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ANALYSIS OF THE INELASTIC SCATTERING OF 50 MeV PROTONS

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Abstract: Differential cross sections and asymmetries for 50 MeV proton scattering from vibrational states in ^{64,66,68,70}Zn and ^{114}Cd have been analysed using a collective-model interaction obtained by deforming the entire optical potential. The strong single-phonon states (2^+ and 3^-) are well described, while predictions for weak two-phonon states are less successful.

1. Introduction

The collective-model generalization of the optical model has been very successful in accounting for polarized-proton inelastic scattering data [1–3]) at 18.6, 30 and 40 MeV. These analyses have indicated that it is necessary to deform the complete optical potential including its spin-orbit term.

The present work is concerned with the application of this model to the analysis of proton scattering data for ^{64,66,68,70}Zn and ^{114}Cd at the higher energy of 50 MeV. Details of the experimental arrangements and of the data analysed here have been given previously [4–6]). Asymmetry data and additional forward-angle differential cross-section data for ^{114}Cd are also included in the present work.

2. Simple optical-model analysis of elastic data

Following the difficulty experienced previously [7]) in fitting some of the forward-angle ^{114}Cd cross-section data, additional measurements were carried out in the range 18°–46°. Discrepancies were found around the first diffraction minimum, and substitution of the new data greatly facilitated the present analysis. Unpublished polarization data obtained using an experimental arrangement similar to that described in ref. [5]) were included. Analysis of the zinc data has been made previously [6]), and extension of this to ^{114}Cd produced similar results.

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The optical potential used for the simple (one-channel) analysis of elastic data has the standard form

$$U(r) = V_C f(r, r_C) - Uf(r, r_U, a_U) - iW_V f(r, r_{WV}, a_{WV}) - i4a_{WD}W_D g(r, r_{WD}, a_{WD}) + \left(\frac{\hbar}{m_\pi c}\right)^2 U_{s.o.} \frac{1}{r} g(r, r_{s.o.}, a_{s.o.}) \sigma \cdot l,$$

where $f(r, r_i, a_i)$ has a Saxon-Woods form, $g(r, r_i, a_i)$ a Saxon-Woods derivative and $V_C f(r, r_C)$ the potential for a uniformly charged sphere of radius $r_C A^{\frac{1}{3}}$. The sets of optimum parameter values in table 1 were deduced from simultaneous fits to both differential cross-section and polarization data.

TABLE 1

Optical-model parameter values for calculations in text

Nucleus	^{64}Zn	^{68}Zn	^{114}Cd
U	45.66	42.37	43.16
r_U	1.125	1.16	1.18
a_U	0.780	0.754	0.748
W_V	6.70	6.70	6.70
W_D	2.50	2.52	2.52
$r_{WV, WD}$	1.33	1.324	1.320
$a_{WV, WD}$	0.656	0.673	0.757
$U_{s.o.}$	6.49	6.03	5.71
$r_{s.o.}$	1.020	1.055	1.047
$a_{s.o.}$	0.706	0.700	0.747

All well depths are in MeV and all radii and diffusenesses in fm.

3. Analysis of one-phonon excitations

The ORNL distorted-wave (DW) program JULIE [8]) and the Oxford strong coupling approximation (SCA) program due to Hill [9]) were used to analyse the inelastic data. Both programs use the same type of nuclear matrix element or "form factor", which is obtained by deforming the entire optical potential. There are four contributions to the form factor corresponding to the real central, imaginary central, spin-orbit and Coulomb parts of the optical potential. Earlier analyses have indicated the need for a complex-plus-Coulomb form factor (CFF). The more recent analyses including asymmetry data [1-3,10]) have shown the necessity for a deformed spin-orbit term (DSO).

Distorted-wave predictions for the first quadrupole (2^+) state of ^{64}Zn using complex coupling and complex-plus-spin-orbit coupling are shown in fig. 1. All calculations in the present work include the Coulomb contribution. To obtain optimum agreement with the present inelastic *cross-section* data, it is necessary for the deformation parameter $\beta_\lambda^{s.o.}$ associated with the spin-orbit term to take a value 1.5

times that associated with the other terms (β_λ). The present optical potentials give somewhat better fits to the elastic cross sections than to the polarizations, and these results complement the findings [3]) at 40 MeV. There, polarization-biassed potentials

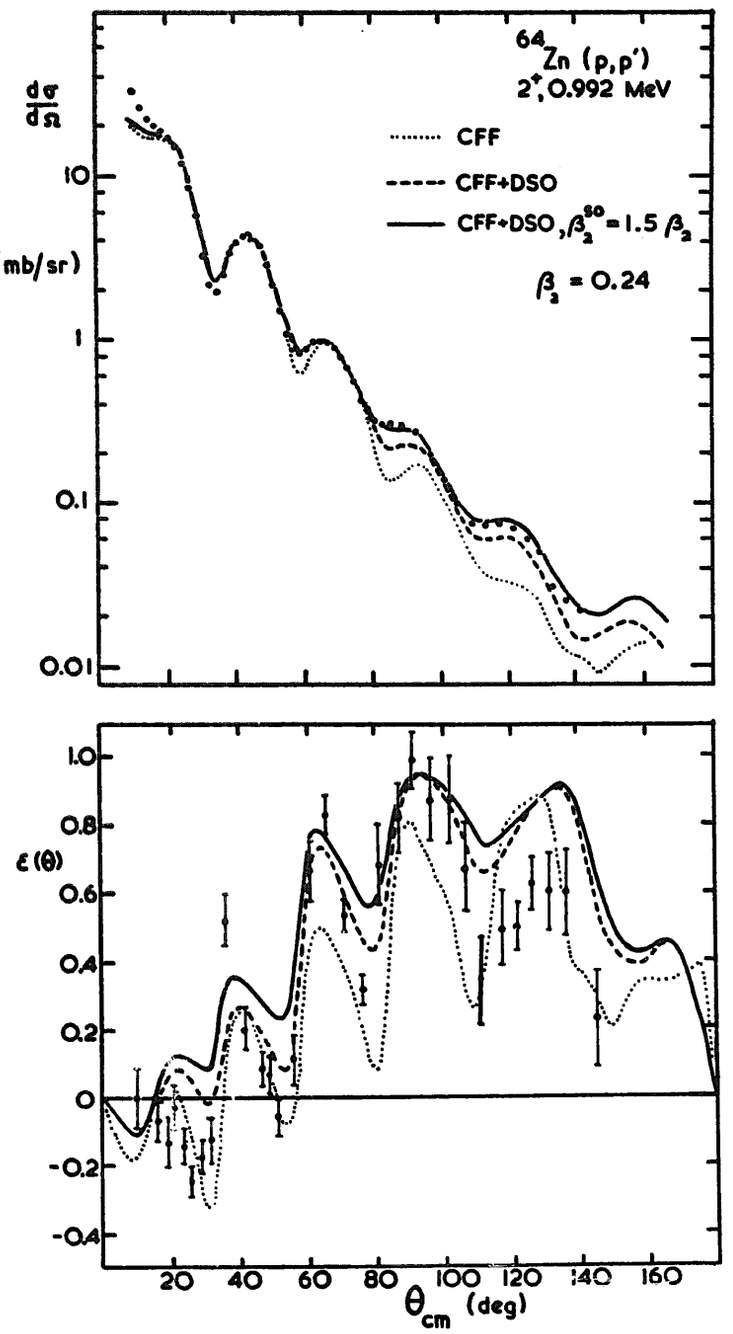

Fig. 1. Comparison between experiment and distorted-wave predictions for the lowest (2^+) quadrupole excitation of ^{64}Zn. Complex coupling and Coulomb excitation plus varying strengths of the deformed spin-orbit contribution are used.

were used, and optimum agreement with the inelastic *asymmetry* data required a deformed spin-orbit contribution with $\beta_\lambda^{s.o.} = 1.5\,\beta_\lambda$. We have also found optical parameters which improve the elastic polarization and inelastic asymmetry fits,

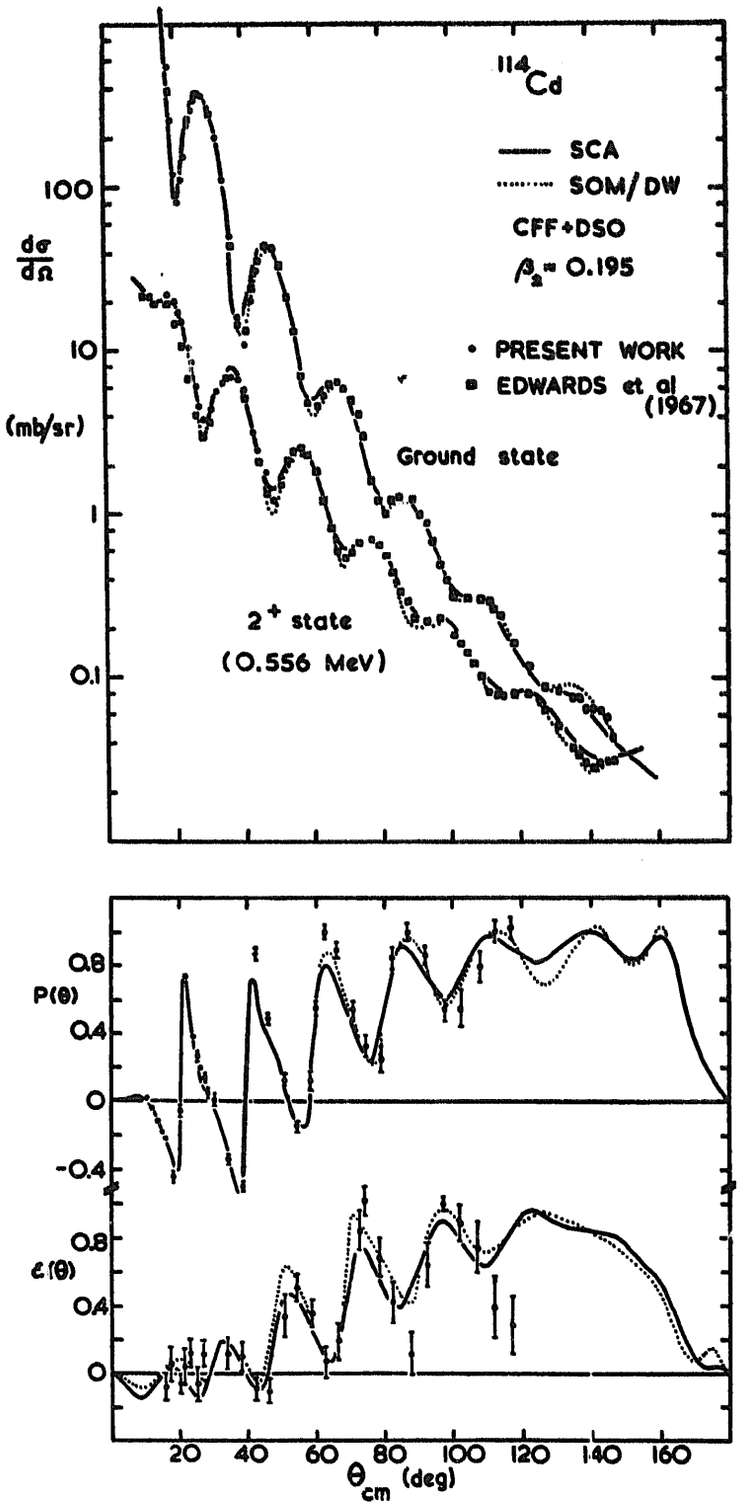

Fig. 2. Comparison of experiment, strong coupling approximation (SCA) predictions and simple optical model (SOM) and distorted-wave (DW) predictions for the ground state and lowest quadrupole excitation of ^{114}Cd. The optical parameters used for the SCA curves differ from those given in table 1; the real central potential depth U is increased to 44.4 MeV and the imaginary surface term depth decreased to 2.10 MeV. Complex-plus-deformed-spin-orbit coupling is used. For the DW predictions, the spin-orbit deformation parameter ($\beta_2^{s.o.}$) has to be 1.5 times that corresponding to the central potential (β_2). Coulomb excitation is included.

but, as found previously [3]), this invariably worsens the cross-section fits. More large-angle elastic polarization data are desirable to deduce optimum, polarization-biassed potentials at this energy. The impossibility of obtaining simultaneous optimum fits to elastic cross-section and polarization data and hence to inelastic cross-section and asymmetry data surely indicates some deficiency in the present form of the optical potential.

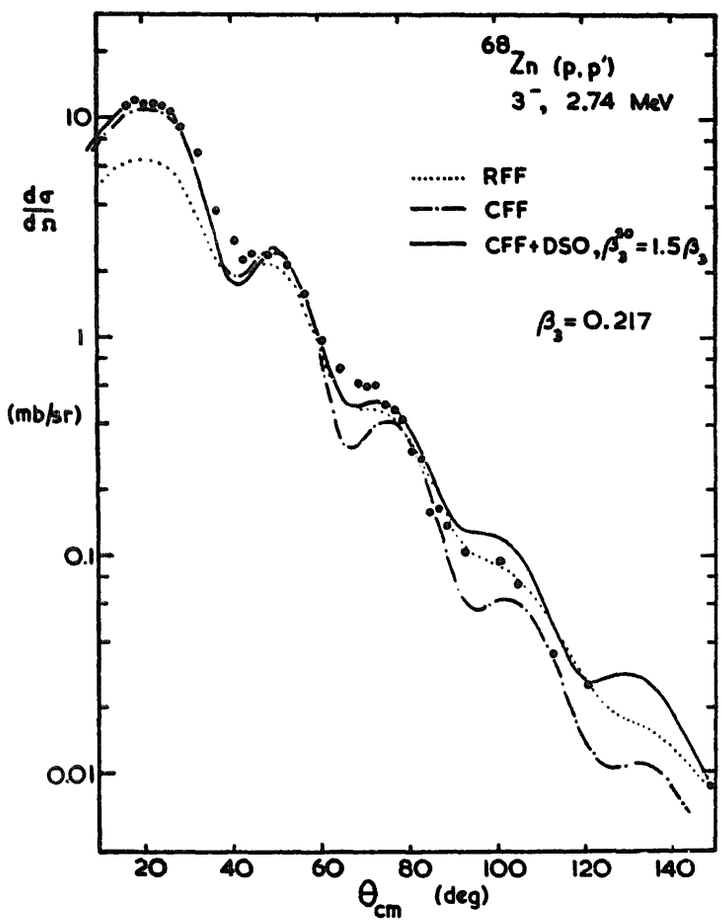

Fig. 3. Comparison between experiment and distorted-wave predictions for the lowest octupole excitation in ^{64}Zn. Deforming only the real part of the optical potential gives the RFF curve, deforming both the real and imaginary parts gives the CFF curve, and deformation of the complete potential gives the CFF+DSO curve.

SCA calculations using the fully deformed optical potential were carried out for the first 2^+ excitations of all these nuclei. Two channels were included, i.e. the ground and 2^+ states. Excellent fits were obtained for all these cross sections when the imaginary well-depth W_D was reduced by about 15 % to compensate for the fact that the scattering to the 2^+ level was now calculated explicitly. It was also found necessary to increase the real central well-depth U by 1–2 %. The fits to the ^{114}Cd data are shown in fig. 2 and compared with the one-channel elastic (SOM) and inelastic (DW) predictions. The DW and SCA predictions agree closely, and both

show a distinct preference for the new data in the range 20°–28°. The adjustment of parameters can to some extent mask deficiencies in the collective-model interaction, but the agreement with the data is significantly better than that obtained previously [7]) for ^{64}Zn and ^{114}Cd, where only complex coupling was employed.

It is interesting to note that the SCA fit to the inelastic cross section does not require a value of $\beta_2^{s.o.}$ different from β_2. However, the significance of this is difficult to assess since neither the DW nor the SCA treatment with the present spin-orbit form factor gives a good fit to the inelastic asymmetry data at forward angles.

TABLE 2

Summary of deformation parameters β_λ for 50 MeV protons

Nucleus	$J\pi$	β	Method	Energy (MeV)	Ref.
Zn(natural)	2^+	0.235	(p, p'),	17	a)
		0.26	(d, d'),	26	b)
	3^-	0.23	(n, n'),	14	c)
^{64}Zn	2^+	0.25	B(E2)		d)
		0.243	(p, p'),	50	e)
		0.235	(p, p'),	50	f)
	3^-	0.235	(p, p'),	50	f)
^{66}Zn	2^+	0.227	B(E2)		d)
		0.228	(p, p'),	50	f)
	3^-	0.25	(p, p'),	50	f)
^{68}Zn	2^+	0.205	B(E2)		d)
		0.217	(p, p'),	50	f)
	3^-	0.217	(p, p'),	50	f)
^{70}Zn	2^+	0.229	B(E2)		d)
		0.229	(p, p'),	50	f)
	3^-	0.216	(p, p'),	50	f)
^{114}Cd	2^+	0.193	B(E2)		d)
		0.192	(p, p'),	50	e)
		0.22	(p, p'),	55	g)
		0.195	(p, p'),	50	f)

a) Ref. [24]). b) Ref. [29]). c) Ref. [30]). d) Ref. [31]). e) Ref. [7]).
f) Present work. g) Ref. [32]).

Distorted-wave calculations of the differential cross sections for the octupole (3^-) levels of the zinc isotopes also indicate some preference for a completely deformed optical potential with $\beta_3^{s.o.} = 1.5\,\beta_3$ (fig. 3), but the predicted distributions are too oscillatory. This difficulty has also been experienced in the analysis of 30 MeV data [2]). Unfortunately, there are no 50 MeV asymmetry data available for these 3^- levels.

The deformation parameters β_λ predicted by the present analysis are given in table 2 and compared with values obtained in other work including measurements on electromagnetic transitions. The over-all agreement is very satisfactory.

4. Modification of the deformed spin-orbit term

The forward-angle asymmetry-fit cannot easily be improved by changes in optical-potential parameters, which suggests that the collective interaction, the deformed spin-orbit term in particular, needs modification. The prescription presently adopted for the spin-orbit interaction is obtained [3]) by deforming the radial part of the Thomas term in the optical potential. This results in the following form factor used in the

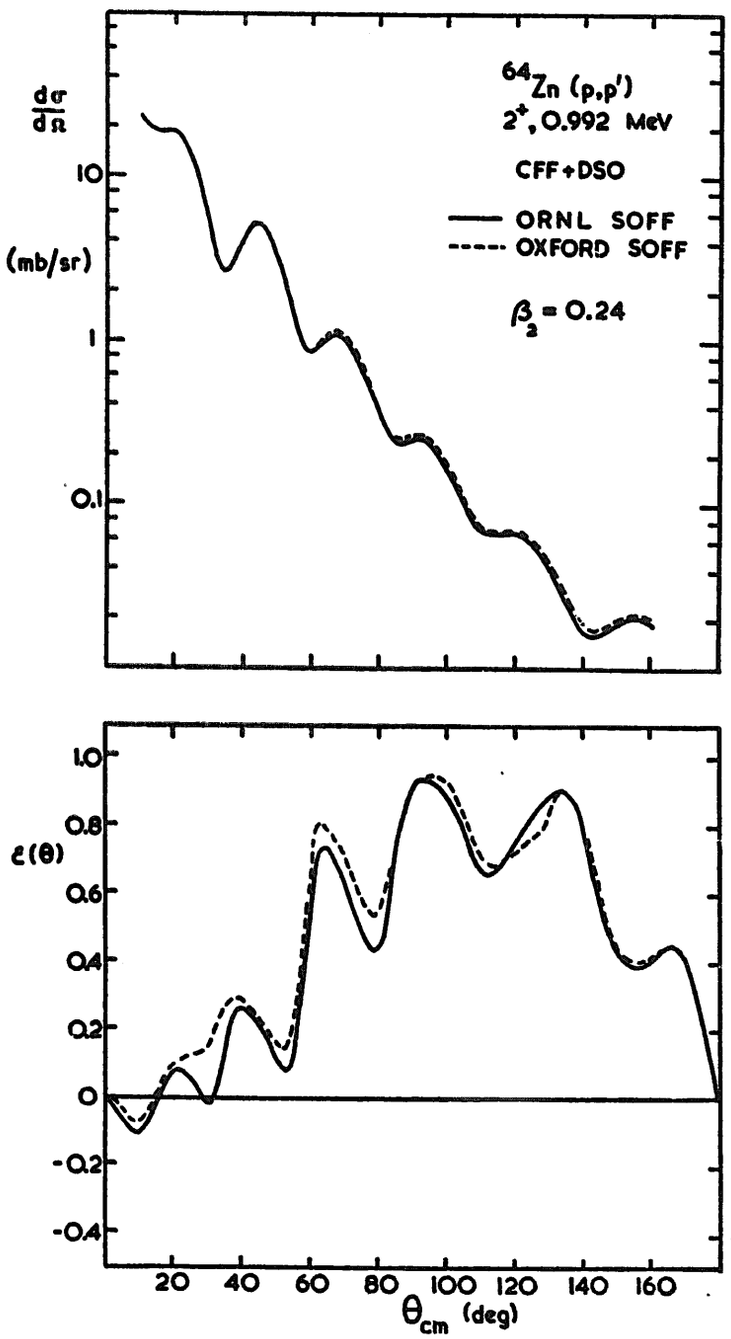

Fig. 4. Distorted-wave predictions for ^{64}Zn lowest 2^+ excitation using the ORNL and original Oxford forms for the deformed spin-orbit term. The coupling is complex and includes Coulomb excitation.

above calculations

$$f_{s.o.}(r) = \left(\frac{\hbar}{m_\pi c}\right)^2 U_{s.o.} \frac{1}{r}\left(\frac{d}{dr}\right) g(r, r_{s.o.}, a_{s.o.}) \cdot \tfrac{1}{2}(Y\sigma \cdot l + \sigma \cdot l\, Y). \quad (1)$$

The original Oxford program used a slightly different form factor [1]) than that obtained in the usual way

$$f_{s.o.}(r) = \left(\frac{\hbar}{m_\pi c}\right)^2 U_{s.o.} \left(\frac{d}{dr}\right)\frac{1}{r}\, g(r, r_{s.o.}, a_{s.o.}) \cdot \tfrac{1}{2}(Y\sigma \cdot l + \sigma \cdot l\, Y). \quad (2)$$

The cross-section predictions with these two form factors are very similar (fig. 4), but the asymmetry predictions have more marked differences; the forward angle asymmetry fit is somewhat worse with the Oxford program. Similar results were obtained for ^{68}Zn and ^{114}Cd.

We have also performed DW calculations where the radial shape of the spin-orbit form factor (1) was changed from

$$\frac{1}{r}\left(\frac{d}{dr}\right) g(r, r_{s.o.}, a_{s.o.}) \text{ to } (r_{s.o.} A^{\frac{1}{3}})^{-1} \left(\frac{d}{dr}\right) g(r, r_{s.o.}, a_{s.o.}).$$

This produced almost no difference in the predictions, and we are thus led to agree with the result at lower energies [1]) that such radial variations in the spin-orbit interaction cannot account for discrepancies in the inelastic asymmetry fits.

The 50 MeV asymmetry data for the zinc isotopes are a little more precise at forward angles than the 40 MeV results [3]) for medium-weight nuclei and should provide a good test for a more elaborate spin-orbit interaction, particularly that obtained by deforming the complete Thomas potential and retaining the non-radial contributions [10]). This indeed improves the forward-angle asymmetry agreement at 40 MeV; the asymmetry predictions at larger angles, and the cross-section predictions are little affected. However the point made above about some deficiency in the present optical potential still holds.

5. Extension to 18.6 MeV data

Some difficulties with the optical potential were also experienced in calculations of proton scattering at energies near 18.6 MeV, where we have carried out an analysis similar to that of ref. [3]). Our findings are by and large coincident with those of Glashausser *et al.* [1]). At this lower energy, we are able to improve the fits to both the elastic polarization [11]) and differential cross-section [12]) data considerably over those obtained [see refs. [1,11])] with parameters near the Perey [13]) values; and superior predictions were then obtained for the inelastic asymmetry data [1]). The new potential had radius parameters similar to those found [3,14]) at 40 MeV but lower values of the imaginary and spin-orbit diffusivities: $a_{WV} \approx a_{WD} \approx a_{s.o.} \approx 0.45$ fm.

However the reaction cross-section values obtained in this process were invariably 10–20 % higher than those expected from measurements [15]) at nearby energies; this effect can be observed in the results given in ref. [1]). Part of this anomaly may be due to idiosyncrasies in the elastic cross-section data, since different geometrical parameters were required to optimize the fits to the cross-section data [12]) at 18.6 MeV from those required at 17.9 and 18.2 MeV [ref. [16])] and at 17.3 MeV [ref. [17])]. Another possibility is that the scattering at this energy from medium-weight nuclei is not entirely free of resonance effects.

6. Two-phonon excitations

The simple vibrational model does not predict the existence of many of the weak levels observed in these and other nuclei. The second 0^+ state in ^{64}Zn, the third 2^+ state in ^{68}Zn and ^{70}Zn and the non-normal-parity states of ^{64}Zn [ref. [18])] for example, can be understood from the quantitative development by Davydov and his co-workers [19]) of the effect of a breakdown of nuclear axial symmetry on Bohr's original collective-model considerations [20]). In this theory, the "γ-vibrational" bands, which include the non-normal-parity states, appear as rotational levels by treating the deformed nucleus as an asymmetric rotator in an adiabatic approximation. Further calculations along these lines by Aisenberg and Suárez [21]) include the coupling between the oscillations corresponding to the deformation parameter β and the shape parameter γ. The expectation values γ_0 for ^{64}Zn, ^{66}Zn and ^{114}Cd (29°, 30° and 24°, respectively) required in order to predict the observed sequence of positive-parity levels imply a large axial asymmetry. Kumar and Baranger [22]) used coupled rotations and anharmonic vibrations in their calculations of the magnitude of the anomalous quadrupole moment of the first 2^+ state ^{114}Cd. It is to be expected, therefore, that good fits to the scattering from many of the weak levels will require an extension of the simple Bohr Hamiltonian for vibrations similar to that undertaken by Tamura [23]) for coupled rotational and vibrational bands in ^{24}Mg.

Strong interference effects between the multiple and direct modes of excitation of two-phonon states [24]) preclude the use of the distorted-wave method. At the same time, neither the present data nor the applicability of the two-phonon model justify an exhaustive analysis, which requires a large amount of computing time.

In this work, a goodness-of-fit criterion Δ_σ was defined in order to facilitate comparison between the curves

$$\Delta_\sigma = \frac{1}{n}\sum^{n}\left(\frac{\sigma_{\mathrm{exp}} - \frac{1}{N}\sigma_{\mathrm{th}}}{\delta\sigma_{\mathrm{exp}}}\right)^2,$$

where n is the number of experimental points $\sigma_{\mathrm{exp}} \pm \delta\sigma_{\mathrm{exp}}$ corresponding to the theoretical values σ_{th}.

The optical parameters were obtained from fits to the elastic data in which all the parameters were varied. These searches resulted in an imaginary potential with the radius of the volume part (r_{WV}) much less than the radius of the surface derivative term (r_{WD}) [refs. [6,25])]. This yielded a reduction in Δ_σ of 25–40 % compared with

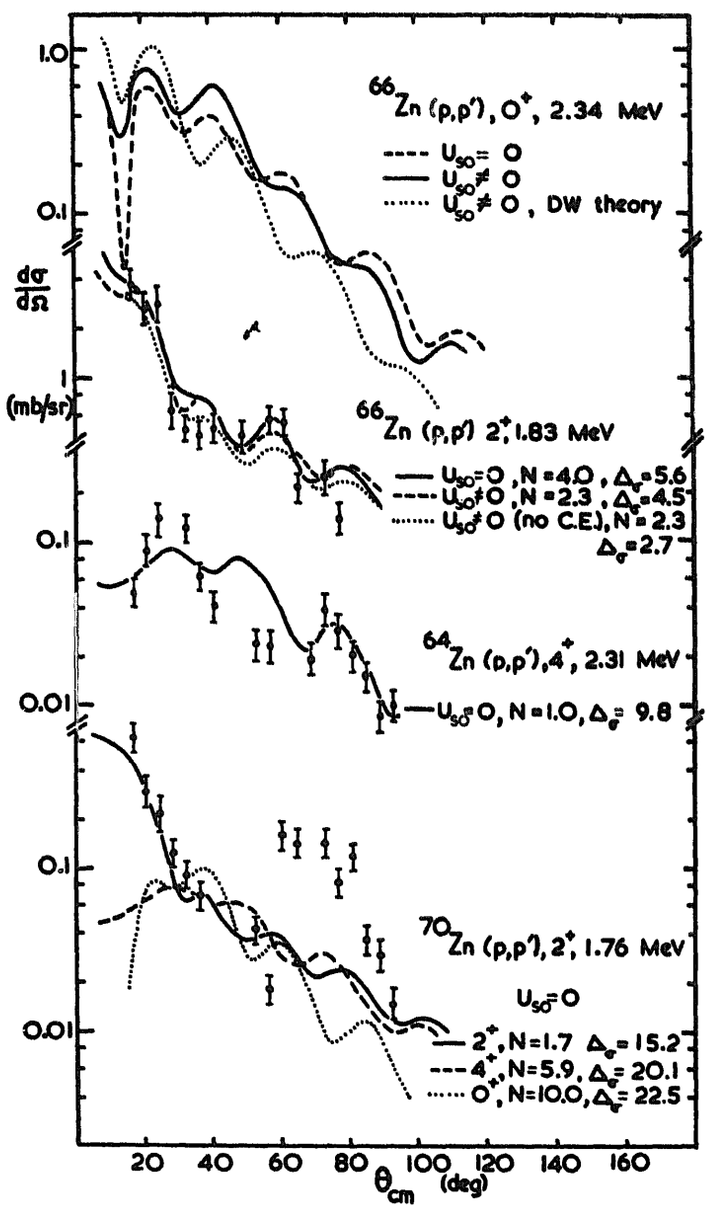

Fig. 5. Experimental and theoretical cross sections for two-phonon states in ^{64}Zn, ^{66}Zn and ^{70}Zn. Except where indicated, the coupling scheme used is 0_0, 2_1^+, (0_2^+ *or* 2_2^+ *or* 4_2^+). The entire optical potential (CFF+DSO) is deformed, and in some cases the spin-orbit interaction is neglected by using optical parameters with $U_{s.o.} = 0$ (table 3).

similar fits in which r_{WV} was kept equal to r_{WD}. It is interesting to note that the imaginary potential shape defined by these parameters is very similar to that obtained by Greenlees *et al.* [26]) from other considerations. The potential well-depths were then

adjusted to fit the 0_0^+ and 2_1^+ data using the SCA program. Calculations were also performed with a "spin-independent" optical potential (i.e., $U_{s.o.}$ set equal to zero). This type of potential was obtained using the same procedure as above except that polarization and asymmetry data were not included in the searches. This type of potential yielded elastic fits in which Δ_σ was increased by 50 %, and two-phonon fits in which Δ_σ was not changed in more than 25 %. This is shown for the 0_2^+ and 2_2^+ states of ^{66}Zn in fig. 5.

Table 3 shows the final sets of parameters used in the two-phonon calculations. Other workers [27,28]) have found it necessary to reduce β_λ values for the two-phonon states below the single-phonon β_λ value. This was not possible with the Oxford program, which instead was allowed to normalize the theoretical cross sections to fit the data. The normalization factor N was found to be greater than unity for some

TABLE 3

Optical-model parameters used in the two-phonon calculations (fig. 5)

	^{66}Zn	^{68}Zn	^{70}Zn
U	34.90	38.25	43.64
r_U	1.24	1.18	1.16
a_U	0.73	0.75	0.74
W_V	18.14	33.41	4.9
r_{WV}	0.70	0.49	1.09
a_{WV}	0.59	0.62	0.28
W_D	4.65	5.25	4.2
r_{WD}	1.13	1.28	1.23
a_{WD}	0.76	0.76	0.88
$U_{s.o.}$	0	6.46	0
$r_{s.o.}$		1.01	
$a_{s.o.}$		0.73	
β_2	0.186	0.23	0.196

The parameters were found by fitting the ground state and first excited state with either $U_{s.o.} = 0$ or $U_{s.o.} \neq 0$. Well-depths are in MeV and radii and diffusenesses in fm.

of the 2_2^+ cross sections (fig. 5), while a factor less than unity was necessary in order to fit the 0_2^+ state in ^{68}Zn. The 4_2^+ state in ^{64}Zn (fig. 5) was well fitted with a normalization factor of unity.

Some of the experimental cross sections showed large oscillations near 60° which could not be reproduced. Other coupling schemes (e.g. 0_0^+, 6_1^+, 4_2^+) also failed to improve the fits at these points. If these anomalies are neglected, the forward-angle data (16°–56°) are sufficient to choose between either a 0_2^+ or 4_2^+ state and a 2_2^+ state. The Δ_σ values obtained for the 1.76 MeV state in ^{70}Zn over this restricted angular range were 9.4(2^+), 17.9(4^+) and 16.9(0^+). On this basis, a spin and parity of 2^+ can be assigned to the 1.56 and 1.76 MeV states of ^{70}Zn.

The β_2 values required in the spin-independent calculations were 80 % of those found in DW and SCA fits to 2_1^+ levels which included a spin-orbit interaction.

This is attributed to the surface-peaking of the spin-orbit potential which tends to increase the reflection coefficients, hence reducing the absorption.

7. Conclusions

The collective model has proved as successful in describing the angular distributions of one-phonon excitations for 50 MeV protons as it has for lower-energy protons. Some difficulties still remain however. The theory predicts too much oscillation in the octupole cross-section distributions. The forward-angle inelastic asymmetry is not well described by the interaction used here, although this difficulty might well be minimized by the use of a more elaborate treatment of the spin-orbit interaction [10]). The difficulty found in fitting elastic cross-section and polarization data simultaneously is reflected in the inelastic predictions and indicates a deficiency in the present form of the optical-model potential. In the two-phonon work, the fits obtained are not so conclusive and indicate the need for more experimental data. At the same time, extensions to the model are required in order to study a greater variety of excitations. Work is continuing on the development of SCA and DW methods to calculate explicitly the effect of gamma vibrations on these cross sections.

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