

Method, apparatus, and experimental results are presented from an investigation which is part of a major study of the low temperature compressibilities of pure gases

COMPRESSIBILITIES FOR HELIUM AT TEMPERATURES FROM 70 TO 120°K AND PRESSURES TO 690 atm

J. A. SULLIVAN and R. E. SONNTAG†

A PROGRAMME has been undertaken at the University of Michigan to extend the low temperature compressibility measurements of pure gases to higher pressures and to investigate the P - v - T behaviour of various gas mixtures. The work presented here is a portion of this undertaking. The P - v - T behaviour in the argon-nitrogen system has been investigated,¹ as has that for neon,² and work is currently in progress on the extension of neon data to temperatures below 70° K.

The Burnett Method

The method used to collect the data in this work was originally proposed by Burnett³ in 1936. This technique makes it possible to obtain accurate compressibility data for pure gases and gas mixtures without having to determine the volume and mass of the test gas at each experimental point. The method has been used successfully in recent years by numerous investigators⁴⁻¹⁴ over wide ranges of temperature and pressure.

In using this method two volumes are placed in an accurately controlled temperature environment and the larger volume is charged with the test gas to some initial pressure P_0 . With the smaller volume evacuated each time, the gas is expanded in a series of steps to a low pressure. The information generated by this procedure is thus a series of pressures

$$P_0, P_1, P_2, \dots, P_{j-1}, P_j \quad \dots (1)$$

all measured for a single isotherm. An analysis of the method shows that the compressibility for each pressure point can be computed from the expression

$$Z_j = \frac{P_j \prod_{i=1}^j N_i}{P_0/Z_0} \quad \dots (2)$$

where the N_i are defined by

$$N_i = N_0 C_i \quad \dots (3)$$

The quantity N_0 is called 'the equipment constant' and is defined as

$$N_0 = \frac{V_I + V_{II}}{V_I} \quad \dots (4)$$

where V_I is the volume of the larger cell and V_{II} the volume of the smaller cell.

The C_i are correction terms for expansion of the cell volumes due to pressurization. The equipment constant is evaluated by extrapolating a plot of P_{j-1}/P_j versus P_j to zero pressure, and the fill constant (P_0/Z_0) is evaluated by extrapolating a plot of

$$P_j \prod_{i=1}^j N_i \text{ versus } P_j$$

to zero pressure.

For this research, where small volumes of the test gas were not at the test temperature (dead-space volumes), the expressions presented above must be modified as described in references 11 and 12. In addition, the values of N_0 and P_0/Z_0 can be adjusted by consideration of the non-linear behaviour of the experimental points on a plot of $v_j(Z_j - 1)$ versus $1/v_j$ as described in reference (5) or (15).

This experiment was designed and data collected with the intention of using the data reduction procedure of reference (5). In doing so, it was found that the results for helium in this range are extremely sensitive to the parameter P_0/Z_0 , in addition to N_0 . This led to the development of the double-optimization technique described in reference (15), which was then used to reduce the helium data. The experimental data-point coverage was not the optimum required for this new analytical technique; this has introduced some scatter into the second virial coefficient results, but has only a small effect on the values of compressibilities, the results of principal interest in this investigation.

† University of Michigan, Ann Arbor, Michigan, U.S.A. Received 8 August 1966. J.A.S. is now at Colorado State University, Fort Collins, Colorado, U.S.A.

Experimental Apparatus

The Burnett cells used in this work were heavy-walled cylinders, machined end-to-end in a 3-in. o.d. \times 12.375 in. bar of 347 stainless steel. The cell diameters were 1.5 in., with the volume of the larger cell 211.5 cm³ and that of the smaller cell 41.1 cm³. The resulting value of N_0 is about 1.194 for the temperature range of this investigation. The cell block was clad with 0.25 in. copper to aid in the attainment of temperature uniformity.

The primary temperature measurement was made with a Leeds and Northrup Type 8164 platinum resistance thermometer completely inserted in a hole drilled into the cell material between the two chambers. The resistance was read on a type G-2 Mueller bridge. A copper-constantan difference thermocouple referenced to the thermometer was located inside each cell, and others were placed on the outside of the cell block to detect any possible temperature gradients. The e.m.f. values were read on a Leeds and Northrup Wenner potentiometer. Thus it was possible to detect cell gradients of the order of ± 0.005 degK. The absolute temperatures reported in this work are considered constant and accurate to ± 0.015 degK.

The Burnett cells were suspended on 0.040 in. diameter wires in the lower portion of a cryostat insulated with multilayer insulation as indicated in Figure 1. An automatically controlled resistance heater made from 28 gauge Chromel-A wire was located around the cells and an additional heater was placed in the upper portion of the cryostat. The cryostat was filled with liquid nitrogen for temperatures from 70 to 100° K, and with liquid argon for the 120° K isotherm. The temperature was maintained constant by controlling the vapour pressure of the bath fluid. The bath was stirred by means of the magnetic stirring arrangement indicated in Figure 1. With this arrangement the bath temperature could be maintained within ± 0.005 degK of the desired temperature for periods exceeding 30 min with only minor adjustments of the controls.

The Burnett cells were connected to an inlet valve, expansion valve, and exhaust valve that were located outside the cryostat. In addition, a differential pressure indicator and the capillary tubing used to connect it to the inlet line of the large cell were also located outside the cryostat. The small volumes in the connecting lines, valves, and DPI constituted 'dead-space volumes' which had to be taken into account in analysing the data. The temperatures of the dead space volumes were measured with copper-constantan thermocouples, two of which were located on the lines leading from the valves to the cells. Two additional thermocouples were used to measure the temperature of the DPI and the valves. The temperatures of the dead-space volumes were recorded for each test point and averaged to obtain the dead-space temperature used in the data reduction. The total dead-space volume in this apparatus was about 0.8 cm³.

The pressure was measured with a Ruska Type 2400-1 dead-weight gauge, with the oil side of the dead-weight gauge separated from the test gas by the Ruska Type 2416-1 differential pressure indicator. From an error analysis of the overall pressure measurement system, the absolute accuracy of the pressure measurement is considered to be

$$\Delta P = 1.26 \times 10^{-4} P + 0.004 \quad (\text{lb/in}^2) \quad \dots (5)$$

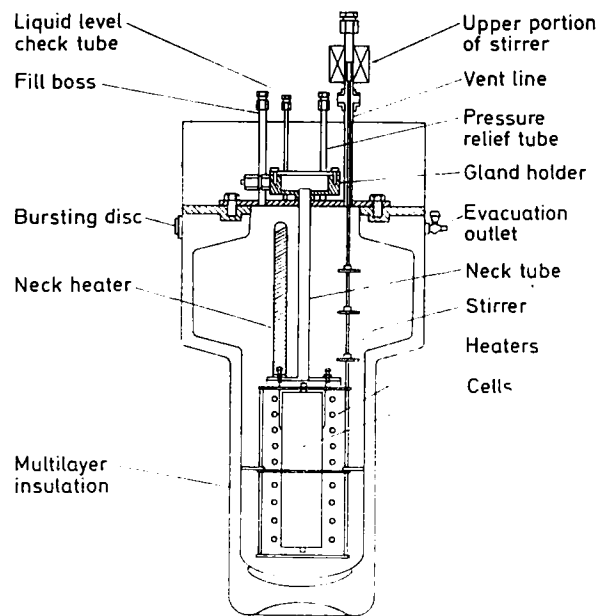


Figure 1. Schematic drawing of the cryostat

Experimental Procedure

After evacuating the smaller cell to 20–30 μ Hg and expanding the test gas from V_I to V_{II} , the temperature of the cells was frequently checked, and the vapour pressure controls and heater controls were manipulated so as to force the cells to remain at the desired temperature during the time required to take a point (20–30 min after expansion). The difference thermocouples were also monitored and when the difference thermocouples in the cells had indicated readings less than 0.25 μ V for 5 min the expansion valve was closed and the final null of the DPI was carried out. The cell temperature reading and the temperatures of the dead-space volumes were then recorded. Following this, the barometer reading was recorded and the gas expanded from the smaller cell and evacuation initiated. The room temperature, oil temperature, and exact weight used on the dead-weight gauge were then recorded. The average total time for one data point was about 70 min.

Experimental Results

Compressibilities

The isotherms for helium were investigated using high purity helium supplied by the U.S. Bureau of Mines helium plant at Amarillo, Texas. The purity of the helium conformed to the specifications listed in Table 1. The experimentally determined values of the compressibilities for helium are listed in Table 2 along with the computed values of the specific volumes. The number of experimental points for each isotherm is composed of the points for a complete run from 690 atm down to about 17 atm and the points for a partial run (6–8 points) at high pressures. The compressibilities for the data points of a partial run were obtained by fitting the reduced data for the full run to a fifth or sixth order polynomial in pressure and computing the compressibility factor for the last measured pressure of the partial run. By making the assumption that N_0 is the same for the partial run as for

TABLE 1. HELIUM PURITY

Contaminant	Amount (p.p.m.)
Hydrogen	<0.4
Methane	0.0
Water	<0.5
Neon	<14.0
Nitrogen	<4.0
Oxygen	<0.8
Argon	Trace
Carbon dioxide	<0.1

the full run, the value of the fill constant for the partial run can be determined from

$$P_0/Z_0 = \frac{P_i}{Z_i} \prod_{i=1}^l N_i \dots (6)$$

Knowing P_0/Z_0 and N_0 for the partial run permits the computation of the compressibilities for the run.

Table 3 is a comparison of the experimental compressibilities for helium with published values for selected values of pressure. The values of compressibility listed from the work of Holborn and Otto¹⁶⁻¹⁸ were obtained by interpolating between their isotherms. The values published by Mann¹⁹ were calculated from the Strohbridge equation of state, based on existing data in the temperature range 3-300° K. The values of compressibility listed for this work were obtained by fitting each isotherm to the Berlin expansion and computing the values of compressibility for the even pressures listed in Table 3.

It is seen that in the range 0-100 atm the compressibilities from this work agree with the results of Holborn and Otto to within 0.3 per cent, while the agreement with the calculated results of Mann is somewhat better than 0.15 per cent.

Second Virial Coefficients

Second virial coefficients obtained from the experimental compressibility data at the four isotherms are shown in Figure 2, and compared with those of previous investigators. Although these values exhibit some scatter, the overall agreement with previous work, especially the recent values of White, is satisfactory. As

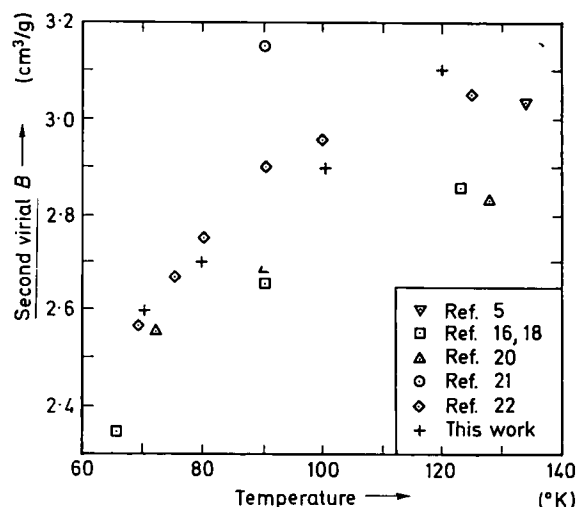


Figure 2. Second virial coefficients

TABLE 2. EXPERIMENTAL RESULTS

T(°K)	P(atm)	Z = Pv/RT	v(cm³/g)	T(°K)	P(atm)	Z = Pv/RT	v(cm³/g)	T(°K)	P(atm)	Z = Pv/RT	v(cm³/g)
70	684.8179	2.38703	5.00180	80	212.1853	1.37949	10.6620	100	78.6122	1.11376	29.0434
	575.2362	2.17780	5.43269		192.5948	1.34369	11.4417		64.5923	1.09325	34.6964
	477.0025	1.98494	5.97132		166.9379	1.29679	12.7394		53.2492	1.07670	41.4500
	408.8116	1.84789	6.48630		152.3615	1.27013	13.6712		44.0156	1.06324	49.5186
	346.0660	1.71950	7.12996		133.0322	1.23480	15.2221		36.4623	1.05223	59.1580
	301.4177	1.62691	7.74528		121.9451	1.21469	16.3356		30.2599	1.04324	70.6741
	259.2827	1.53844	8.51432		107.0731	1.18756	18.1890		25.1502	1.03587	84.4323
	228.6351	1.47372	9.24944		86.8495	1.15102	21.7345		20.9281	1.02978	100.869
	199.1316	1.41104	10.1682		70.8718	1.12237	25.9715		17.4326	1.02477	120.506
	177.2556	1.36451	11.0463		58.1139	1.09975	31.0348				
	155.8527	1.31895	12.1438		47.8305	1.08162	37.0854				
	139.7372	1.28472	13.1928		39.4923	1.06718	44.3159				
	123.7492	1.25079	14.5038		32.6789	1.05524	52.9565				
	111.5619	1.22502	15.7569		27.0986	1.04566	63.2819				
	99.3449	1.19934	17.3229		22.5058	1.03777	75.6209				
	89.9440	1.17961	18.8196		18.7183	1.03142	90.3659				
80.4433	1.15988	20.6902									
65.5593	1.12903	24.7123									
53.7108	1.10481	29.5167									
44.1820	1.08549	35.2552									
36.4628	1.07001	42.1097									
30.1718	1.05755	50.2972									
25.0185	1.04743	60.0767									
20.7789	1.03908	71.7578									
17.2845	1.03240	85.7105									
80	693.3695	2.21331	5.23494	100	685.9706	1.96335	5.86729	120	692.8584	1.81597	6.44750
	601.2871	2.05939	5.61682		596.0191	1.84474	6.34483		518.3546	1.62214	7.69817
	492.1884	1.87656	6.25266		503.0904	1.71951	7.00653		396.7062	1.48243	9.19245
	433.8434	1.77490	6.70927		442.7845	1.63667	7.57728		309.0784	1.37927	10.9776
	362.1074	1.64925	7.46938		379.2859	1.54826	8.36801		244.2293	1.30160	13.1101
	323.3245	1.58022	8.01520		337.2588	1.48891	9.05005		195.1435	1.24208	15.6575
	274.2133	1.49211	8.92375		292.2335	1.42483	9.99488		157.3013	1.19579	18.7003
	247.1254	1.44302	9.57613		261.9459	1.38129	10.8098		127.6941	1.15939	22.3350
					228.9879	1.33360	11.9387		104.2411	1.13043	26.67652
					206.5452	1.30100	12.9125		99.1653	1.12411	27.8852
					181.8324	1.26498	14.2612		85.4788	1.10716	31.8624
					164.8037	1.24004	15.4246		81.3971	1.10206	33.3059
					145.8905	1.21241	17.0360		70.3492	1.08834	38.0568
					132.7315	1.19305	18.4259		67.0455	1.08422	39.7808
					118.0002	1.17145	20.3510		58.0692	1.07302	45.4557
					107.6887	1.15632	22.0117		55.3808	1.06970	47.5147
			96.0702	1.13935	24.3116	45.8487	1.05776	56.7526			
						38.0308	1.04798	67.7868			
						31.5946	1.03990	80.9666			
						26.2807	1.03319	96.7094			
						21.8844	1.02764	115.513			
						18.2393	1.02301	137.974			
						15.2130	1.01918	164.802			
						12.6974	1.01605	196.847			

TABLE 3. COMPARISON OF SELECTED VALUES OF COMPRESSIBILITY WITH PREVIOUSLY PUBLISHED VALUES

T(°K)	70			80			100			120		
	P(atm)	20	50	100	20	50	100	20	50	100	20	50
This work	1-03788	1-09700	1-20060	1-03440	1-08472	1-17445	1-02841	1-07198	1-14508	1-02526	1-06293	1-12519
Holborn and Otto ¹⁶⁻¹⁸	1-03520	1-09383	1-19690	1-03199	1-08423	1-17654	1-02682	1-06938	1-14460	1-02294	1-05906	1-12182
Mann ¹⁹ (Compilation)	1-03653	1-09619	1-20002	1-03381	1-08771	1-18051	1-02938	1-07489	1-15172	1-02579	1-06506	1-13092

was discussed earlier, the experimental data were collected prior to, and in fact necessitated, the development of the modified data reduction technique.¹⁵ Thus it was not possible to run a data-point coverage to permit optimum use of the analytical reduction procedure. The scatter seen in Figure 2 is the result of small uncertainties in the lower-pressure compressibilities—well within the limits of accuracy to be discussed in the final section of this paper.

Fit of the Experimental Data to the Berlin Expansion

Each isotherm of helium was least-squares fitted to the Berlin expansion of compressibility in terms of a polynomial in pressure. The coefficients which were obtained for each isotherm are listed in Table 4 along with the standard deviation of the experimental values of compressibility from those computed from the polynomial. For all of the isotherms the fit is well within the expected error in the compressibilities. The coefficients listed in Table 4 were used to compute values of the compressibility and specific volume of helium for even values of pressure. The results of these computations are presented in Table 5.

Estimate of Errors

For the *j*th expansion, the compressibility factor *Z_j* is given by equation (2). It is possible to represent the error in this value as

$$\frac{\Delta Z_j}{Z_j} = \frac{\Delta P_j}{P_j} + E_{N_0, P_0/Z_0} + E_T + E_E \dots (7)$$

which includes error in pressure measurement, the interrelated errors associated with the data reduction constants *N₀* and *P₀/Z₀*, error in temperature measurement, and that due to incomplete evacuation of the small cell between expansions.

A set of typical values for these errors at 80° K indicates the estimated accuracy of the experimental results. The first term in equation (7) is approximately 1.26×10^{-4} in the range of this investigation, and the last term, *E_E*, is 0.04×10^{-4} . Since the remaining two terms are strongly pressure dependent, values will be considered at about 20, 100, 300, and 600 atm. The error *E_{N₀, P₀/Z₀}* can be expressed analytically, but is not conveniently separated due to the interrelation between errors in *N₀* and *P₀/Z₀*. From the data reduction procedure,^{12,15} it is estimated that *N₀* is accurate to ± 0.00005 and *P₀/Z₀* to ± 0.02 atm. By a series of perturbations, the maximum value of *E_{N₀, P₀/Z₀}* is found to be 4.1, 7.6, 9.7, 10.6×10^{-4} at about 20, 100, 300, and 600 atm, respectively. The remaining term, *E_T*, depends on

$$\frac{(\partial Z_j / \partial T) P_j}{Z_j}$$

and is found from the results to be 0.05, 0.2, 0.7, 1.1×10^{-4} at these same pressures.

Consequently, the maximum errors in compressibility at 80° K are estimated to be 0.055 per cent at 20 atm, 0.091 per cent at 100 atm, 0.117 per cent at 300 atm, 0.13 per cent at 600 atm. This accuracy is felt to be quite acceptable for compressibility data and subsequent computation of thermodynamic properties.

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TABLE 4. BERLIN COEFFICIENTS

$$Z = A' + B'P + C'P^2 + D'P^3 + E'P^4 + F'P^5 + G'P^6 \text{ (Pressure in atmospheres)}$$

Temperature (°K)	Standard deviation	A'	B' × 10 ³	C' × 10 ⁶	D' × 10 ⁹	E' × 10 ¹²	F' × 10 ¹⁵	G' × 10 ¹⁸
70	0.265 × 10 ⁻³	1.00061	1.80897	2.96340	-13.2222	30.1403	-35.1756	16.2242
80	0.601 × 10 ⁻³	1.00360	1.46855	3.96361	-20.0961	50.4692	-62.0908	29.4334
100	0.396 × 10 ⁻⁴	0.99954	1.43890	0.234588	-0.726644	0.216244	0.818254	-0.683837
120	0.316 × 10 ⁻⁴	1.00001	1.26602	-0.161639	0.240920	-0.512839	0.340978	-

TABLE 5. VALUES OF COMPRESSIBILITY FOR EVEN VALUES OF PRESSURE (ν in cm³/g)

P(atm)		70° K	80° K	100° K	120° K	P(atm)		70° K	80° K	100° K	120° K
10	v	146.22155	167.05729	207.85583	249.10711	350	v	7.08152	7.62285	8.82666	10.03474
	Z	1.01899	1.01866	1.01395	1.01265		Z	1.72724	1.62686	1.50702	1.42774
20	v	74.46615	84.81940	105.40945	126.10496	360	v	6.96705	7.49226	8.66184	9.83647
	Z	1.03788	1.03440	1.02841	1.02526		Z	1.74787	1.64468	1.52114	1.43951
30	v	50.56913	57.43824	71.26328	85.10180	370	v	6.85861	7.36869	8.50574	9.64872
	Z	1.05722	1.05072	1.04290	1.03785		Z	1.76846	1.66248	1.53521	1.45126
40	v	38.63447	43.76747	54.19194	64.59847	380	v	6.75572	7.25157	8.35766	9.47068
	Z	1.07694	1.06752	1.05743	1.05040		Z	1.78901	1.68028	1.54926	1.46298
50	v	31.48312	35.57814	43.95036	52.29512	390	v	6.65794	7.14038	8.21700	9.30158
	Z	1.09700	1.08472	1.07198	1.06293		Z	1.80951	1.69805	1.56327	1.47467
60	v	26.72221	30.12752	37.12354	44.09180	400	v	6.56488	7.03466	8.08320	9.14076
	Z	1.11733	1.10225	1.08657	1.07543		Z	1.82997	1.71581	1.57724	1.48633
70	v	23.32633	26.24037	32.24788	38.23138	410	v	6.47619	6.93399	7.95575	8.98761
	Z	1.13789	1.12004	1.10117	1.08791		Z	1.85038	1.73353	1.59118	1.49797
80	v	20.78287	23.32922	28.59160	33.83529	420	v	6.39153	6.83798	7.83422	8.84157
	Z	1.15865	1.13803	1.11579	1.10036		Z	1.87073	1.75123	1.60509	1.50957
90	v	18.80710	21.06784	25.74815	30.41545	430	v	6.31063	6.74628	7.71817	8.70216
	Z	1.17956	1.15619	1.13043	1.11278		Z	1.89103	1.76888	1.61897	1.52114
100	v	17.22827	19.26060	23.47362	27.67899	440	v	6.23320	6.65857	7.60725	8.56892
	Z	1.20060	1.17445	1.14508	1.12519		Z	1.91126	1.78648	1.63281	1.53268
110	v	15.93775	17.78311	21.61278	25.43955	450	v	6.15901	6.57455	7.50112	8.44144
	Z	1.22173	1.19279	1.15973	1.13756		Z	1.93144	1.80403	1.64662	1.54420
120	v	14.86316	16.55251	20.06214	23.57287	460	v	6.08783	6.49396	7.39946	8.31935
	Z	1.24294	1.21118	1.17439	1.14992		Z	1.95154	1.82151	1.66040	1.55568
130	v	13.95446	15.51152	18.75008	21.99295	470	v	6.01946	6.41654	7.30198	8.20229
	Z	1.26419	1.22960	1.18905	1.16225		Z	1.97157	1.83892	1.67415	1.56714
140	v	13.17589	14.61926	17.62542	20.63833	480	v	5.95372	6.34207	7.20844	8.08996
	Z	1.28548	1.24801	1.20371	1.17456		Z	1.99153	1.85625	1.68786	1.57856
150	v	12.50130	13.84580	16.65064	19.46397	490	v	5.89043	6.27035	7.11858	7.98207
	Z	1.30678	1.26641	1.21836	1.18685		Z	2.01141	1.87350	1.70155	1.58996
160	v	11.91107	13.16874	15.79760	18.43606	500	v	5.82943	6.20119	7.03220	7.87835
	Z	1.32809	1.28478	1.23301	1.19912		Z	2.03120	1.89065	1.71521	1.60132
170	v	11.39021	12.57099	15.04479	17.52876	510	v	5.77060	6.13443	6.94909	7.77856
	Z	1.34939	1.30312	1.24764	1.21136		Z	2.05092	1.90770	1.72883	1.61266
180	v	10.92711	12.03926	14.37548	16.72197	520	v	5.71379	6.06990	6.86905	7.68247
	Z	1.37068	1.32141	1.26226	1.22358		Z	2.07054	1.92464	1.74243	1.62397
190	v	10.51258	11.56309	13.77644	15.99981	530	v	5.65889	6.00749	6.79192	7.58988
	Z	1.39194	1.33965	1.27687	1.23578		Z	2.09009	1.94148	1.75600	1.63525
200	v	10.13931	11.13415	13.23713	15.34960	540	v	5.60580	5.94706	6.71754	7.50060
	Z	1.41317	1.35785	1.29145	1.24796		Z	2.10954	1.95822	1.76953	1.64651
210	v	9.80136	10.74568	12.74899	14.76104	550	v	5.55442	5.88853	6.64575	7.41445
	Z	1.43438	1.37600	1.30602	1.26012		Z	2.12891	1.97485	1.78304	1.65774
220	v	9.49391	10.39218	12.30501	14.22574	560	v	5.50466	5.83179	6.57641	7.33126
	Z	1.45554	1.39410	1.32057	1.27225		Z	2.14821	1.99138	1.79652	1.66894
230	v	9.21296	10.06910	11.89943	13.73674	570	v	5.45647	5.77678	6.50940	7.25088
	Z	1.47667	1.41216	1.33509	1.28436		Z	2.16742	2.00783	1.80997	1.68012
240	v	8.95520	9.77267	11.52743	13.28825	580	v	5.40977	5.72344	6.44459	7.17318
	Z	1.49776	1.43018	1.34958	1.29644		Z	2.18657	2.02419	1.82338	1.69127
250	v	8.71783	9.49971	11.18497	12.87540	590	v	5.36450	5.67174	6.38187	7.09801
	Z	1.51882	1.44816	1.36405	1.30850		Z	2.20566	2.04048	1.83677	1.70240
260	v	8.49851	9.24755	10.86863	12.49409	600	v	5.32063	5.62164	6.32112	7.02527
	Z	1.53983	1.46611	1.37849	1.32054		Z	2.22470	2.05674	1.85012	1.71352
270	v	8.29522	9.01391	10.57550	12.14079	610	v	5.27811	5.57313	6.26226	6.95483
	Z	1.56081	1.48403	1.39290	1.33255		Z	2.24370	2.07298	1.86344	1.72461
280	v	8.10625	8.79681	10.30309	11.81252	620	v	5.23693	5.52622	6.20518	6.88659
	Z	1.58174	1.50193	1.40728	1.34454		Z	2.26269	2.08922	1.87673	1.73568
290	v	7.93013	8.59459	10.04923	11.50667	630	v	5.19706	5.48093	6.14979	6.82045
	Z	1.60264	1.51981	1.42163	1.35651		Z	2.28168	2.10552	1.88997	1.74674
300	v	7.76557	8.40576	9.81209	11.22101	640	v	5.15850	5.43729	6.09601	6.75632
	Z	1.62350	1.53767	1.43594	1.36844		Z	2.30070	2.12191	1.90318	1.75778
310	v	7.61145	8.22906	9.59002	10.95356	650	v	5.12124	5.39536	6.04375	6.69411
	Z	1.64432	1.55553	1.45023	1.38036		Z	2.31977	2.13845	1.91635	1.76881
320	v	7.46680	8.06335	9.38162	10.70263	660	v	5.08529	5.35520	5.99294	6.63375
	Z	1.66511	1.57337	1.46448	1.39224		Z	2.33893	2.15519	1.92947	1.77982
330	v	7.33076	7.90765	9.18563	10.46671	670	v	5.05068	5.31691	5.94351	6.57515
	Z	1.68585	1.59121	1.47869	1.40410		Z	2.35821	2.17220	1.94255	1.79083
340	v	7.20255	7.76108	9.00097	10.24447	680	v	5.01743	5.28057	5.89538	6.51826
	Z	1.70656	1.60904	1.49288	1.41593		Z	2.37765	2.18955	1.95558	1.80183