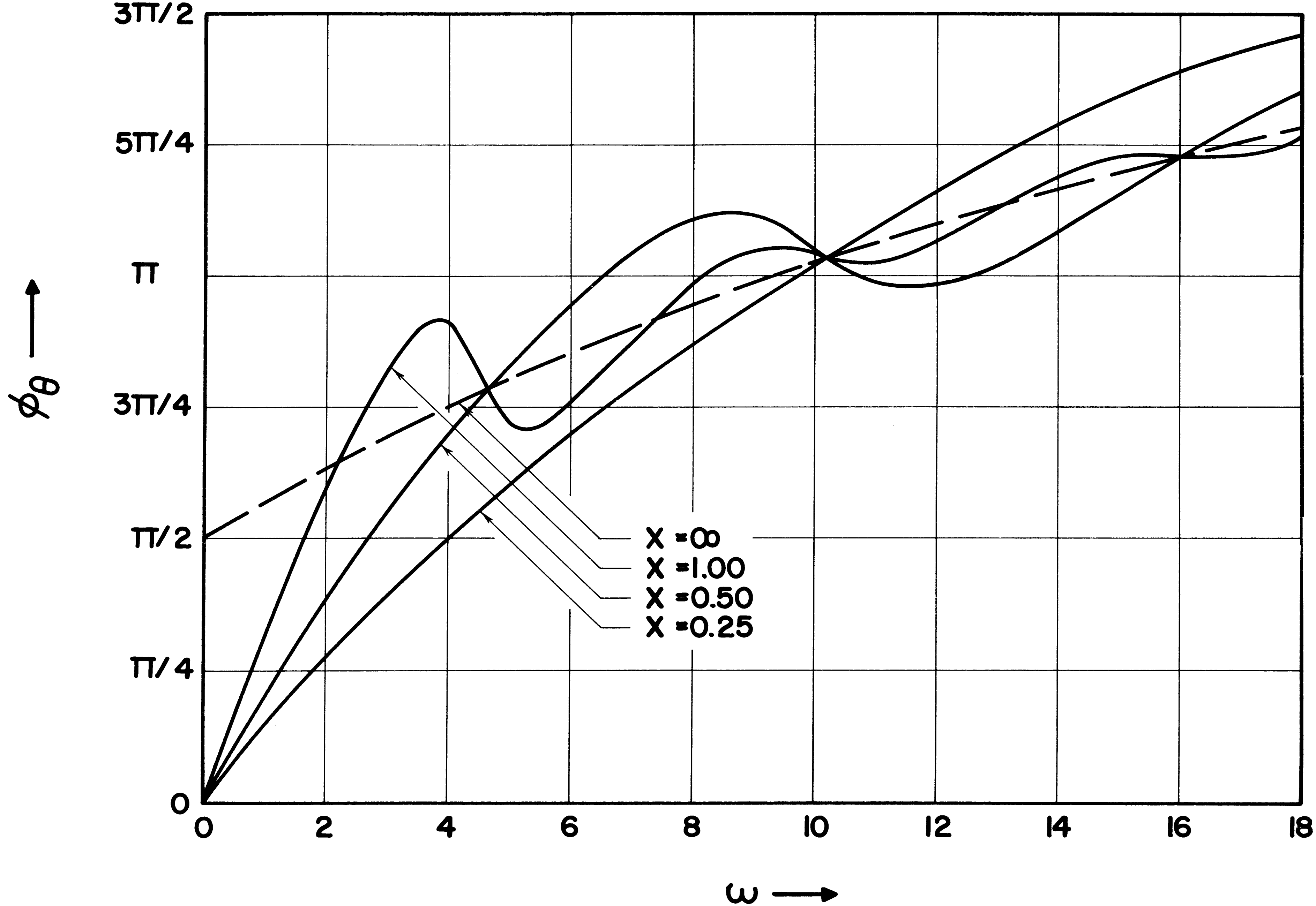

Figure 3. Phase in Rod Temperature versus Angular Velocity (r=0)

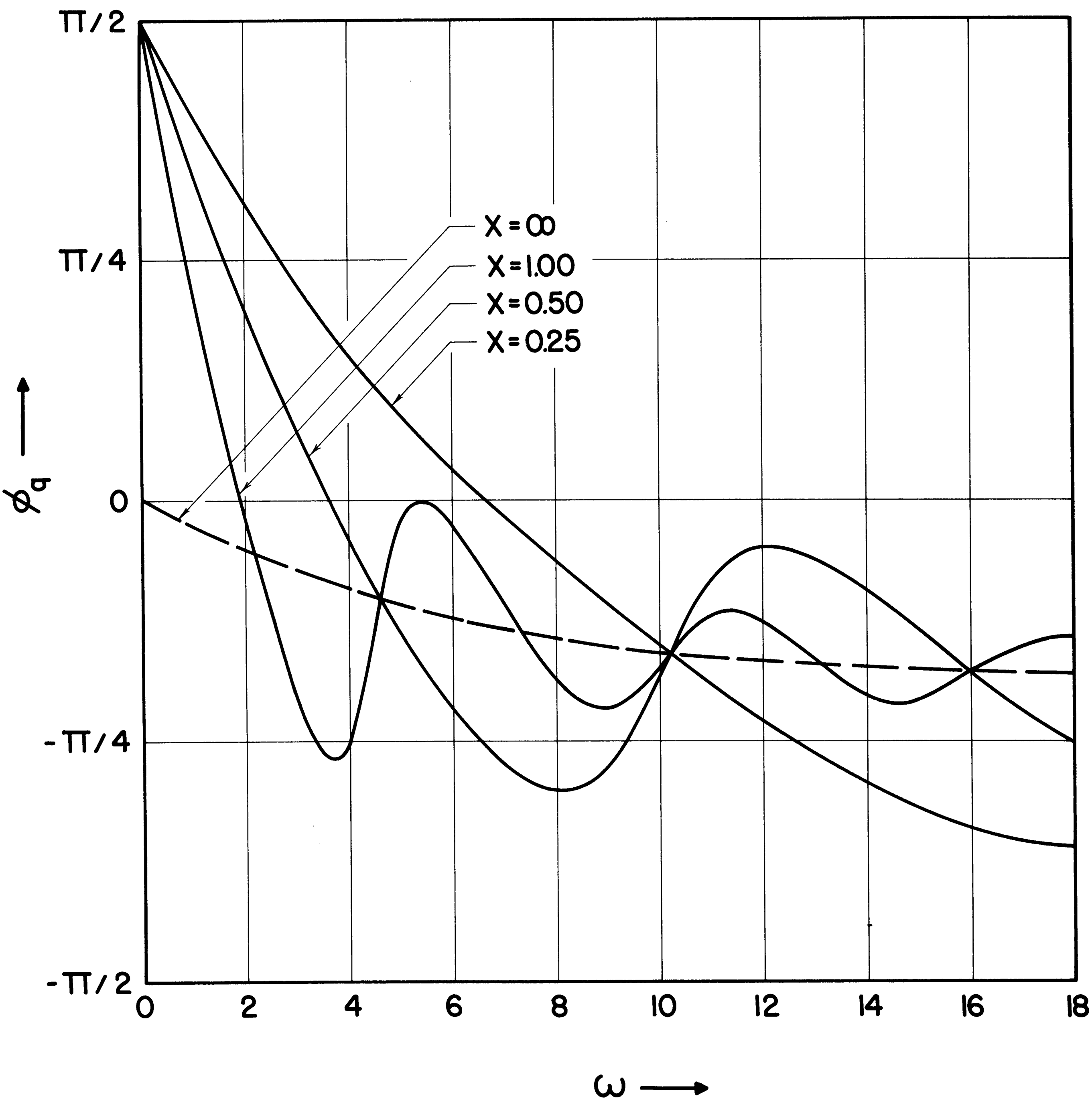

Figure 4. Phase in Interface Heat Flux versus Angular Velocity

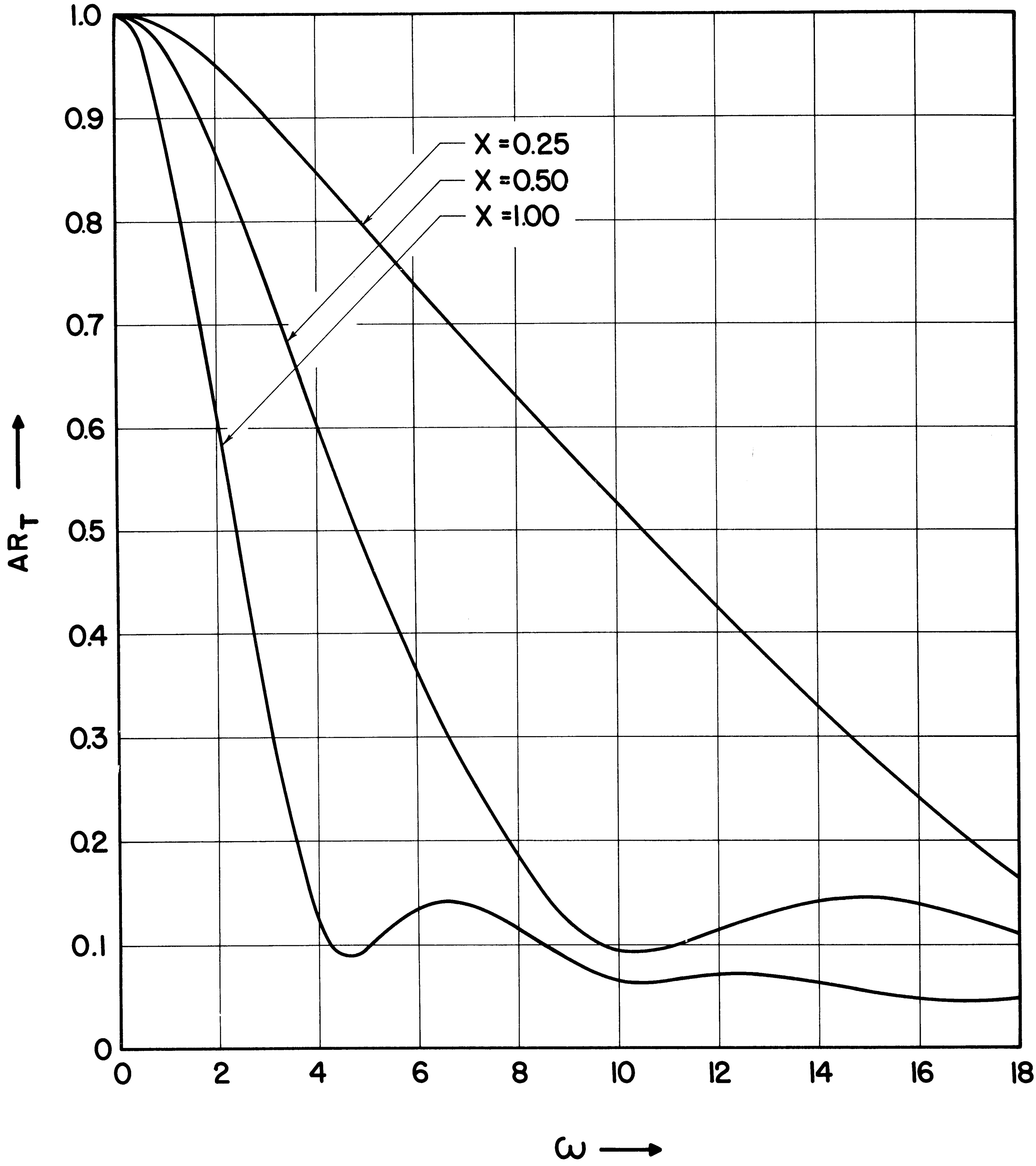

Figure 5. Amplitude Ratio for Coolant Temperature versus Angular Velocity (r=0)

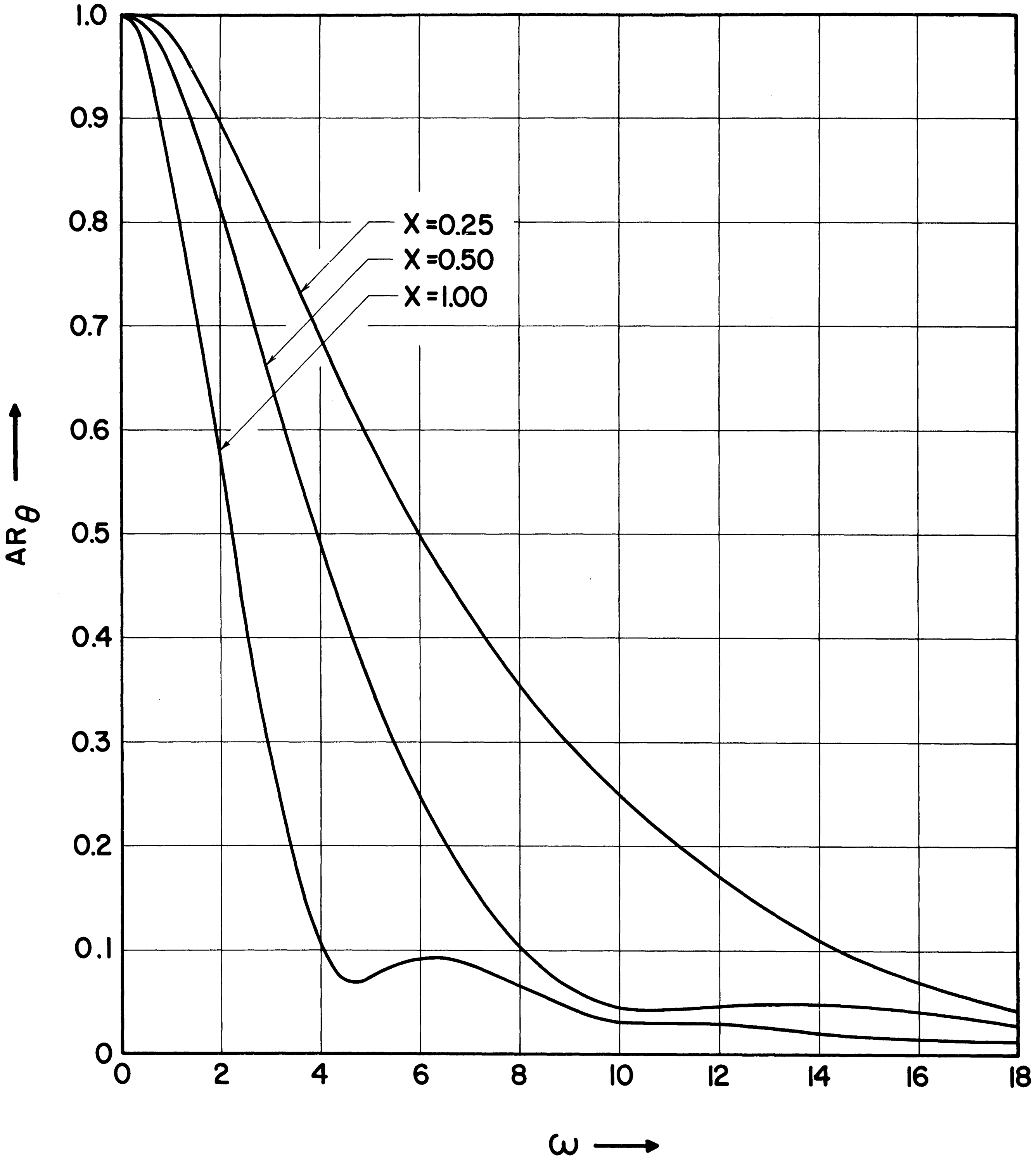

Figure 6. Amplitude Ratio for Rod Temperature versus Angular versus Angular Velocity (r=0)

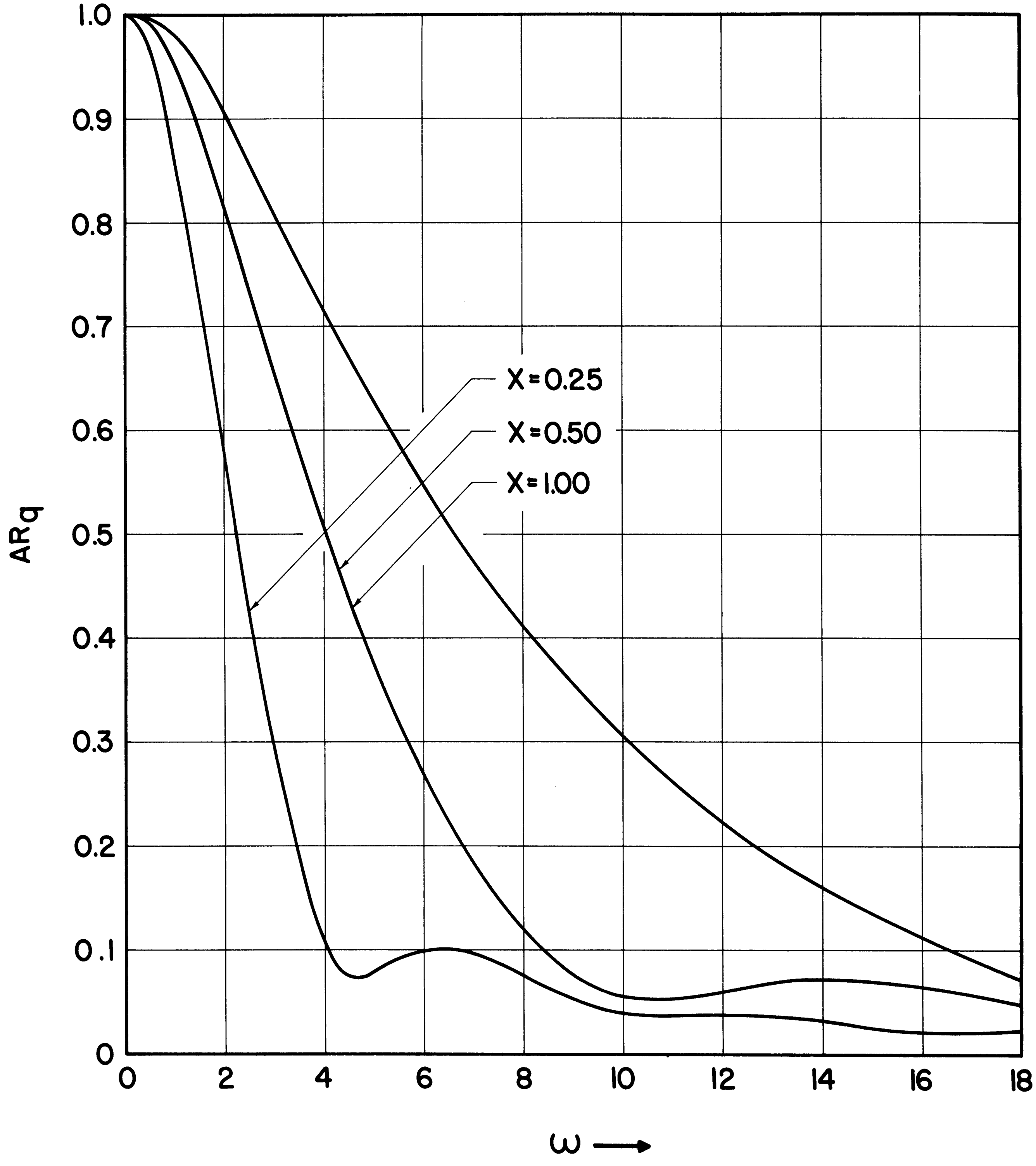

Figure 7. Amplitude Ratio for Interface Heat Flux versus Angular Velocity