

chemistry) of photochemical phenomena; the utilization of photochemical techniques in synthesis is treated in passing but is not emphasized. After an introduction, which seemed rather disjointed, the two succeeding chapters survey atomic spectra and structure, and the structure and spectra of small (usually diatomic) molecules; photophysical (light emission, energy transfer, energy degradation) and photochemical (decomposition, photosensitized reactions) processes are discussed in terms of the spectroscopy of the species involved. Purely physical reactions of photo-excited polyatomic molecules are discussed in considerable detail.

The reviewer gained the impression that contemporary interpretation has been presented in too dogmatic a fashion. If, as is to be anticipated, some modifications of current views will arise because of future work, much of this chapter could become obsolete. Perhaps here was the place for a bit more skepticism in the treatment.

Chapter 5 contains an up-to-date, but relatively uncritical, survey of observations of the photochemical facts concerning a large number of polyatomic molecules. Both the novice and the experienced research worker will find this chapter an indispensable key to the literature on the photochemistry of the molecules cited. The chapter on the "Determination of Mechanism" contains much material on reaction rate theory. This should now be regarded as extraneous in a work on photochemistry, but otherwise it is an excellent survey of approaches available to disentangle the secondary from the primary processes.

The authors are to be commended for their 123 pages on experimental methods. Here are described and discussed essentially all the basic pieces of equipment that are necessary for the newcomer to begin work. The more experienced will find a most convenient collection of working formulas. In addition he will find several cautionary notes concerning measurements of quantum yields which all too frequently either have been unrecognized or ignored in much of the published literature on photochemistry.

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INTERVAL ANALYSIS, by Ramon E. Moore.  
145 pages, diagrams, 6 × 9 in. New Jersey,  
Englewood Cliffs, Prentice-Hall, 1966.  
Price, \$9.00.

Many practical problems require numerical computation with inexact data because the magnitudes involved are determined experimentally by approximate measurements. Usually there is some knowledge of the accuracy of the measurement which may be indicated by a bound for its error.

The usual point arithmetic—in which the numbers used can be represented by points on a scale—is inadequate, without some modification, for drawing precise conclusions about many of the results of arithmetic operations in which the magnitudes of the measurements can not be completely represented by points.

If however with  $a \leq b$  it is known that measurement  $x$  is no smaller than  $a$  and no larger than  $b$ , then the measurement can be represented by the closed interval  $[a, b]$  which consists of all the point values of  $x$ ,  $a \leq x \leq b$ , and is formally defined by the set  $\{x \mid a \leq x \leq b\}$ . This dual number is called an *interval number* and analysis with interval numbers is called *interval analysis*. The customary (point) number is then a special (degenerate) case of an interval number with  $b = a$ .

Professor Moore, building on the results of earlier authors and using modern digital computation, presents an integrated treatment of interval numbers which covers not only the basic operations of interval arithmetic but also matrix computations with intervals, interval integrals, an interval method for integral equations, etc.

One modification of the use of point numbers for the solution of problems featuring measurements with inherent errors, applicable to formulae with arguments having bounds for the inherent error, is the expression of the increment of the function in terms of the arguments and their inherent errors.

The differential of the function is frequently used as an approximation to the increment. Alternatively probability theory can be applied, in situations in which certain assumptions regarding the distribution of the inherent errors seem suitable, to discover how these errors are propagated under calculation.

But probably the most useful modification,

applicable to more general problems, is that of significant figures in which the decimal number resulting from a calculation is expressed to no more places than is warranted by the precision of the result. This modification calls for the interpretation of a decimal number as an interval rather than a point. Thus the digital 31.96 is interpreted as the interval number [31.955, 31.965].

This is an ingenious device and with the additional rules which have been worked out for the basic arithmetical operations, has been very useful in building an interval analysis using the structure of point analysis. But this interval analysis is severely restricted since only a relatively small class of interval numbers can be represented because (a) the interval featured can have only the size of the unit of the last decimal position and (b) the mid value of the interval number may not correspond to any point number in the subset of digital numbers available.

Thus while the interval number [0.985, 0.995], with interval length 0.010, is completely expressed by the point number 0.99, the interval number [0.989, 0.991], which has interval length of only 0.002, is also expressed by the same point number. The interval number [0.984, 0.986], also having interval length 0.002, is not represented by 0.985 nor by 0.98 nor by 0.99 but rather by 1.0, with an interval interpretation of [0.95, 1.05] and an interval length of 0.1. This interval is 50 times larger than the interval it is supposed to identify!

The author illustrates the fact that erroneous conclusions can come from the use of point arithmetic interpreted as interval arithmetic, even with multiple precision techniques, for problems with a small number, as well as for problems with a large number, of sequential computations.

What is needed, particularly in this age of improved calculational aids, is an analysis for interval numbers and this is what the author gives us. The operational laws of interval arithmetic are then presented. The distributive law does not always hold for interval arithmetic though a property of subdistributivity is helpful.

Because the number of decimal positions in precise interval arithmetic may grow very rapidly with successive operations, rounded interval arithmetic is introduced with the resultant intervals given by bounds rather

than limits. Chapter 4 is devoted to a matrix topology and features the concept of the "distance" between two intervals. Next the author discusses matrix computations with intervals as applied to direct methods of solving linear equations and inverting matrices, and explains E. Hansen's method for obtaining sharp bounds for the inverse of an interval matrix.

In discussing values and ranges of values of real functions, Moore introduces the concept and the use of the "centered form" and discusses briefly the determination and use of extreme values of rational functions as well as the values and ranges of irrational functions.

Other chapters deal with interval analysis for such topics as interval contractions and root finding, integrals, integral equations, and the initial-value problem in ordinary differential equations. Final discussions cover machine generation of Taylor coefficients, numerical results with Taylor expansion of order  $K$ , and coordinate transformations for the initial-value problem for systems of ordinary differential equations, for which a flow chart is shown in an appendix. Another appendix illustrates the various kinds of arithmetic used. References (mostly current) are given in each chapter to pertinent sources.

This book is a valuable introduction to interval analysis. It is a pioneering work on a subject which needs further development. As stated on p. 45, the concept of "centered form" is in an initial stage; and on pp. 67-69, that additional research is needed on the interval-vector version of Newton's method. The organization of this material and its presentation should do much toward stimulating further interest.

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ANNUAL REVIEW OF ASTRONOMY AND ASTROPHYSICS, VOL. IV, edited by L. Goldberg. 513 pages, diagrams, illustr.,  $6 \times 9$  in. Palo Alto, Cal. Annual Reviews, Inc., 1966. Price, \$8.50.

With the extensive astronomical literature being published today, it is difficult, if not impossible, to keep up with all the recent developments. Therefore, a comprehensive review in fields apart from one's specialties