NOTE

# BAND STRENGTH AND LINE HALF-WIDTH OF THE $10 \cdot 4 \mu \mathrm{CO}_{2}$ BAND 

S. R. Drayson and C. Young*

High Altitude Engineering Laboratory, Department of Aerospace Engineering, University of Michigan
(Received 31 March 1967)


#### Abstract

Experimental absorption data using a $10.4 \mu \mathrm{CO}_{2}$ laser have been re-analysed. The band intensity of the $10.4 \mu\left(10^{\circ} 0-00^{\circ} 1\right) \mathrm{CO}_{2}$ band is $0.0184 \mathrm{~cm}^{-1}\left(\mathrm{~atm} \mathrm{~cm}_{273}{ }^{\circ} \mathrm{K}\right)^{-1} \pm 5 \%$ at $300^{\circ} \mathrm{K}$ and the self broadened Lorentz half-width of the $\mathrm{P}(20)$ line is $0.096 \mathrm{~cm}^{-1} \pm 5 \%$ at 1 atm and $300^{\circ} \mathrm{K}$.


The advent of the laser has made possible new techniques for determining intensities and collisional half-widths of lines in molecular absorption bands. McCubrin et al. ${ }^{(1)}$ have recently made measurements with a $\mathrm{CO}_{2}$ laser on the $\mathrm{P}(20)$ line of the $10-4 \mu$ band of $\mathrm{CO}_{2}$. Using the laser as an essentially monochromatic source with the frequency of the line-centre, they measured the transmission of the beam through a $\mathrm{CO}_{2}$ absorption cell. The cell was initially empty and small amounts of pure $\mathrm{CO}_{2}$ were added until the pressure reached about 150 torr. The experiment was very carefully conducted, the data having only a small amount of scatter.

McCubbin et al. analysed their data as follows. At all pressures the influence of neighbouring lines at the centre of the $\mathrm{P}(20)$ line is negligible. At very low pressures the line tends to the Doppler profile, and at the line-centre we have

$$
\ln \tau=-\frac{S}{\alpha_{D}}\left(\frac{\ln 2}{\pi}\right)^{1 / 2} u_{0} p
$$

where $\tau$ is the transmittance, $S$ the line intensity, $\alpha_{D}$ the Doppler half-width, $u_{0}$ the length of the absorption cell and $p$ the pressure in atmospheres. The line intensity can be found from the slope of the plot of $\ln \tau$ against $p$, as the pressure tends to zero.

At much higher pressures collisional broadening dominates, and the expression for $\tau$ now becomes:

$$
\ln \tau=-\frac{S}{\pi} \frac{u_{0}}{\alpha_{0}}
$$

where $\alpha_{0}$ is the Lorentz half-width at 1 atm . As the pressure increases, $\tau$ tends to a constant value and $\alpha_{0}$ can easily be found using the value of $S$ already determined.

The difficulty of this method lies in accurately determining the slope of the curve as $p$ tends to zero. The problem is compounded by the fact that, even for pressures as low as

[^0]1 torr the absorption coefficient of the mixed Doppler-Lorentz (or Voigt) profile is about 10 per cent lower than the pure Doppler. At lower pressures the scatter of the experimental data, due to the small absorption, makes it very difficult to accurately determine the slope.

We have re-analysed the data using the mixed Doppler-Lorentz absorption coefficient. In this case

$$
\ln \tau=-\frac{s}{\alpha_{D}}\left(\frac{\ln 2}{\pi}\right)^{1 / 2} u_{0} p \frac{y}{\pi} \int_{-\infty}^{+\infty} \frac{\mathrm{e}^{-t^{2}}}{y^{2}+t^{2}} \mathrm{~d} t
$$

where $y=\frac{\alpha_{0} p}{\alpha_{D}}(\ln 2)^{1 / 2}$.
Calculations using the values deduced by McCubbin et al. showed that the predicted absorption was consistently less than the experimental (see Fig. 1). Several larger values of


Fig. 1. Logarithmic plot of transmittance vs. pressure. Dashed line is theoretical curve of growth using parameters from McCubbin et al. Solid line uses re-estimated values, with error bars indicating $\pm 5$ per cent deviation in line intensity.

From Fig. 1, McCubbin et al. ${ }^{(1)}$
line intensity were tried. The value $S=5.6 \times 10^{-4} \mathrm{~cm}^{-1}\left(\mathrm{~atm} \mathrm{~cm}_{300}{ }^{\circ}\right)^{-1}$ gave a good fit to the experimental points over the entire range of pressure. This corresponds to a selfbroadened Lorentz half-width of $0.096 \mathrm{~cm}^{-1}$ at $300^{\circ} \mathrm{K}$. The curve is shown in Fig. 1, with error bars showing the effect of a 5 per cent change in $S$.

The value of band intensity for the $10.4 \mu$ band $\left(10^{0} 0-00^{\circ} 1\right)$ obtained is $0.0184 \mathrm{~cm}^{-1}$ (atm $\left.\mathrm{cm}_{273} \mathrm{~K}\right)^{-1} \pm 5 \%$ at $300^{\circ} \mathrm{K}$. This lies well within the range of values recently deduced by $\operatorname{Gray}^{(2)}\left(0.019 \pm 0.004 \mathrm{~cm}^{-1}\left(\mathrm{~atm} \mathrm{~cm}_{273^{\circ} \mathrm{K}}\right)^{-1}\right.$ at $\left.300^{\circ} \mathrm{K}\right)$. MADDEN ${ }^{(3)}$ has measured the variation of the self-broadened half-width with rotational quantum number for the $\left(02^{\circ} 0-\right.$ $\left.01^{1} 0\right) \mathrm{CO}_{2}$ band near $600 \mathrm{~cm}^{-1}$. The optical collision cross-section varies with rotational quantum number and we would also anticipate some variations from band to band. However the latter effect is probably quite small. Madden obtained the value $0.102 \mathrm{~cm}^{-1} \pm$ 10 per cent at $300^{\circ} \mathrm{K}$ for the $\mathrm{P}(21)$ line, which agrees with the laser determined value within experimental error.

A second paper by Gerry and Leonard ${ }^{(4)}$ used the same experimental technique and the mixed Doppler-Lorentz absorption coefficient discussed above. However, the larger scatter in their measurements makes it difficult to obtain an accurate fit to their data. In addition to this, both the Doppler and Lorentz half-widths vary by more than 15 per cent over their experimental temperatures. Using their best fit curve, the value of line intensity $2.9 \times 10^{-4} \mathrm{~cm}^{-1}\left(\mathrm{~atm} \mathrm{~cm} 273^{\circ} \mathrm{K}\right)^{-1}$ at $273^{\circ} \mathrm{K}$ and $\alpha_{0}=0.10 \mathrm{~cm}^{-1}$ was obtained. Converted to $300^{\circ} \mathrm{K}$, the line intensity becomes $4.4 \times 10^{-4} \mathrm{~cm}^{-1}\left(\mathrm{~atm} \mathrm{~cm} 300^{\circ} \mathrm{K}\right)^{-1}$ at $300^{\circ} \mathrm{K}$, some 20 per cent lower than the value from McСubin et al.

In addition Gerry and Leonard give a value for the optical collision cross-section which is close to the gas kinetic cross-section for $\mathrm{CO}_{2}$. This is clearly incorrect, since the optical cross-section is appreciably larger than the gas kinetic values and only tends to the gas values for rather large rotational quantum numbers.

The laser method for the determination of line intensities and Lorentz half-widths of optically active molecules appears capable of a high degree of accuracy. Further measurements on foreign broadened gasses, particularly atmospheric constituents, should prove most useful. The band intensity $0.0184 \mathrm{~cm}^{-1}\left(\operatorname{atm} \mathrm{~cm}_{273 \cdot \mathrm{~K}}\right)^{-1} \pm 5 \%$ at $300^{\circ} \mathrm{K}$ and the selfbroadened Lorentz half-width of the $\mathrm{P}(20)$ line $0.096 \mathrm{~cm}^{-1} \pm 5 \%$ at atmospheric and $300^{\circ} \mathrm{K}$ probably represent the most accurate determination to date.

## REFERENCES

1. T. K. McCubbin, Jr., R. Darone and J. Sorrell, Appl. Phys. Lett. 8, 118 (1966).
2. L. D. Gray, JQRST 7, 143 (1967).
3. R. P. Madden, J. Chem. Phys. 35, 2083 (1961).
4. E. T. Gerry and D. A. Leonard, Appl. Phys. Lett. 8, 227 (1966).

[^0]:    * Dr. Young is now at Dept. of Physics, University of New Brunswick, Fredericton, New Brunswick, Canada.

